# Prediction of Eight Ears in Drawn Cup Based On a New Anisotropic Yield Function

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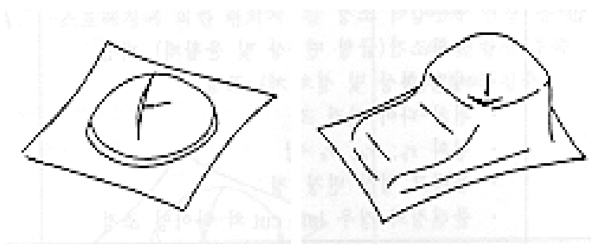
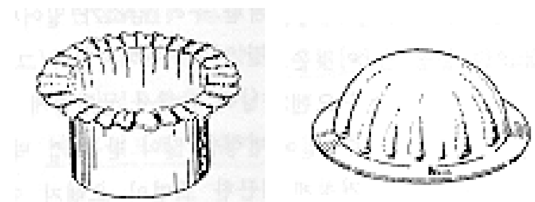
## Instability in Sheet Metal Forming Processes



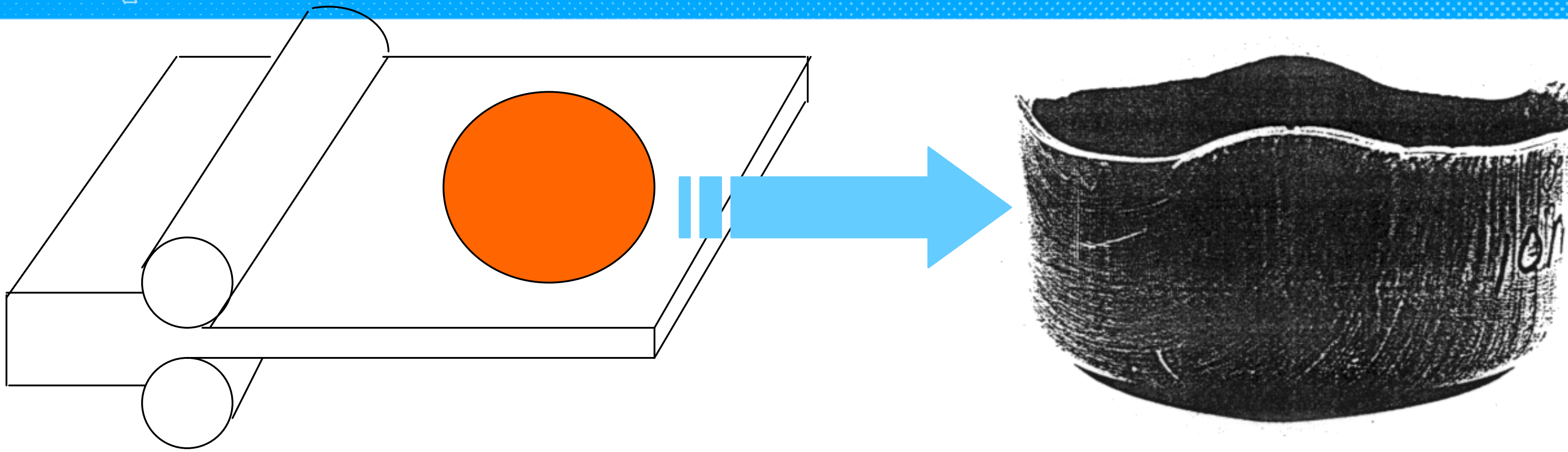


**Tensile Instability**

**Compressive Instability**

**Defects in Products**

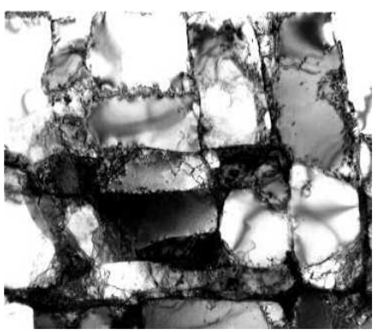
# Anisotropy in Metal



## Source of Anisotropy

* **Deformation modes** (Dislocation glide, Twinning)
* **Microstructure** (Grain structure, Dislocation structures, Second-phases, Solutes)

**Dislocation structures**



1050-O

<111>

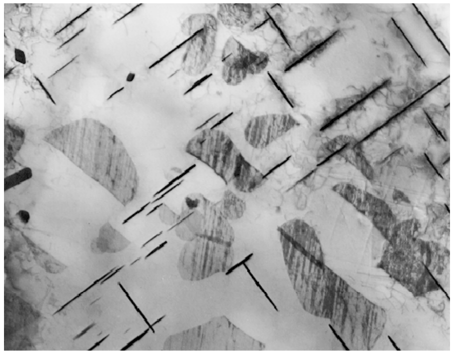
1μm

6022-T4

<111>

1μm

**Θ' precipitates**



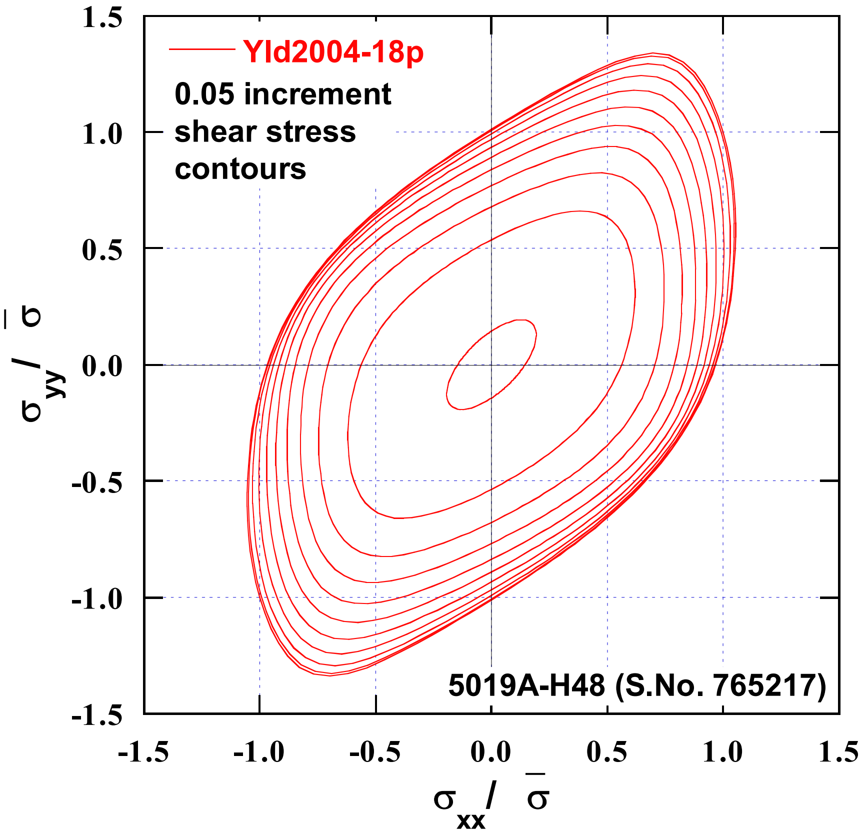
Al-3%Cu

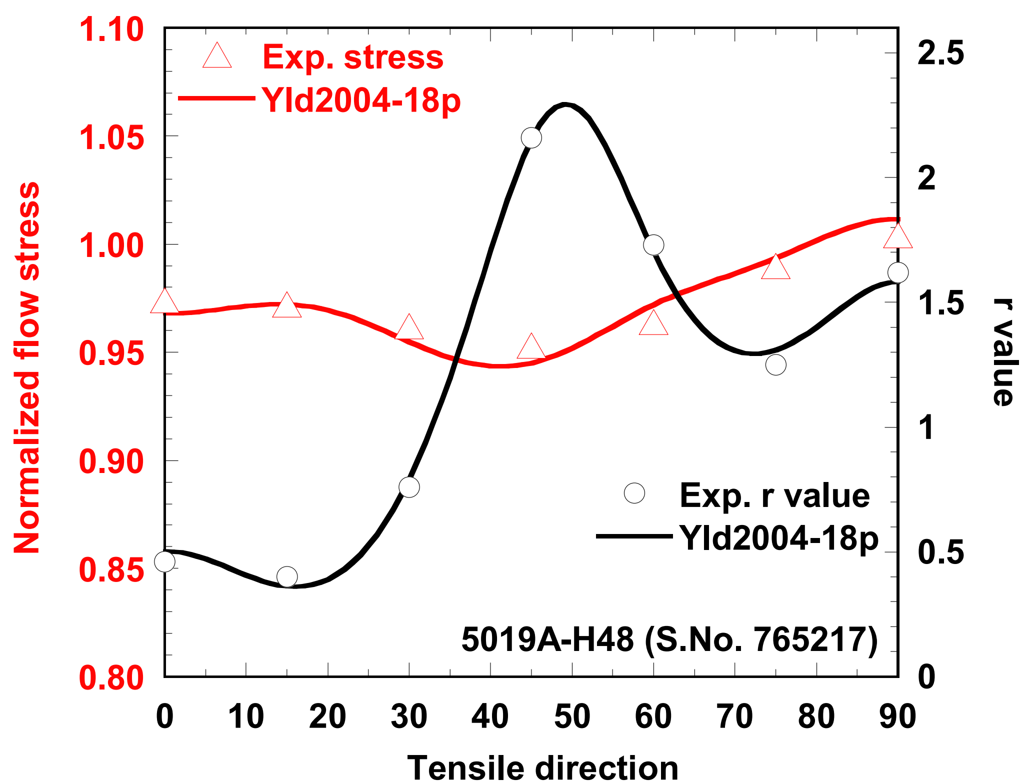
<010> <100>

1 μ m

# Yld2004 (Barlat et al. (2004)) Prediction for AL 5019

$$\phi = \phi(\mathbf{\Sigma}) = \left|\tilde{S}'_1 - \tilde{S}''_1\right|^a + \left|\tilde{S}'_1 - \tilde{S}''_2\right|^a + \left|\tilde{S}'_1 - \tilde{S}''_3\right|^a +$$
$$\left|\tilde{S}'_2 - \tilde{S}''_1\right|^a + \left|\tilde{S}'_2 - \tilde{S}''_2\right|^a + \left|\tilde{S}'_2 - \tilde{S}''_3\right|^a +$$
$$\left|\tilde{S}'_3 - \tilde{S}''_1\right|^a + \left|\tilde{S}'_3 - \tilde{S}''_2\right|^a + \left|\tilde{S}'_3 - \tilde{S}''_3\right|^a = 4\bar{\sigma}^a$$



- **Stress Update**

$$\Delta\hat{\sigma} = \hat{C}\Delta\hat{\varepsilon}^e = \hat{C}(\Delta\hat{\varepsilon} - \Delta\hat{\varepsilon}^p)$$

