An Efficient Constitutive Model for Shape Memory Alloy Materials

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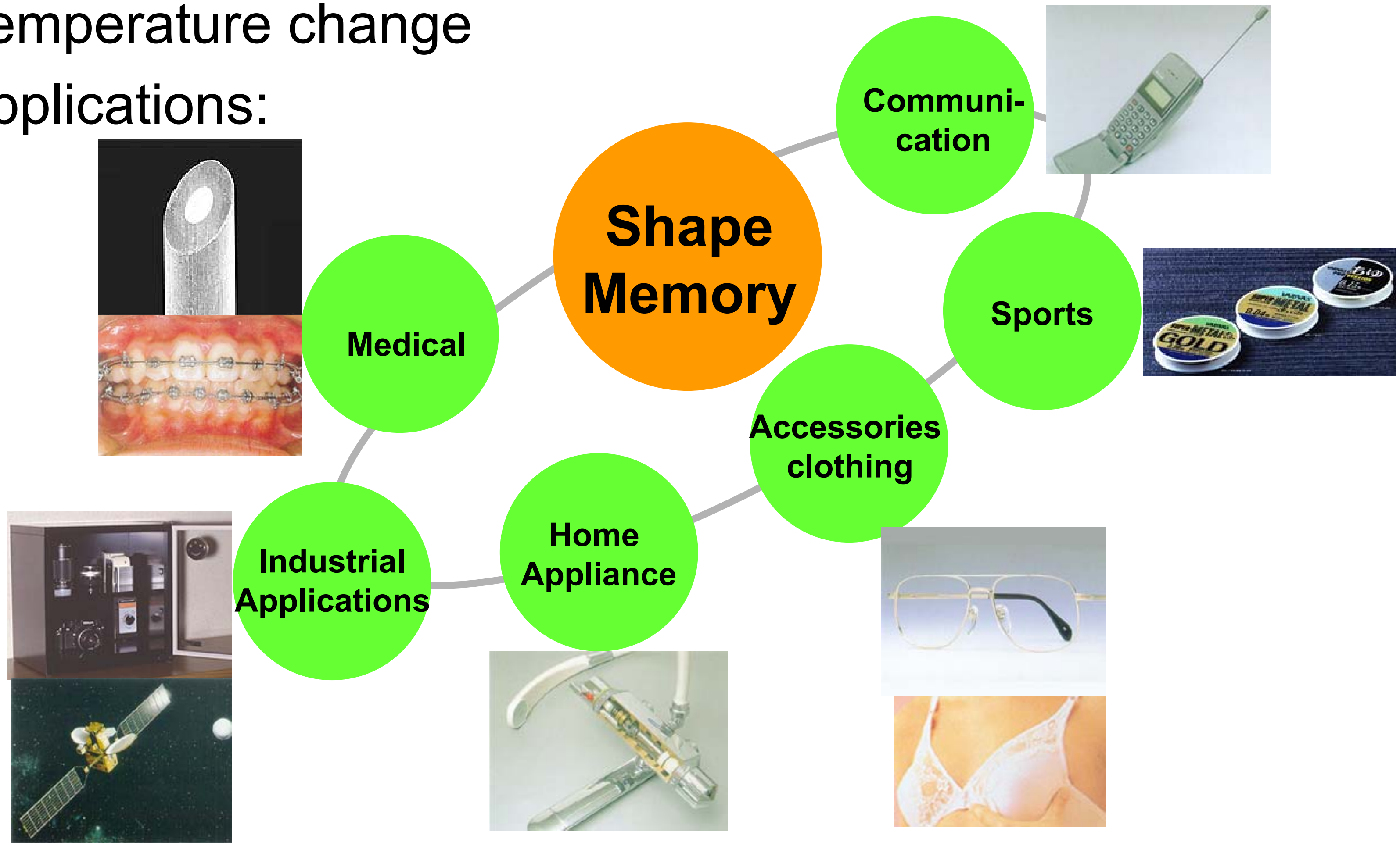
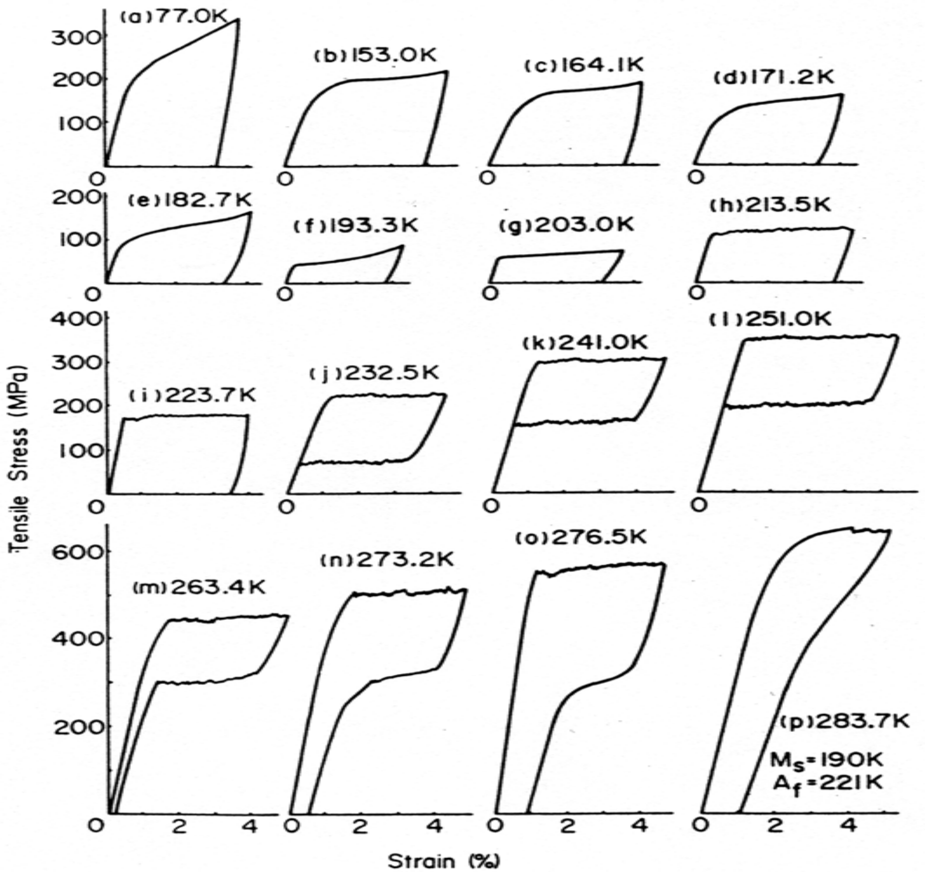
Shape Memory Alloys

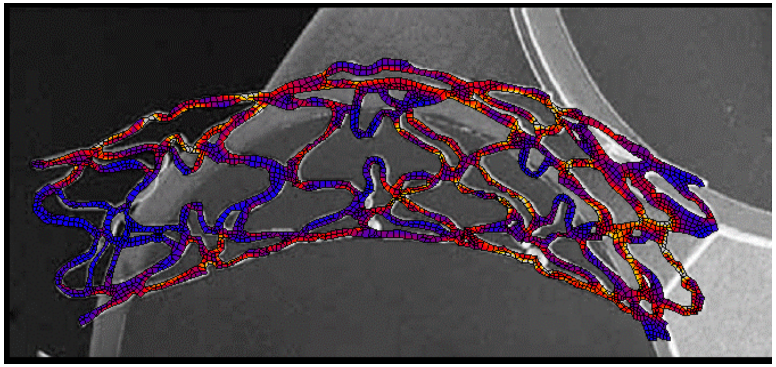
Materials that return to some shape upon appropriate temperature change

Applications:



Nickel-Titanium alloys to show shape memory effect that severely deformed specimens, with residual strains up to 15%, regained their original shape after a loading cycle at a certain temperature.

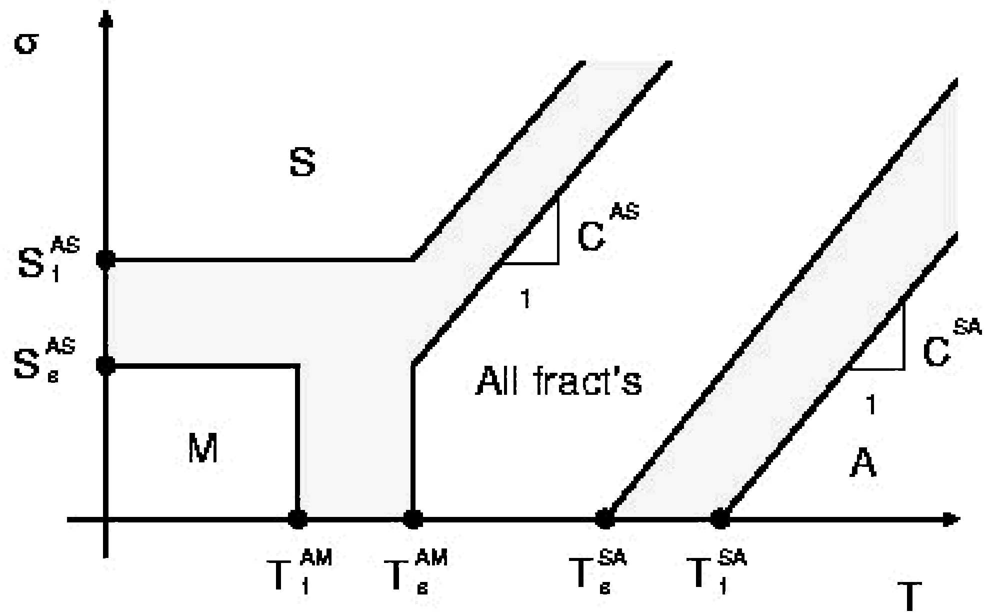




NiTi Stent

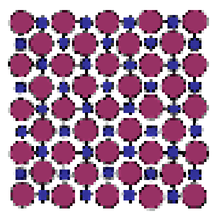
Shape memory effect is a consequence of a crystallographically reversible solid-solid phase transformation occurring in particular metal alloys (Ni – Ti, Cu based alloys).

This transition occurs between a crystallographically more-ordered phase (called *austenite*) and a crystallographically less-ordered phase (*martensite*).

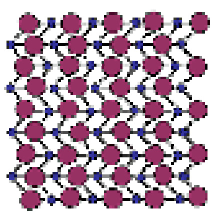


Stability for Martensite and Austenite Phases

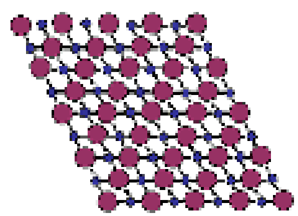
Austenite
- High temperature phase
- Cubic Crystal Structure



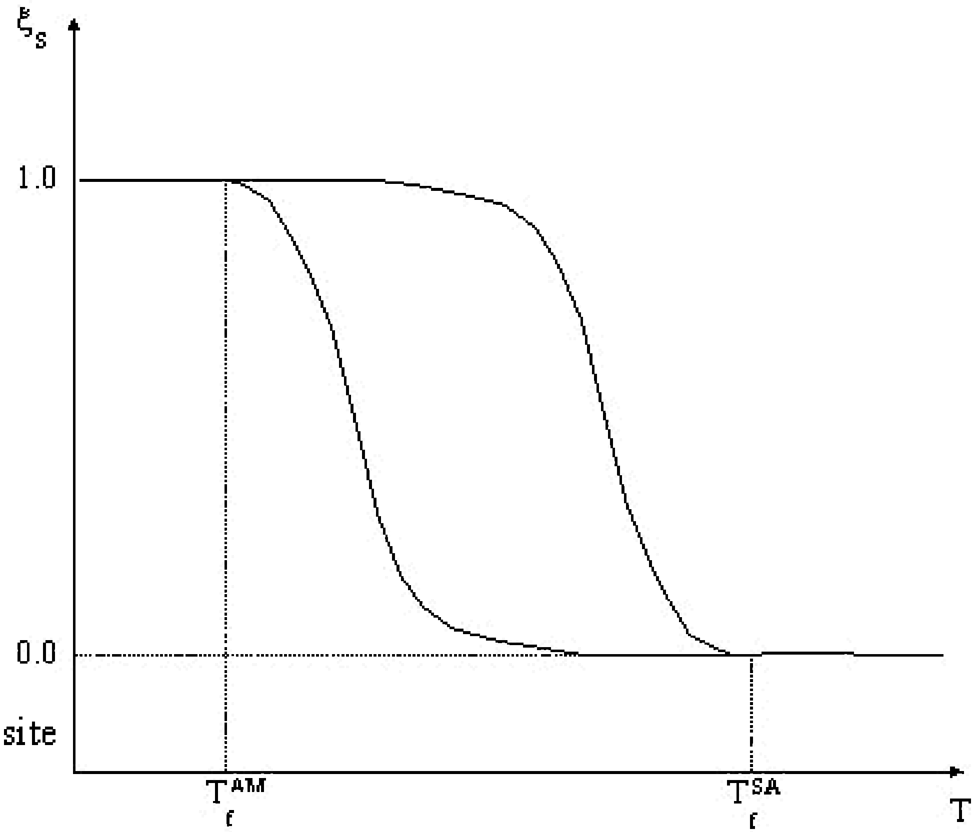
Martensite
- Low temperature phase
- Monoclinic Crystal Structure



