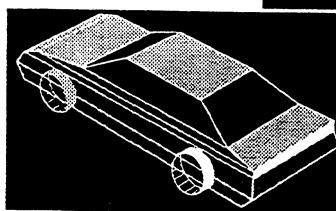
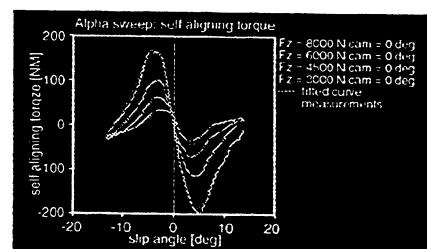
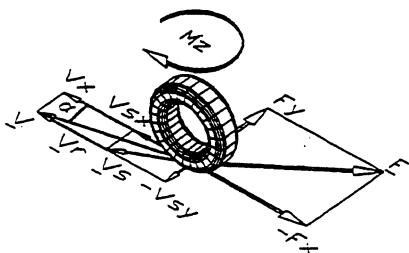


# Delft-Tyre

## a design and analysis tool...



- A design and analysis tool, to support testing and modelling of tyre behaviour, to be used with any dynamic system simulation code



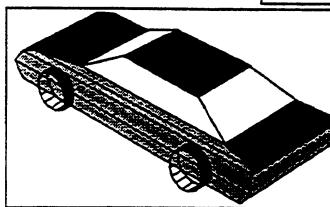
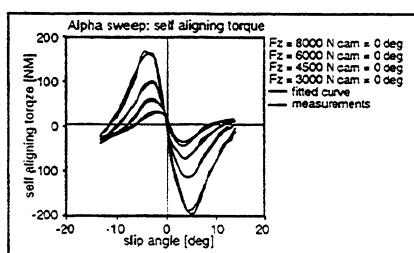
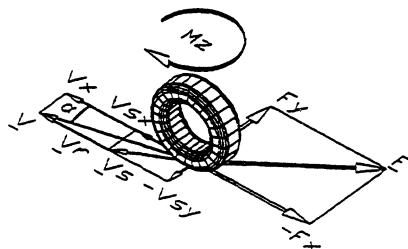
# Delft-Tyre

## a design and analysis tool...

Delft-Tyre



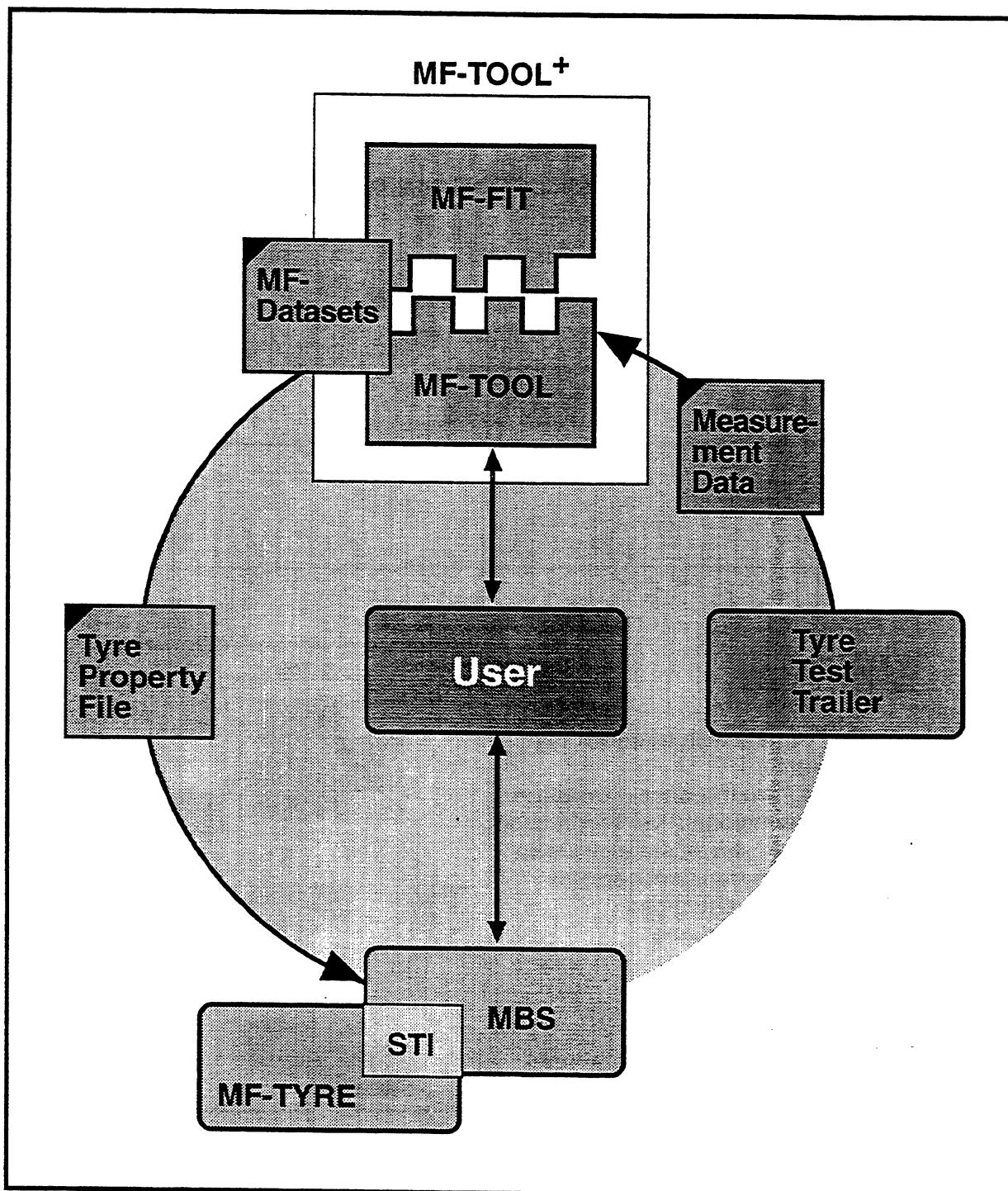
A design and analysis tool, to support testing and modelling of tyre behaviour, to be used with any dynamic system simulation code