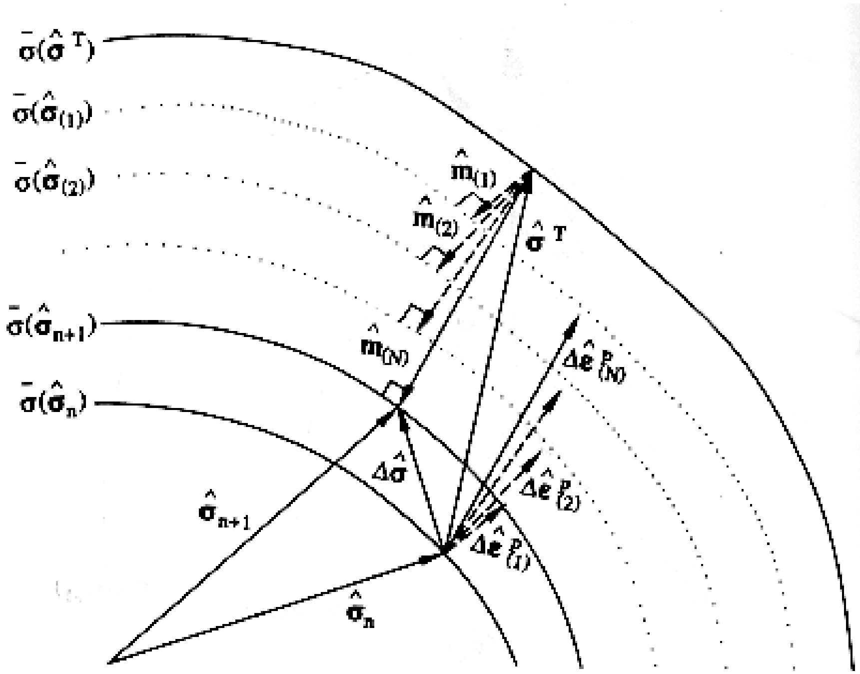
- **Consistency Condition**

$$\Phi(\sigma, \bar{\varepsilon}^p) = \bar{\sigma}(\sigma) - \rho(\bar{\varepsilon}^p) = 0$$

- **Stress-Strain Relation**

$$\rho(\bar{\varepsilon}^p) = k(\varepsilon_0 + \bar{\varepsilon}^p)^n$$

- **Multi-Step Return Mapping**



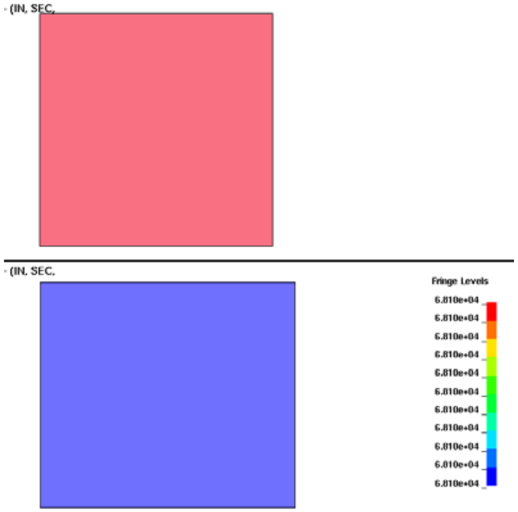

$$\Phi_0(\gamma_{(0)} = 0) = \bar{\sigma}(\hat{\sigma}^T) - \rho(\bar{\varepsilon}^P_n) = \Phi_0$$

$$\Phi_1(\gamma_{(1)}) = \bar{\sigma}(\hat{\sigma}^T - \gamma_{(1)}\hat{\mathbf{C}}^e\hat{\mathbf{m}}_{(1)}) - \rho(\bar{\varepsilon}^P_n + \gamma_{(1)}) = \Phi_1$$

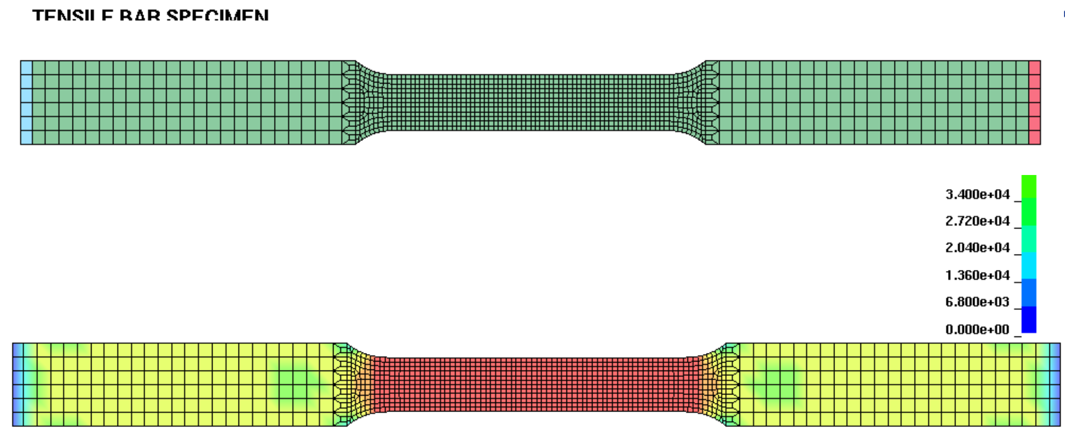
$$\Phi_1(\gamma_{(N)}) = \bar{\sigma}(\hat{\sigma}^T - \gamma_{(N)}\hat{\mathbf{C}}^e\hat{\mathbf{m}}_{(N)}) - \rho(\bar{\varepsilon}^P_n + \gamma_{(N)}) = \Phi_N = 0$$

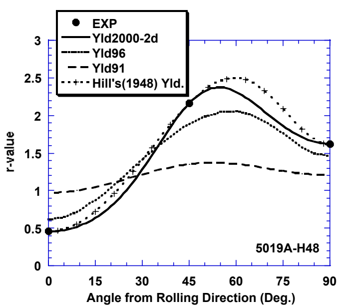
- **Stable Convergence for Large Step Increment**

## One element test



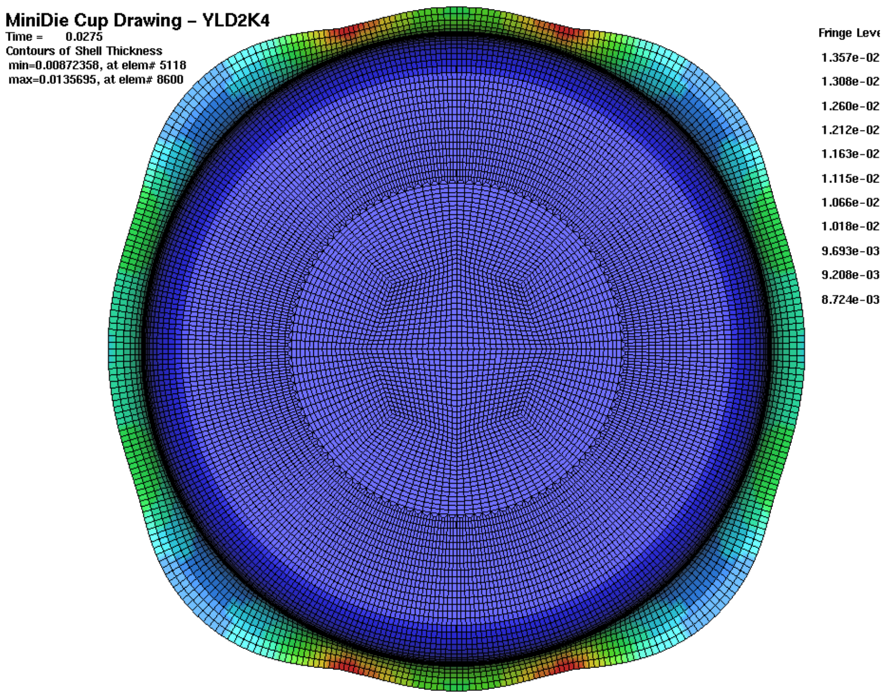

## Tensile bar test

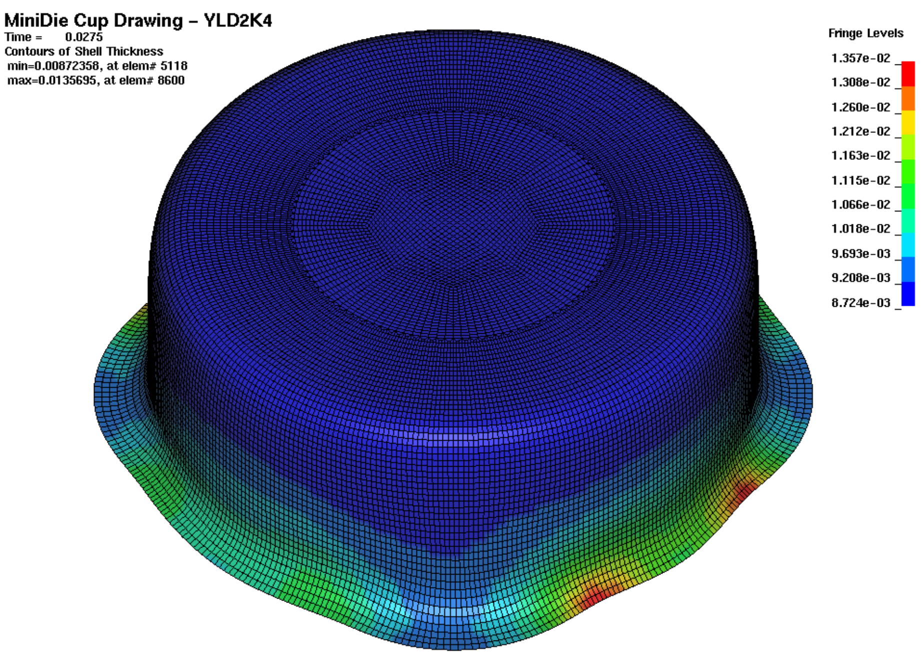


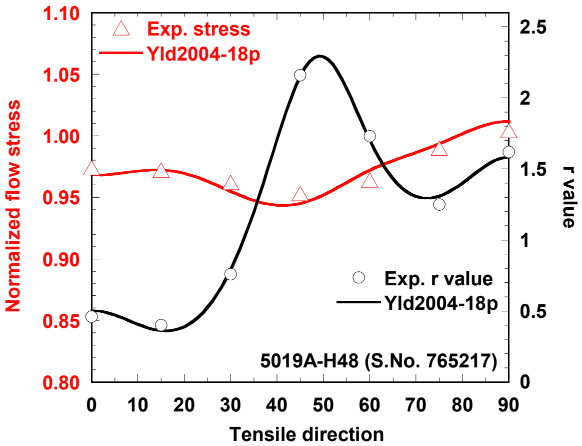

| | Theory (Yld2000) | Single Element | Error (%) | Tensile Bar | Error (%) |
|---|---|---|---|---|---|
| 0 | **0.460** | 0.461 | 0.2 | 0.460 | 0.0 |
| 15 | **0.637** | 0.637 | 0.0 | 0.638 | 0.1 |
| 30 | **1.281** | 1.277 | 0.3 | 1.288 | 0.5 |
| 45 | **2.160** | 2.152 | 0.3 | 2.162 | 0.1 |
| 60 | **2.310** | 2.299 | 0.4 | 2.316 | 0.2 |
| 75 | **1.859** | 1.851 | 0.4 | 1.865 | 0.3 |
| 90 | **1.620** | 1.616 | 0.2 | 1.621 | 0.1 |

