Simulation of Modal Vibration Pattern Variations Due to Gyroscopic Effects in an Active Vibration Controlled Structure

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ABSTRACT

A method to model and simulate an active vibration controlled structure, while subject to external inertia and gyroscopic effects is presented. Finite Elements is used to model the structure in detail, and modal reduction is used to reduce the model size. The Finite Element model is also used to extract a modal reduced Gyroscopic matrix [$\underline{G}$]. This rate dependent matrix is then included into the dynamics model. By reformulating the model in state-space, a rotational rate dependent dynamics matrix [A], drive actuator matrix [B], and sensor matrix [C] are developed. These matrices can then be readily integrated into a closed loop control system.

This method is demonstrated on a hemispherical resonating gyroscopic where the modal patterns are influenced by the Coriolis forces from an externally applied rotation. MSC.Patran is used to model the significant components using solid geometry and the surrounding structure is modeled with lumped mass and stiffness. An MSC.Nastran finite element analysis is used to construct the state space dynamics matrices [A], [B], and [C] including the modally reduced gyroscopic matrix [$\underline{G}$]. These matrices are then assembled and used in a closed loop feedback control in MATLAB/SIMULINK. The vibrating gyroscope is then subjected to external inertia and rotation loads. The inclusion of the gyroscopic loads is verified by observing the shift in the modal pattern due to a given rotational input and the results are compared to measured values.

NOMENCLATURE

[A]	State dynamics matrix of dimension 2n X 2n
[B]	Actuator matrix of dimension 2n X m
[C]	Sensor matrix of dimension p X 2n
[D]	Assembled element damping matrix
{F}	Forcing function in physical coordinates
[G]	Assembled element gyroscopic matrix
[$\underline{G}$]	Modal reduced gyroscopic matrix [G]
[I]	Identity matrix

[K]	Assembled element stiffness matrix
[M]	Assembled element mass matrix
{P}	Forcing function in modal coordinates
[R,θ,Z]	Fixed cylindrical coordinate system
[X,YZ]	Fixed cartesian coordinate system
m	Number of selected actuator DOF
n	Number of modes
p	Number of sensors
{q}	Physical displacement vector relative to R
α	Input rotational angle (Ω·t)
β	Lag of modal angle pattern with respect to α
{η}	Displacements in modal coordinates
{γ}	State-space vector of modal displacements and velocities
[Φ]	Eigenvectors
1	Externally applied spin rate
{δ}	State-space variable in fixed physical coordinates
ω	Natural frequencies
λ	Eigenvalues of the rotor equation of motion
{ζ}	Linear viscous damping ratio

INTRODUCTION

The Finite Element equation of motion for the resonator including damping and gyroscopic terms can be written in matrix form as:

$$[M]\{\ddot{q}\} + ([D] + \Omega[G])\{\dot{q}\} + [K]\{q\} = \{F\} \tag{1}$$

The [M], [D], and [K], are the traditional mass, damping, and stiffness matrices respectfully. The gyroscopic forces are incorporated by including the Coriolis matrix [G] as a function of the externally applied angular rate Ω. The physical displacement degrees of freedom are represented by the vector {q} and the external force acting on the rotor is vector {F}.

To reduce the size of the problem, the standard equation of motion is expressed in modal coordinates and truncated to a desired number of modes. Rewriting equation (1) in modal coordinates:

$$\{\ddot{\eta}\} + (2[{}^{\backslash}\zeta\omega_{\backslash}] + \Omega[\underline{G}])\{\dot{\eta}\} + [{}^{\backslash}\omega^2_{\backslash}]\{\eta\} = \{P\} \tag{2}$$

The eigenvaulues {ω}, and eigenvectors [Φ], can be readily obtained with a standard Finite Element modal analysis. The modal damping terms {ζ}, can either be obtained by a complex eigenvalue analysis, or experimentally. The reduced order gyroscopic matrix $[\underline{G}]$ can also be obtained from the FE model. If the Coriolis forces are small, they can be modally reduced using the modal transformation matrix [Φ].

$$[\underline{G}] = [\Phi]^T [G][\Phi]\,. \tag{3}$$

The modal reduction method employed here, based on modal superposition, is commonly used when the effect of damping is generally small enough as not to significantly change the eigenvectors [Φ]. Care must be taken to assure that the damping does not significantly alter the eigenvector or eliminating an excessive number of modes loses valuable information.