Delft-Tyre



# Product range



# Objectives

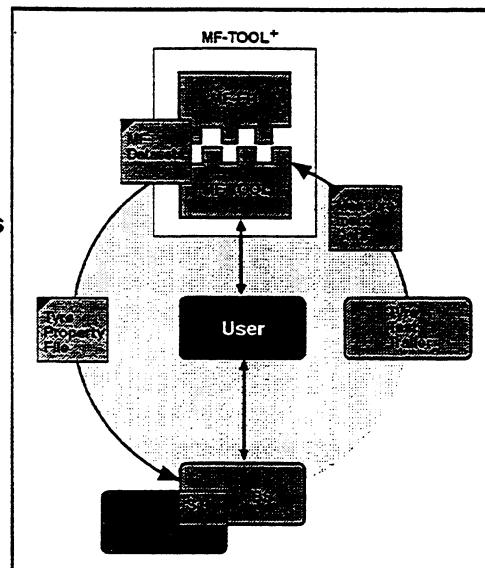
*To offer a complete range of tools for modelling and simulation of tyre behaviour*

- Tyre model with physical parameters
- Integrated with vehicle simulation environment
- Integrated with tyre data processing
- Tyre characteristic manipulation and tools for tyre-vehicle optimization
- Calculation of tyre model parameters from measurement data
- Latest developments
- Internationally accepted standards
- Open and well-known to the user



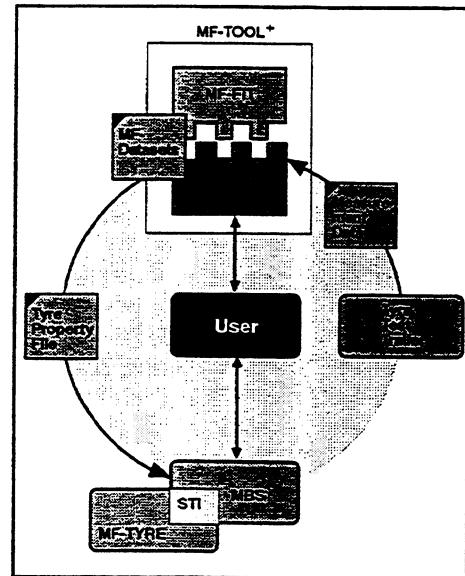
## MF-Tyre

- Magic Formula with transient tyre behaviour
- Standard Tyre Interface, developed within International Tyre Workshops
- From simple steady state pure slip conditions to transient complex combined slip situations.
- Real time applications



