Simulation of Modal Vibration Pattern Variations Due to Gyroscopic Effects in an Active Vibration Controlled Structure #2004-23

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The Charles Stark Draper Laboratory

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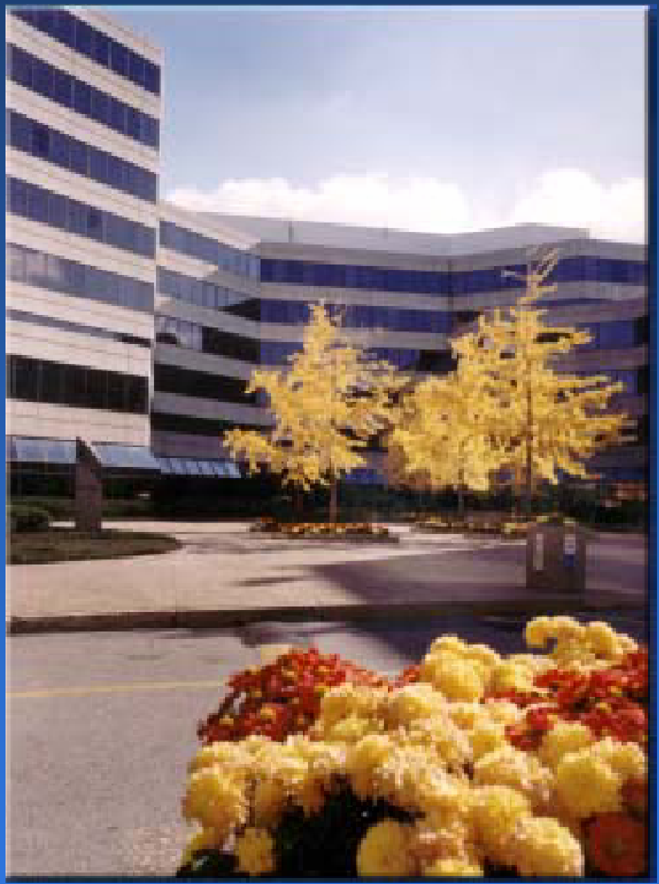
- Headquarters in Cambridge, Massachusetts with 4 satellite locations nationwide
- $244 million in revenues
- 1,041 employees, 2/3 of whom are technical staff and technicians