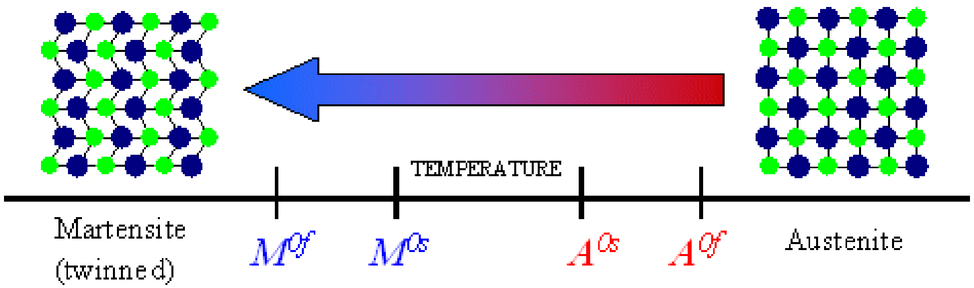
Twinned Martensite

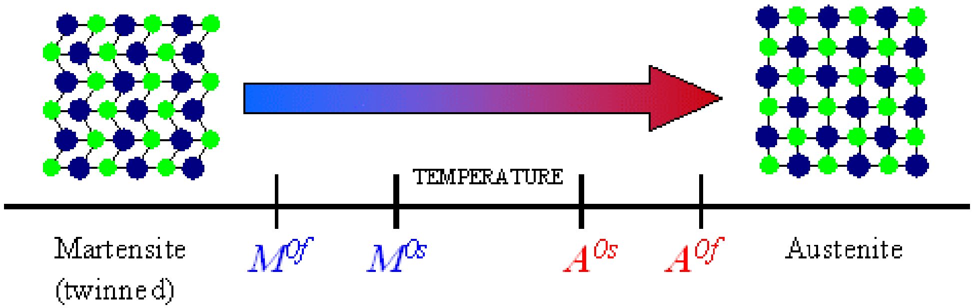


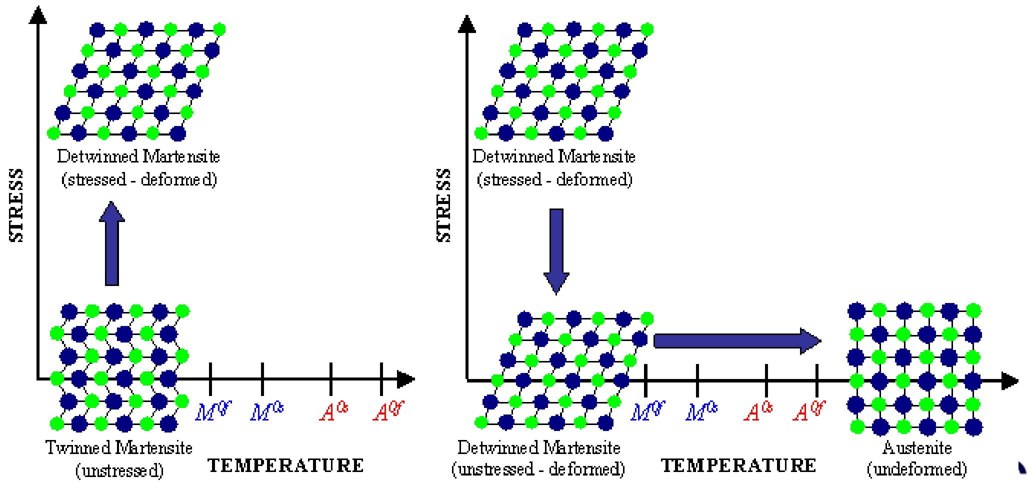
Detwinned Martensite



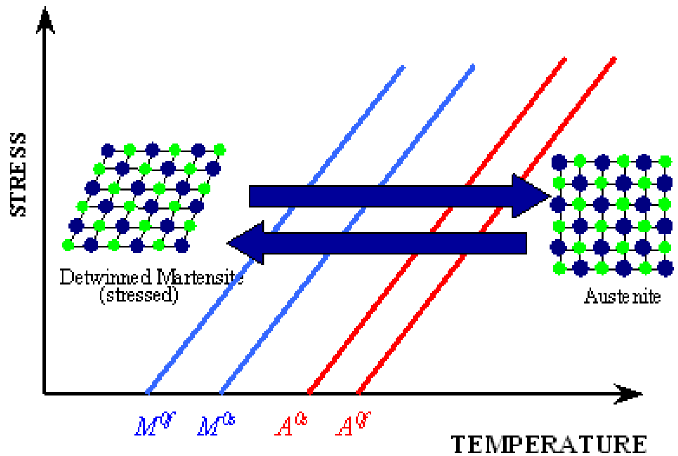
Shape Memory Effect

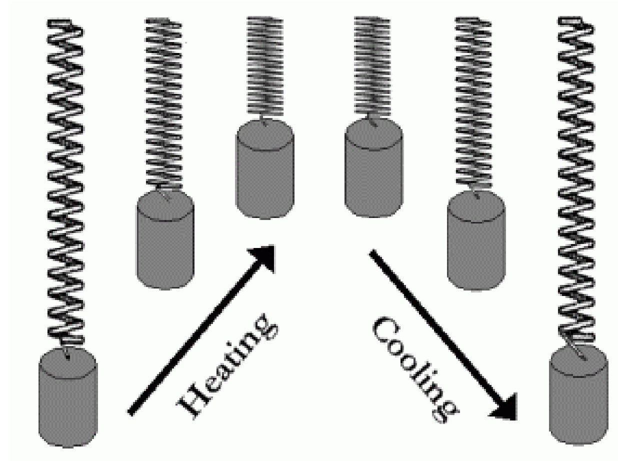




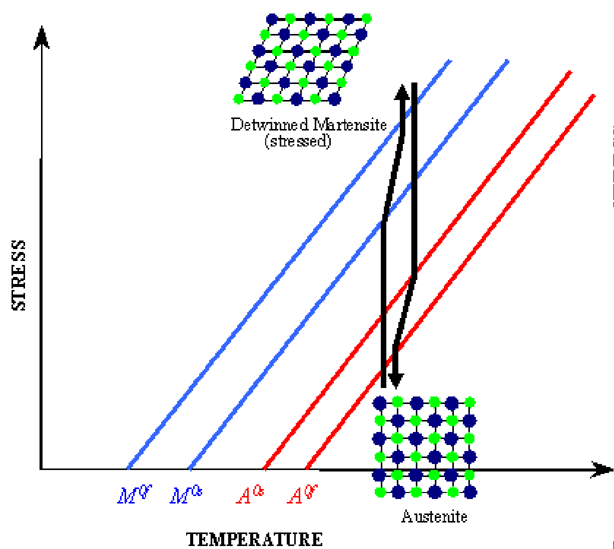


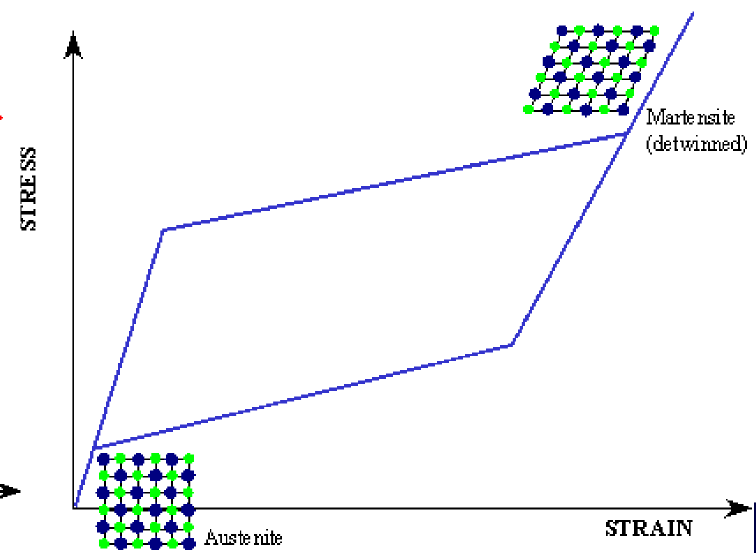
Temperature induced phase transformation





Peudoelastic Stress-Strain Behavior

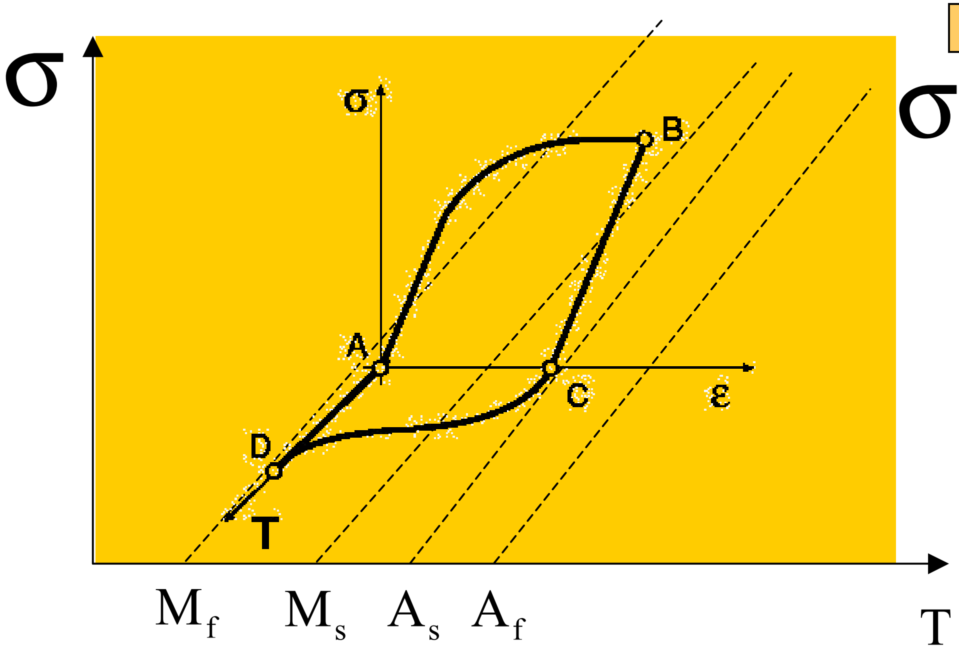




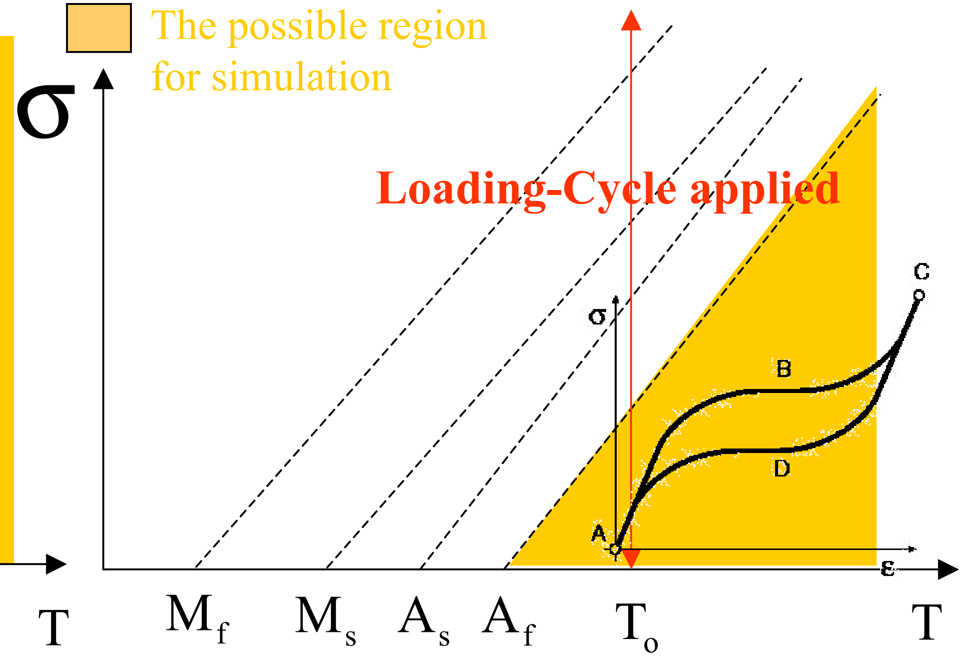
- Thermo-mechanical model
  (Saeedvafa & Assaro's model)