It can be noted that the resulting modal reduced gyroscopic matrix $[\underline{G}]$ remains a non-diagonal skew symmetric matrix, whereas the modal damping [ξω] is a diagonal symmetric matrix. The symmetric nature of the velocity dependent matrix can be shown to remove energy from the system while the skew symmetric matrix is energy neutral. Thereby gyroscopic forces neither add nor subtract energy while the modal damping matrix is dissipative.

To include the dynamics in a state space model, equation (2) is rewritten in state-space:

$$\{\dot{\alpha}\} = [A]\{\alpha\} + [B]\{F\}\,, \tag{4}$$

where the state space vector is:

$$\{\gamma\} = \begin{bmatrix} \eta \\ \hline \dot{\eta} \end{bmatrix} \tag{5}$$

and the state space vector can be output in physical coordinates as:

$$\{q\} = [C]\{\gamma\}. \tag{6}$$

The dynamics matrix [A], and the actuator and sensor transformation matrices [B], and [C] are then:

$$[A] = \begin{bmatrix} [0] & [I] \\ -[\omega^2] & -(2[\zeta\omega]) - \Omega[\underline{G}] \end{bmatrix} \tag{7a}$$

$$[B] = \begin{bmatrix} [0] \\ [\underline{\Phi}]^T \end{bmatrix} \tag{7b}$$

$$[C] = \left[[\underline{\Phi}] \quad [0] \right]. \tag{7c}$$

The size of the dynamics matrix [A] is a function of the number of modes n, kept in the model. From inspection of equation (7), the dynamics matrix [A] is of the order (2n X 2n). The actuator matrix [B] is a function of the number of the actuator DOF, r. For r actuators, the [B] matrix is (r X 2n). Similarly, the sensor matrix [C] is a function of the number of sensor DOF, s. For s sensors, the [C] matrix is (2n X s). The modal reduced gyroscopic matrix [$\underline{G}$] is a subset of [A] and is (n X n).

APPLICATION TO A GYROSCOPE

Hemispherical or cylindrical resonating gyroscopes belong to a group of vibrating gyroscopes that are used to measure angular rotation rate. Similar to spinning mass gyroscopes, these instruments use the Coriolis effect from a rotating frame of reference. The significant difference is that vibrating gyroscopes use the momentum of a resonating elastic structure instead of a spinning mass. The advantage of the vibrating gyroscopes is that they have no motors or bearings and have the potential for infinite service life without the need for maintenance.

The resonating gyroscopes are typically a hemispherical or cylindrical shell that is vibrated in the N=2 mode. During the first half cycle of vibration, the circular rim deforms to an ellipsoid and then back to a circular shape. The second half of the cycle, the same ellipsoid shape takes place 90 degrees away in azimuth. This results in a standing modal wave of four nodes and four antinodes.

When the gyroscope is physically rotated about its input axis, the modal pattern rotates a percentage of the applied rotation. This percentage is always constant and has been found to lag the physical rotation from about 30% for a hemispherical shape to approximately 40% for a cylindrical shape. This lagging precession, or scale factor, is attributed to the Coriolis force and is the fundamental principle in which the resonating gyroscopes operate.

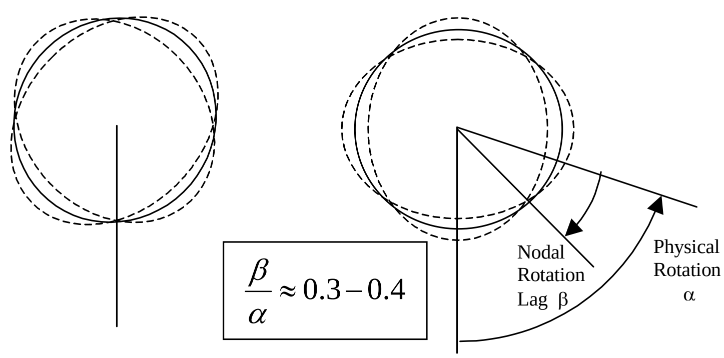

Figure 1. N=2 Modal Pattern

Electrostatic drive electrodes placed along the outside circumference rim of the resonator are used to drive the standing wave. Similarly, capacitive sensors are also placed along the circumference to sense the resonator motion. By logically combining the motion information and location from the capacitive sensors, the modal pattern can be determined. By measurements of the shift in the N=2 pattern, the amount of external rotation can then be determined.

