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Title: Dynamic Stress and Durability Analyses: Time Vs Frequency Domain Approaches

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ABSTRACT

An automotive component experiences many types of vibration: from simple sinusoidal to complex random excitation. Using Computer Aided Engineering (CAE), durability due to vibratory stresses can be simulated and the fatigue life predicted using MSC Nastran and FE-Fatigue. The input loads used for these predictions are derived from tests, or surrogate or CAE-predicted in either time or frequency domain formats.

Periodic input for fatigue analysis can be used for predicting fatigue life in the time domain by using either the direct or the modal transient approach. This process, however, is time consuming, sometimes non-convergent and requires more disc space.

Random input loads are prevalent in automotive components. CAE based Fatigue analysis using test loads - which contain random inputs and generally exhibit a normal distribution pattern- can be analyzed in the frequency domain using Power Spectral Density (PSD). This is performed in Nastran by utilizing the frequency response module. The methodology is fast and utilizes less disc space, when compared to that of the time domain solution sequence.

In this paper, dynamic stresses using Finite Element Analysis (FEA) are computed and durability analyses are performed in both time and frequency domains for test inputs. The fatigue results are compared and discussed.

INTRODUCTION AND BACKGROUND

Fatigue or Durability analysis is traditionally a test based activity; wherein components are tested under actual operating loads. The drawback with testing is that it can be performed only after the prototype has been built and should design problems surface it would be difficult to redesign, as the design by then is finalized. For this reason, FEA based fatigue analysis is increasingly popular and its up-front usage becoming the norm [1].

In general there are two approaches for FEA based Fatigue analysis [2]. The traditional one is the time domain approach, wherein input loads are expressed with respect to time. Almost all components are traditionally analyzed in the time domain for both structural integrity and fatigue life. However, an effective method is the frequency domain approach wherein loads are expressed in terms of PSDs and the fatigue life is predicted.

Vibration loads experienced by an automobile component contain harmonic and random events. In the time domain; it is a motion associated with many frequencies and the amplitude of these frequencies vary randomly with time [3]. It is random because the second sample measured from the test setup will be different from the first. The random component could be arbitrary and is associated with geometric and material non-linearities, impacts, variations in combustion pressures, sample variance, measurement noise etc. This random component can be described statistically. Using Fourier analysis technique, random loading of finite length can be simulated in terms of a set of sine wave functions, each having a set of values for amplitude, frequency and phase. This representation is deterministic in nature as each sine wave can be precisely determined at any point of time.

Extended Fourier transform allow these loads to be represented by a spectral formulation like a PSD function. The loads are still a function of the frequency and are therefore in the frequency domain. For a majority of engineering problems, if one form of the load is specified then the other form can be derived. These transformations are useful for structural durability formulations when the loading is Ergodic, Gaussian and Stationary Random. Fortunately most engineering problems satisfy these requirements [4].

In this paper two examples are computed for vibratory stress and the fatigue life is predicted using both the time and frequency (PSD) domain approaches and the results are compared and discussed.

STRUCTURAL ANALYSIS

Transient Structural Analysis in Time Domain:

Transient response approach is the generally preferred method in CAE for predicting the dynamic stress and hence the fatigue life of a structure subject to time-varying excitation loads. Modal transient response is an efficient formulation to compute the transient stress response. This formulation utilizes the mode shapes of the structure to reduce the matrix size, uncouple the equations of motions and make the numerical integration more efficient [5].

The first step in the formulation is transformation of the variables from the physical coordinates $\{u\}$ to the modal coordinates $\{\xi\}$ by

$$\{u(t)\} = [\phi]\{\xi(t)\} \qquad 1.0$$

Where, the mode shapes vectors are defined by the matrix $[\phi]$

The general equations of motion is defined by the equation below

$$[M]\{\ddot{u}(t)\}+[K]\{u(t)\}=\{P(t)\} \qquad 2.0$$

If the physical coordinates are substituted by the modal coordinates, the following equation is obtained:

$$[M][\phi]\{\xi(t)\}+[K]\{\phi\}\{\xi(t)\}=\{P(t)\} \qquad 3.0$$

This is now the equation of motion in terms of the modal coordinates and the equations could be de-coupled by pre multiplying $[\phi]^T$ to obtain

$$[\phi]^T[M][\phi]\{\xi\}+[\phi]^T[K][\phi]\{\xi\}=[\phi]^T\{P(t)\} \qquad 4.0$$