Closed Loop Gyro Model

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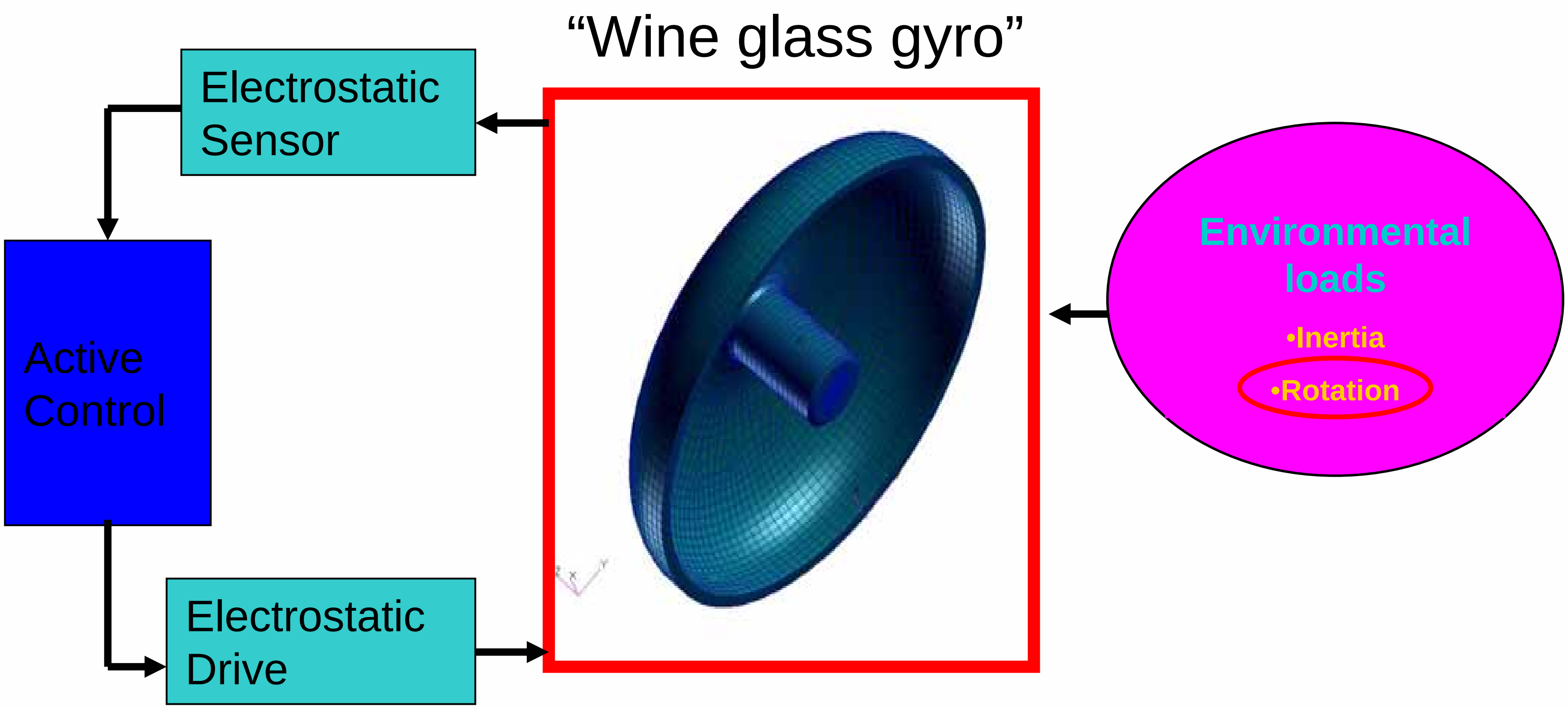
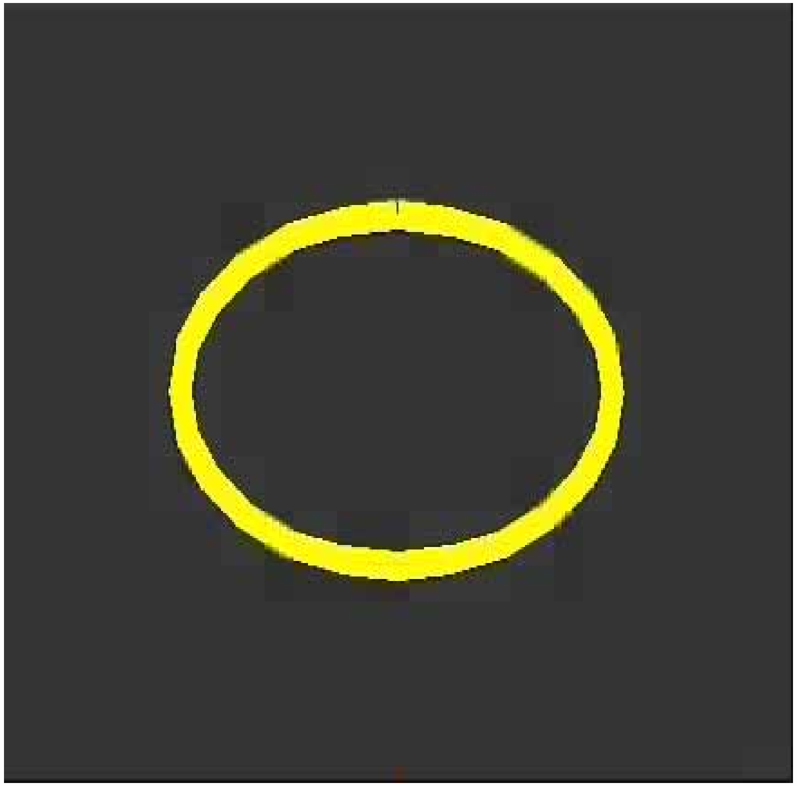


"Wine Glass" Gyro



The standing wave pattern of vibration neither remains fixed in inertial space nor remains fixed to the "wineglass angle"

$$Lag = SF \sim -0.3$$

Mathematical Model

Equation-of-Motion Including Damping and gyroscopic Forces

$$[M]\{\ddot{q}\} + ([D] + \Omega[G])\{\dot{q}\} + [K]\{q\} = \{F\}$$

Rewritten in Modal Coordinates

$$\{\ddot{\eta}\} + (2[\,{}^{\backslash}\zeta\omega_{\backslash}\,] + \Omega[\underline{G}])\{\dot{\eta}\} + [\,{}^{\backslash}\omega_{\backslash}^{2}\,]\{\eta\} = \{P\}$$