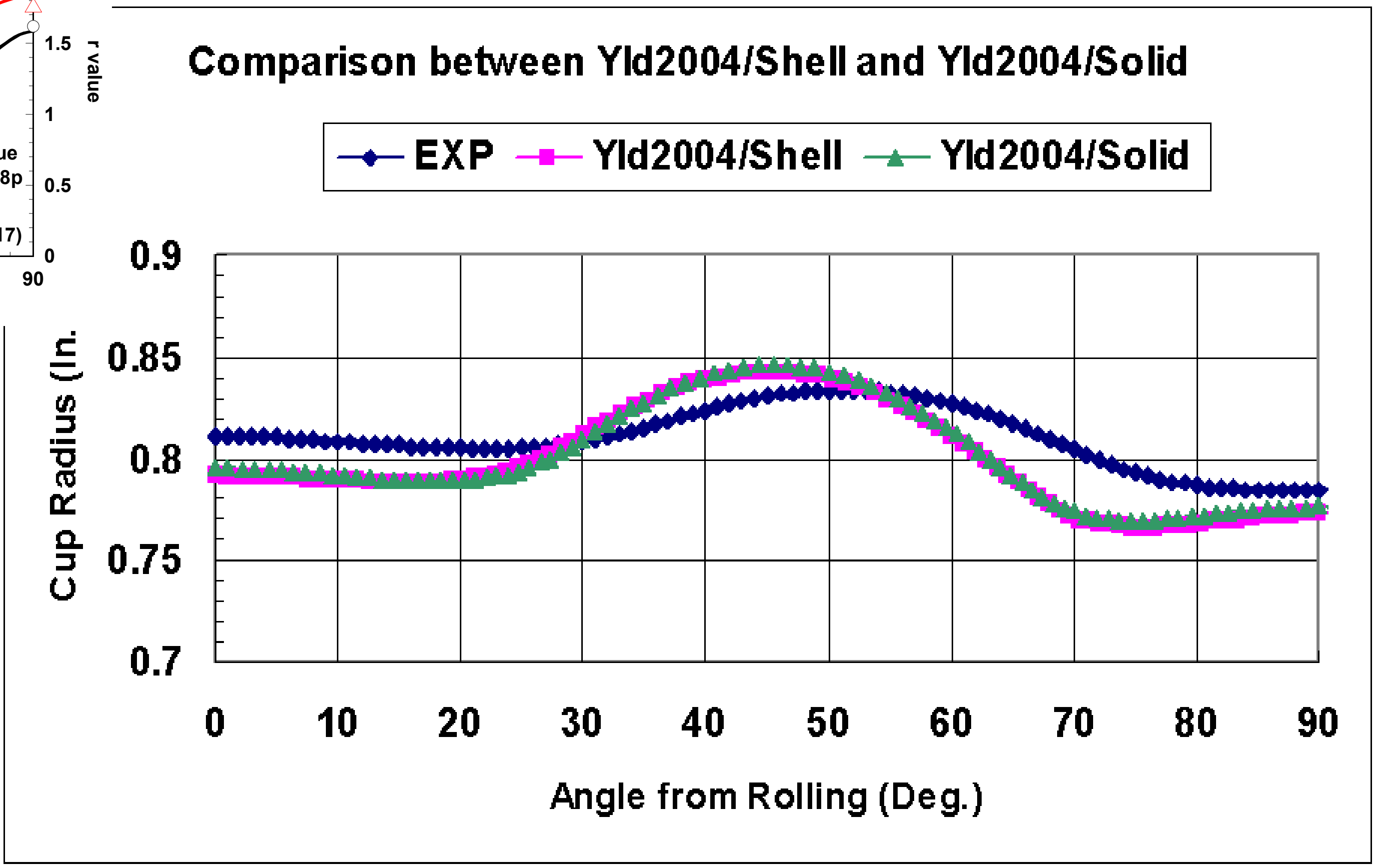
# Mini Die Drawing with Yld2004 : Thickness Contour for AL 5019



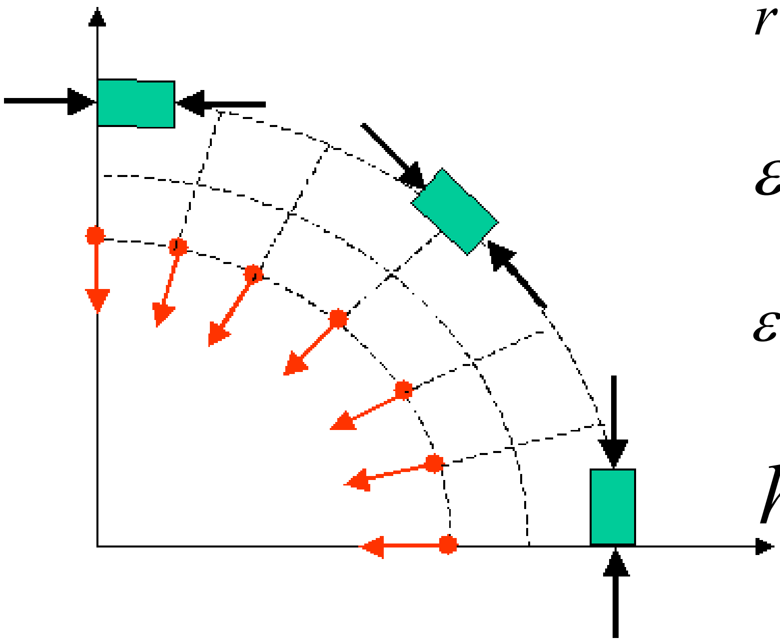



# Earing Profile (shell vs. solid): Yld2004 Prediction for AL 5019



# The relationship between r-value and earing



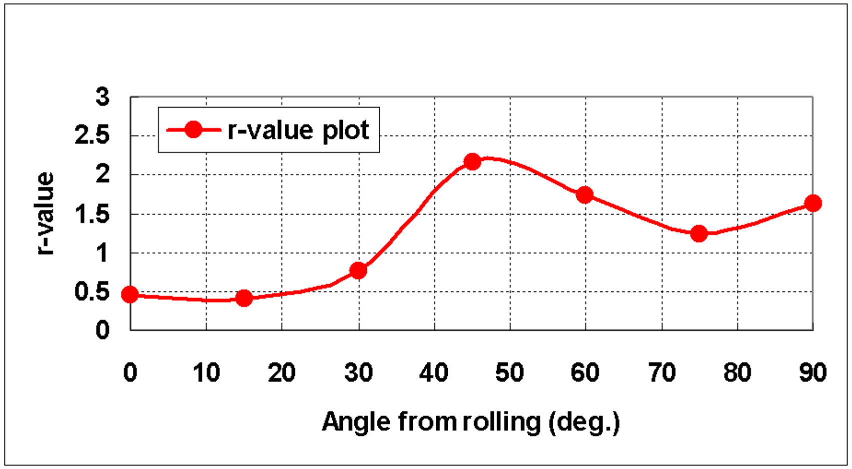
$$r = \frac{\varepsilon_w}{\varepsilon_t} \longrightarrow r_{\theta+90} = \frac{\varepsilon_r}{\varepsilon_t} = -\frac{\varepsilon_r}{\varepsilon_r + \varepsilon_\theta}$$

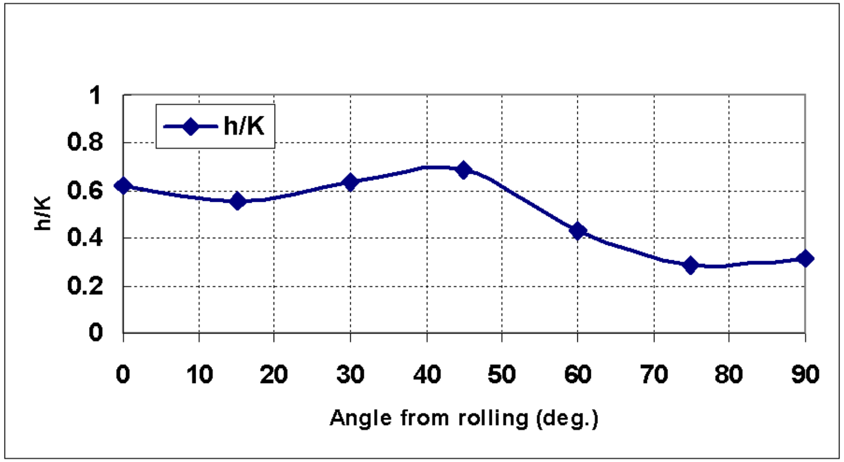
$$\varepsilon_\theta : \varepsilon_r : \varepsilon_t \,|_\theta = -(r_{\theta+90} + 1) : r_{\theta+90} : 1$$

$$\varepsilon_r \,|_\theta = \frac{E r_{\theta+90}}{(r_{\theta+90} + 1)} \quad where \quad \varepsilon_\theta = \ln\left(\frac{R_{cup}}{R_{blank}}\right) = -E$$

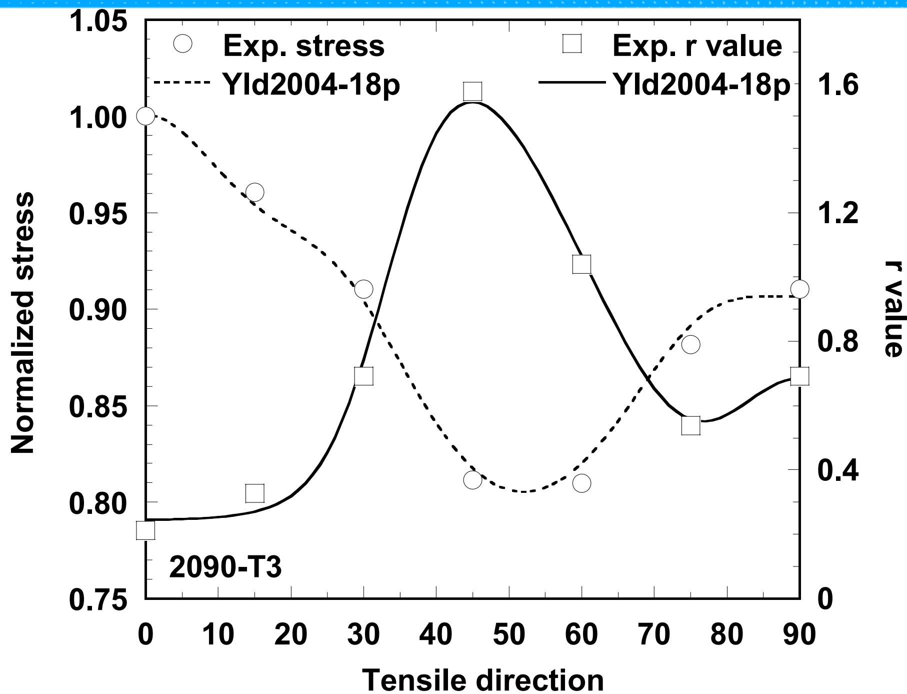
$$h\,|_\theta \propto \varepsilon_r \longrightarrow \boxed{h\,|_\theta / K = \frac{r_{\theta+90}}{(r_{\theta+90} + 1)}}$$

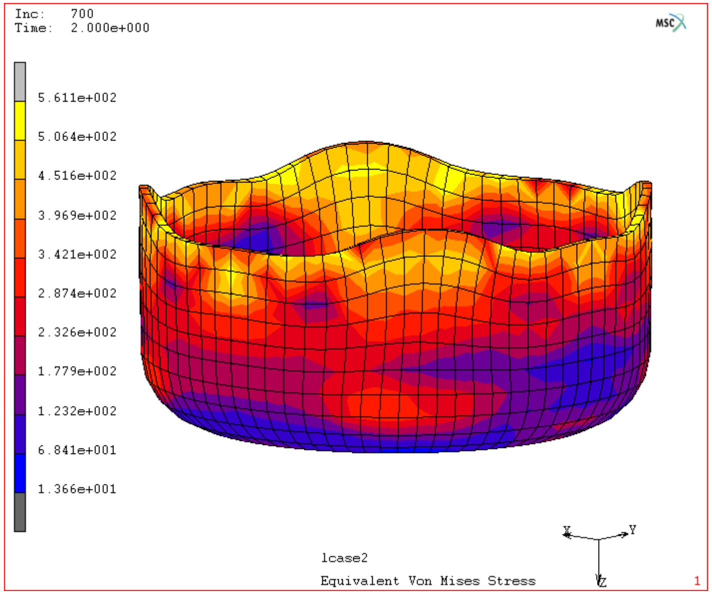
**r-value plot** ←(Mirror Image)→ **Cup Radius or Height**