Where:

$[\phi]^T[M][\phi]$ = modal (generalized) mass matrix
$[\phi]^T[K][\phi]$ = modal (generalized) stiffness matrix
$[\phi]^T\{P\}$ = modal force vector

The orthogonal property of the mode shapes could be used to formulate the equations of motion as a set on uncoupled SDOF systems in terms of its generalized mass and stiffness matrices that are essentially diagonal matrices. In this uncoupled form, the equations of motion are written as

$$m_i\xi_i(t)+k_i\xi_i(t)=p_i(t) \qquad 5.0$$

Where
m_i = i-th modal mass
k_i = i-th modal stiffness
p_i = i-th modal force

The individual modal responses $\xi_i(t)$ can be computed and the physical degrees of freedom responses could be recovered as the summation of the modal responses.

$$\{u(t)\}=[\phi]\{\xi(t)\} \qquad 6.0$$

When the damping matrix [B] is present the generalized damping matrix is no longer a diagonal matrix.

$$[\phi]^T[B][\phi]\neq \text{diagonal} \qquad 7.0$$

In the modal transient formulation when the damping matrix [B] is present, the solution is attained in terms of the modal coordinates using the direct transient numerical integration approach as shown below:

$$[A_1]\{\xi_{n+1}\} = [A_2] + [A_3]\{\xi_n\} + [A_4]\{\xi_{n-1}\} \qquad 8.0$$

Where

$$[A_1] = [\phi^T][\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} + \frac{K}{3}]$$

$$[A_2] = \frac{1}{3}[\phi]^T\{P_{n+1} + P_n + P_{n-1}\}$$

$$[A_3] = [\phi^T][\frac{2M}{\Delta t^2} - \frac{K}{3}][\phi]$$

$$[A_4] = [\phi]^T[-\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} - \frac{K}{3}][\phi]$$

In the case of modal damping, the decoupled equations of motion are maintained and each mode has damping of b_i .

$$m_i\xi_i(t) + b_i\xi_i(t) + k_i\xi_i(t) = p_i(t) \qquad 9.0$$

or

$$\xi_i(t) + 2\xi_i\omega_i\xi_t(t) + \omega_i^2\xi_i(t) = \frac{1}{m}p_i(t) \qquad 10.0$$

Where

$\xi_i = \frac{b_i}{2m_i\omega_i}$ = modal damping ratio

$\omega_i^2 = \frac{k_i}{m_i}$ = modal frequency (eigenvalue)

Random Vibration Analysis in Frequency Domain (Power Spectral Density - PSD):

In MSC Nastran, Random Response analysis is treated as a data reduction procedure that is applied to the results from Frequency Response Analysis. First, the frequency response analysis is performed for each sinusoidal loading condition $\{P_a\}$ in a separate subcase for each applied load point and direction and the values recovered for a requested set of frequencies ω_i . Each load in an individual subcase represents a unique random load source, individually associated with each location and direction. Typically, these loads are chosen to be of unit value representing loads like "g" loads or unit pressures [5].

The application of these frequency response techniques to analyze random processes requires that the system be linear and that the excitation be stationary. An important quantity in the random analysis is the autocorrelation function $R_j(\tau)$ of a physical variable u_j which is defined by

$$R_j(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T u_j(t) u_j(t-\tau) dt \qquad 11.0$$

Note that $R_j(0)$ is the time average value of u_j^2, and is an important quantity in the analysis of structural failure. The one-sided power spectral density $S_j(\omega)$ of u_j is defined by

$$S_j(\omega) = \lim_{T \to \infty} \frac{2}{T} \left| \int_0^T e^{-i\omega t} u_j(t) dt \right|^2 \qquad 12.0$$