To model this system, a Finite Element model of a resonator was created in MSC.Nastran utilizing 8 node solid elements. The model was created in cylindrical coordinates with the Z-axis along the axial stem of the resonator. The sensor and drive electrodes were also modeled in the same fashion. The resulting solid Finite Element model had approximately 75,000 DOF.

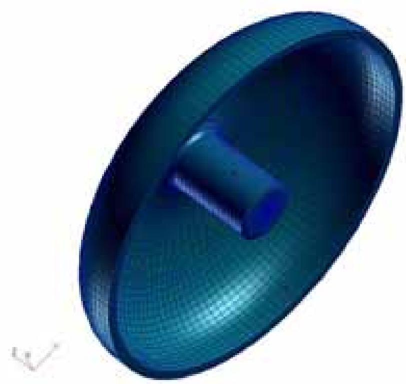

Figure 2 – Hemispherical resonator FE model

A node and element representing the remainder the mass and CG of the case and flange was added to the model. The entire structure was then mounted on a six DOF spring with the representative stiffness of the case and flange. A support node using a large mass is fixed in all DOF except the X and Y direction. Inertia loads can then be put on the model in the X and Y direction by applying forces on this support node.

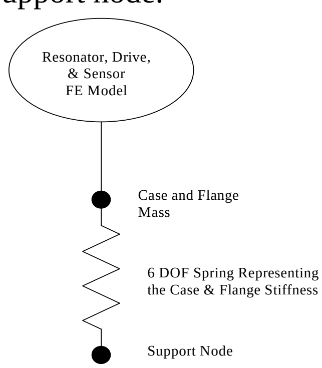

Figure 3 – Lumped mass and stiffness model

To drive and sense the model with electrostatics, the gap information at the sensors and actuators were created using scalar points and multi point constraints. The MPC (Multi-Point Constraint) elements are used to calculate the difference in the radial displacement from the hemispherical resonator to the sensor and actuators. The modal eigenvector at these scalar point are then used in the construction of the actuator [B], and sensor [C] matricies.

The Coriolis matrix [G] is created in a NASTRAN static analysis, SOL 101, by using a modified version of the MSC DMAP Alter "segyro". Since differential stiffening effects are not essential in this simulation, the DMAP alter was modified to only create the Corilois. matrix. The spin rate set on the RFORCE entry is set to unity so that in the simulation, the gyroscopic terms can be scaled the based on the rotational rate.

On a modal, SOL 103, A DMAP alter was written to perform the modal reduction in equation (3) and output the modal reduced gyroscopic matrix along with the eigenvectors to an OUTPUT4 file. Additional information to map the DOF and geometry to define the actuators and sensor locations are written to an OUTPUT2 file. A FORTRAN program, was used to extract the [A], [B], [C], and [$\underline{G}$] matrices and output to a MATLAB readable format.

The extracted eigenvectors were plotted to ensure that all desired modes were captured. The first two rigid body modes were eliminated and the higher frequency modes consisting of a variety of resonator and stem bending modes including the N=2 operational mode were kept. Note that two N=2 modes are expected because of the symmetry in the X and Y plane.