# 6 Ears Prediction for AL 2090 based on Yld2004

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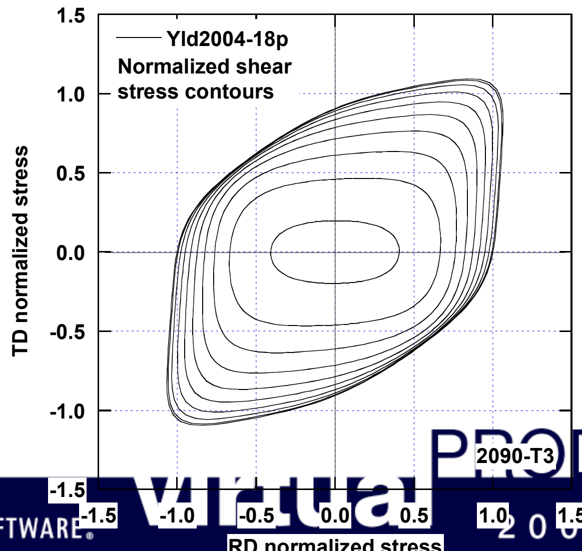
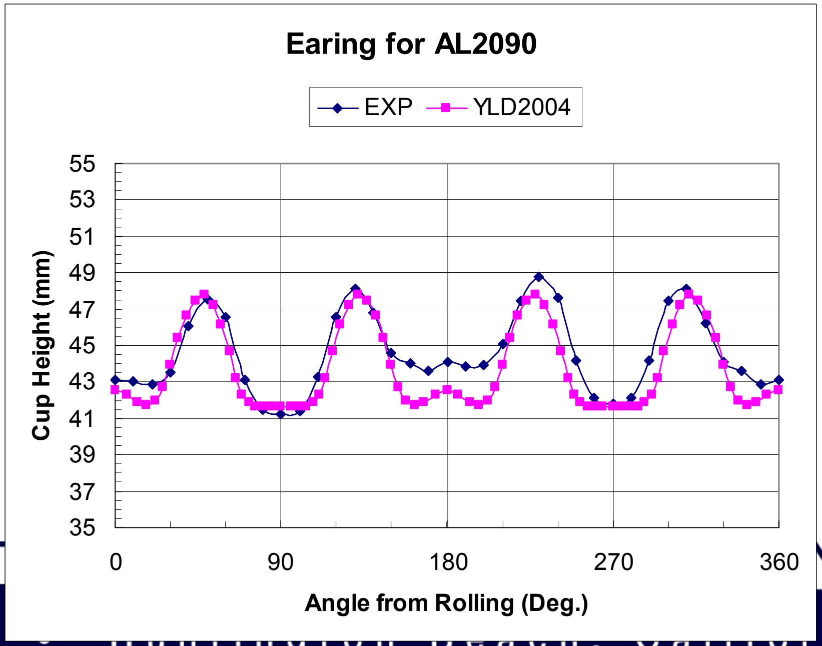
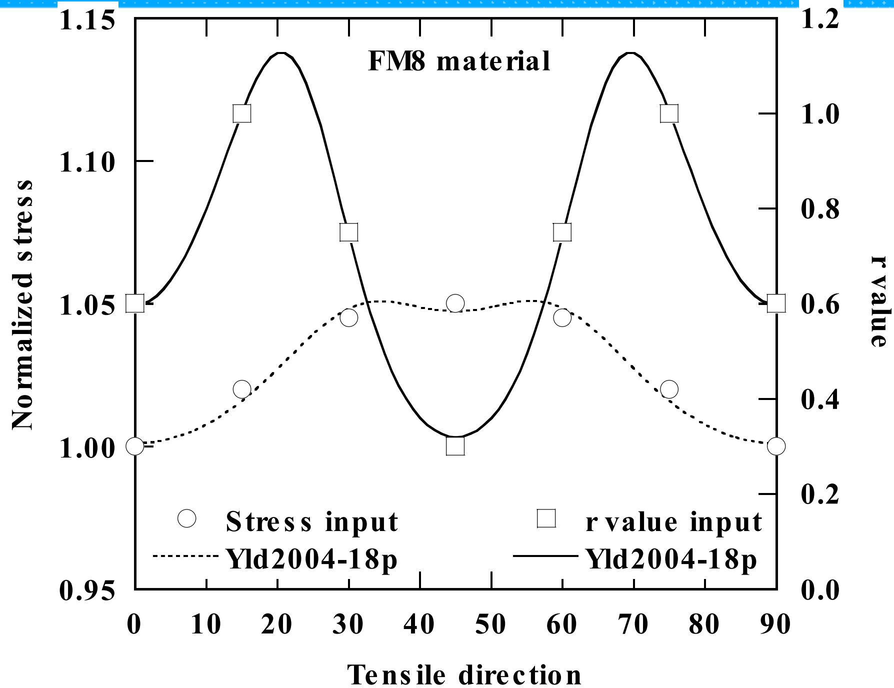
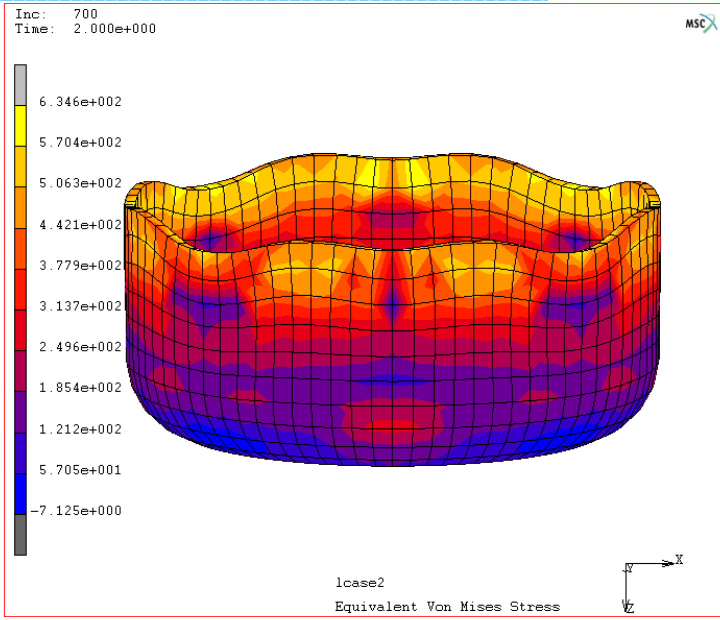
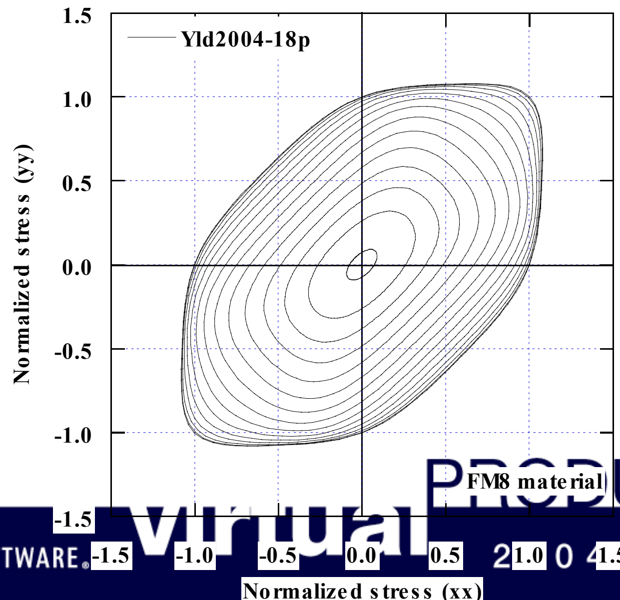


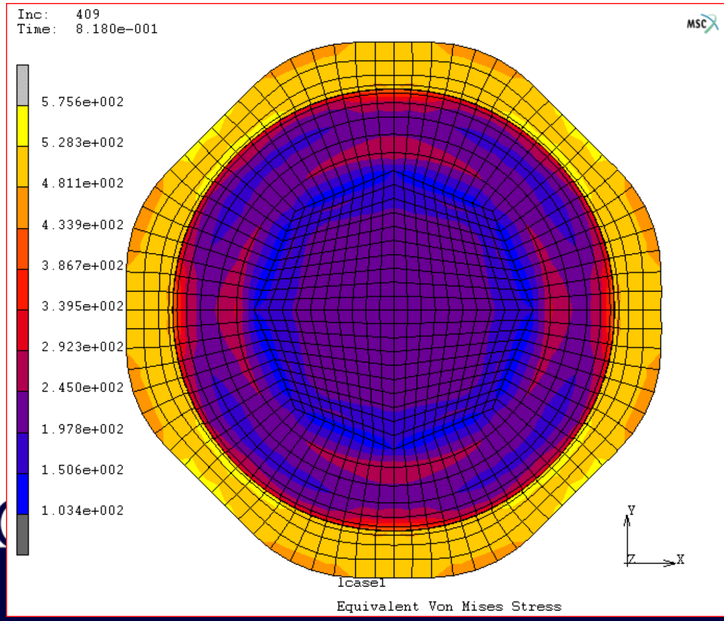



# 8 Ears Prediction based on Yld2004



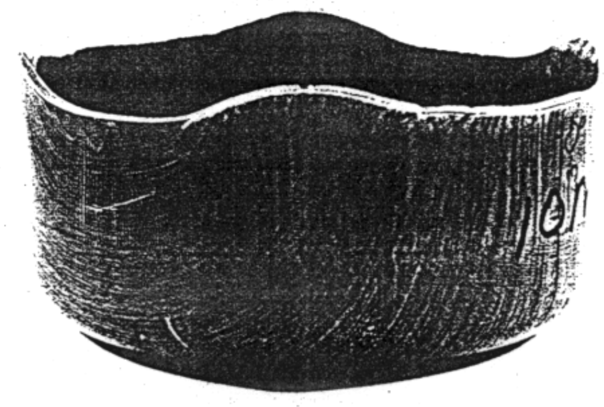



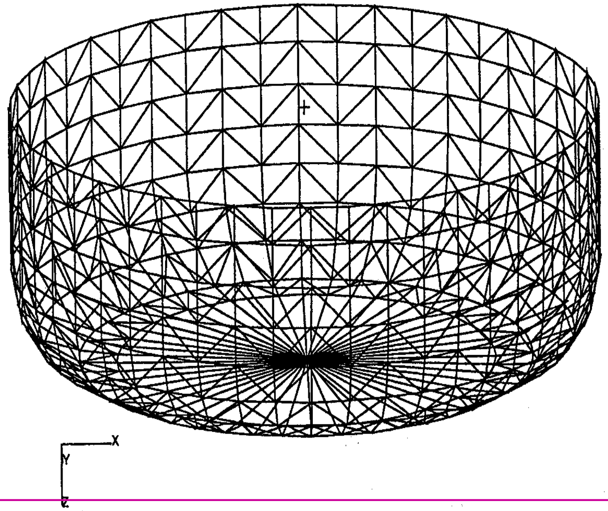
# Earless Target Cup Design ?