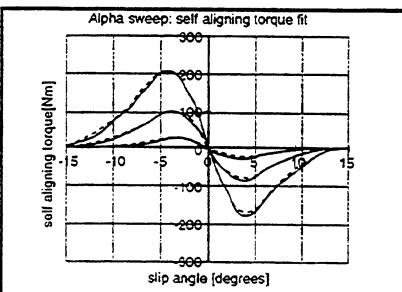
# MF-Tool

- Visualisation of tyre characteristics and tyre physical properties
- Adaption and manipulation of tyre characteristics using a graphical interface (user-defined tyre datasets)
- Tyre dataset database
- Preparation of tyre datasets for simulation



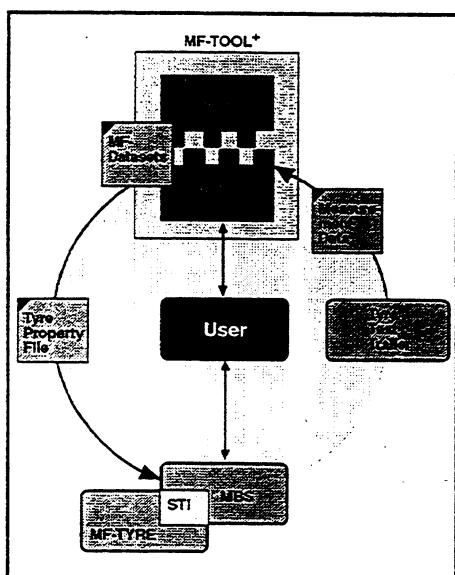
Delft-Tyre

# MF-Tool<sup>+</sup>



## MF-Tool including MF-Fit

- Calculation of tyre model parameters
- Control of MF-Fit
- Graphical comparison measurement data
- Quality check of fit results
- Tyre data in standardized TYDEX format

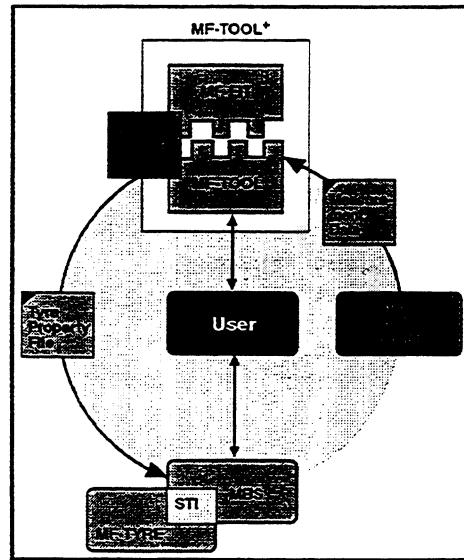


Delft-Tyre

## MF-Datasets



- Tyre library  
Regular updates and new datasets
- Tailor made datasets



**Delft-Tyre**

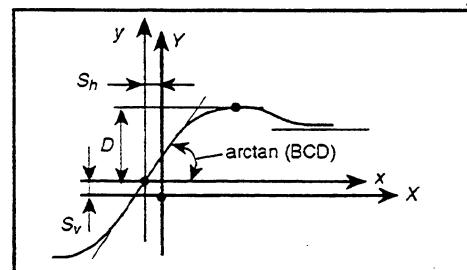
## MF-Tyre

- **Steady state:**  
**MAGIC FORMULA TYRE MODEL**

$$y = D \sin [\operatorname{Carctan}\{Bx - E(Bx - \arctan(Bx))\}]$$

$$Y(x) = y(x) + S_v$$

$$x = X + S_h$$



- **Transient and oscillatory aspects:**  
**FIRST ORDER LAGS and GYROSCOPIC COUPLE**

**Delft-Tyre**

# MF-Tyre

## MAIN PROPERTIES of NEW VERSION:

### *Steady state pure slip*

- aligning torque based on pneumatic trail

### *Steady state combined slip*

- forces using weighting functions
- aligning torque based on pneumatic trail

### *Dynamic state*

- first order lag
- relaxation lengths in x- and y- directions
- gyroscopic couple about z- axis
- start from zero velocity