The modal damping ratio ζ, are input individually for each mode as shown in equation (7a). A damping ratio equal to 0.0005 was used which results in a transmissibility quality factor of 1,000.

$$Q = \frac{1}{2\varsigma} = \frac{1}{2(.0005)} = 1{,}000. \tag{8}$$

These matrices can now be input into a state-space simulation. To facilitate this the product of the spin rate times the Coriolis matrix [G] is added to the A matrix.

$$[A] = \begin{bmatrix} [0] & [I] \\ -[\omega^2] & -(2[\zeta\omega]) \end{bmatrix} + \begin{bmatrix} [0] & [0] \\ [0] & -\Omega[\underline{G}] \end{bmatrix} \tag{9}$$

The resulting Bode plot of the transfer function at zero input rate is shown in Figure 4 with the amplitude expressed in decibels. The first two peaks (~ 450Hz & 550 Hz) result from the semi rigid body modes of the instrument. The third peak (775 Hz) is the conning of the hemisphere on the stem. The largest peak at 1000Hz, is the N=2 operational mode of the gyroscope.

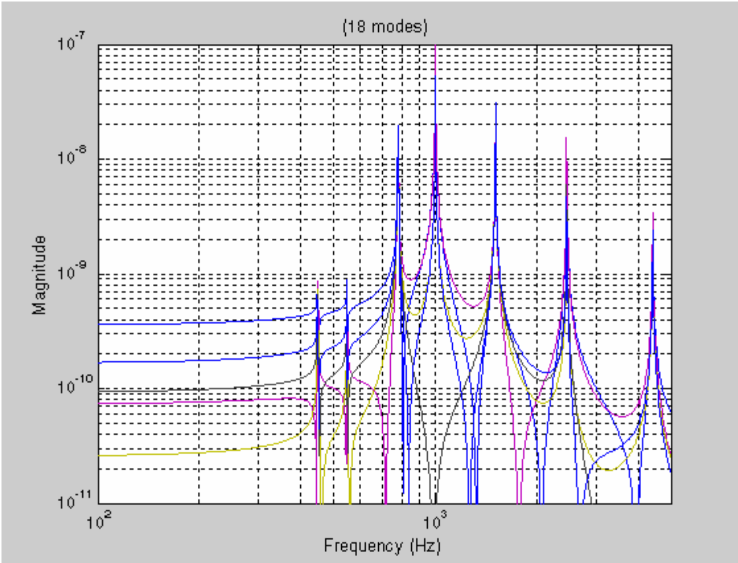

Figure 4 – Bode plot of the Transfer function at Zero Spin rate

Plots can now also be constructed at any given input rotational rate. As the rate increases, the resonant peaks split into two peaks. This is consistent with rotordynamic theory as the two peaks represent the backward and forward wave frequencies. The bifurcation of the 1 kHz N=2 peak with a rotational input rate is shown in Figure 5.

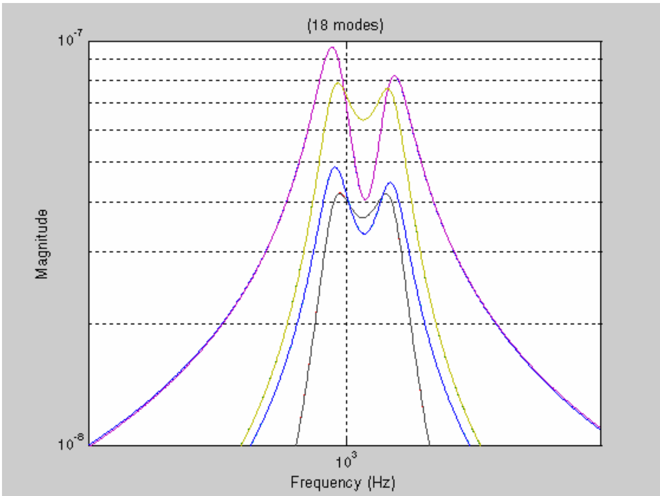

Figure 5 – Bifurcation of the N=2 peak with rotation

The MATLAB simulation plant model was constructed using a mex file S-function block. The mex file, re-calculates the dynamics matrix [A] based on the rate input Ω, and outputs the X and Y displacements {q} based on the input forces {F}.

Input to the plant model was included by adding a driving force that supplies the forcing function at two of the opposing drive actuators. The output of the plant model was fed to a block containing the pickoff logic, which calculates the modal pattern rotation from the sensor displacements.