Using State-Space to reduce to first order

$$\{\dot{\alpha}\} = [A]\{\alpha\} + [B]\{F\}$$

$$\{\alpha\} = \begin{bmatrix} \eta \\ \hline \dot{\eta} \end{bmatrix}$$

Mathematical Model Cont'd

Final Plant Control Matrices – Constructed from a MSC.Nastran FE analysis

$$[A] = \begin{bmatrix} [0] & [I] \\ -[\omega^2] & -(2[\zeta\omega]) - \Omega[\underline{G}] \end{bmatrix}$$

$$[\underline{G}] = [\Phi]^T [G][\Phi]$$

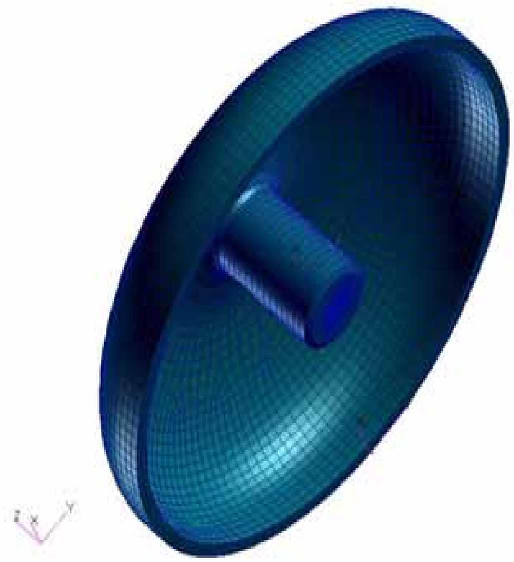
Modal Damping values

Gyroscopic forces are function of Ω

$$[B] = \begin{bmatrix} [0] \\ [\underline{\Phi}]^T \end{bmatrix}$$

$$[C] = \begin{bmatrix} [\underline{\Phi}] & [0] \end{bmatrix}$$

MSC.Patran FE Model



Resonator "wine glass"

- 8 Node Solid Elements
- ~60,000 DOF

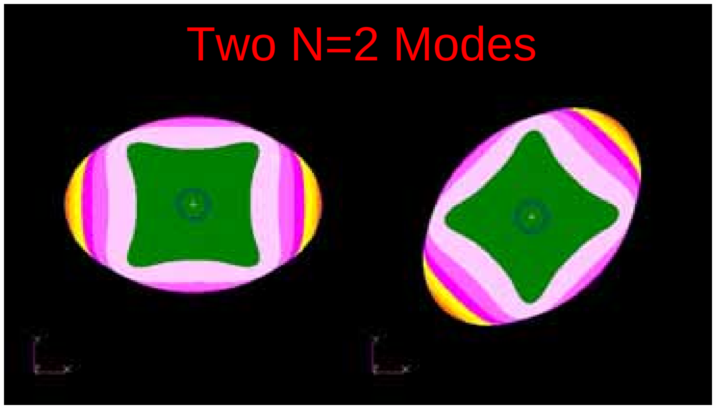
Sensors & Actuators

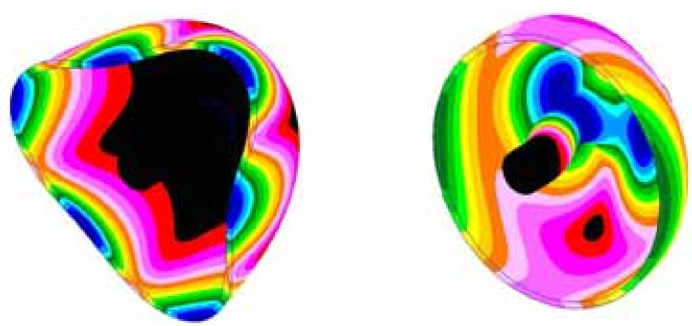
- 8 Node Solid Elements
- Gap- Scalar points & MPC's
- ~15,000 DOF

Case & Flange

- Beam Elements
- Lumped Mass
- ~ 100 DOF

MSC.Nastran (v70.5)





Various Other Modes

MSC.Nastran (SOL101)

- RF alter
  - Create ~ 60,000 DOF Coriolis matrix [G]
  - Spin rate of unity
  - Matrix saved for restart

