MODELIZING STRUCTURAL HYSTERETIC DAMPING BY MEANS OF MDI ADAMS

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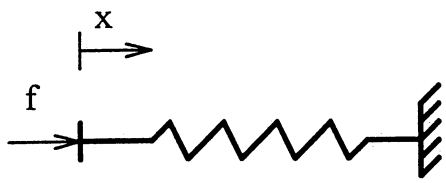
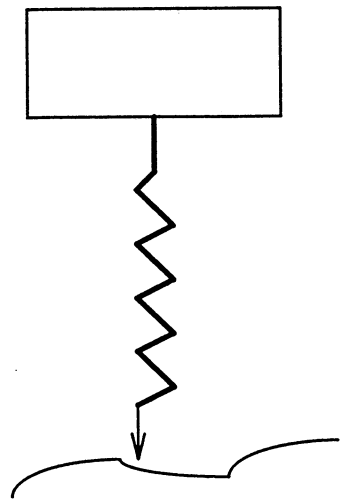
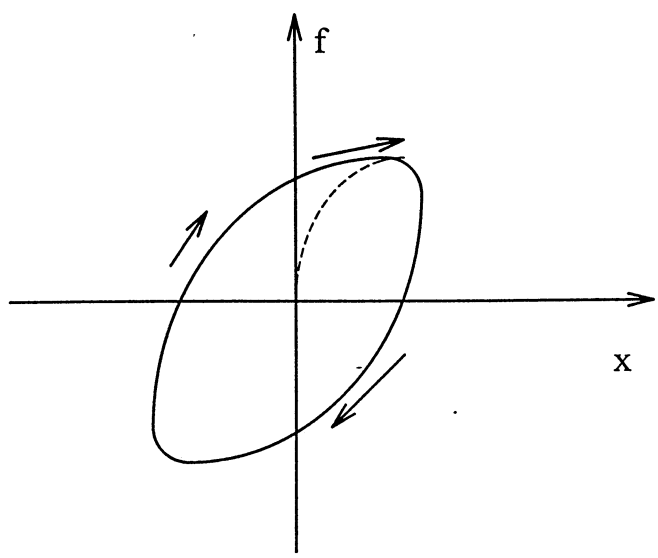
ABSTRACT

Modelizing structural hysteretic damping is an important topic in modern technical dynamics. It is well known how to deal with this problem by making use of fractional derivatives. In the present paper a new technique is proposed requiring the introduction of a new state variable which is easy to implement in MDI ADAMS.







Let us consider the hysteretic cycle of the figure where we have a hysteretic spring-damper loaded by a force f(t)on the left and blocked on the right. We shall call x(t) the displacement of the left side. ''t'' is the time.Let:

1) $x(t) = X \sin(\omega t)$

2) $f(t) = F \sin(\omega t + \varphi)$

3) $dx/dt = \omega X \cos(\omega t) = v(t)$

4) $d^2x/dt^2 = -\omega^2 X \sin(\omega t) = a(t)$

with obvious meaning of the symbols and letters. We can then write

5) $f(t) = F \sin(\omega t) \cos(\varphi) + F \cos(\omega t) \sin(\varphi)$

As well known:

6) $F \cos(\varphi) = k X$ and $F \sin(\varphi) = kX \, tg(\varphi)$

We can then rewrite (5):

7) $f(t) = kX \sin(\omega t) + k \, tg(\varphi) X \cos(\omega t)$

and substituting (3) in (7) we get:

8) $f(t) = k (x(t) + tg(\varphi) v(t)/\omega)$

At this point it must be said that our aim is to express the force f(t) as a function of state variables only in order to modelize the spring-damper in ADAMS. To get this we still have to express the ''$1/\omega$'' factor as a function of state variables. One could think to do this as:

9) $1/\omega = |x(t)/a(t)|^{1/2}$

but as it is shown in ref. 1 for ex., forces cannot in general be expressed as functions of accelerations so that we must introduce a new state variable which we define as the solution of the following differential eq.:

10) $dy/dt = -hy + x(t)$

first of all note that if h is zero the new variable y is simply the time integral of the strain x. The constant h is introduced to eliminate the well known zero frequency problem of hysteretic damping: note that if h is zero, y goes to infinity when x is constant. In the following we shall choose h much lower than the first eigenfrequency of the model but not negligible-with respect to it. We shall then express:

11) $1/\omega = |y(t)/v(t)|^{1/2}$

and substituting this in (8) we get easily:

12) $f(t) = k\,(x + tg(\varphi)\,(v(t)/|v(t)|)\;|y(t)\,v(t)|^{1/2})$

the initial condition for y is assumed to be

$$(t = 0) \Rightarrow (y = 0)$$

so that the cycle starts with the dotted curve displayed in the figure. We modelize the simple mass spring damper system shown in the figure which is excited at the base and we compare the transmissibilities of the system with the well known formula:

13) $\text{gain} = (1 + i\,tg(\varphi)) / (1 - (\omega/\omega_0)^2 + i\,tg(\varphi))$

where the gain is the ratio of the mass displ. over the base displ. and ω_0 is the eigenfrequency of the system. It is easy to show that the gain modulus is always 1 if

14) $\omega = \omega_0 \sqrt{2}$

we shall assume:

15) $tg(\varphi) = 0.12$

in all subsequent calculations.
In Tables 1 and 2 we can see the case of sinusoidal base excitation at ω_0 first and at $\omega_0 \sqrt{2}$ after: the result is in very close agreement with (13).

In Tab. (3) and (4) we can see the case of pulse base excitation: here the gain is computed as the ratio of two fourier transforms and it is compared with (13). The reader can verify that it is even difficult to distinguish one curve from the other.

In Tab 5 and 6 a different pulse is shown. The response is shown in Tab. 7 for a system with

16) $\omega_0 = 3.16$ Hz

the gain is shown in Tab. 10.