The loop was closed by adding an active gain loop and a phase loop. The active gain loop is a slower loop that adjusts the amplitude of the force so that the peak-to-peak displacements at the gap are constant. The phase loop provides a 90-degree phase shift of the displacement, so that the force acts in the velocity phase. Additionally the force is applied only on the positive velocity (hemisphere expanding, gap decreasing) since electrostatic force is only attraction.

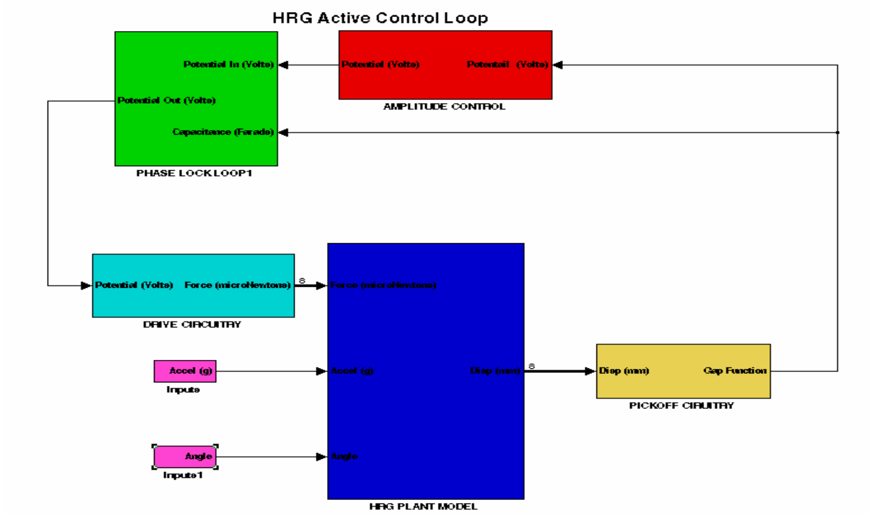

Figure 6 – Simulink Model

The entire block diagram of the simulation model is shown in Figure 6. Additionally two blocks were added to the plant model to apply the environmental loads of inertia and angular rate.

SIMULATION RESULTS

Simulation runs were completed to demonstrate the model. All runs start at time t = -.3 seconds since it takes approximately 0.3 seconds to achieve a reasonable steady state condition. A time step of .0000002 seconds was chosen that at ~1kHz operation yields 5,000 points per cycle.

The first run starts with the hemisphere oscillating from the drive at 0.0 degrees. At time t = 0.0 seconds, the actuating force shifts to the drive located at 22.5 degrees. The modal pattern then starts to shift asymptotically to 22.5 degrees as shown in Figure 7. From Figure 7, the time constant (τ=1-1/e) of this shift is ~ 0.3 seconds. Therefore the maximum rate the gyro in this model could null the modal pattern is approximately 9 rpm.

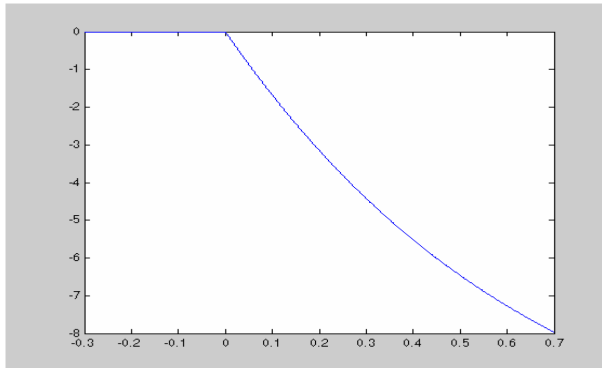

Figure 7 – Actively Forcing A Shift In The Modal Pattern

A second simulation looks at the effect of an applying an external inertia load. Initially the resonator is vibrating in an N=2 pattern as shown by the resonator displacement at the sensors in Figure 8a. A step impulse load is then applied which excites other various modes as shown in Figure 8b.