- Mechanical model :
  (Auricchio's model) based on super-elasticity

Thermo-mechanical model



Mechanical model

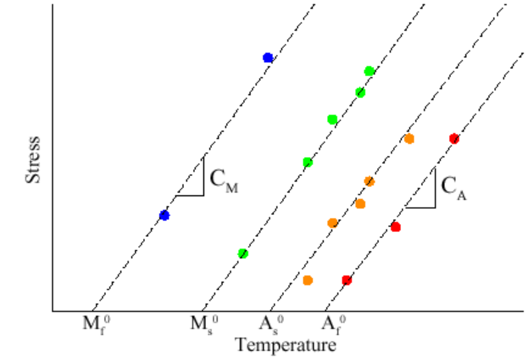


Formulation : Thermo-Mechanical Shape Memory Alloy

$$\dot{\varepsilon} = \dot{\varepsilon}^{EL} + \dot{\varepsilon}^{Th} + \dot{\varepsilon}^{PL} + \dot{\varepsilon}^{Ph} \left(= \dot{\varepsilon}^{TRIP} + \dot{\varepsilon}^{TWIN} \right)$$

where

$$\dot{\varepsilon}^{EL} = \mathbf{L}^{-1} : \dot{\sigma} \;, \quad \dot{\varepsilon}^{Th} = \alpha \dot{T} \mathbf{I} \,, \quad \dot{\varepsilon}^{PL} = \dot{\lambda} \frac{3\dot{\sigma}'}{2\sigma_{eq}}$$



$$\dot{\varepsilon}^{TRIP} = \dot{f}^{(+)} \underline{g}(\sigma_{eq}) \underline{\varepsilon^{T}_{eq}} \frac{3}{2} \frac{\sigma'}{\sigma_{eq}} + \dot{f}^{(+)} \underline{\varepsilon^{T}_{v}} \mathbf{I} + \dot{f}^{(-)} \varepsilon^{Ph}$$

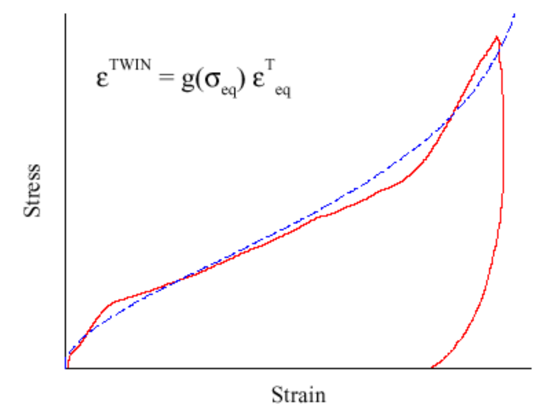
$$\dot{\varepsilon}^{TWIN} = f \, \underline{\dot{g}}(\sigma_{eq}) \underline{\varepsilon^{T}_{eq}} \frac{3}{2} \frac{\sigma'}{\sigma_{eq}} \{\sigma_{eq}\} \{\sigma_{eq} - \underline{\sigma^{g}_{eff}}\}$$

where

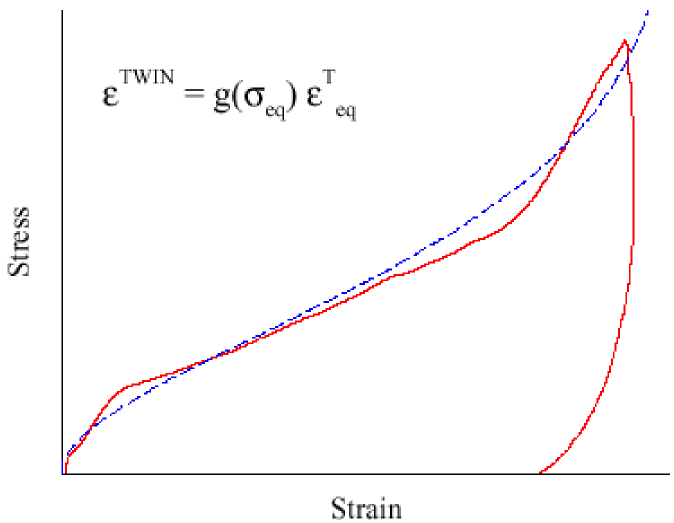
$$\dot{f} = \dot{f}^{(+)}(T, M_s, M_f, C_m, \sigma_{eq}) + \dot{f}^{(-)}(T, A_s, A_f, C_a, \sigma_{eq})$$

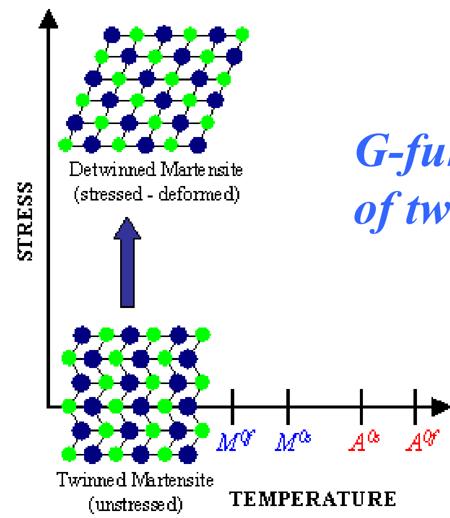
$$\{x\} = 0.5 * (x + |x|) / |x|$$

— : input value

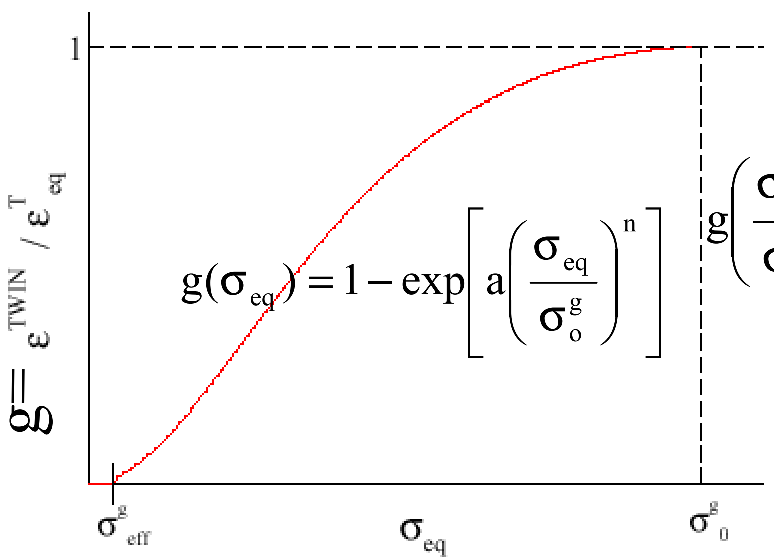


Formulation(Continued) : Thermo-Mechanical Shape Memory Alloy





G-function fits the formation of twin stress below M_f

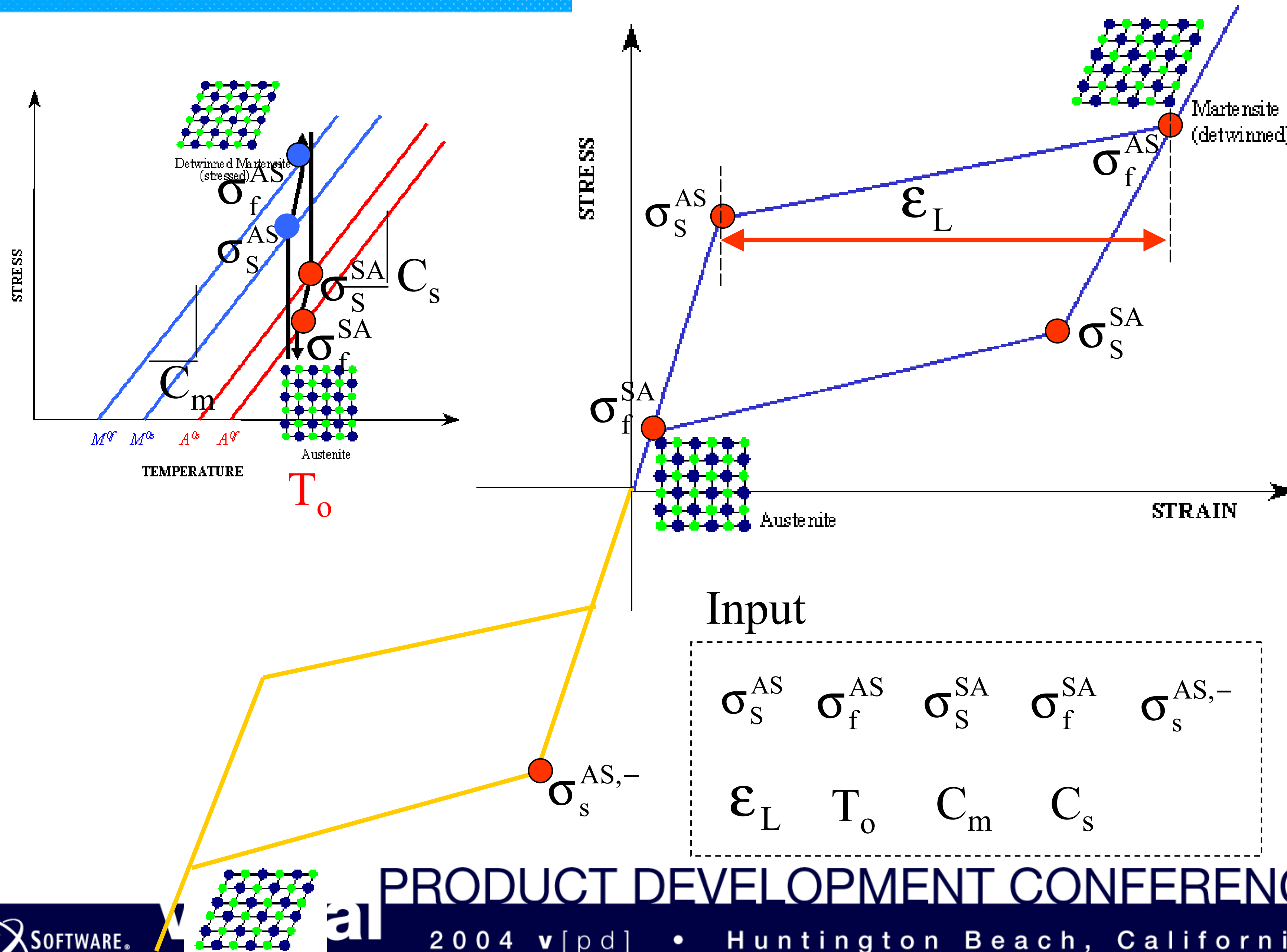


More general form for g-function

$$g\left(\frac{\sigma_{eq}}{\sigma^{g}_{o}}\right) = 1 - \exp\left[g_a\left(\frac{\sigma_{eq}}{\sigma^{g}_{o}}\right)^{g_b} + g_c\left(\frac{\sigma_{eq}}{\sigma^{g}_{o}}\right)^{g_d} + g_e\left(\frac{\sigma_{eq}}{\sigma^{g}_{o}}\right)^{g_f}\right]$$

$$g_a < 0, \quad g_b = 3.0, \quad g_c \geq 0,$$
$$g_d = 2.75, \quad g_e \geq 0, \quad g_f = 3.0$$

Input Data for Mechanical Shape Memory Alloy



Input

σ_S^{AS} σ_f^{AS} σ_S^{SA} σ_f^{SA} $\sigma_s^{AS,-}$

ε_L T_o C_m C_s

Kinematics :

$$\mathbf{F} = \mathbf{F}^{e}\mathbf{F}^{trT}$$

$$\mathbf{b} = \mathbf{F}\mathbf{F}^{T} = \sum_{A=1}^{3} (\lambda_A)^2 \mathbf{n}^A \otimes \mathbf{n}^B \quad \text{Then ,} \quad J = \lambda_1 \lambda_2 \lambda_3 \quad \text{and} \quad \bar{\lambda}_A = (J)^{-1/3} \lambda_A$$

$$\theta = \log(J), \quad e_A = \log(\bar{\lambda}_A)$$

Drucker-Prager type loading function

$$F(\tau) = \left\| \sum_{A=1}^{3} t_A \mathbf{n}^A \otimes \mathbf{n}^B \right\| + 3\alpha p$$

$$\left| \begin{array}{l} t_A = 2G(e_A - \varepsilon_L \xi_S n_A) \\ p = K(\theta - 3\alpha \varepsilon_L \xi_S) \end{array} \right.$$

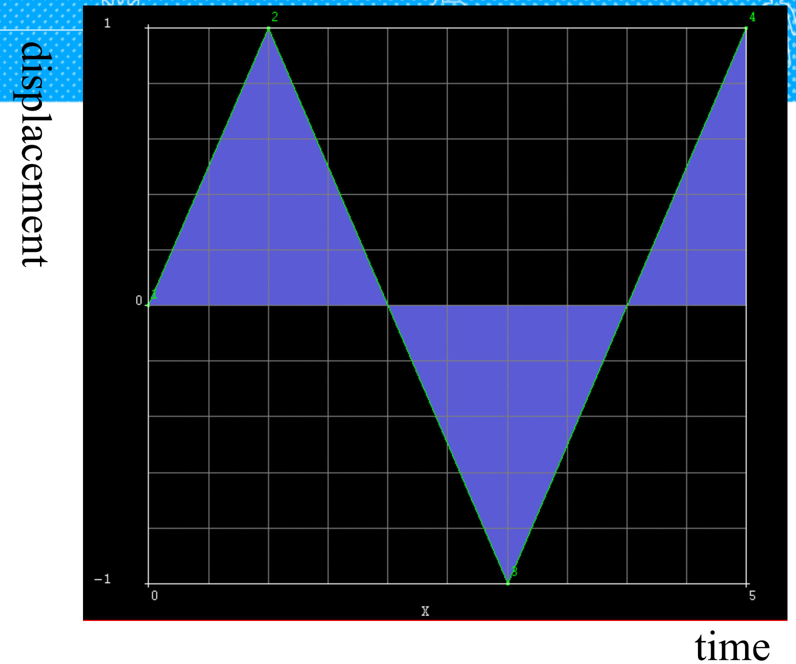
$$R^{AS} = (F - R_f^{AS})\lambda_s - H^{AS}(1 - \xi_S)(F - F_n) = 0 \quad (A \rightarrow M)$$

$$R^{SA} = (F - R_f^{SA})\lambda_s - H^{SA}\xi_S(F - F_n) = 0 \quad (M \rightarrow A)$$