**General cup shape after drawing**

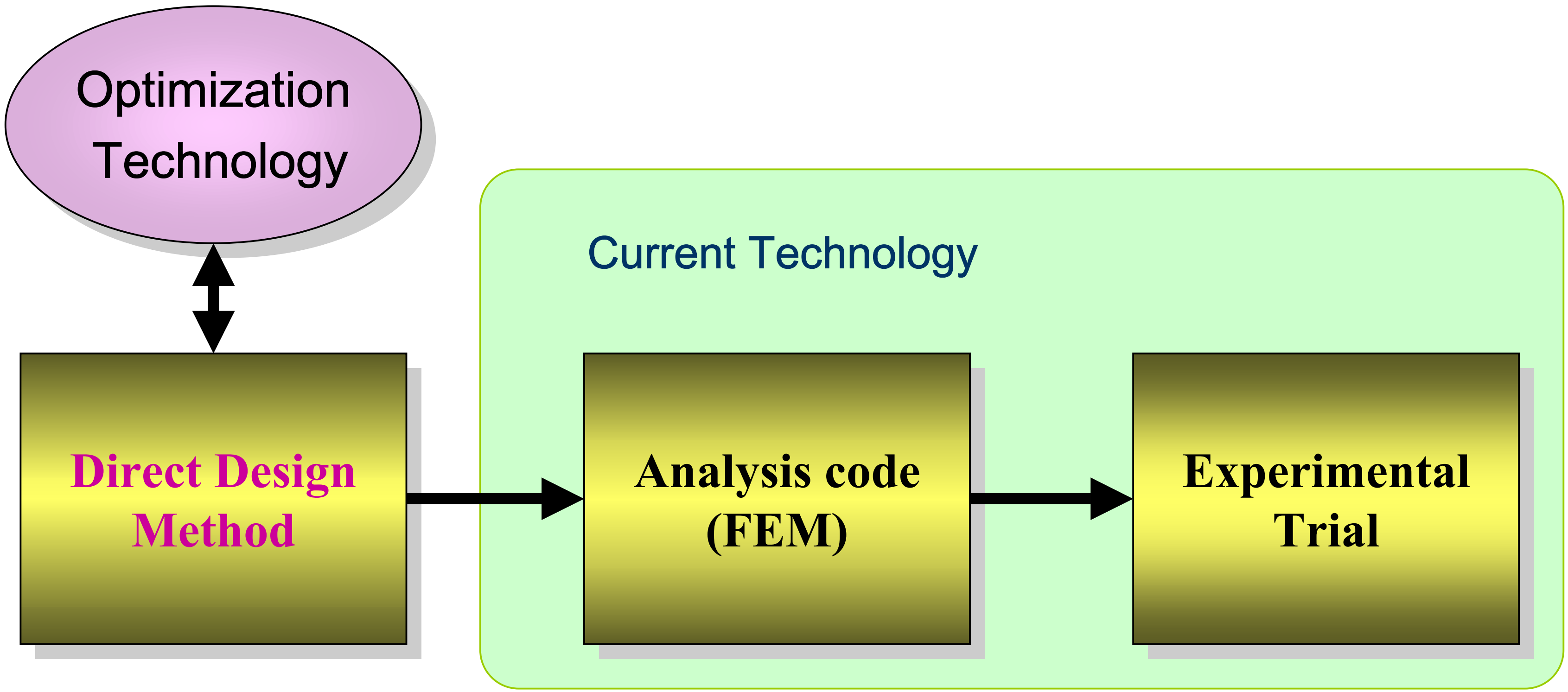


◆⟶

*Earless target cup (?)*



# Optimum Blank Design



# Example : Travel from Seoul to Busan

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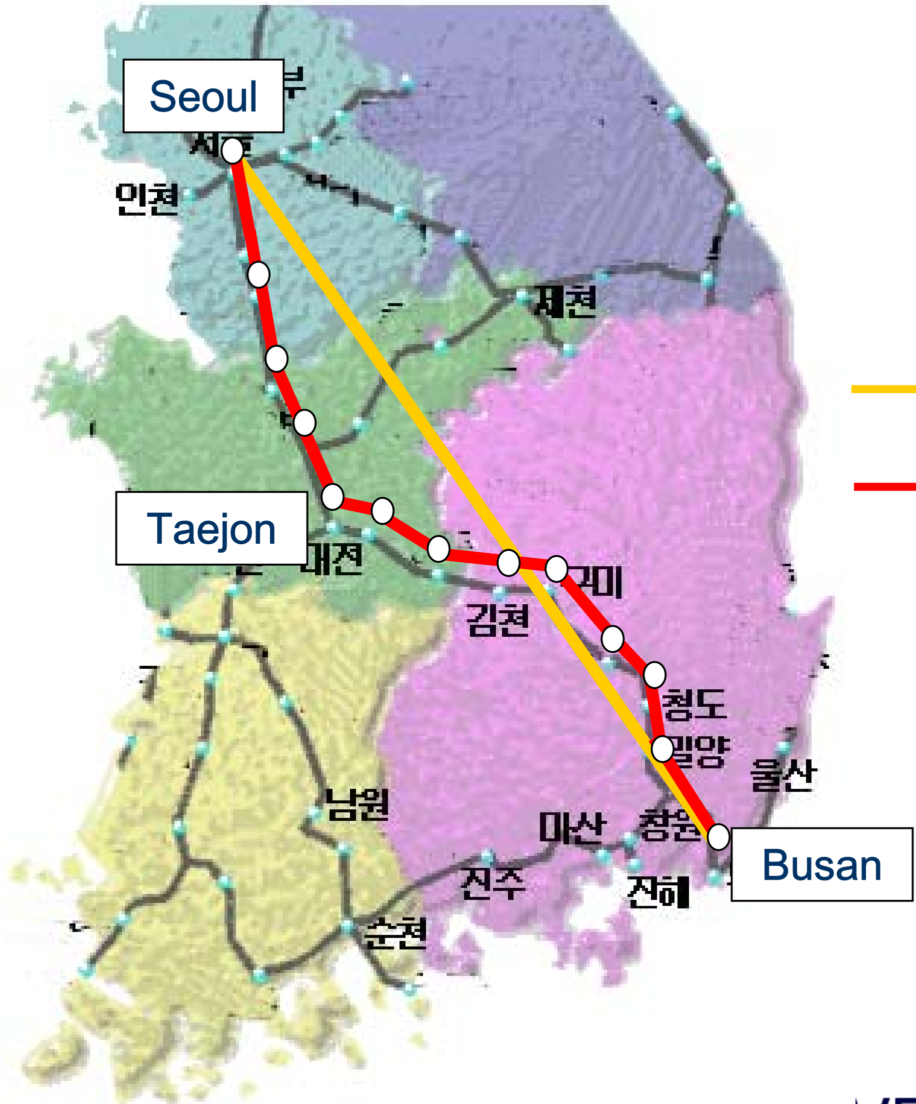