### *Non-dimensional model parameters*

#### *Scaling factors*



# Magic Formula

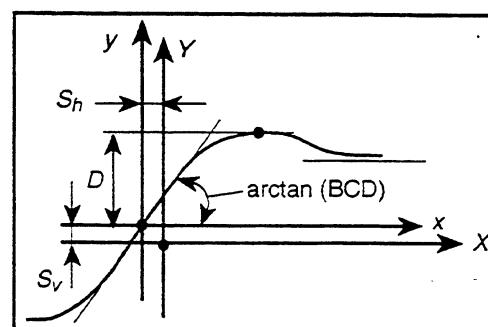
$$y = D \sin [\text{Carctan}\{Bx - E(Bx - \arctan(Bx))\}]$$

$$Y(x) = y(x) + S_v$$

$$x = X + S_h$$

where Y: output variable  $F_x$  or  $F_y$   
X: input variable  $\alpha$  or  $\kappa$

and    B: stiffness factor  
      C: shape factor  
      D: peak factor  
      E: curvature factor  
       $S_h$ : horizontal shift  
       $S_v$ : vertical shift



# Magic Formula / Pure slip

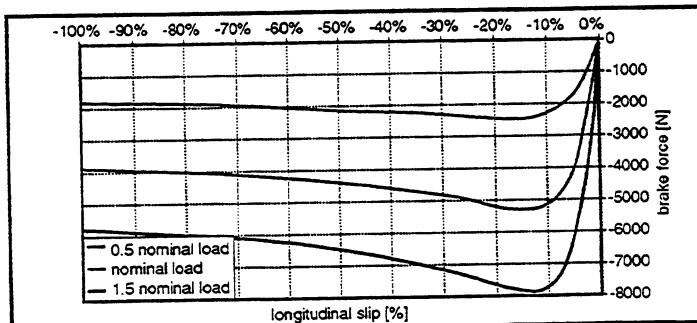
## *Longitudinal force*

$$F_{x_0} = D_x \sin [C_x \arctan \{B_x \kappa_x - E_x (B_x \kappa_x - \arctan (B_x \kappa_x))\}] + S_{vx}$$

$$D_x = (p_{Dx1} + p_{Dx2} df_z) F_z \lambda_{\mu x}$$

$$K_x = BCD_x = F_z (p_{Kx1} + p_{Kx2} df_z) \exp (-p_{Kx3} df_z) \lambda_{Kx}$$

etc.



# Magic Formula / Pure slip

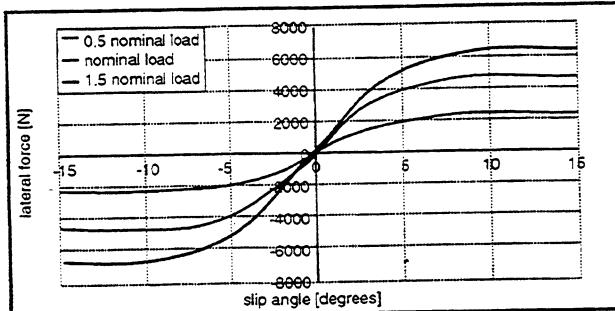
## *Lateral force*

$$F_{y_0} = D_y \sin [C_y \arctan \{B_y \alpha_y - E_y (B_y \alpha_y - \arctan (B_y \alpha_y))\}] + S_{vy}$$

$$D_y = (p_{Dy1} + p_{Dy2} df_z) F_z \lambda_{\mu y}$$

$$K_y = BCD_y = p_{Ky1} F_{zo} \sin [2 \arctan \{\frac{F_z}{p_{Ky2} F_{zo} \lambda_{Fzo}}\}] (1 - p_{Ky3} |\gamma_y|) \lambda_{Fzo} \lambda_{Ky}$$

etc.