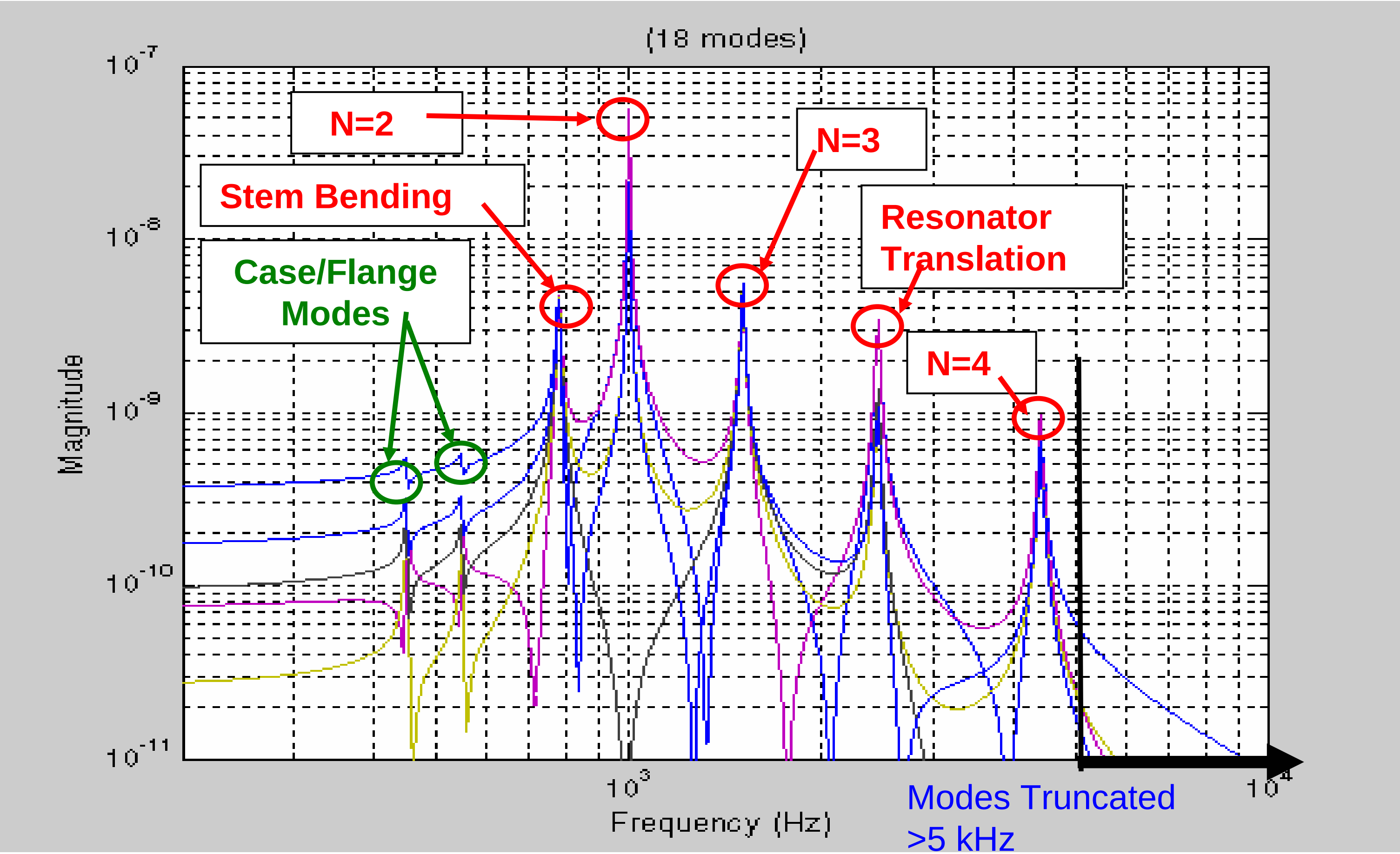
MSC.Nastran (SOL103)

- Modal restart from sol 101
- RF alter
  - Perform modal reduction of the gyroscopic matrix $[\underline{G}]$ using the first 50 modes

$$[\underline{G}] = [\Phi]^T [G][\Phi]$$

  - Output ω, Φ, and $[\underline{G}]$ matrices
  - Output geometry (data blocks) to construct the [B] and [C] matrices
    - GPL – Grid point list, internal sort
    - EQEXIN – equivalence, external sort
    - BGPDT – Grid point definition R, θ, Z