Using theory of Fourier integrals it may be shown that the autocorrelation function and the power spectral density are Fourier transforms of each other (transform pairs). Thus, autocorrelation function can be defined in terms of frequency response functions.

$$R_j(\tau) = \frac{1}{2\pi} \int_0^\infty S_j(\omega) \cos(\omega\tau) d\omega \qquad 13.0$$

From the means square theorem, the Root Mean Square (RMS) magnitude, u_j is

$$\bar{u}_j^2 = R_j(0) = \frac{1}{2\pi} \int_0^\infty S_j(\omega) d\omega \qquad 14.0$$

The transfer function theorem states that, if $H_{ja}(\omega)$ is the frequency response of any physical variable u_j due to an excitation source Q_a, which may be a point force, a loading condition or some other form of excitation, then

$$u_j(\omega) = H_{ja}(\omega).Q_a(\omega) \qquad 15.0$$

Where $u_j(\omega)$ and $Q_a(\omega)$ are Fourier transforms of u_j and Q_a.

The power spectral density of the response $S_j(\omega)$, is related to the power spectral density of the source, $S_a(\omega)$ by

$$S_j(\omega) = \left| H_{ja}(\omega) \right|^2 .S_a(\omega) \qquad 16.0$$

The above equation permits the statistical properties (like autocorrelation function) of the system response under random excitation to be evaluated via the technique of frequency response. Another useful result is that, if sources Q_1, Q_2, Q_3 etc. are statistically independent, i.e., if the cross-correlation function

$$R_{ab}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T q_a(t) q_b(t-\tau) d\tau \qquad 17.0$$

is null between any pair of source a, b, then, the power spectral density of the total response is equal to the sum of the power spectral densities of the responses due to the individual sources. Thus

$$S_j(\omega) = \sum_a S_{ja}(\omega) = \sum_a \left| H_{ja}(\omega) \right|^2 S_a(\omega) \qquad 18.0$$

If the sources are statistically correlated, the degree of correlation can be expressed by a cross-spectral density, S_{ab} and the spectral density of the response may be evaluated from

$$S_j = \sum_a \sum_b H_{ja} H^*_{jb} S_{ab} \qquad 19.0$$

Where H^*_{jb} is the complex conjugate of H_{jb}.

Important parameters output by Nastran are the number of Zero Crossings and the RMS value of the response.

FATIGUE LIFE ESTIMATION

Time Domain Fatigue Life Estimation – General Procedure

Fatigue life estimation of any component under vibration begins with the prediction of the stress or the strain response due to applied load. If the response of the structure exhibits constant amplitude stress or strain with a complete cycle reversal then the fatigue life prediction can be done using a simple S-N diagram. Real loadings rarely conform to this ideal constant amplitude situation and therefore an empirical approach is used to estimate life due to varying amplitudes of stress or strain. Palmgren-Miner rule is generally used for computing the accumulated damage. This linear relationship assumes that the damage caused by portions of a stress or strain history can be calculated and accumulated to predict the total damage. A ratio is calculated for each stress range, equal to the number of actual cycles for that particular stress range ‘n’ divided by the allowable number of cycles to failure at that stress ‘N’ (obtained from the S-N curve), and summed up. Failure is assumed to occur when the sum of these ratios for the entire stress and time histories equals to 1.0. An equivalent alternating stress using Goodman correction is used if mean stress is present [6].

If the response time history is irregular with time, then rainflow cycle counting is used to decompose the irregular time history into equivalent sets of block loading. The number of cycles in each block is usually recorded in a stress range histogram. This can then be used in the Palmgren-Miner calculation.

Frequency Domain Fatigue Life Estimation – General Procedure

In the frequency domain, the narrow band (Raleigh) or the broad band (Dirlik) method could be used to predict the fatigue life. In the narrow band, Raleigh distribution is used for the stress probability distribution function; whereas in the broad band approach the Gaussian (Normal) distribution is used for the stress probability distribution function. Dirlik's empirical solution is the recommended method at GM Powertrain.

Dirlik (1985) had generated an empirical closed form expression for the PDF (stress probability distribution function) after extensive computer simulations using the Monte Carlo technique [7].