# Magic Formula

Pure slip

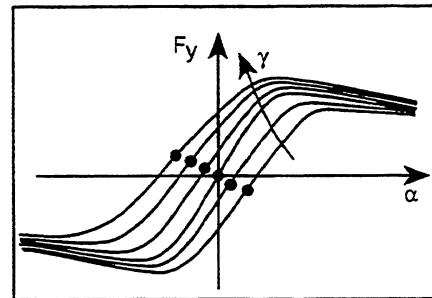
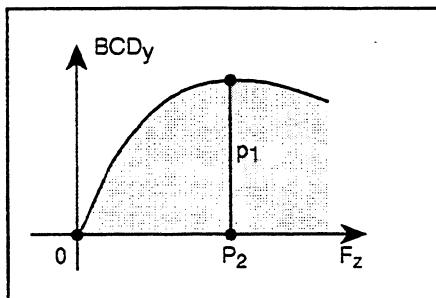
## Asymmetry

Curvature factor E

Cornering stiffness

$$E = E_o + \Delta E \operatorname{sign}(x)$$

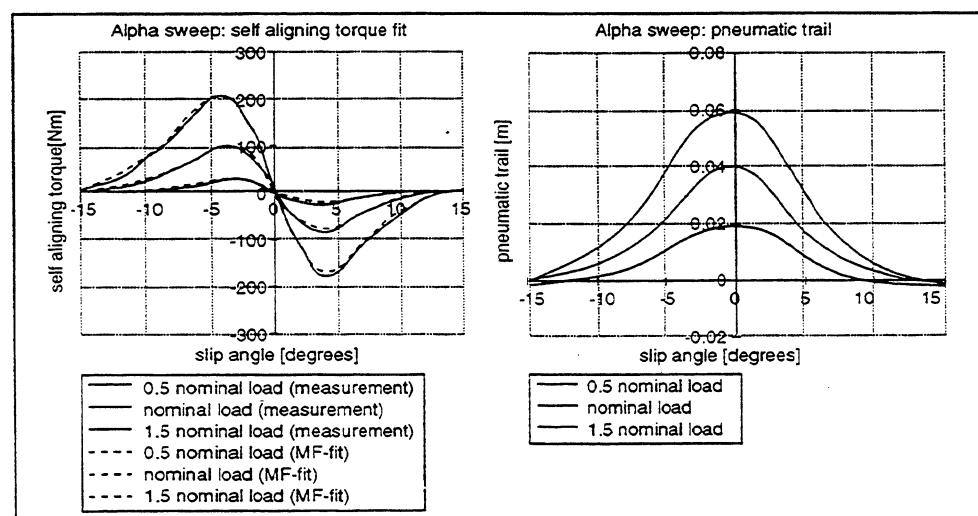
$$BCD_y = p_1 \sin [2 \arctan (\frac{F_z}{p_2})] (1 - p_3 |\gamma|)$$



Delft-Tyre

# Magic Formula

Aligning torque



Delft-Tyre

## Magic Formula / Pure slip

*Aligning torque*

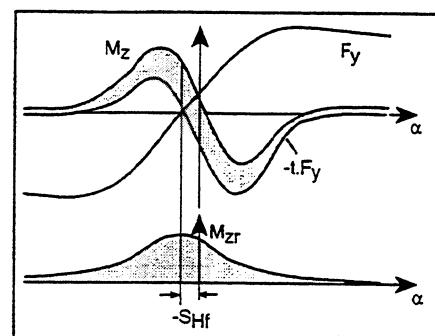
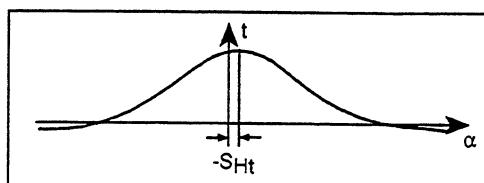
$$M_{zo} = -t F_{yo} + M_{zr}$$

*Pneumatic trail*

$$t(\alpha_r) = D_t \cos [C_t \arctan \{B_t \alpha_t - E_t (B_t \alpha_t - \arctan (B_t \alpha_t))\}] ; \alpha_t = \alpha + S_{Ht}$$

*Residual torque*

$$M_{zr}(\alpha_r) = D_r \cos [\arctan (B_r \alpha_r)] ; \alpha_r = \alpha + S_{Hr}$$



Delft-Tyre

## Magic Formula / Combined slip

Weighting functions G multiplied with pure slip functions

Weighting function have a hill shape

Cosine version of Magic Formula is used

$$G = D \cos [\arctan (Bx)]$$

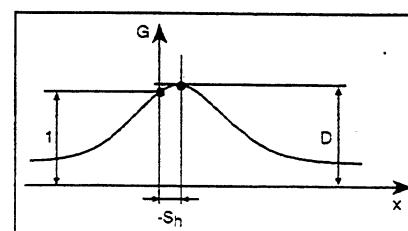
x is either  $\kappa$  or  $\alpha$  (possibly shifted)