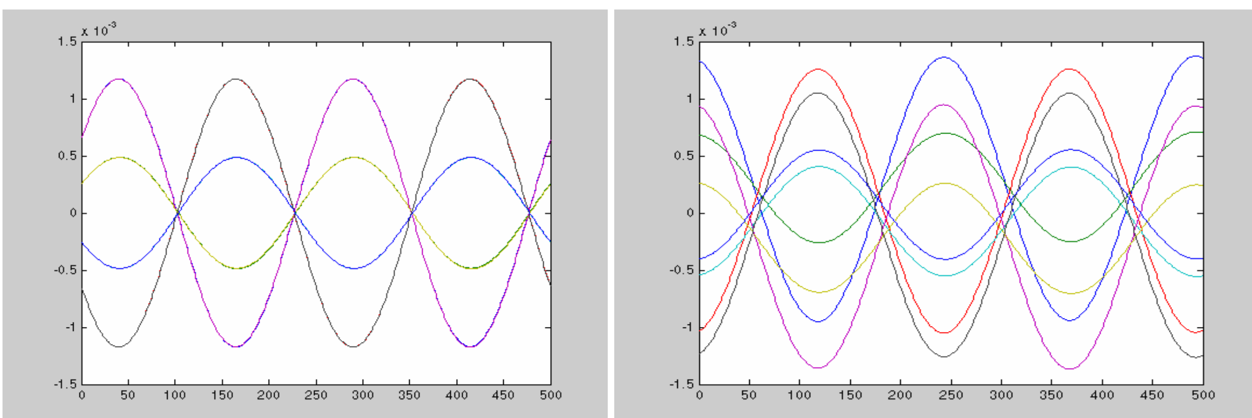

Figure 8a & 8b – Hemisphere Response Due To Inertia Step Load

A third simulation was used to determine the scale factor of the gyro. As discussed earlier, this is the amount the modal pattern shift lags behind the rotational input. At time t = 0.0 seconds, an input angular rate of -1 rad/sec was applied. The modal pattern was found to shift -1.682 degrees in 0.1 seconds. Since the gyro rotates –.1π/180 degrees in 0.1 seconds the scale factor is found to be 0.294   As shown in Figure 5, the shift in the nodal pattern is not linear. This is a direct result of the actuator forces being applied at 0 degrees. The actuator force tends to restore the modal vibration pattern back to 0 as shown in the previous case. Since this restoring force is small, especially at 0 degrees, it is ignored in calculating the scale factor. Removing this effect would result in a very small increase in the calculated SF.

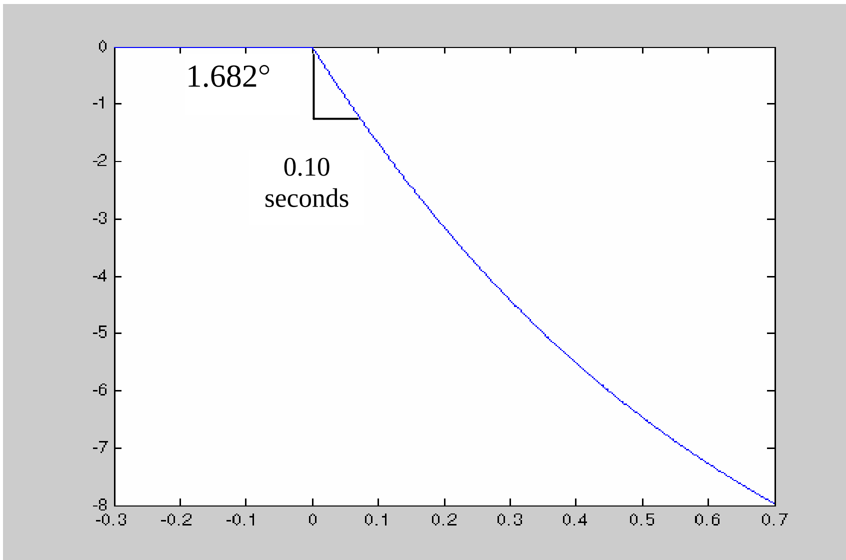

Figure 9 – Gyroscopic Scale Factor – Modal Pattern Shift Due to Angular Input

SUMMARY

A reduced order gyroscopic state-space transfer-function model of a structure has been presented. The simulation model runs as expected and yields very reasonable results, It appears to be adequate to capture the essential closed loop dynamics of a rotating structure including the Coriolis effects.

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