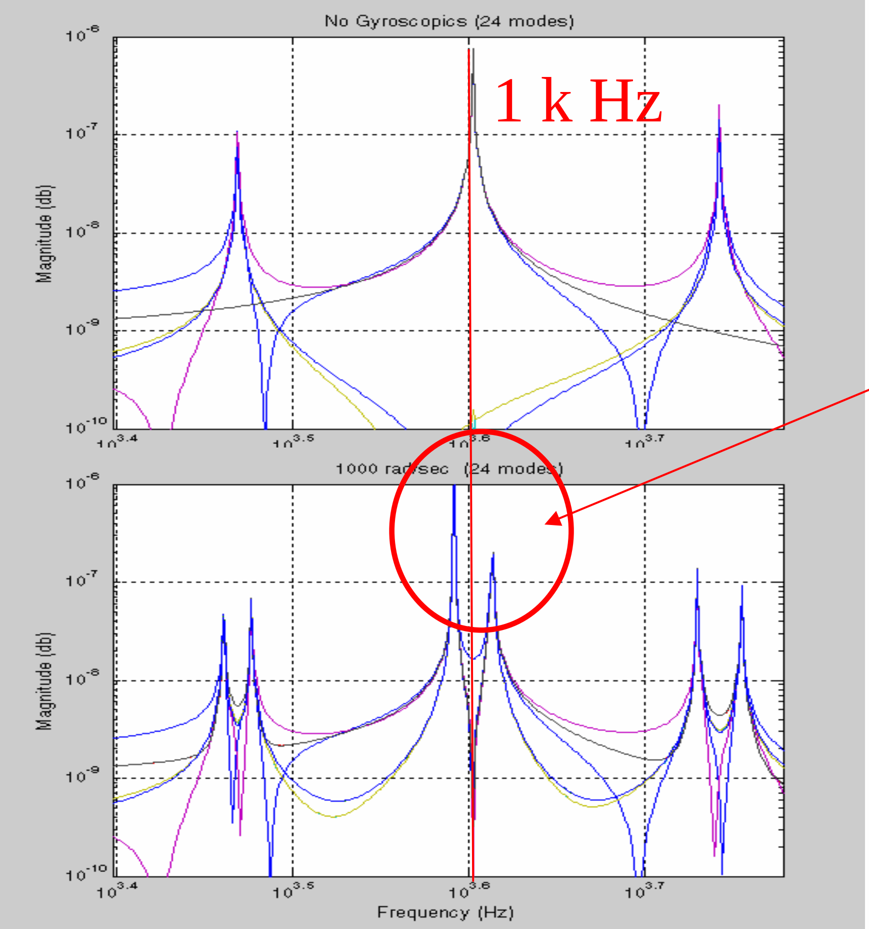
Transfer Function



Coriolis Effects

$\Omega = 0$

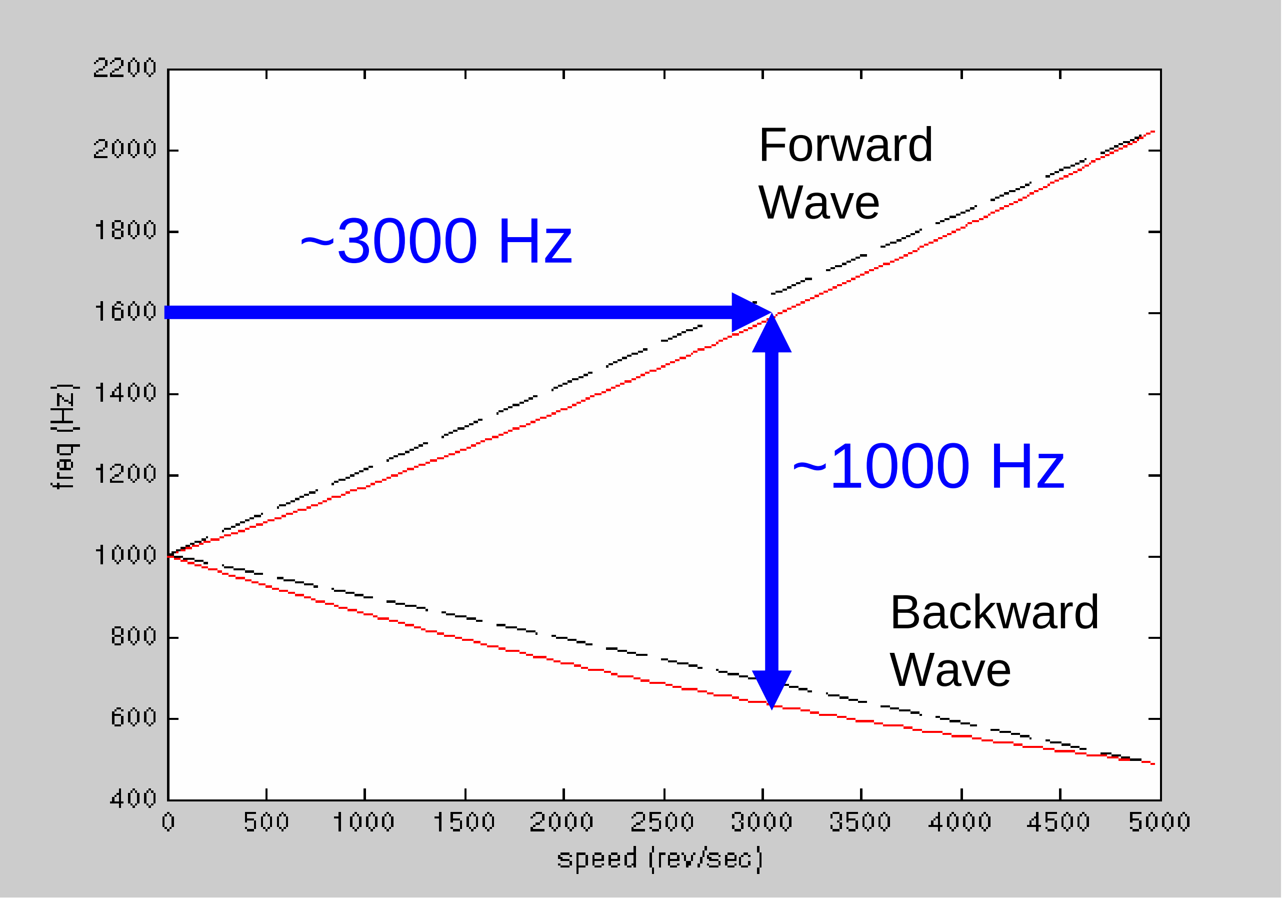
$\Omega \neq 0$



Coriolis Effects