D represents the peak value

C determines the height of the hill's base

B influences the sharpness of the hill

E is not needed to improve the fit



Delft-Tyre

## Magic Formula Combined slip

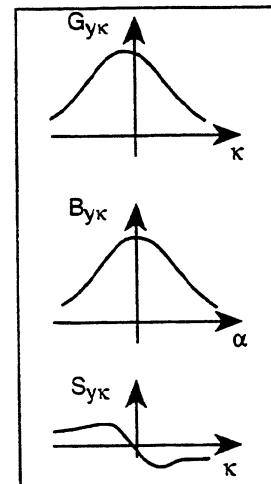
### Side force

$$F_y = G_{y\kappa} F_{yo} + S_{vy\kappa}$$

$$G_{y\kappa} = \frac{\cos[C_{y\kappa} \arctan\{B_{y\kappa}(\kappa + S_{Hy\kappa})\}]}{\cos[C_{y\kappa} \arctan(B_{y\kappa} S_{Hy\kappa})]}$$

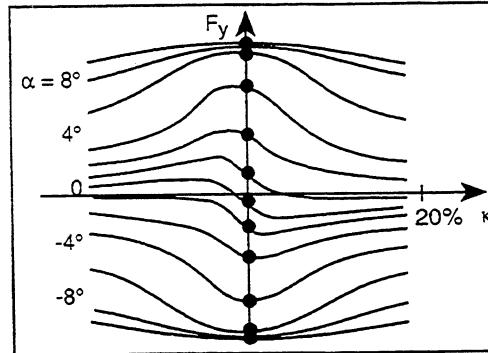
$$B_{y\kappa} = r_{By1} \cos [\arctan \{r_{By2}(\alpha - r_{By3})\}]$$

$$S_{vy\kappa} = \sin [r_{vy5} \arctan (r_{vy6}\kappa)]$$



## Magic Formula Combined slip

- $F_{yo}$  is the side force at pure side slip
- $B_{y\kappa}$  influences the sharpness of the hill
- $S_{vy\kappa}$  is the vertical shift due to ' $\kappa$ -induced' plysteer



# Magic Formula

## Combined slip

*Longitudinal force*

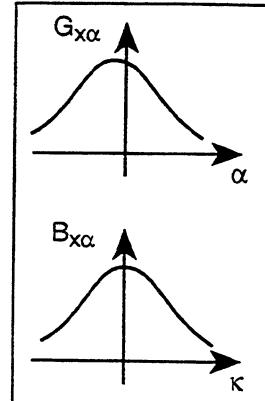
$$F_x = G_{x\alpha} F_{xo}$$

$$G_{x\alpha} = \frac{\cos[C_{x\alpha} \arctan\{B_{x\alpha}(\alpha + S_{Hx\alpha})\}]}{\cos[C_{x\alpha} \arctan(B_{x\alpha} S_{Hx\alpha})]}$$