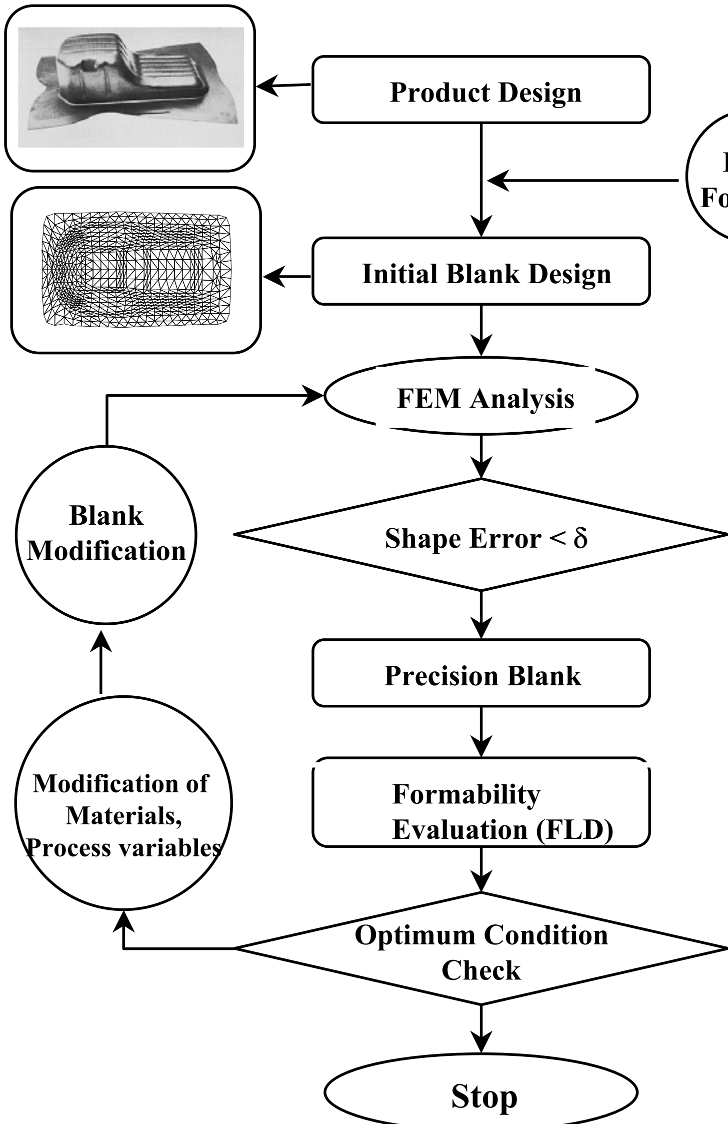

Direct Design Method

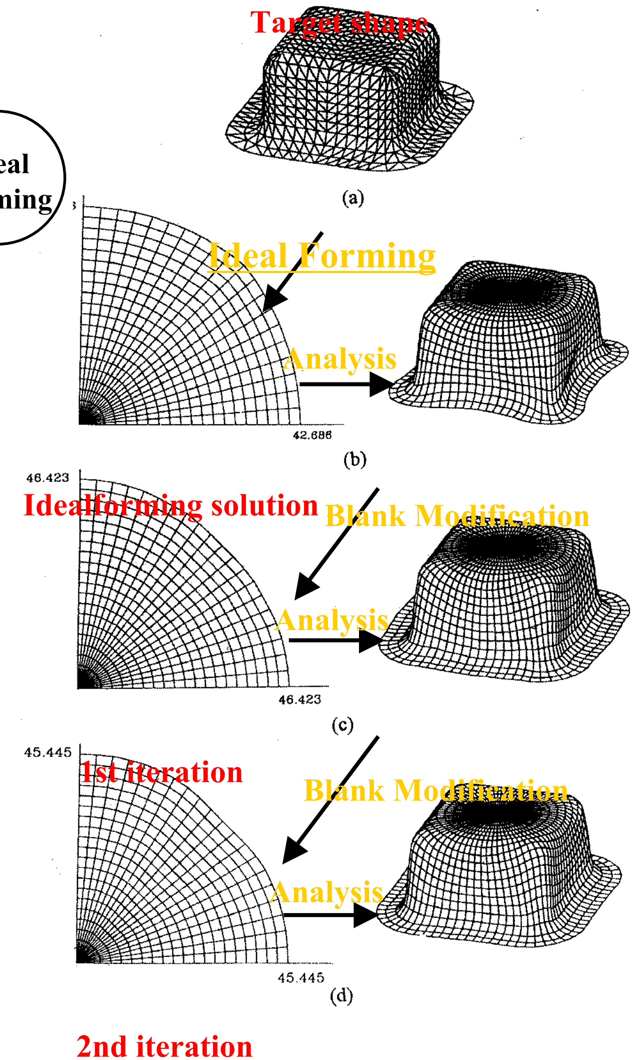
Analysis Method

# Iterative Blank Design using Inverse Method and Analysis Code

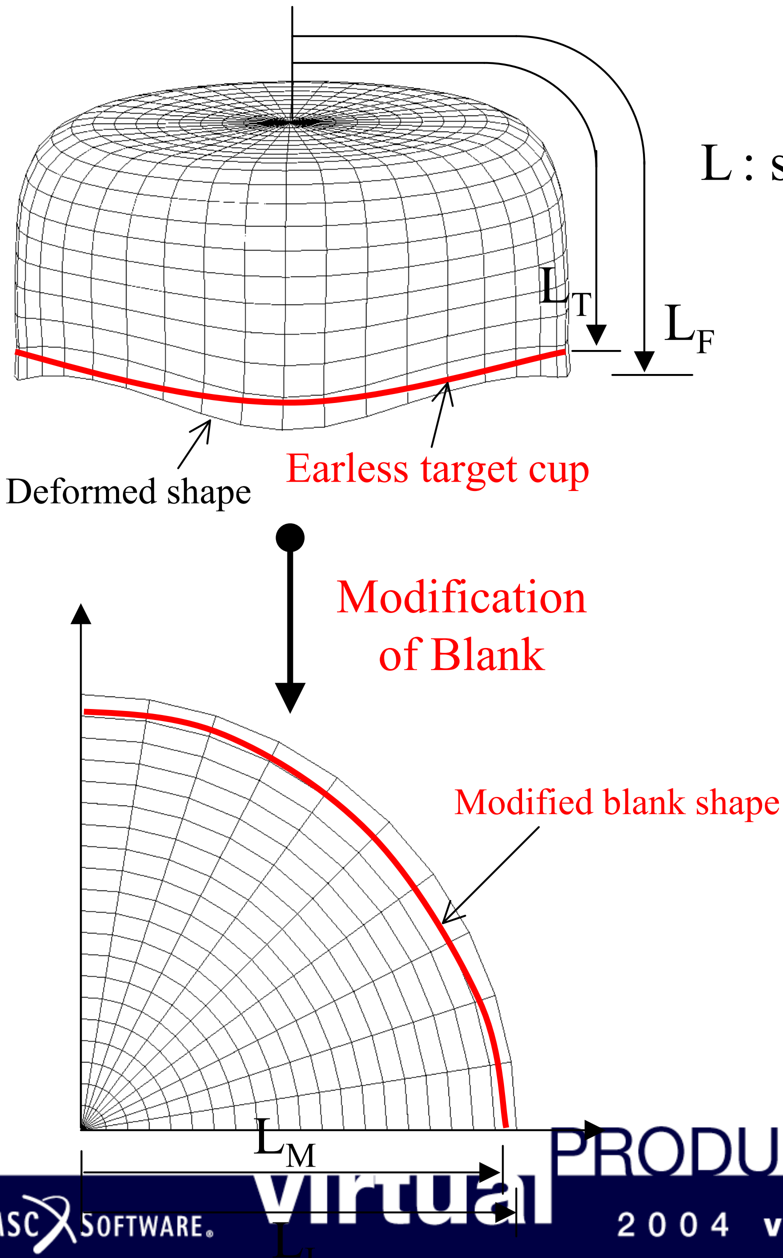
**Combination strategy for optimum blank design :**

1. Ideal Forming design code for initial estimation
2. Analysis for detail modification with deformation path method



# Blank Modification Algorithm for Analysis Code




L : summation of element lengths in radial direction

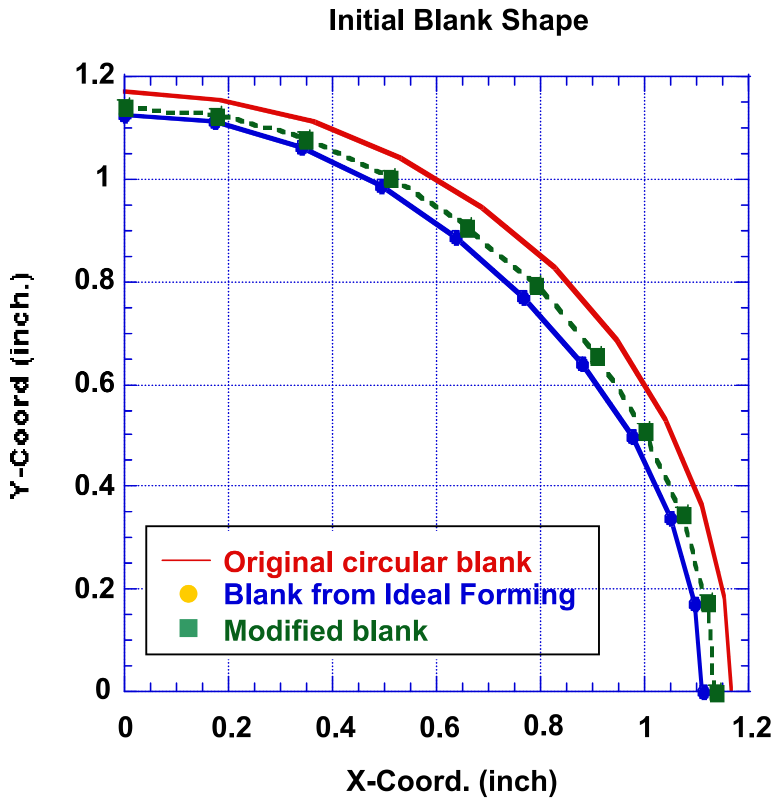
- $L_I$ : for initial geometry
- $L_F$ : for final geometry
- $L_T$ : for target geometry
- $L_M$ : for modified initial geometry

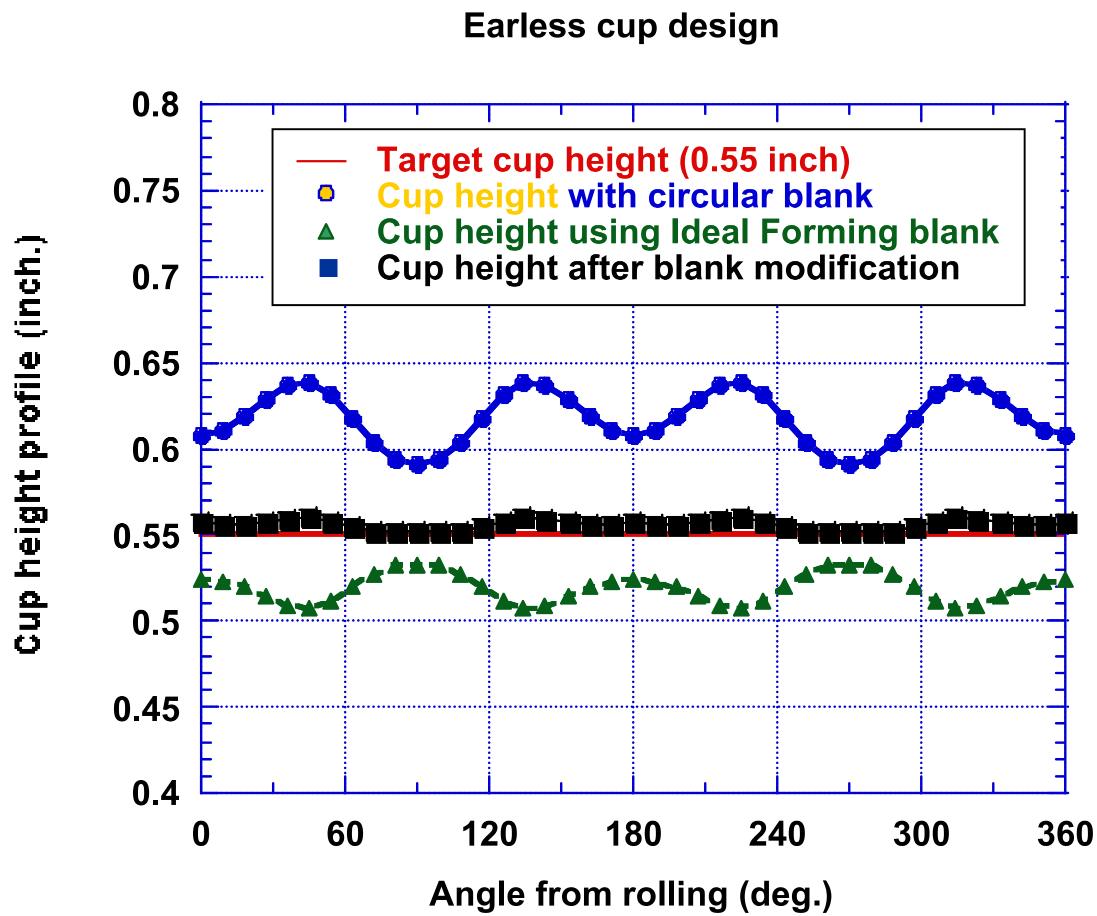
$$\ln \frac{L_F}{L_I} = \ln \frac{L_T}{L_M} \longrightarrow \frac{L_F}{L_I} \approx \frac{L_T}{L_M}$$