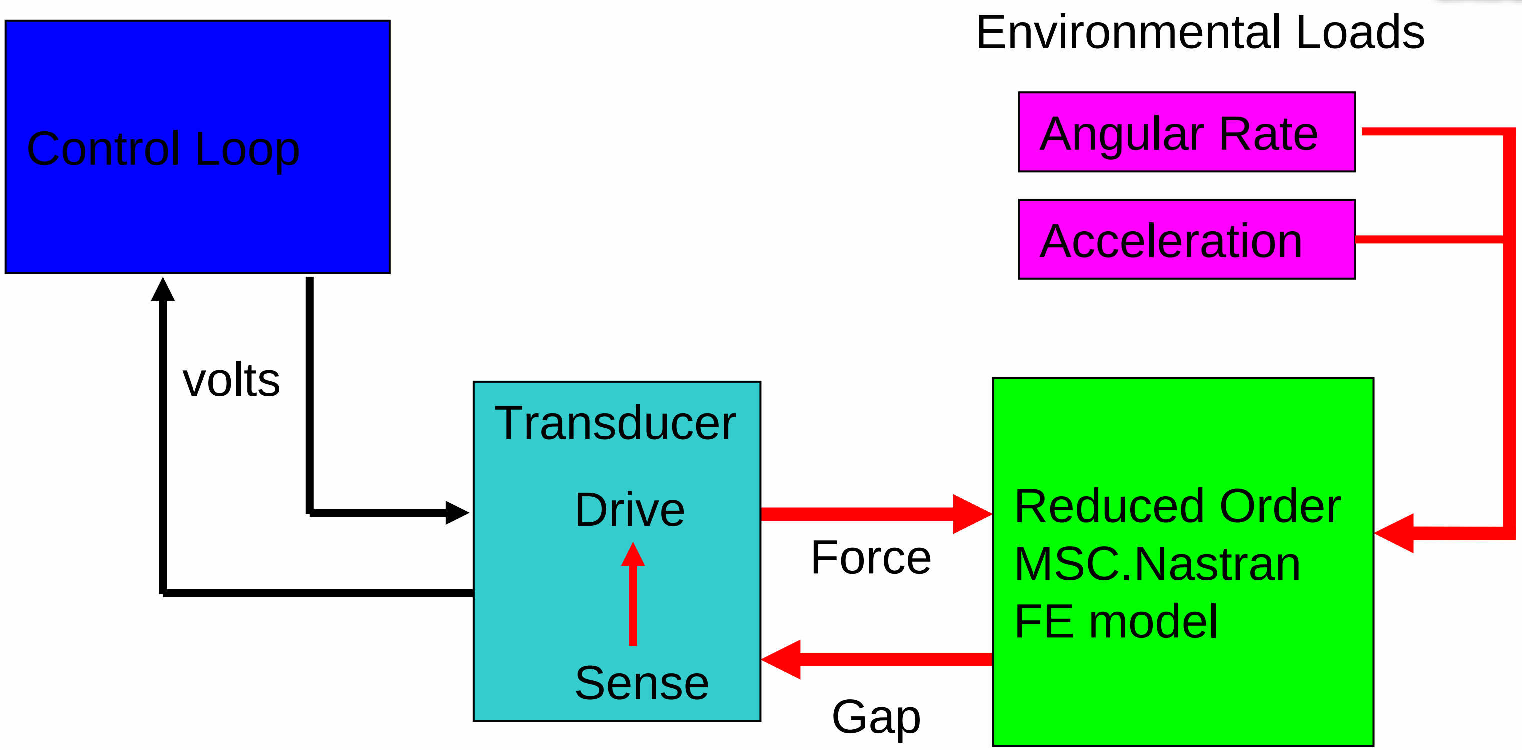
The gyroscopic forces from the spin rates, result in a bifurcation of the mode into forward and backward components