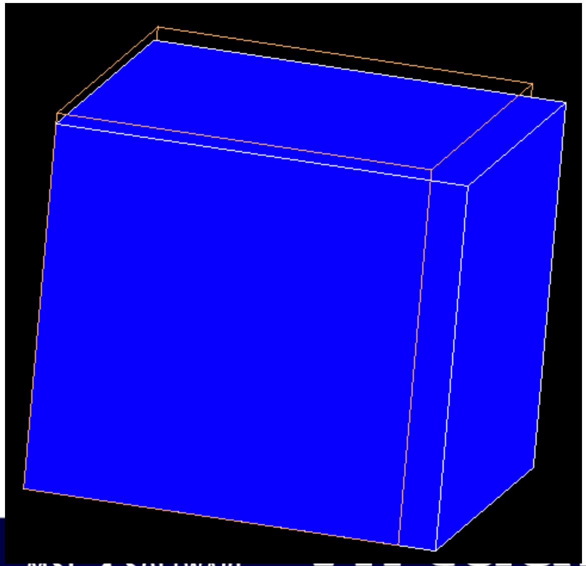
where

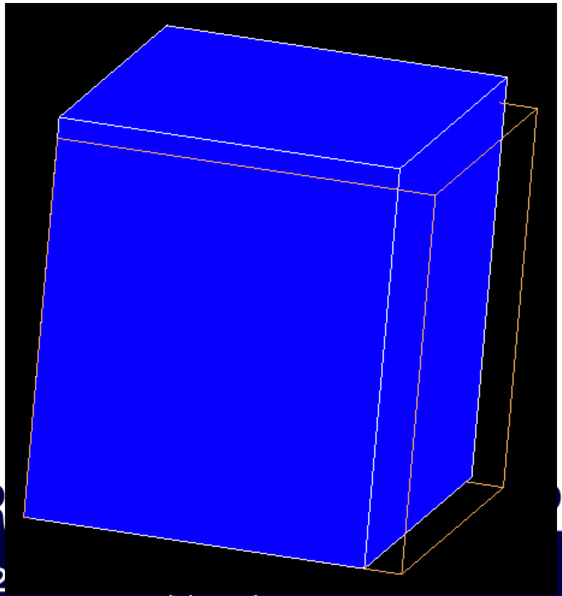
$$\lambda_S = \int_{t_n}^{t} \dot{\xi} dt = \xi_S - \xi_{S,n} \qquad R_f^{SA} = \left[\sigma_f^{SA} \left(\sqrt{\frac{2}{3}} + \alpha \right) \right] \qquad R_f^{AS} = \left[\sigma_f^{AS} \left(\sqrt{\frac{2}{3}} + \alpha \right) \right]$$

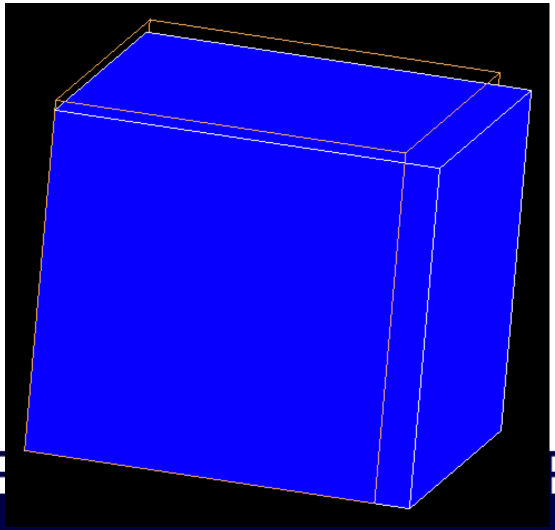
Cyclic Tension-Compression Tests for Mechanical SMA model



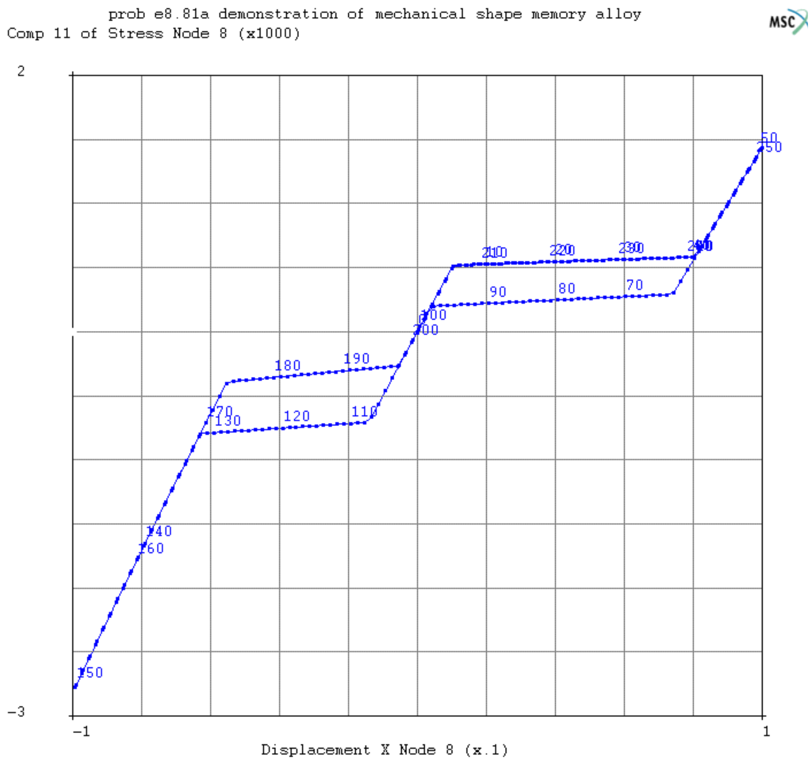
Tension(10%) → Compression(10%) → Tension (10%)



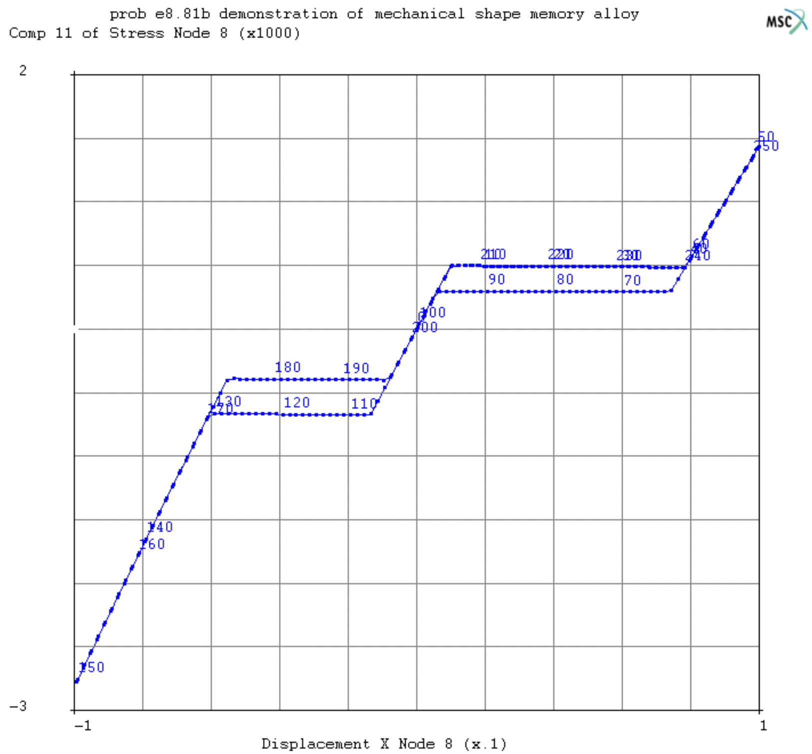




* Cyclic Tension-Compression Tests for Mechanical SMA model

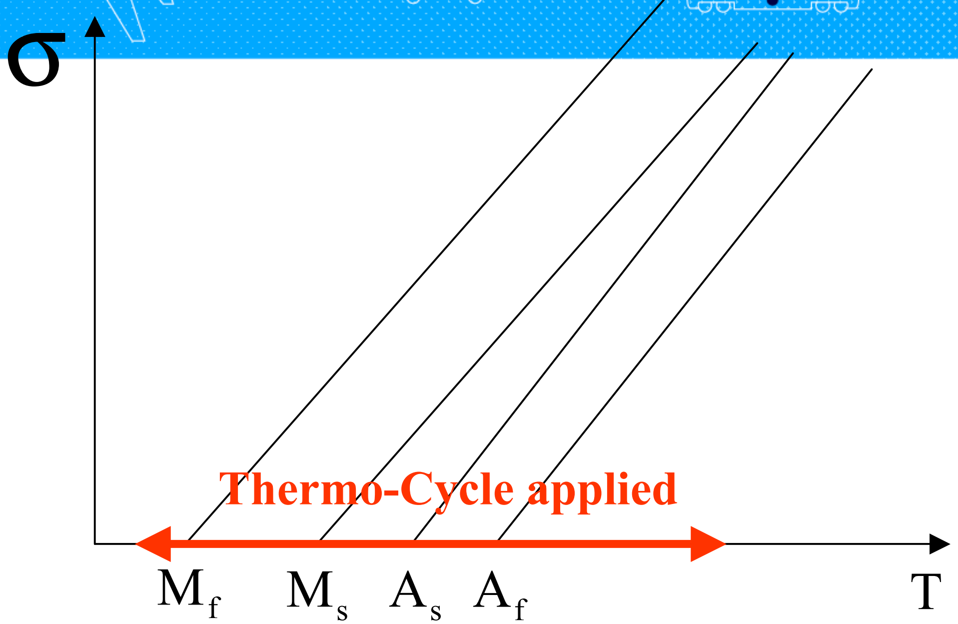


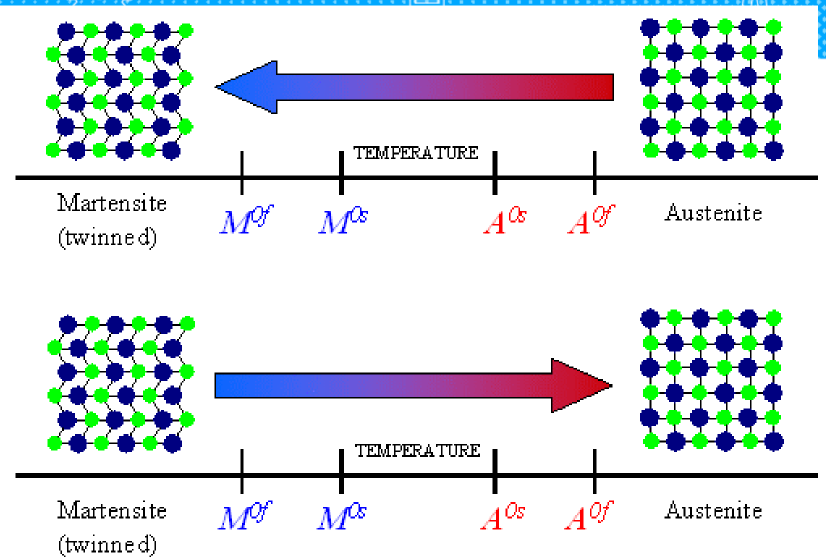
sigAS_s=520, sigAS_f=600
sigSA_s=300, sigSA_f=200



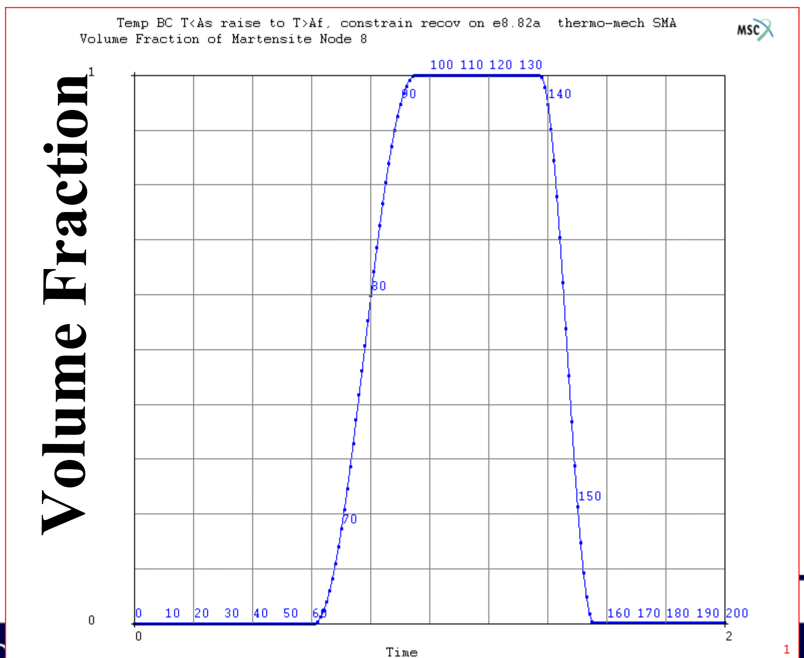
sigAS_s=500, sigAS_f=500
sigSA_s=300, sigSA_f=300