$$\therefore L_M = \frac{L_I}{L_F} L_T$$

# Earless Cup Design Result

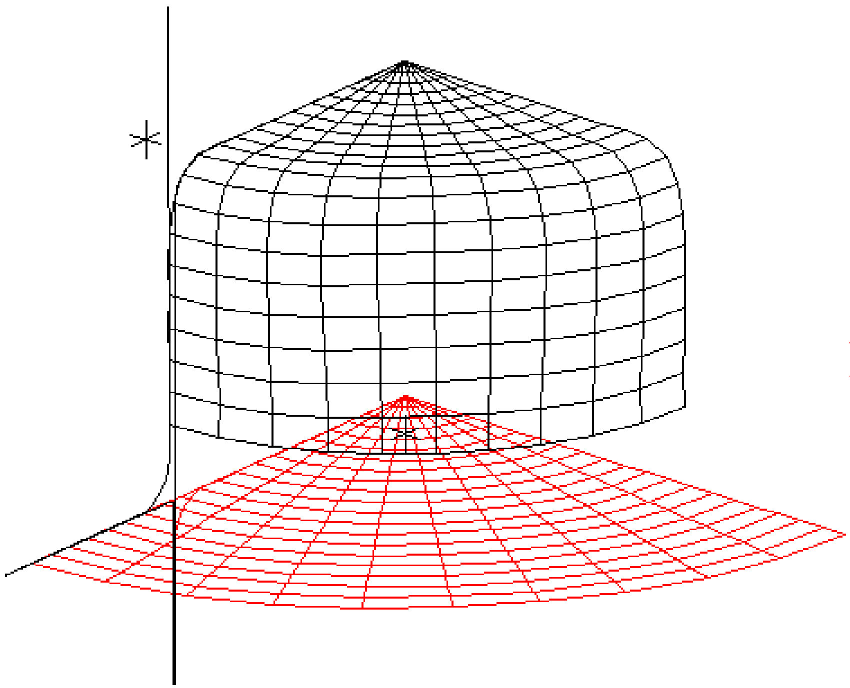
## Mini Die Drawing

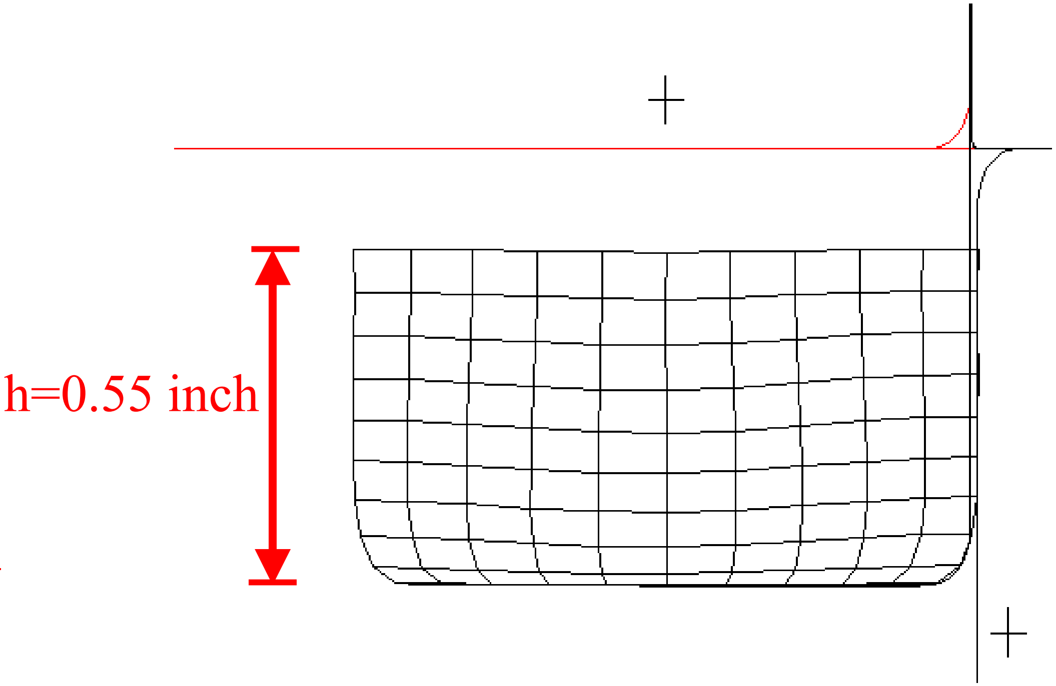


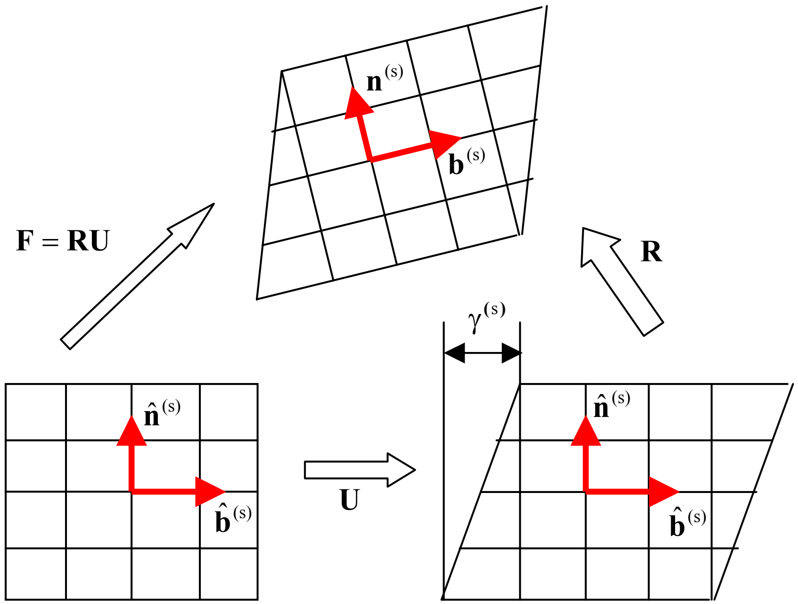
# Earless Cup Design Result

**Mini Die Drawing**





## Polycrystal model based on incremental deformation theory




$$\hat{\mathbf{D}} = \hat{\mathbf{D}}^e + \hat{\mathbf{D}}^p = \mathbf{R}^T\mathbf{D}\mathbf{R} = (\dot{\mathbf{U}}\mathbf{U}^{-1})_s$$

$$\hat{\mathbf{D}}^p = \sum_{(s)} \dot{\gamma}^{(s)} \frac{1}{2}(\hat{\mathbf{b}}^{(s)}\hat{\mathbf{n}}^{(s)} + \hat{\mathbf{n}}^{(s)}\hat{\mathbf{b}}^{(s)}) = \sum_{(s)} \dot{\gamma}^{(s)}\hat{\mathbf{P}}_s$$

$$\mathbf{b}^{(s)} = \mathbf{R}\cdot\hat{\mathbf{b}}^{(s)} \textit{ and } \mathbf{n}^{(s)} = \hat{\mathbf{n}}^{(s)}\mathbf{R}^T$$

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### * Grain Level Stress

$$\hat{\dot{\sigma}} = \hat{\mathbf{C}}(\hat{\mathbf{D}} - \hat{\mathbf{D}}_p) = \hat{\mathbf{C}}(\hat{\mathbf{D}} - \sum_{(s)} \dot{\gamma}^{(s)}\hat{\mathbf{P}}_s)$$

### * Slip System Level Stress

$$\tau^s_{t+\Delta t} = \tau^s_t + \Delta t\, \dot{\tau}^s$$

**where**

$$\tau^s_t = \hat{\sigma}_t : \hat{\mathbf{P}}_s \qquad \dot{\tau}^s = \hat{\mathbf{C}}\hat{\mathbf{P}}_s : \mathbf{D}^* = \hat{\mathbf{C}}\hat{\mathbf{P}}_s : (\hat{\mathbf{D}} - \hat{\mathbf{D}}^p) =$$

$$\hat{\mathbf{C}}\hat{\mathbf{P}}_s : (\hat{\mathbf{D}} - \sum_{(a)} \dot{\gamma}^{(a)}\hat{\mathbf{P}}_a)$$

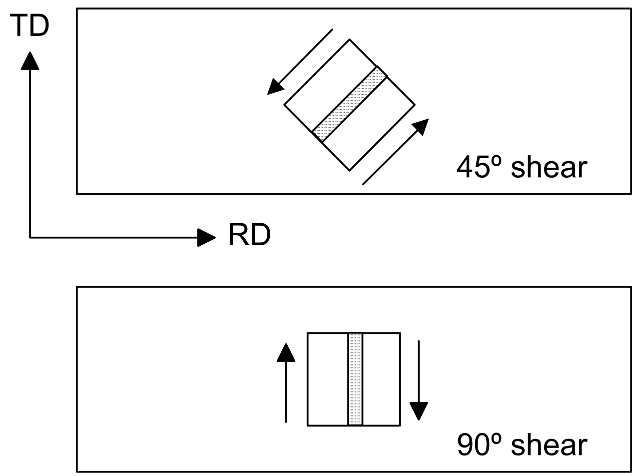
### * Hardening in Slip System Level

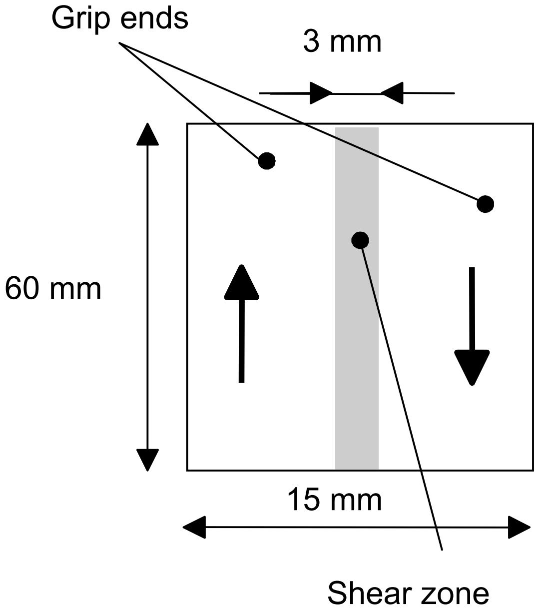
$$\tau^s_c = g^s \left( \frac{|\dot{\gamma}^s|}{\gamma_o} \right)^m \mathrm{sign}(\dot{\gamma}^s)$$

**Nonlinaer Equilibrium Equation**

$$E^s(\dot{\gamma}^{(s)}) = \tau^s_{t+\Delta t} - \tau^s_c = 0$$

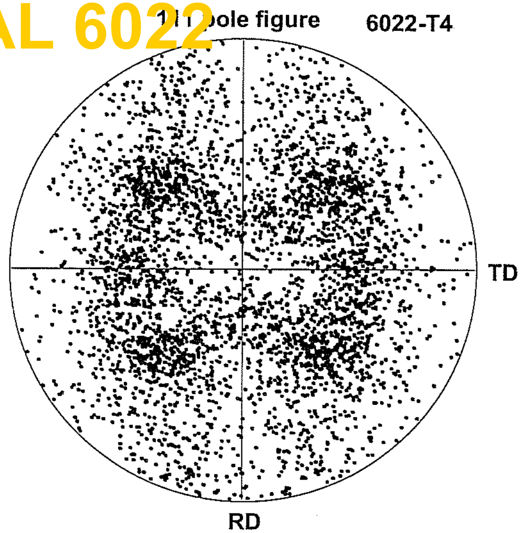
# Shear Test Simulation with Polycrystal Approach