$$C_{x\alpha} = r_{Cx1}$$

$$B_{x\alpha} = r_{Bx1} \cos [\arctan \{r_{Bx2} \kappa\}]$$

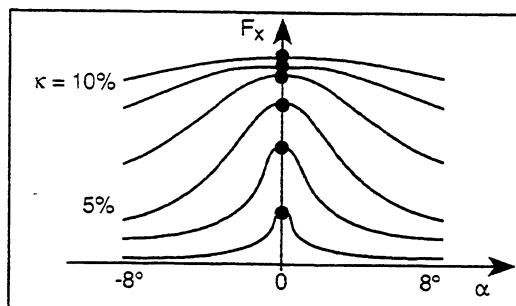
$$S_{Hx\alpha} = r_{Hx1}$$



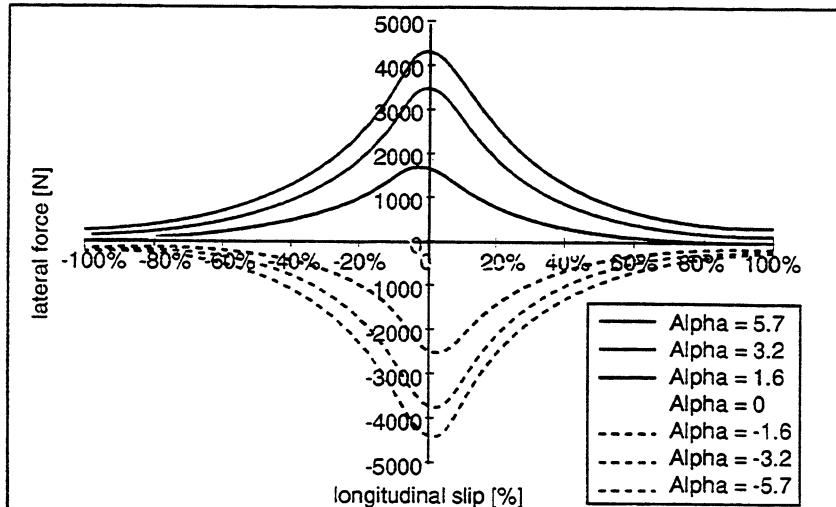
# Magic Formula

## Combined slip

- $F_{xo}$  is the longitudinal force at pure longitudinal slip
- $B_{x\alpha}$  influences the sharpness of the hill
- $C_{x\alpha}$  influences the height of the hill base
- $S_{Hx\alpha}$  is the horizontal shift of the hill peak



# Magic Formula



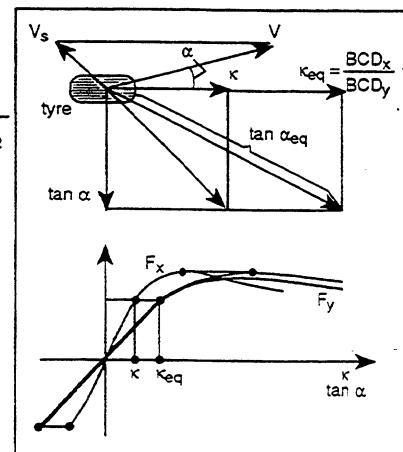
# Magic Formula Combined slip

## Aligning torque

$$M_z = -t(\alpha_{t,eq}) F_y + M_{zx} (\alpha_{r,eq}) + s(F_y \gamma)^* F_x$$

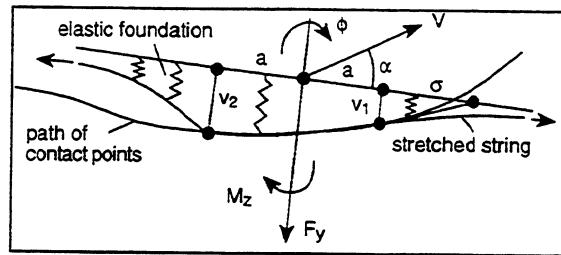
$$\alpha_{t,req} = \text{sign}(\alpha_{t,r}) \cdot \arctan \sqrt{\tan^2(\alpha_{t,r}) + \left( \frac{BCD_x}{BCD_y} \right)^2 \kappa^2}$$

$$s = \{s_{sz1} + s_{sz2} \left( \frac{F_y}{F_{zo}} \right) + (s_{sz3} + s_{sz4} df_2) \gamma\} R_o \lambda_s$$



## Transient aspects

$$\frac{1}{V_x} \frac{dv_1}{dt} + \frac{v_1}{\sigma} = \tan(\alpha) - a\phi$$



Multiplication by  $V_x$  and  $\sigma$ , neglect turn slip  $\phi$

$$\sigma \frac{dv_1}{dt} + V_x v_1 = -\sigma V_{sy}$$

NOW POSSIBLE TO START AND STOP!

$$\text{deflection angle } \alpha' \quad \tan(\alpha') = \frac{v_1}{\sigma}$$



## Dynamic Combined Slip

- slip speeds  $V_{sx,y}$  input instead of  $\kappa$  and  $\alpha$
- longitudinal and lateral deformations  $u$  and  $v$
- relaxation lengths  $\sigma_\kappa$  and  $\sigma_\alpha$
- speed of rolling  $V_r$

$$\sigma_\kappa \frac{du}{dt} + |V_r| u = -\sigma_\kappa V_{sx} \quad \sigma_\alpha \frac{dv}{dt} + |V_r| v = -\sigma_\alpha V_{sy}$$