gain and phase are shown in Tab. 8 and 9 of a system with

17) $\omega_0 = 10$ Hz

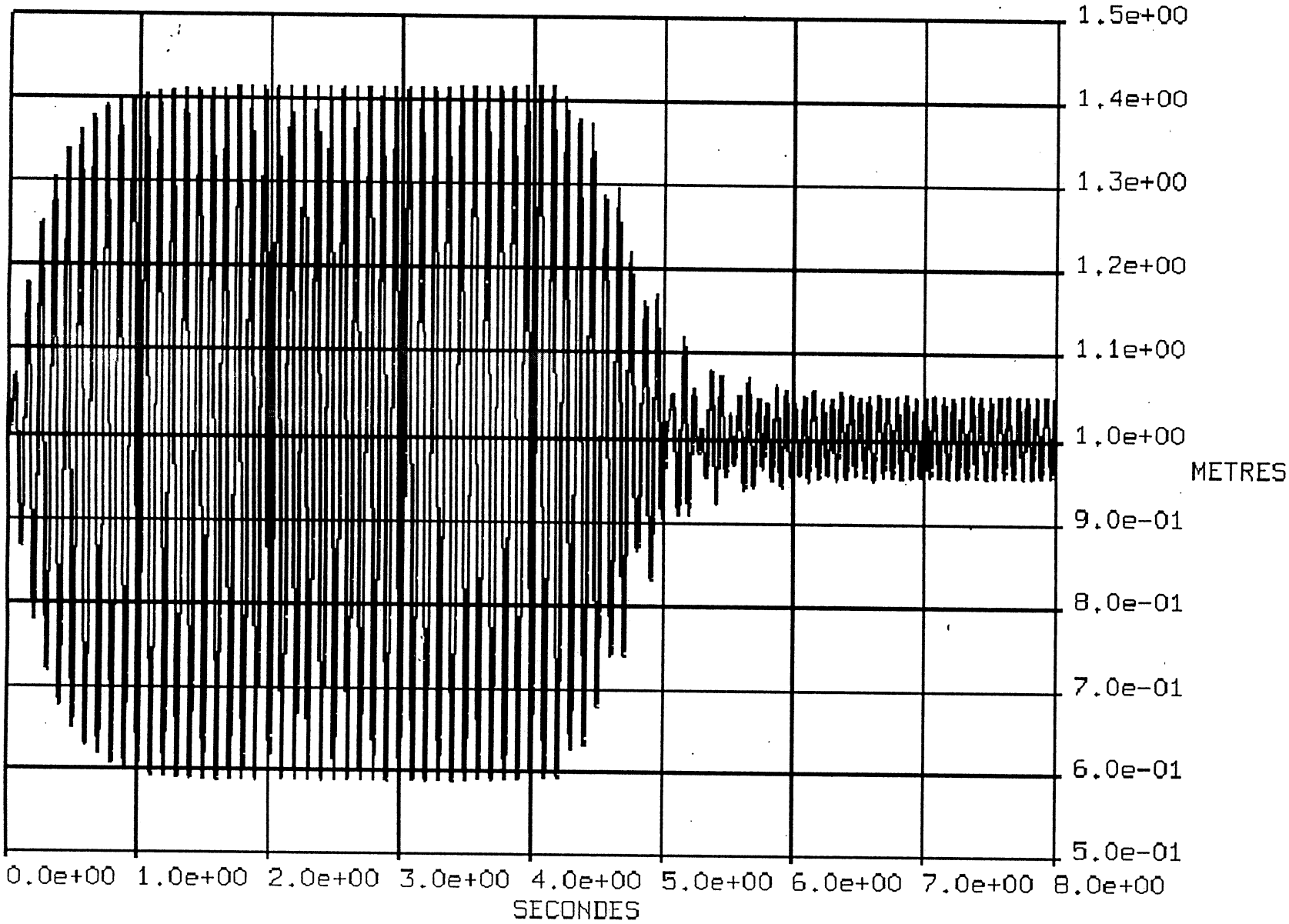
CONCLUSIONS

A non linear system has been proposed whose behaviour is very close to that of a linear mass spring-damper mechanical system with hysteretic damping and base excitation. A new state variable has been proposed which is necessary to the definition of the damping forces. MDI ADAMS permits easy implementation of this technique by its functions and differential eq. cards.

BIBLIOGRAPHY

1) L. A. PARS : A TREATISE ON ANALYTICAL DYNAMICS

REPONSE ADAMS



```
HYSTERETIC DAMPING
part/03,ground
part/02,mass = 0., cm = 2, ip = 0,0,0,qg = 0,0,0,
part/01,mass = 253.30295, cm = 3, ip = 0,0,0,qg = 0,0,0,vz = 0,
marker/1, part = 3, qp = 0,0,-1,
marker/2, part = 2, qp = 0,0,0,
marker/3, part = 1, qp = 0,0,0,
marker/4, part = 2, qp = 0,0,-1,
joint/9,i = 2, j = 1, translational,
joint/10,i = 3, j = 4, translational,
motion/201,joint = 9, trans
,fu = 1 + (1 - havsin(time,4,0,5,1)) * sin(62.831854 * time)  * 0.05
,+ havsin(time,4,0,5,1) * sin(1.41 * 62.831854 * time) * 0.05
diff/11, ic = 0, fu = - 2. * dif(11) + dm(3,4) - 1
sforce/41, i = 3, j = 4, tr,
,fu = - 1000000 * (dm(3,4) - 1)
, - 120000 * if(vr(3,4): - sqrt( - vr(3,4)), 0, sqrt(vr(3,4)))
, * if(dif(11) : sqrt( - dif(11)), 0, sqrt(dif(11)))\
output/grs
request/51,d,i=3,j=1,
end
```