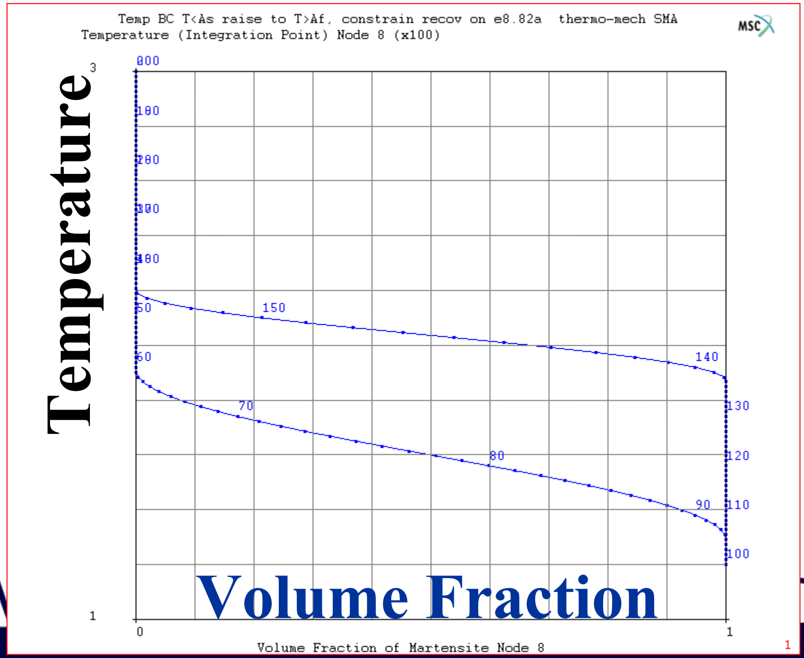
* Thermo-Mechanical Model Verification : Thermo-Cycle(1)



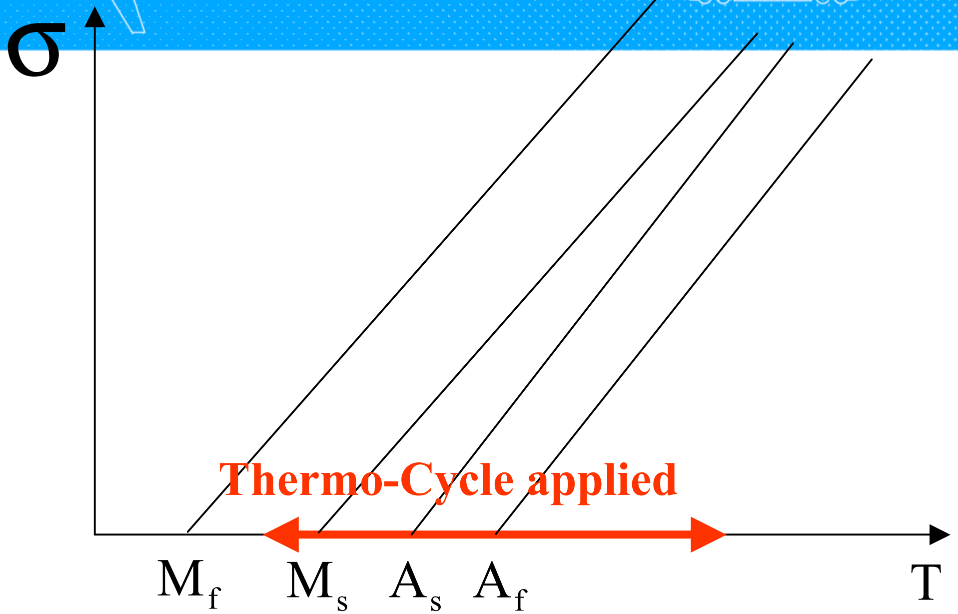


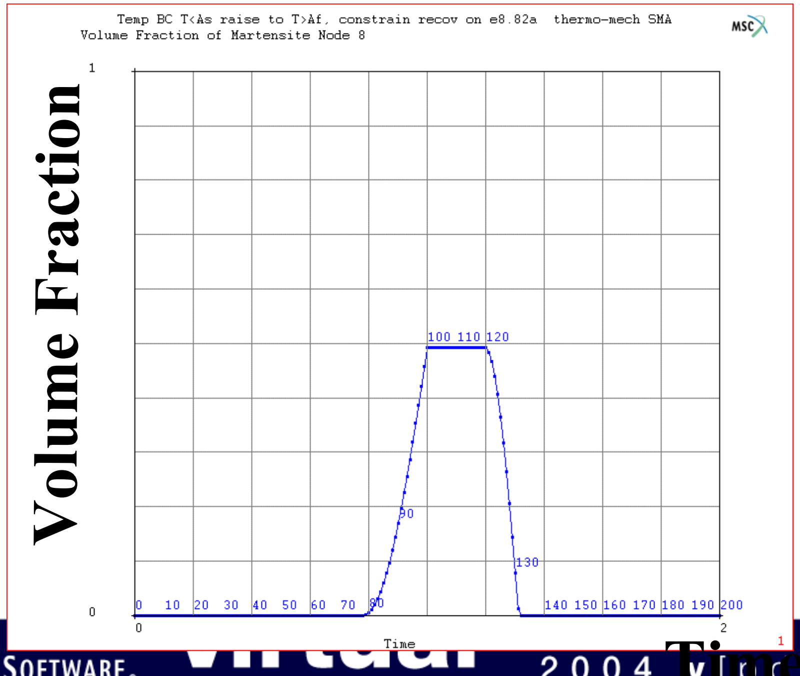
Physical meaning and SMA Structure

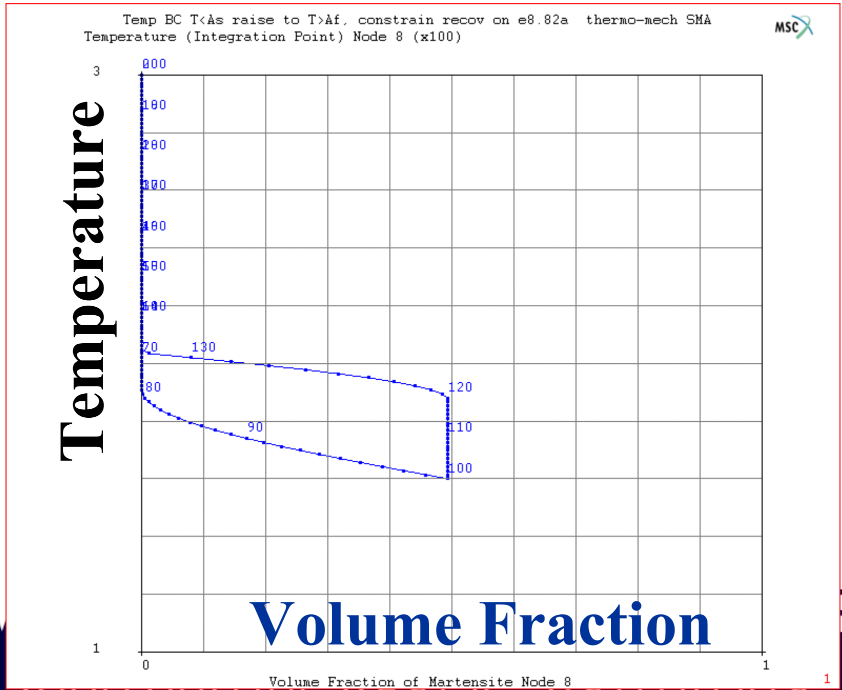




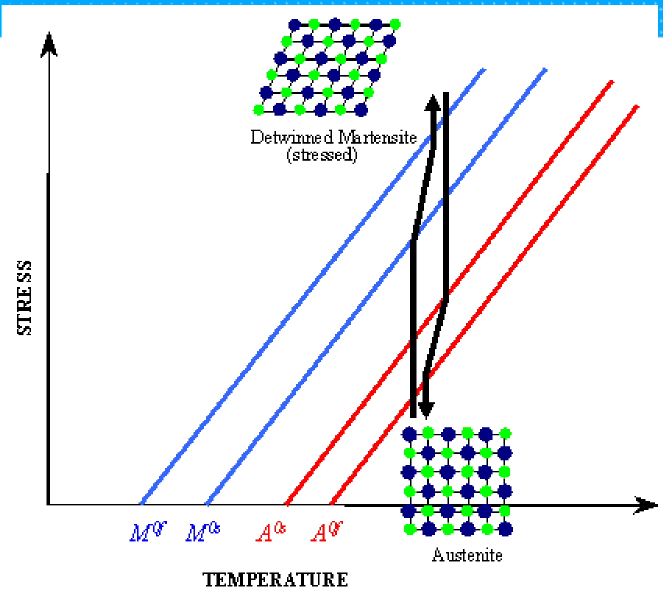
* Thermo-Mechanical Model Verification : Thermo-Cycle (2)

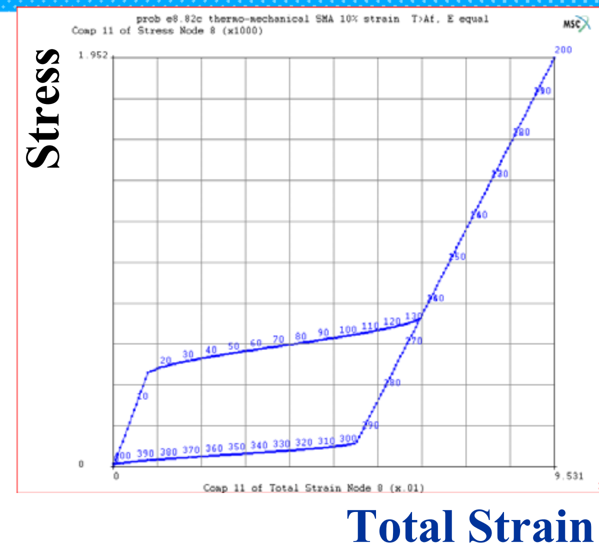


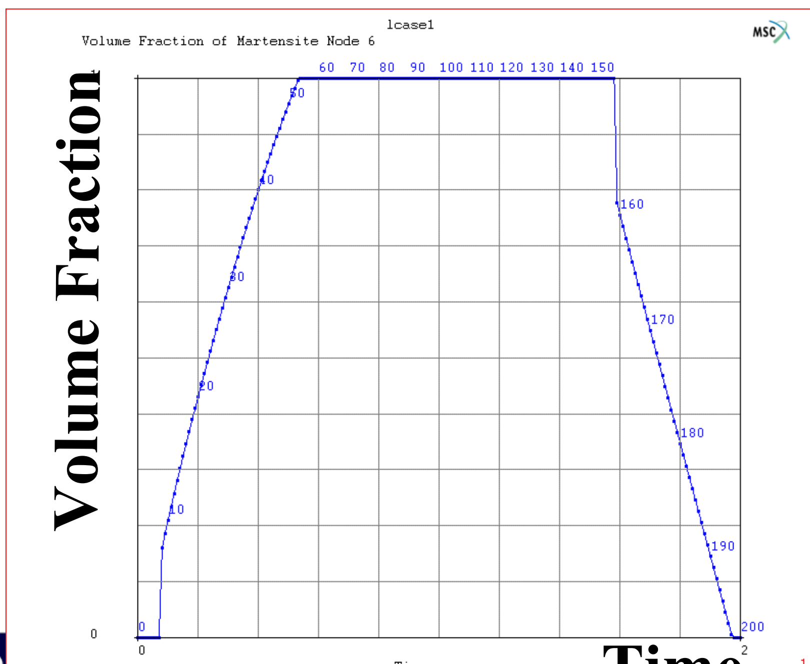


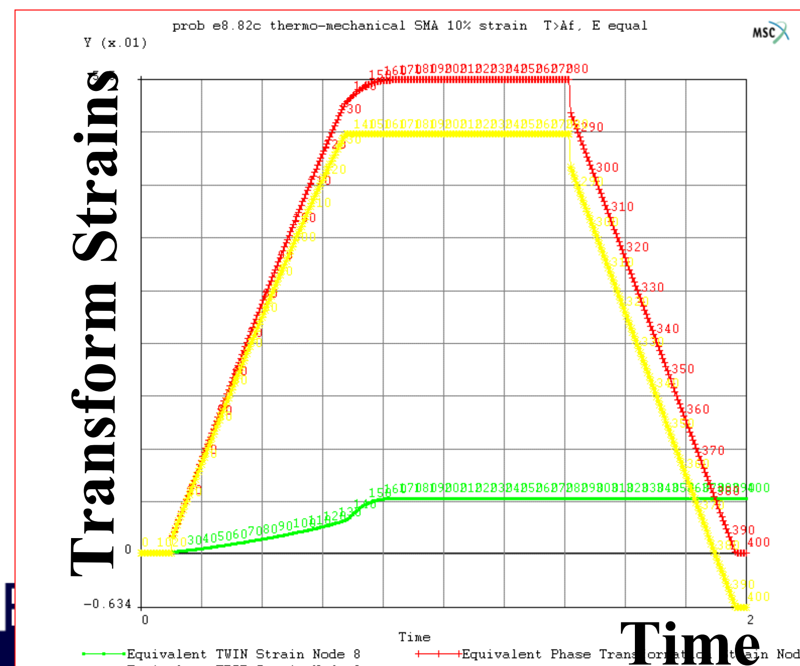


* Thermo-Mechanical Model Verification : Mechanical Cycle (1)