Dirlik's equation for the Stress PDF is given below.

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2(m_0)^{1/2}} \qquad 20.0$$

Where D_1, $D2$, $D3$, Q and R are functions of m_0, m_1, m_2 and m_4. Z is normalized variable and is equal to $\frac{S}{2(m_0)^{1/2}}$.

Where, m_n is the n^th moment of the area of the PSD.

EXAMPLES

Two example problems are shown here to demonstrate the technique and to compare the life predictions using the time and frequency based solutions.

Example I

Secondary Air Injection Pump Assembly (SAI)

Figure 1 shows the SAI Assembly that failed at the pipe location under test conditions. The pipe was fabricated from S-304 Stainless steel and was subjected to operating temperatures of 500 Deg C and an additional vibratory input. The Finite Element (FE) model of the SAI pump assembly is shown in Figure 2. This FE model consists of 53476 second order tetrahedral elements and 98177 nodes. The SAI pipe is attached at one end to the outlet port by rigid elements (RBE2s) while the other end is attached to the

supporting bracket through rigid elements (RBE2s). Finally, the SAI pump assembly is supported by the cylinder head at three locations as shown in Figure 2.
Accelerations at the support locations in translation degrees of freedom in the time domain were measured. For predicting the fatigue life using the time domain approach, modal transient analysis (SOL 112) in MSC.Nastran was used and the time history loads were entered through TABLED1 cards.

For predicting life using the frequency domain approach, Auto-PSD Spectrum of the measurements were generated from the time history and incorporated in TABRND1 cards and the modal frequency response solution (SOL 111) in Nastran was used. However for logistical reasons, Cross-PSD Spectrum input was not used in this analysis.

Analysis Results and Comparisons:

40 seconds of test load data were generated in the time domain, and transient stresses for time histories 0 to 10 seconds, 10 to 20 seconds, 20 to 30 seconds and 30 to 40 seconds were solved for separately due to the constraints imposed by the requirement of time resolution for Nyquist frequency and disk space. Fatigue analyses were performed using FE-Fatigue (nCode) software.

For transient dynamic, S-N fatigue analyses were performed incorporating Goodman's mean stress correction. From results of fatigue analyses it was found that the damage occurs only for the loads in time history 20 to 30 seconds. The predicted fatigue life was 4.02E05 seconds.

For PSD stresses, Dirlik's method was used and the calculated fatigue life was predicted at 4.4E06 secs, which is comparable to the predicted life of 4.02E05 seconds computed earlier using transient analysis.

The two approaches were able to predict the life and the failure locations. The component was subsequently redesigned. Using CAE, the component was re-analyzed and the results indicated that there were no fatigue failures. The damage plot of the modified system is shown in Figure 3. This was confirmed during subsequent durability tests and the component was incorporated in the GM Powertrain program. This CAE process was standardized and deployed at GM Powertrain for fatigue life predictions using PSD functions in the frequency domain.

A second example is shown below highlighting the application of this technique.

<u>Example II</u>

Exhaust System Assembly

The FE model of an exhaust system assembly is shown in Figure 6. This FE model consists of total 281510 second order tetrahedral elements and 514433 nodes.

Dynamic loads in time domain were measured and are shown in Figure 4. The Auto-PSD for this time loading is shown in Figure 5. Structural and fatigue analyses similar to the previous example were performed.

Analysis Results and Comparisons:

2 seconds of test data were available in the time domain. Transient analyses of the time history from 0 to 1 second and from 1 to 2 second were performed separately. From the fatigue analysis it was found that the fatigue life for 0 to 1 second duration and 1 to 2 second duration histories were 2.16E05 and 1.88E05 seconds respectively.

From PSD analysis, the fatigue life was predicted at 7.97E05 seconds and compares well with the life predictions using the transient analyses.