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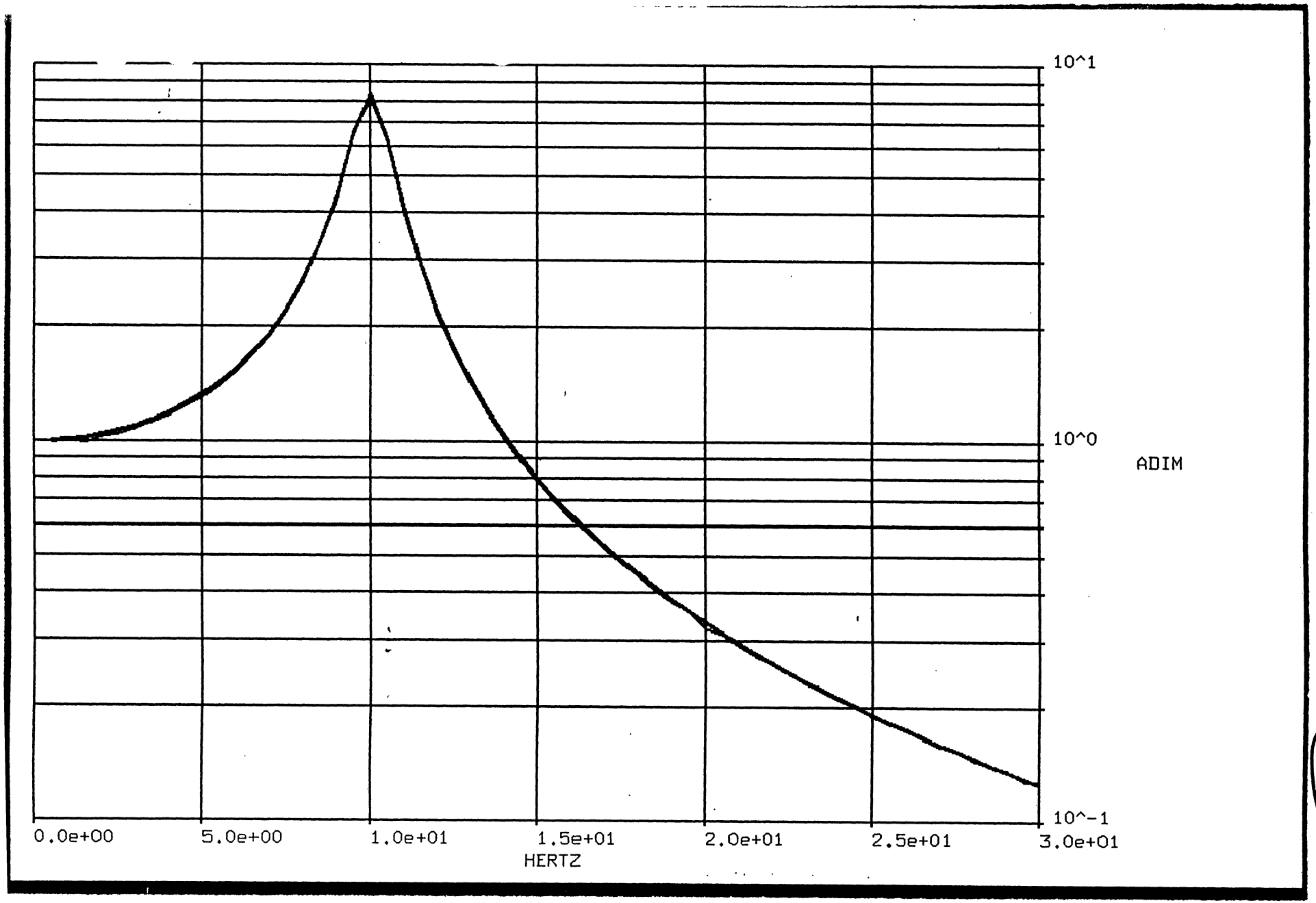
10^1
10^0
10^-1
ADIM
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5.0e+00
1.0e+01
1.5e+01
2.0e+01
2.5e+01
3.0e+01
HERTZ

```
HYSTERETIC DAMPING
part/03,ground
part/02,mass = 0., cm = 2, ip = 0,0,0,qg = 0,0,0,
part/01,mass = 253.30295, cm = 3, ip = 0,0,0,qg = 0,0,0,vz = 0,
marker/1, part = 3, qp = 0,0,-1,
marker/5, part = 3, qp = 0,0,0,
marker/2, part = 2, qp = 0,0,0,
marker/3, part = 1, qp = 0,0,0,
marker/4, part = 2, qp = 0,0,-1,
joint/9,i = 2, j = 1, translational,
joint/10,i = 3, j = 4, translational,
motion/201,joint = 9,
,fu =
,step(time,0,0,0.01,0.1) - step(time,0.01,0,0.02,0.1) + 1
diff/11, ic = 0, fu = - 2. * dif(11) + dm(3,4) - 1
sforce/41, i = 3, j = 4, tr,
,fu = - 1000000 * (dm(3,4) - 1)
, - 120000 * if(vr(3,4): - sqrt( - vr(3,4)), 0, sqrt(vr(3,4)))
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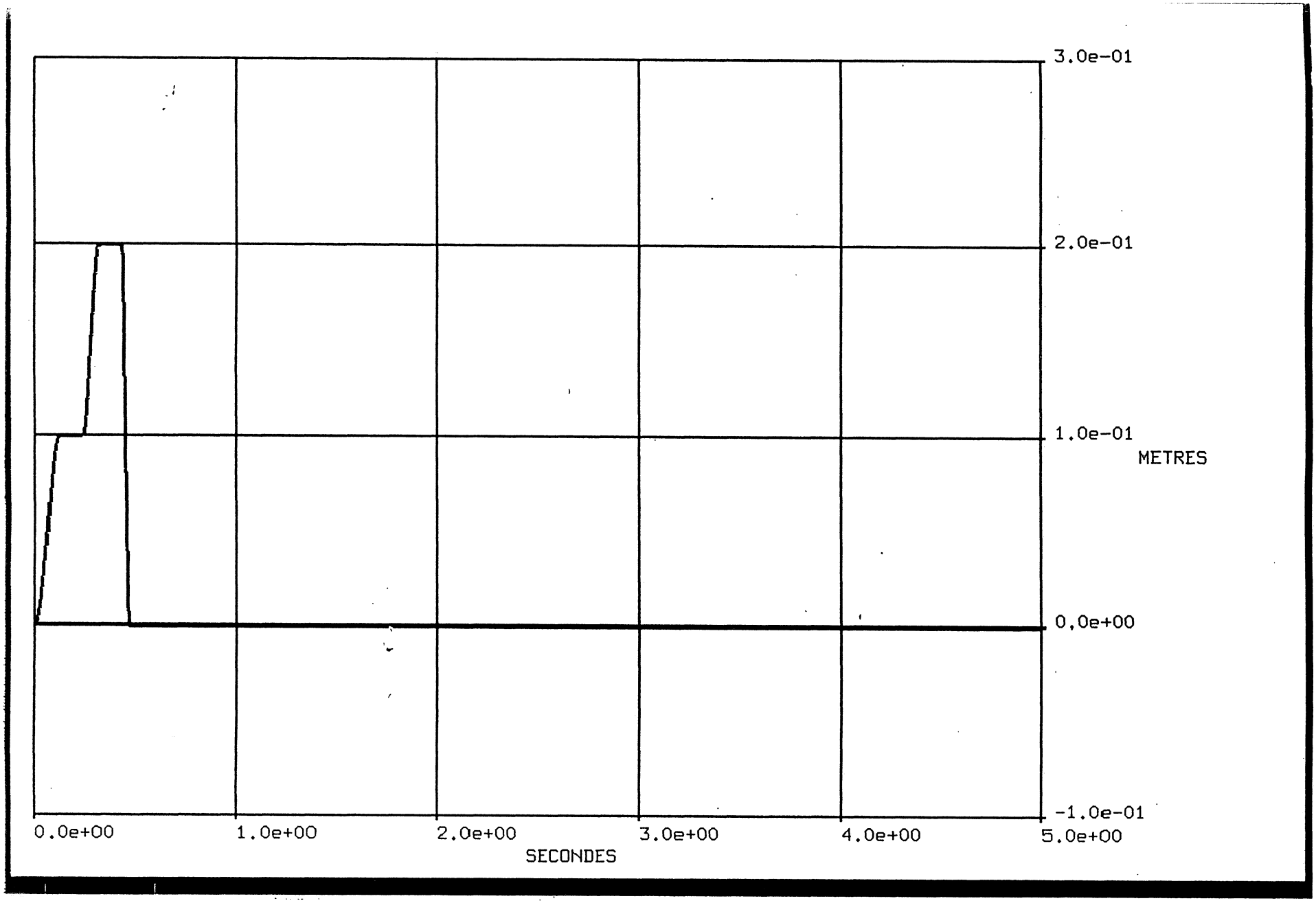
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2.0e-01
1.0e-01
METRES
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2.0e+00
3.0e+00
4.0e+00
5.0e+00
SECONDES

6