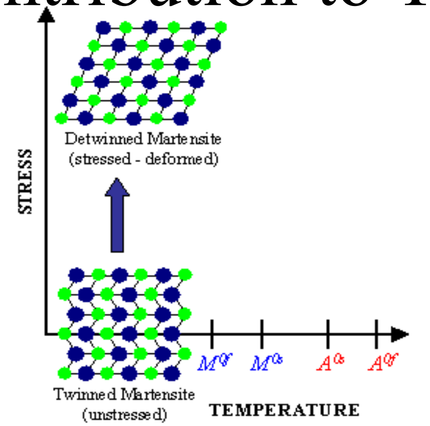






•Thermo-Mechanical Model Verification : G-function's major effect (1)

(Contribution to Twinning Strain : Starting with $V_f = 1.0$)

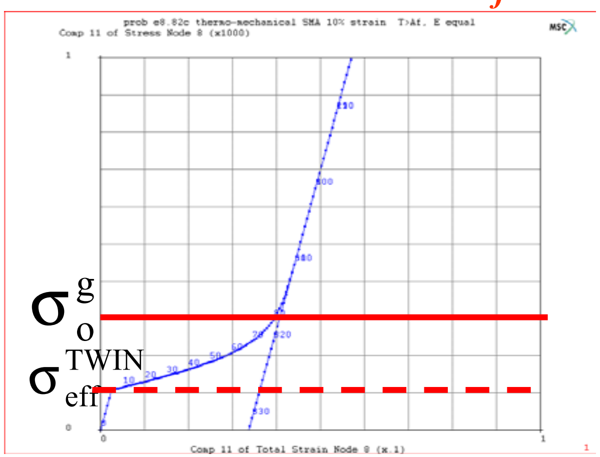


$$g(\sigma_{eq}) = 1 - \exp\left[a\left(\frac{\sigma_{eq}}{\sigma_o^g}\right)^n\right]$$

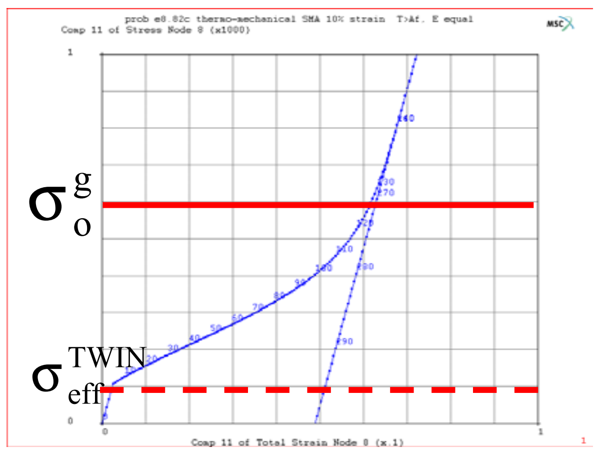
$$\dot{\varepsilon}^{TWIN} = f\,\dot{g}(\sigma_{eq})\varepsilon^T_{eq}\,\frac{3}{2}\,\frac{\sigma'}{\sigma_{eq}}\{\sigma_{eq}\}\{\sigma_{eq} - \sigma^{Twin}_{eff}\}$$

G-function values (a,n and σ_o^g) should be selected to fit Exp. Curve ***Under M_f***

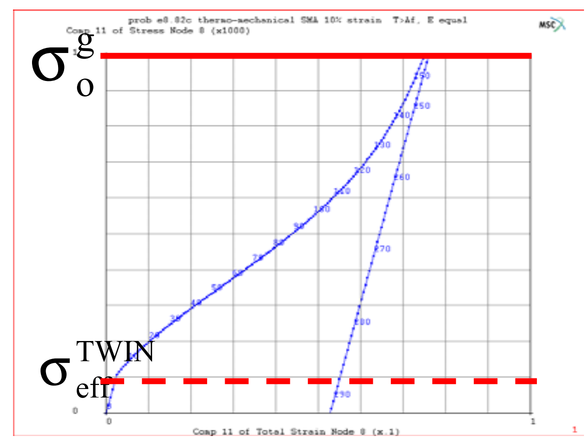
(Under M_f, Phase Transformation Strain = Twin Strain)



$\sigma_o^g = 300$ Mpa



$\sigma_o^g = 600$ Mpa



$\sigma_o^g = 1000$ Mpa

•Thermo-Mechanical Model Verification : G-function's side effect (2)

(Contribution to Trip Strain : Starting with $V_f = 0.0$)

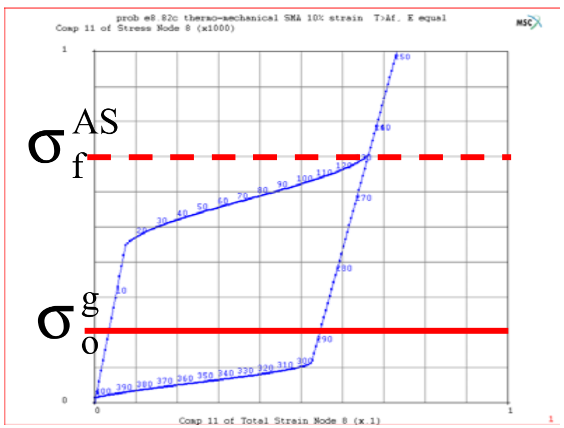
$$g(\sigma_{eq}) = 1 - \exp\left[a \left(\frac{\sigma_{eq}}{\sigma_o^g} \right)^n \right] \qquad \dot{\varepsilon}^{TRIP} = \dot{f}^{(+)} g(\sigma_{eq}) \varepsilon_{eq}^T \frac{3}{2} \frac{\sigma'}{\sigma_{eq}} + \dot{f}^{(+)} \varepsilon_v^T \mathbf{I} + \dot{f}^{(-)} \varepsilon^{Ph}$$

Upper M_f , Phase Transformation Total Strain = Trip Strain + Twin Strain

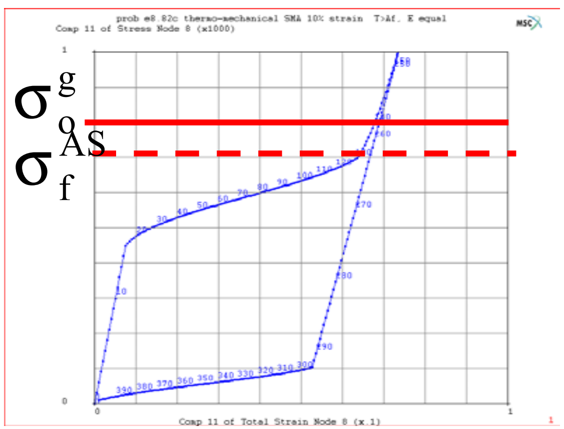
σ_o^g ***effects the strain after the transformation (from Austenite to Martensite).***

However, g-function value (a, n) also effect the strain during the transformation.

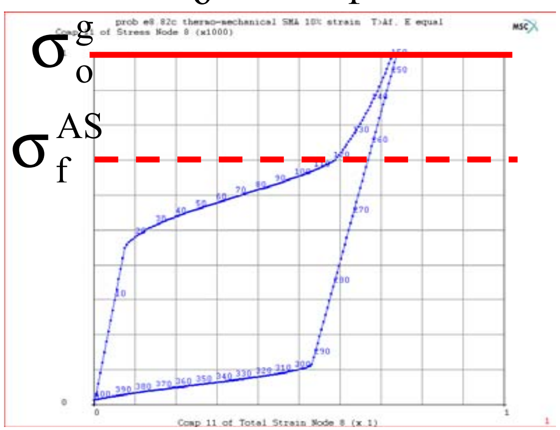
$\sigma_o^g < \sigma_f^{AS}$ (No effect) | $\sigma_o^g > \sigma_f^{AS}$ | $\sigma_o^g > \sigma_f^{AS}$