Forward and Backward Components



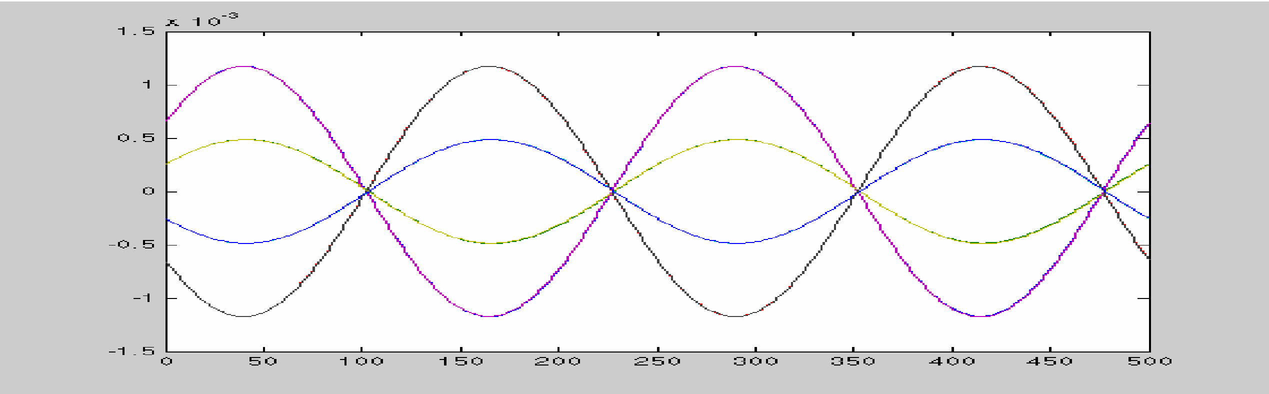
SF = ~ 0.3

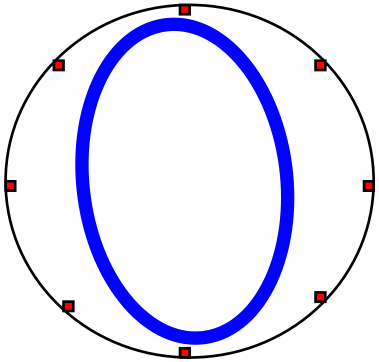
"Wineglass" Simulation Model



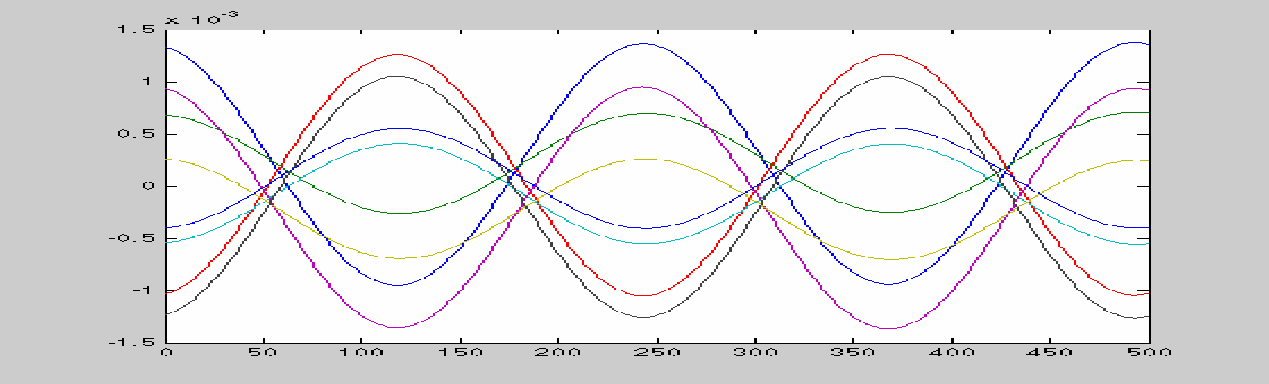
Wineglass response

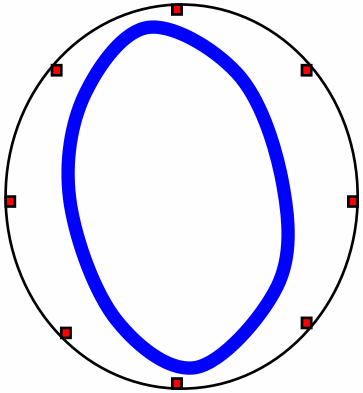
Motion at Sensors (mm) – Steady State





Motion at Sensors (mm) - After a Inertia Load Step Input





Wineglass "Scale Factor"