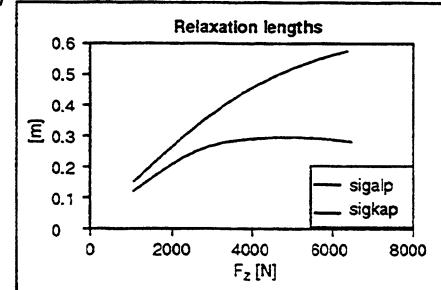


# Dynamic Combined Slip

Relaxation lengths depend on vertical load and camber:

$$\sigma_\kappa = F_z (p_{Tx1} + p_{Tx2} df_z) \exp(-p_{Tx3} df_z) (R_0/F_{z0}) \lambda_{\sigma\kappa}$$

$$\sigma_\alpha = p_{Ty1} \sin [2 \arctan \{ \frac{F_z}{p_{Ty2} F_{z0} \lambda_{Fz0}} \}] (1 - p_{Ky3} \gamma) R_0 \lambda_{Fz0} \lambda_{\sigma\alpha}$$



# Dynamic Combined Slip

Theoretical tyre deformation slip components:

$$\zeta'_x = \frac{u}{\sigma_\kappa} , \quad \zeta'_y = \frac{v}{\sigma_\alpha}$$

and the practical tyre deformation slip quantities:

$$\kappa' = \frac{\zeta'_x}{\text{sign } V_r - \zeta'_x} , \quad \tan(\alpha') = \frac{\zeta'_y}{\text{sign } V_r - \zeta'_x}$$

Forces and moment (steady state formulae)

$$F_x = F_x(\alpha', \kappa', F_z)$$

$$F_y = F_y(\alpha', \kappa', \gamma, F_z)$$

$$M'_z = M'_z(\alpha', \kappa', \gamma, F_z)$$



# Gyroscopic Couple

Moment due to tyre inertia acting about the z-axis

$$M_{z,gyr} = C_{gyr} m_t V_r \frac{dv}{dt}$$

Total aligning torque

$$M_z = M_z' + M_{z,gyr}$$



## Scaling factors

Pure slip

$\lambda_{Fzo}$	nominal load
$\lambda_{\mu x}, \lambda_{\mu y}$	peak friction levels $F_x, F_y$
$\lambda_{Kx}, \lambda_{Ky}$	slip stiffness
$\lambda_{\gamma y}$	camber force stiffness
$\lambda_{\gamma z}$	camber torque stiffness
$\lambda_t$	pneumatic trail
$\lambda_{Mr}$	residual torque



## Scaling factors

### Combined slip

$\lambda_{x\alpha}$	$\alpha$ influence on $F_x$ ( $\kappa$ )
$\lambda_{y\kappa}$	$\kappa$ influence on $F_y$ ( $\alpha$ )
$\lambda_{vy\kappa}$	$\kappa$ induced $F_y$
$\lambda_s$	$M_z$ moment arm of $F_x$

### Transient response

$\lambda_{\alpha\kappa}$	relaxation length for $F_x$
$\lambda_{\alpha\alpha}$	relaxation length for $F_y$
$\lambda_{gyr}$	tyre mass (gyroscopic couple)



## TYDEX Workshops

International group of vehicle and tyre manufacturers  
with the objective to standardize interfaces between  
*vehicle models - tyre models - tyre data*

- Standard Tyre Interface (STI)  
Interface between vehicle model and any tyre model  
to enable simple exchange of tyre models
- TYDEX data format  
Format for simple exchange of tyre test data and  
parameters of tyre models
- International Colloquia on Tyre Modelling



# MF-Datasets

## Library

### Contents (Jan. 1996)

tyre	manufacturer	type	conditions
175/70 R13	Michelin	MXT	dry
195/65 R15	Michelin	MXV3a Energy	dry
195/65 R15	Michelin	Pilot HX MXV3a	dry, wet
195/65 R15	Vredestein	Snowtrac	dry
225/50 R16	Goodyear	Eagle GST	dry, wet
235/75 R15	Goodyear	Invicta GS	dry, wet



# MF-Datasets

### Tyre datasets

#### *Steady state:*

- Pure and combined slip
- Effective tyre radius
- Vertical tyre stiffness

#### *Dynamic:*

- Tire lags depending on load