$\sigma_o^g = 300$ Mpa



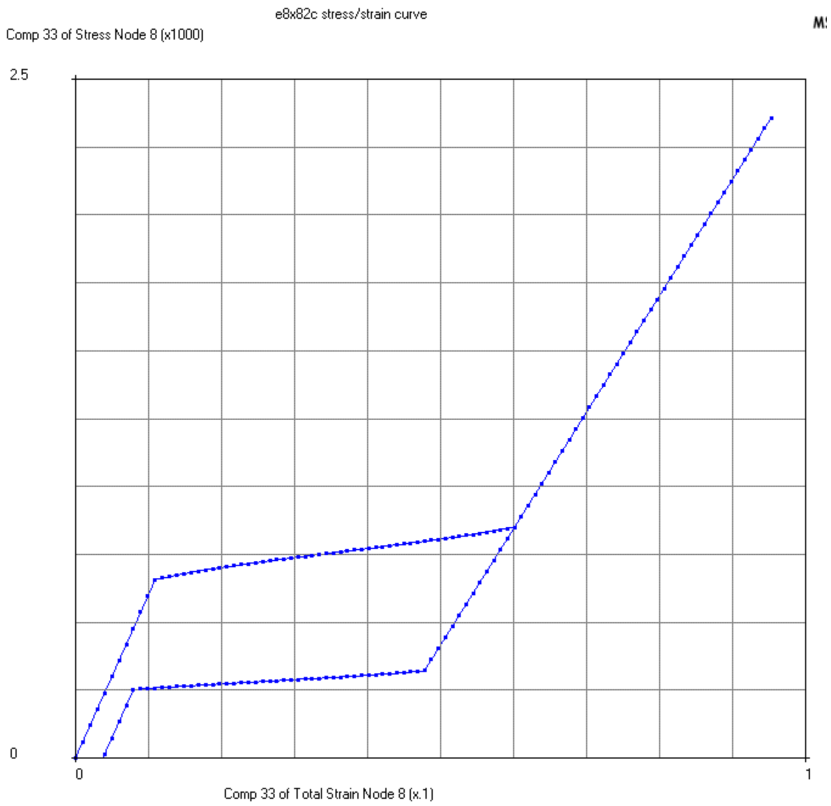
$\sigma_o^g = 800$ Mpa



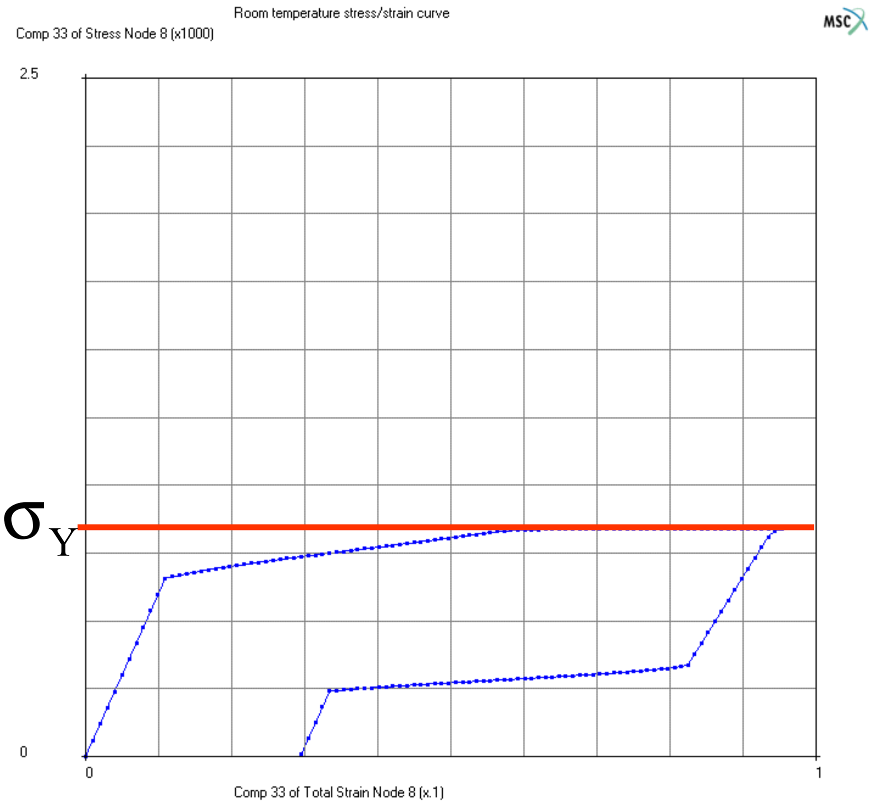
$\sigma_o^g = 1000$ Mpa

•Thermo-Mechanical Model Verification : Permanent Plastic Strain

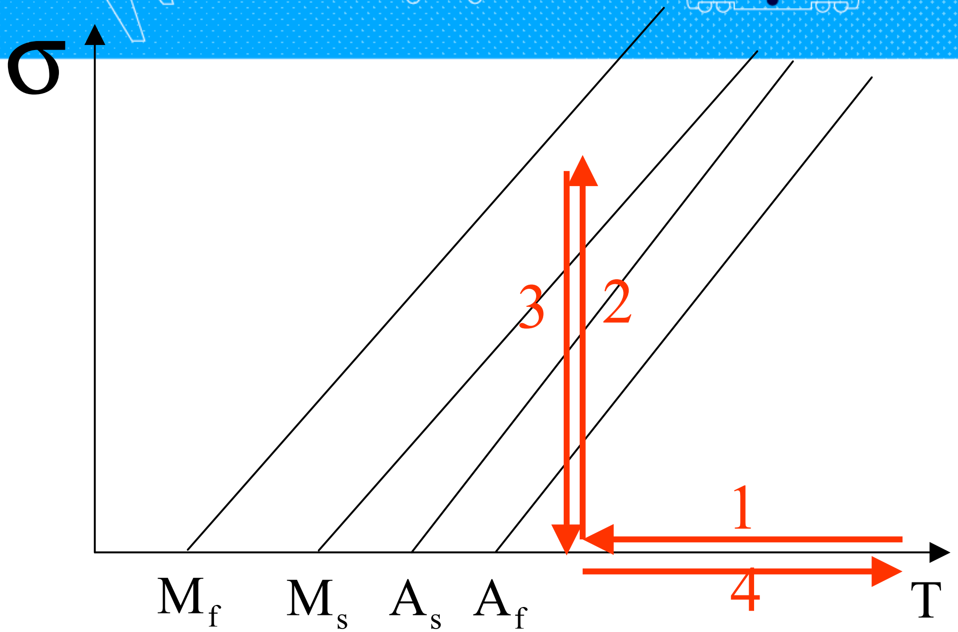
No yielding for plasticity



Yielding for Plasticity : 800 Mpa

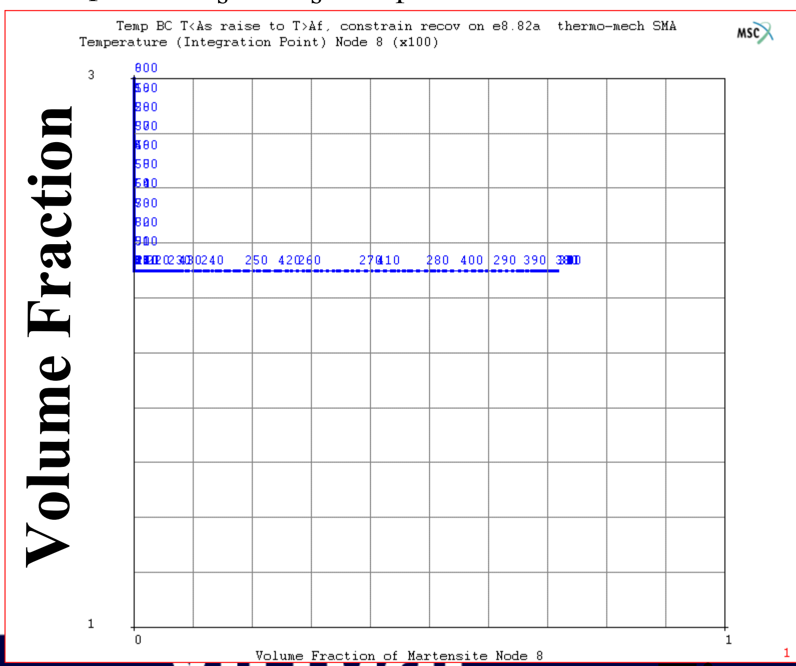


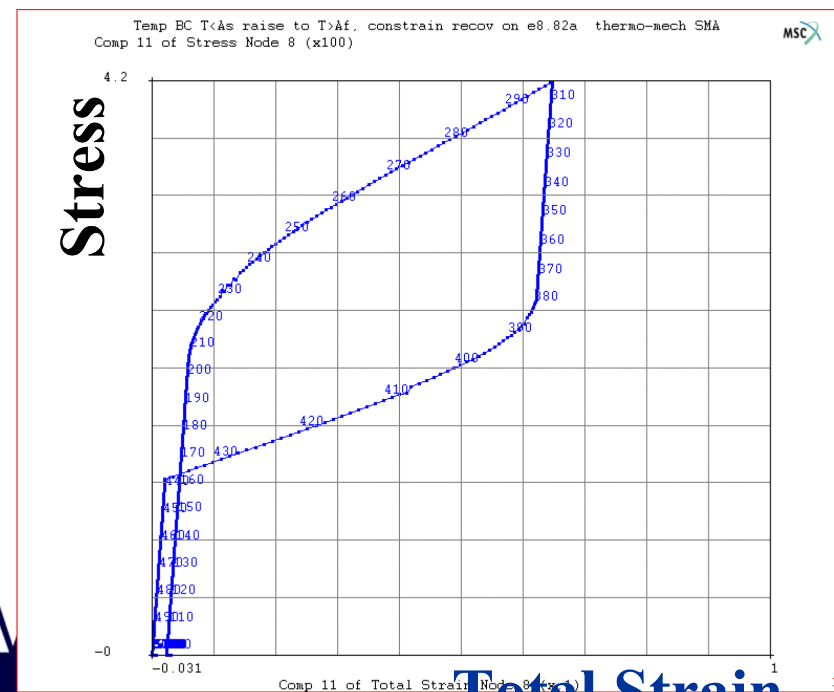
* Thermo-Mechanical Model Verification : Complex Coupled Cycle



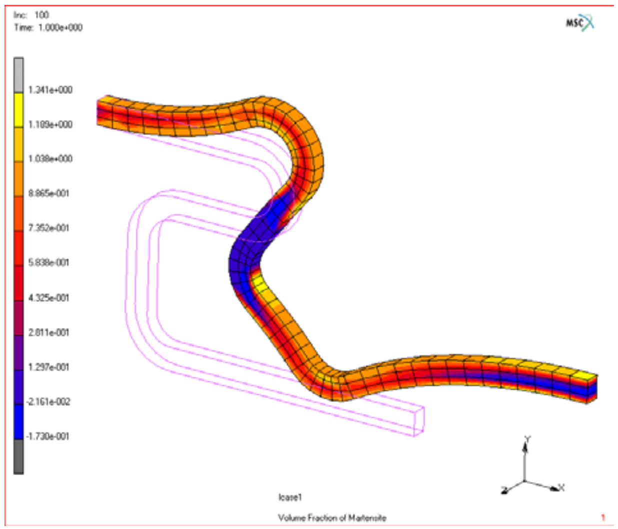
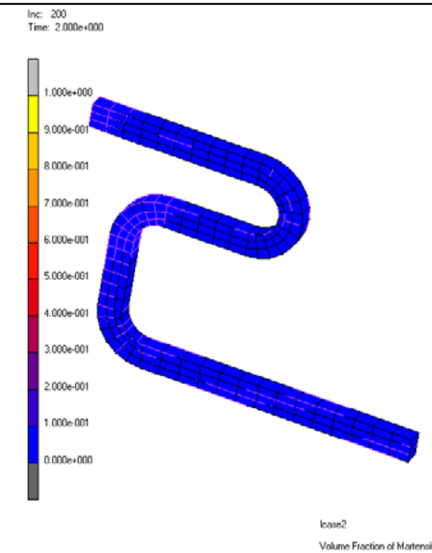
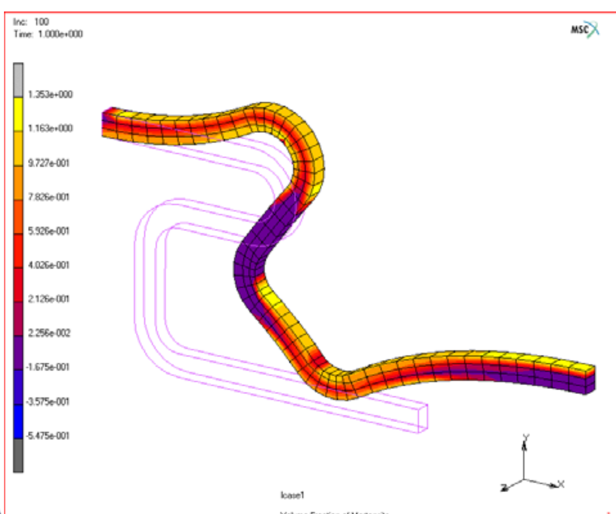
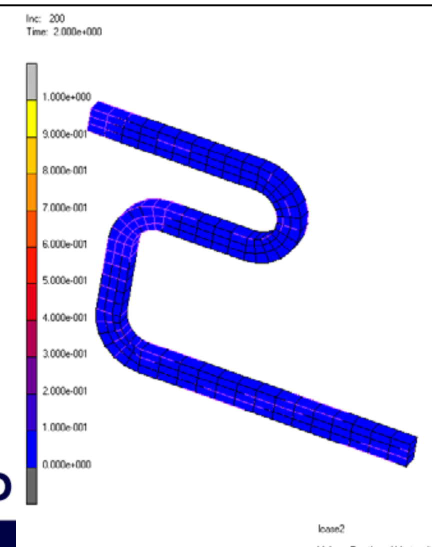
1 and 4 : Thermo Cycle

2 and 3 : Mechanical Cycle
(Loading between M_s and M_f)

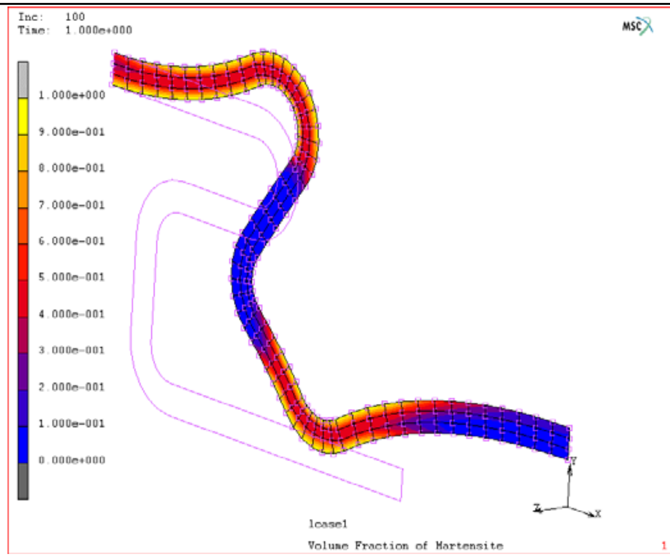
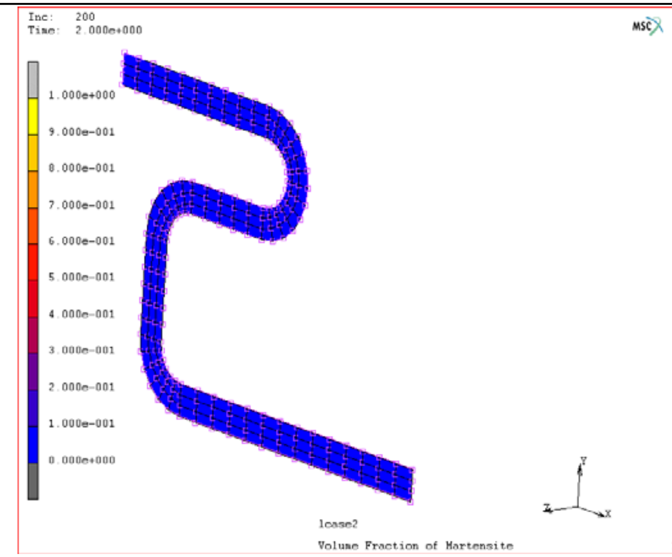
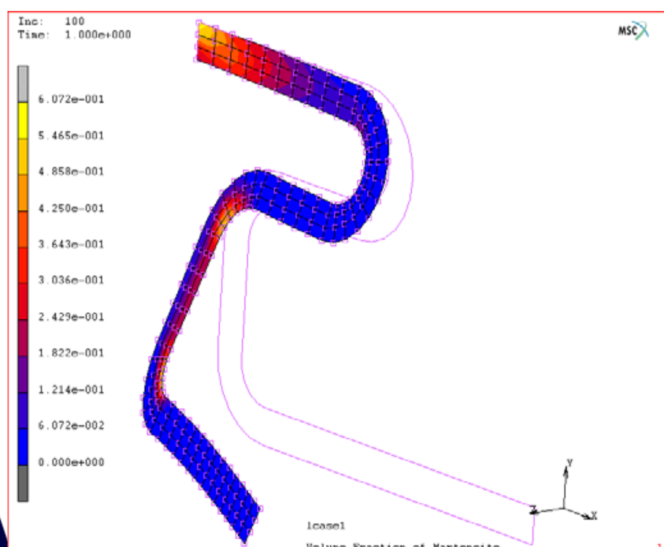
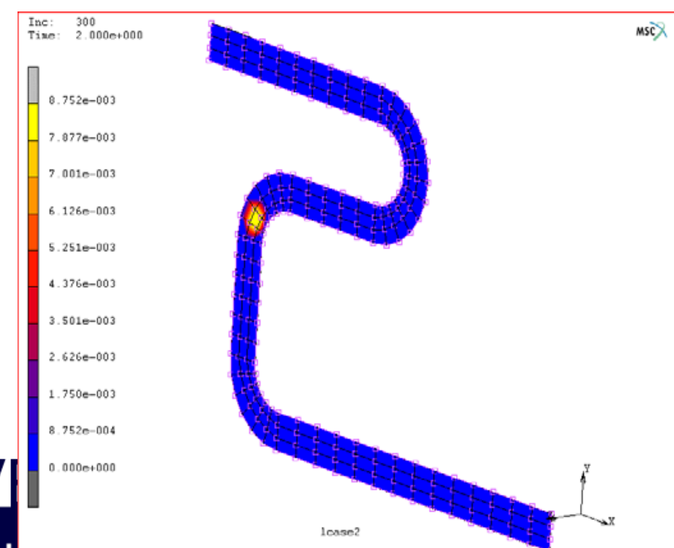