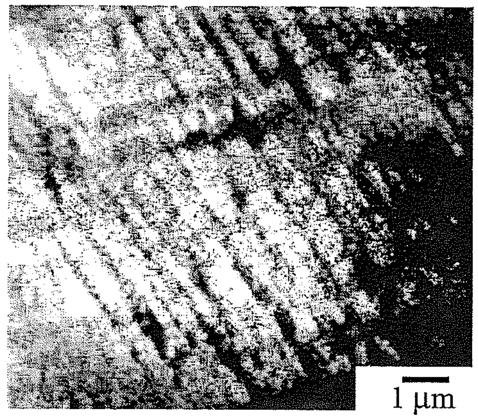



# Numerical Results based on Polycrystal Plasticity

## AL 6022

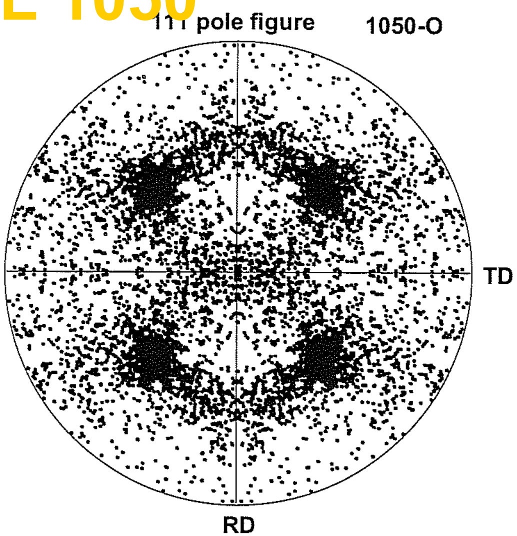



(111) pole figure for 6022-T4

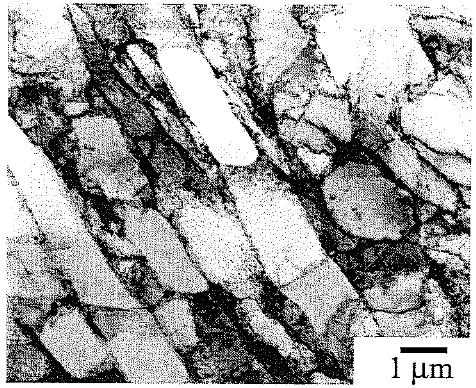



90° shear, γ = 0.35

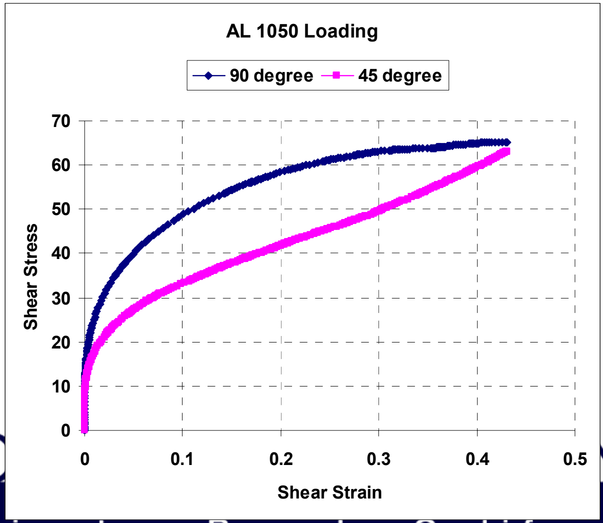

## AL 1050




(111) pole figure for 1050-O

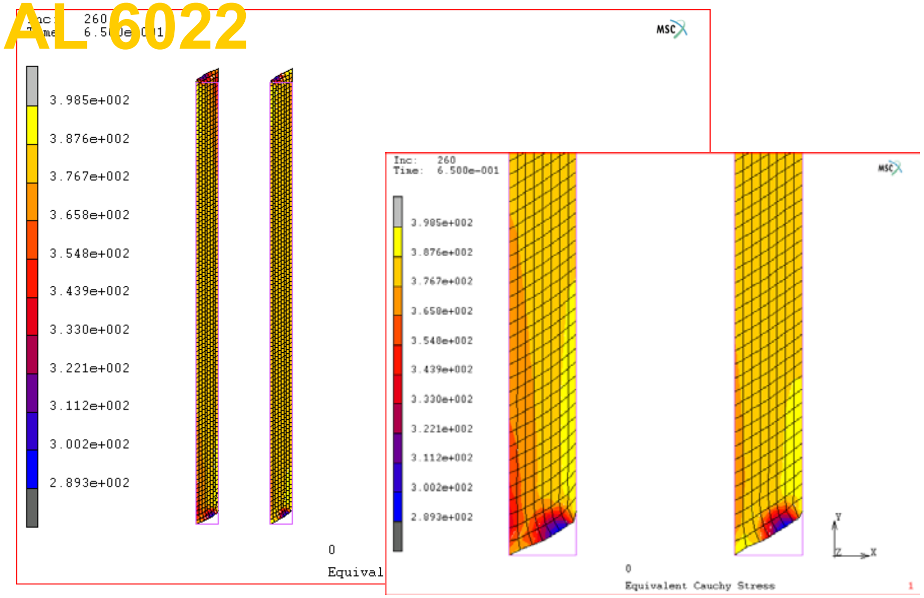



90° shear, γ = 0.30

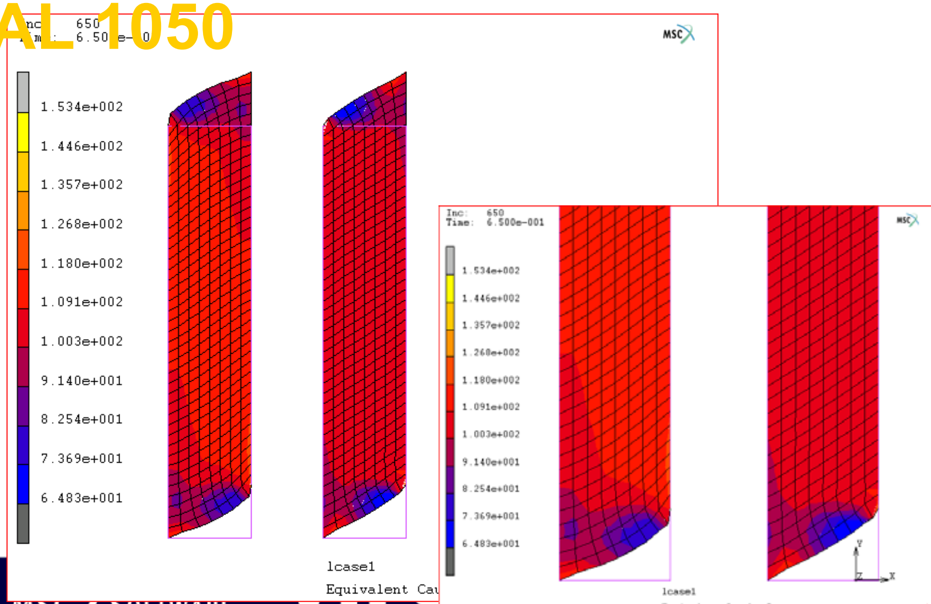


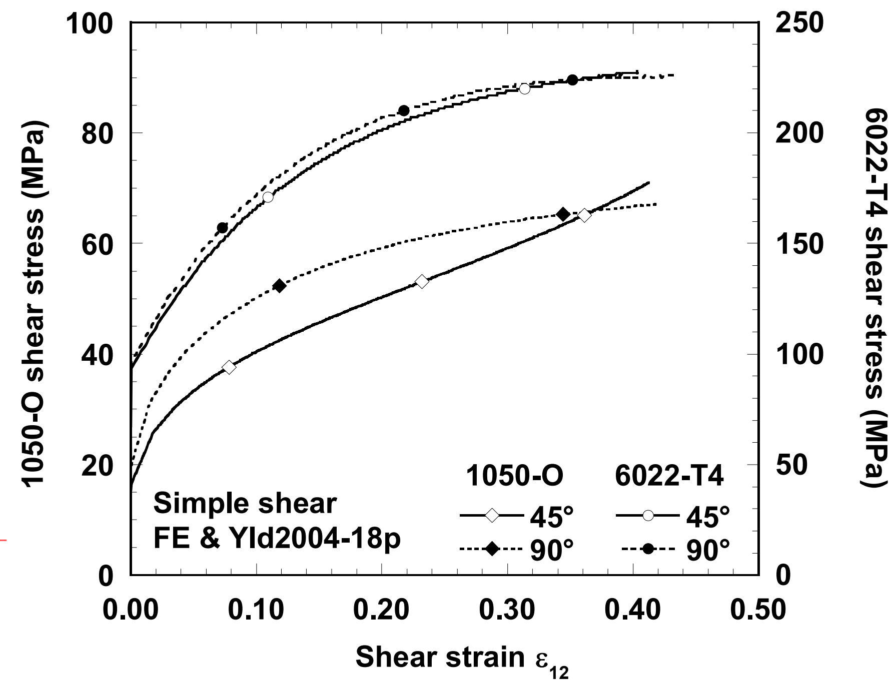
# Numerical Results based on Yld2004

**AL 6022**



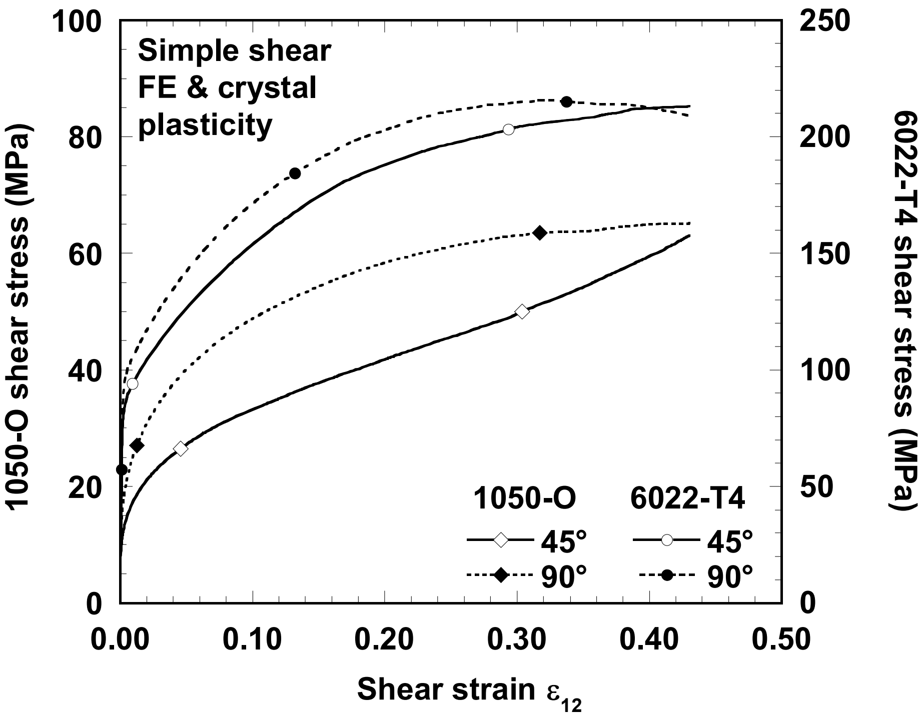

**AL 1050**



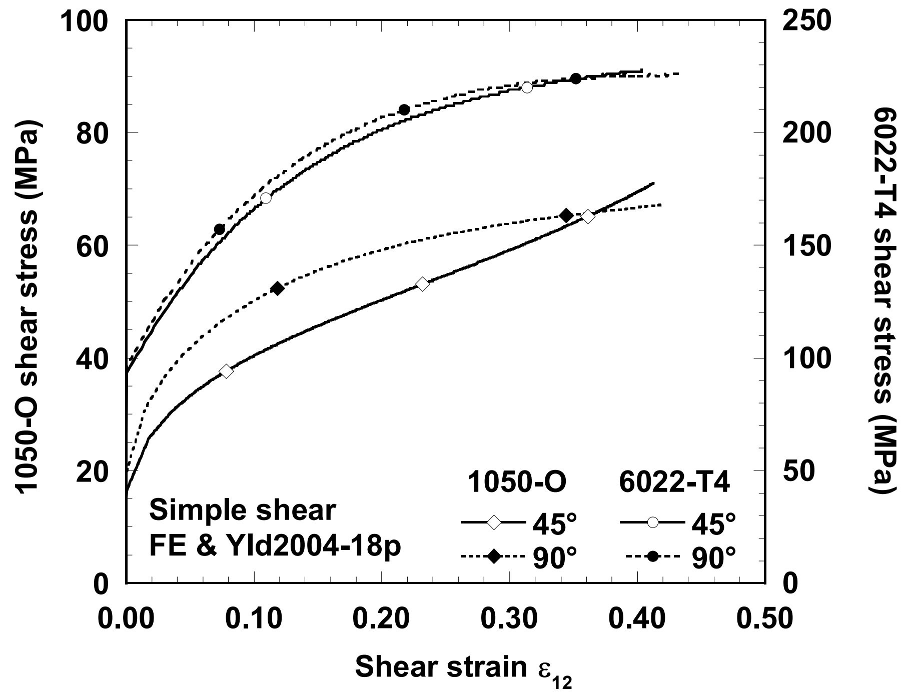



# Comparison between Polycrystal plasticity and Yld2004

## Crystal plasticity




## Yld2004



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**Phenomenological models**

- Anisotropic yield function
- Balance between accuracy and time-efficiency
- Mechanical test data or microstructure modeling results as input

**Microstructural models**

- Microstructure evolution
- Quantitative microstructure data needed as input
- Time-consuming in FE applications