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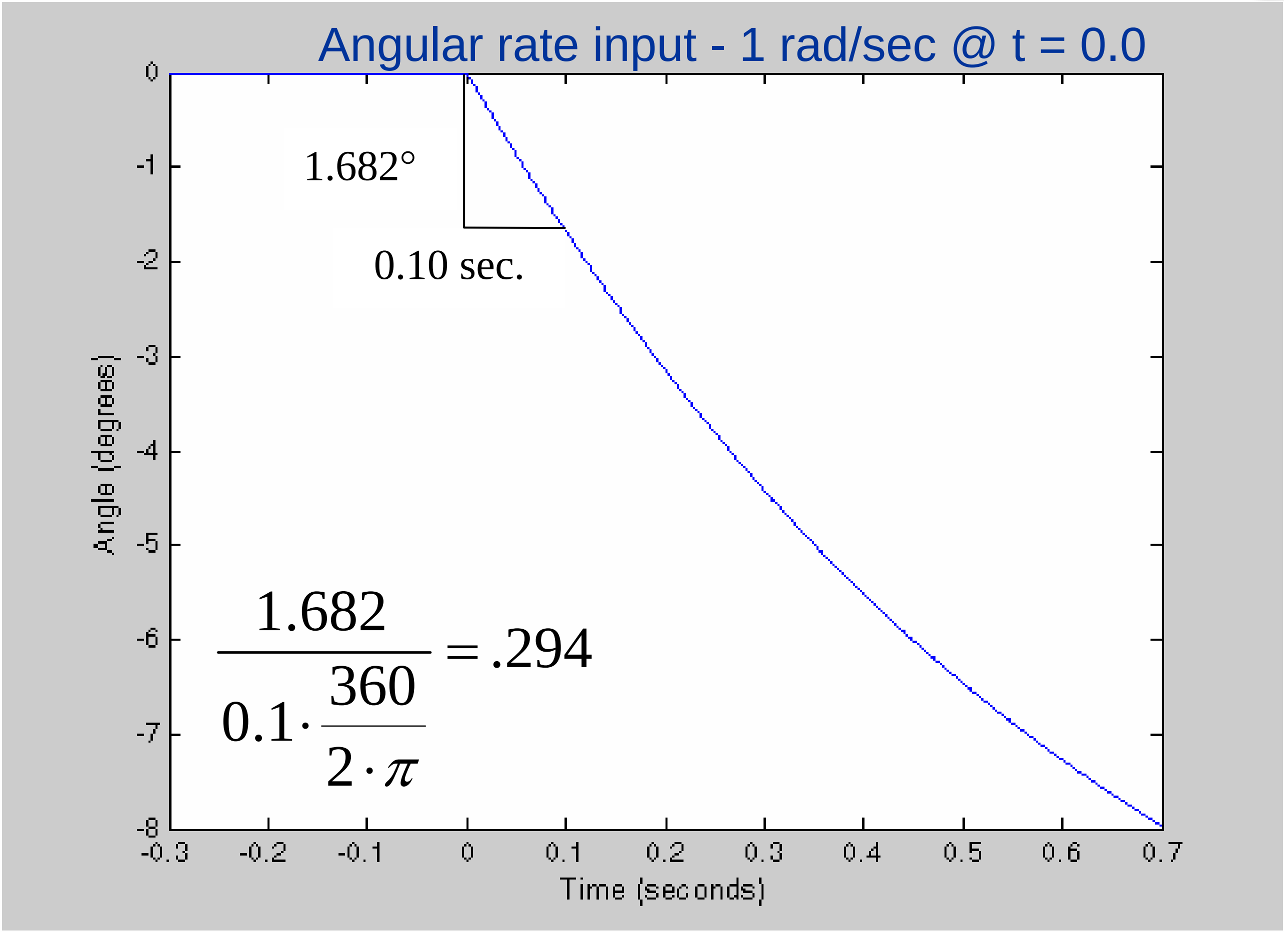

Summary

•An MSC.Nastran FE model was used to construct a State Space model of a dynamic structure, including the reduced order gyroscopic matrix

•The precession frequency of the N=2 mode is the difference between the forward and backward modes.

•The gyroscopic results match known values

•The final State-Space model is able to be integrated with a closed loop control, and run in a time domain simulation analysis