Archwire example- Tension and Relaxation with Solid Element

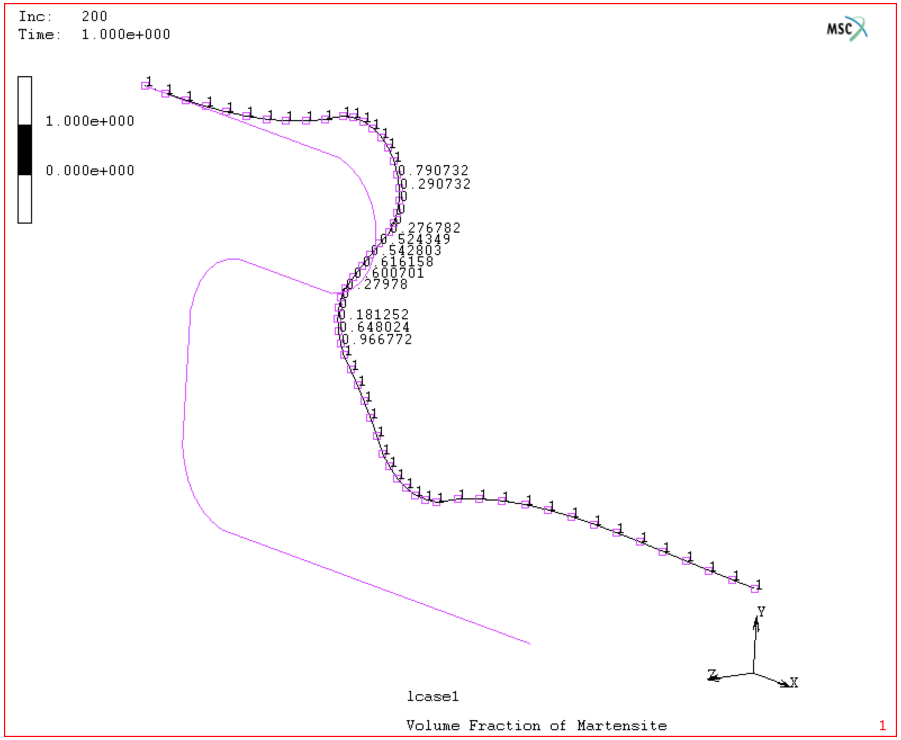
Martensite fraction	Max. deformation	Unloaded
Thermo-mechanical model		
Mechanical model		

Simulation with different type of elements : Shell Element

Martensite fraction	Max. deformation	Unloaded
In-Plane Bending		
Out of Plane Bending		

Martensite fraction contour

Max. deformation



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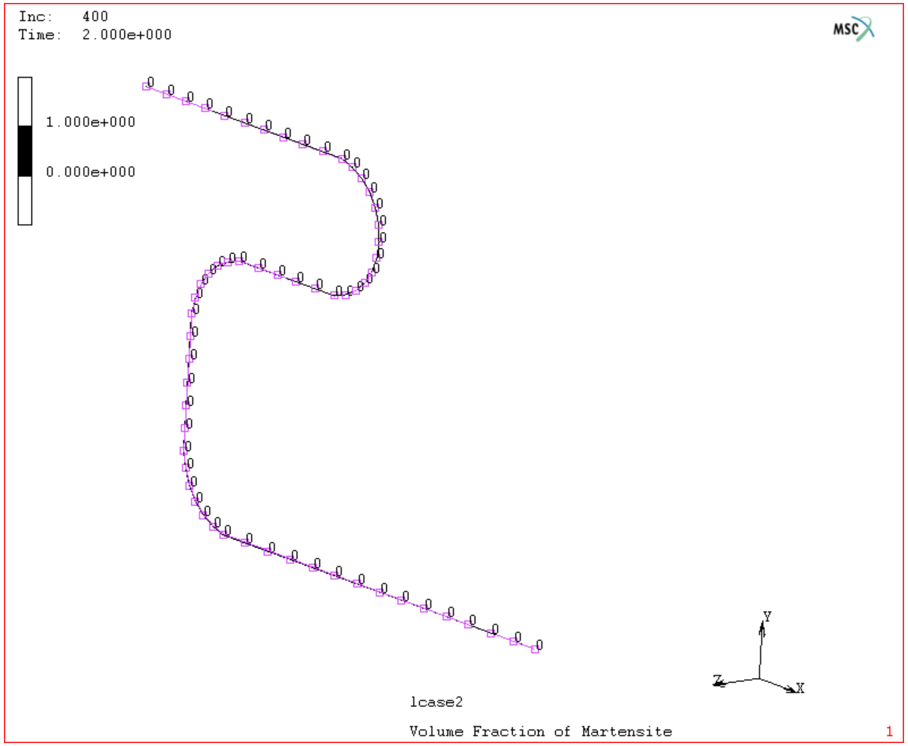
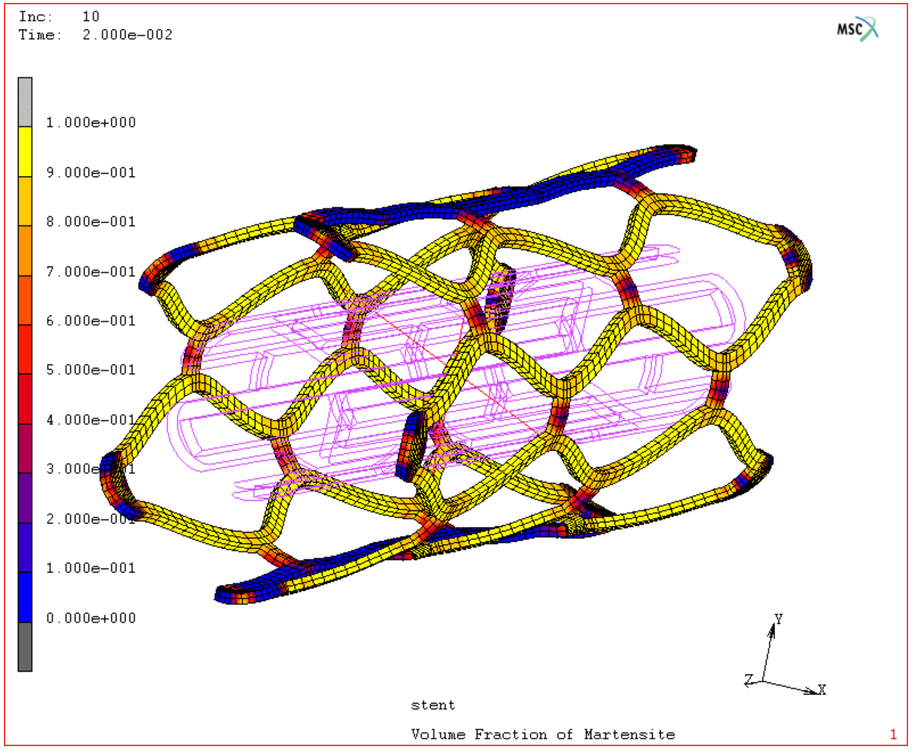


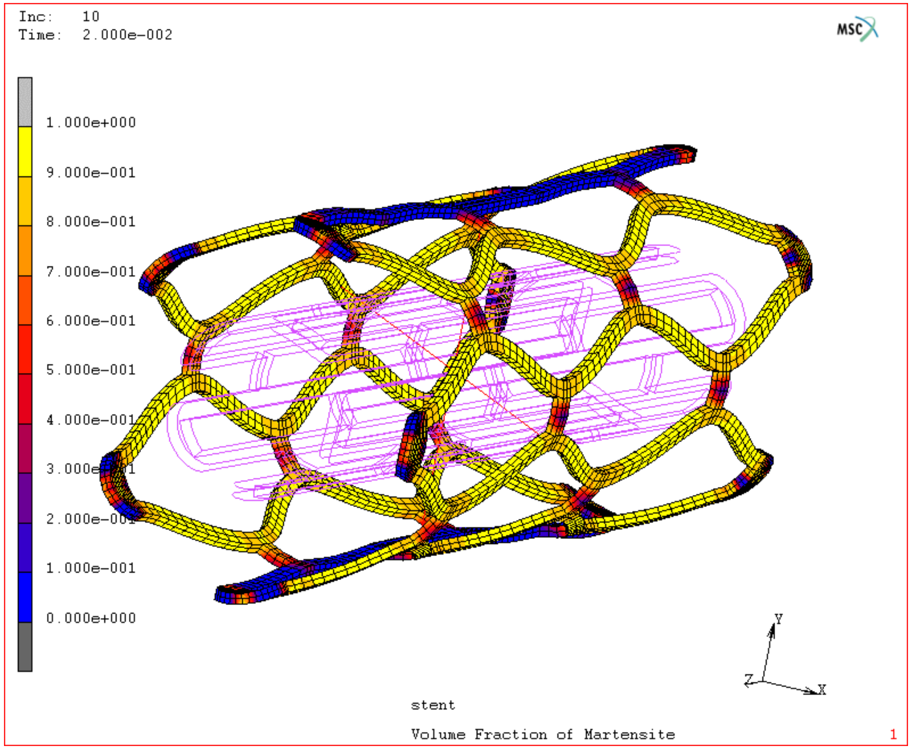
Stent example with Solid Element

Martensite fraction contour

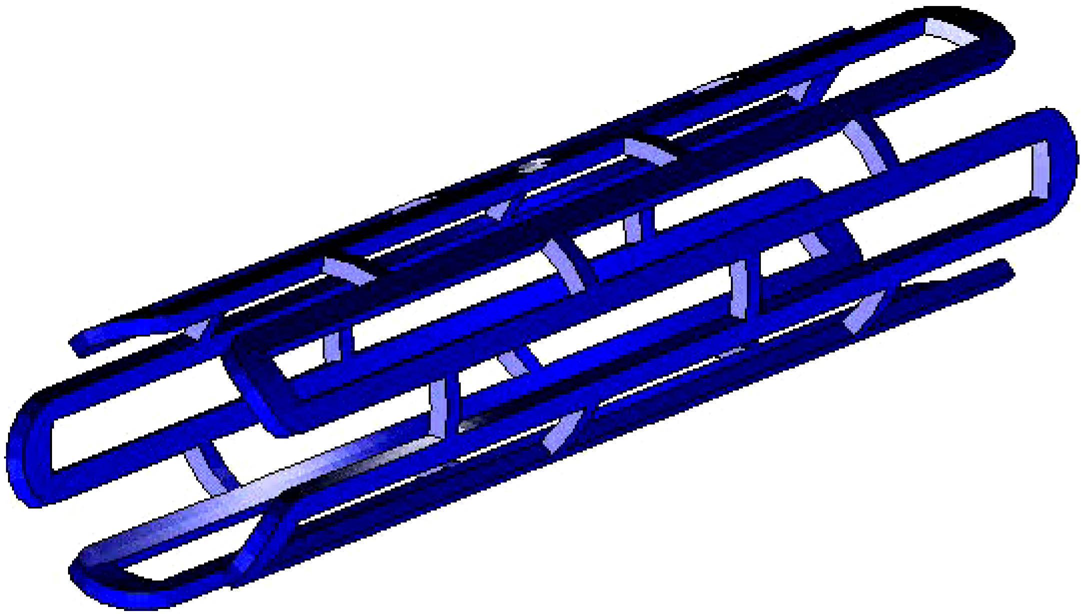
Thermo-mechanical model



Mechanical model



Stent Simulation



Summary :

Implementation of two SMA models in MSC.Marc 2003

1. Thermo-Mechanical SMA Model :
- An Accurate Model including Trip, Twin and Plastic Strains
- Applied to Complex Thermo-Mechanical Coupling Analysis
- Support Solid, Shell and Beam Elements
- General Purpose Applications

2. Mechanical SMA Model
- Peuso-Elasticity Simulation over A_f Temperature
- Simple Input Data
- Support Solid Elements only
- Useful for Initial Verification