Figure 1

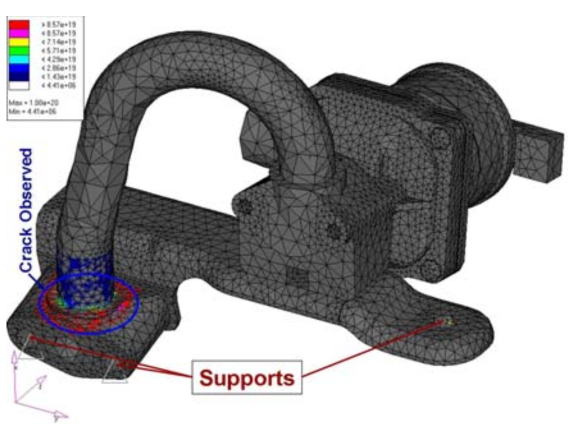

Figure 2

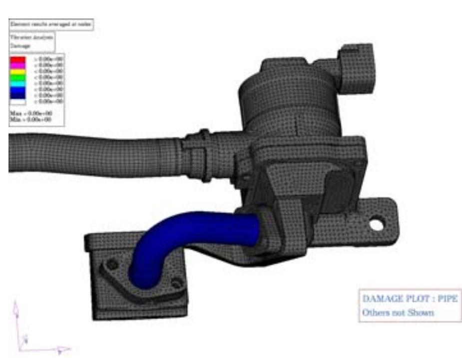

Figure 3

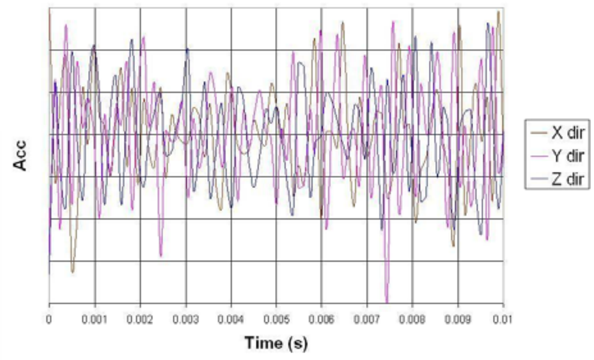

Figure 4

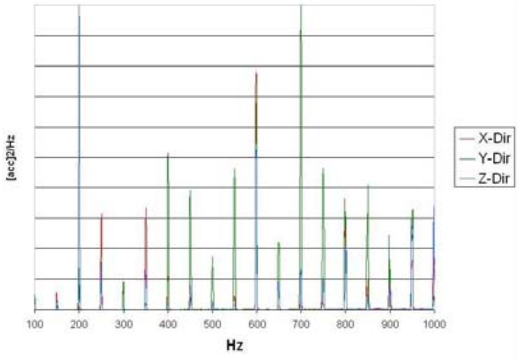

Figure 5

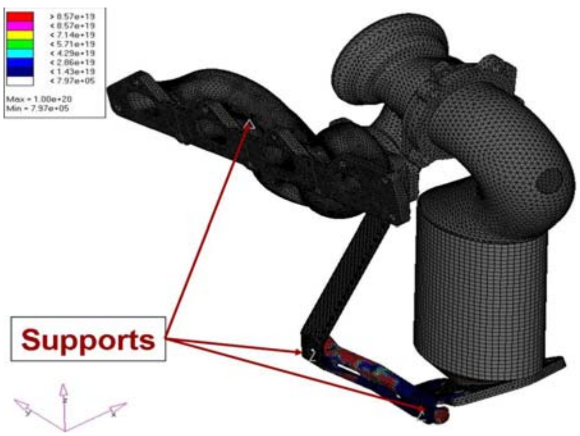

Figure 6

CONCLUSIONS

Two examples of automotive powertrain components were analyzed for durability and the fatigue life was predicted using both the time and frequency domain approaches. From both these example it is observed that the fatigue life predictions using the transient dynamic and random vibration approaches are comparable.

The conclusion from this exercise has been that the frequency domain random vibration fatigue analysis approach is simpler and is devoid of the need to address solution convergence issues- generally associated with the transient dynamic analysis methodology- especially when test load is used.

Consequently, this CAE process was standardized and deployed at GM Powertrain for performing durability analysis of powertrain components.

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