```
HYSTERETIC DAMPING
part/03,ground
part/02,mass = 0., cm = 2, ip = 0,0,0,qg = 0,0,0,
part/01,mass = 253.30295, cm = 3, ip = 0,0,0,qg = 0,0,0,vz = 0,
marker/1, part = 3, qp = 0,0,-1,
marker/5, part = 3, qp = 0,0,0,
marker/2, part = 2, qp = 0,0,0,
marker/3, part = 1, qp = 0,0,0,
marker/4, part = 2, qp = 0,0,-1,
joint/9,i = 2, j = 1, translational,
joint/10,i = 3, j = 4, translational,
motion/201,joint = 9,
,fu =
,step(time,0,0,0.12,0.1) + step(time,0.24,0,0.32,0.1) -
,step(time,0.43,0,0.47,0.2) + 1
diff/11, ic = 0, fu = - 2. * dif(11) + dm(3,4) - 1
sforce/41, i = 3, j = 4, tr,
,fu = - 1000000 * (dm(3,4) - 1)
, - 120000 * if(vr(3,4): - sqrt( - vr(3,4)), 0, sqrt(vr(3,4)))
, * if(dif(11) : sqrt( - dif(11)), 0, sqrt(dif(11)))\
output/grs
request/51,d,i=3,j=5,
request/52,d,i=2,j=5,
end
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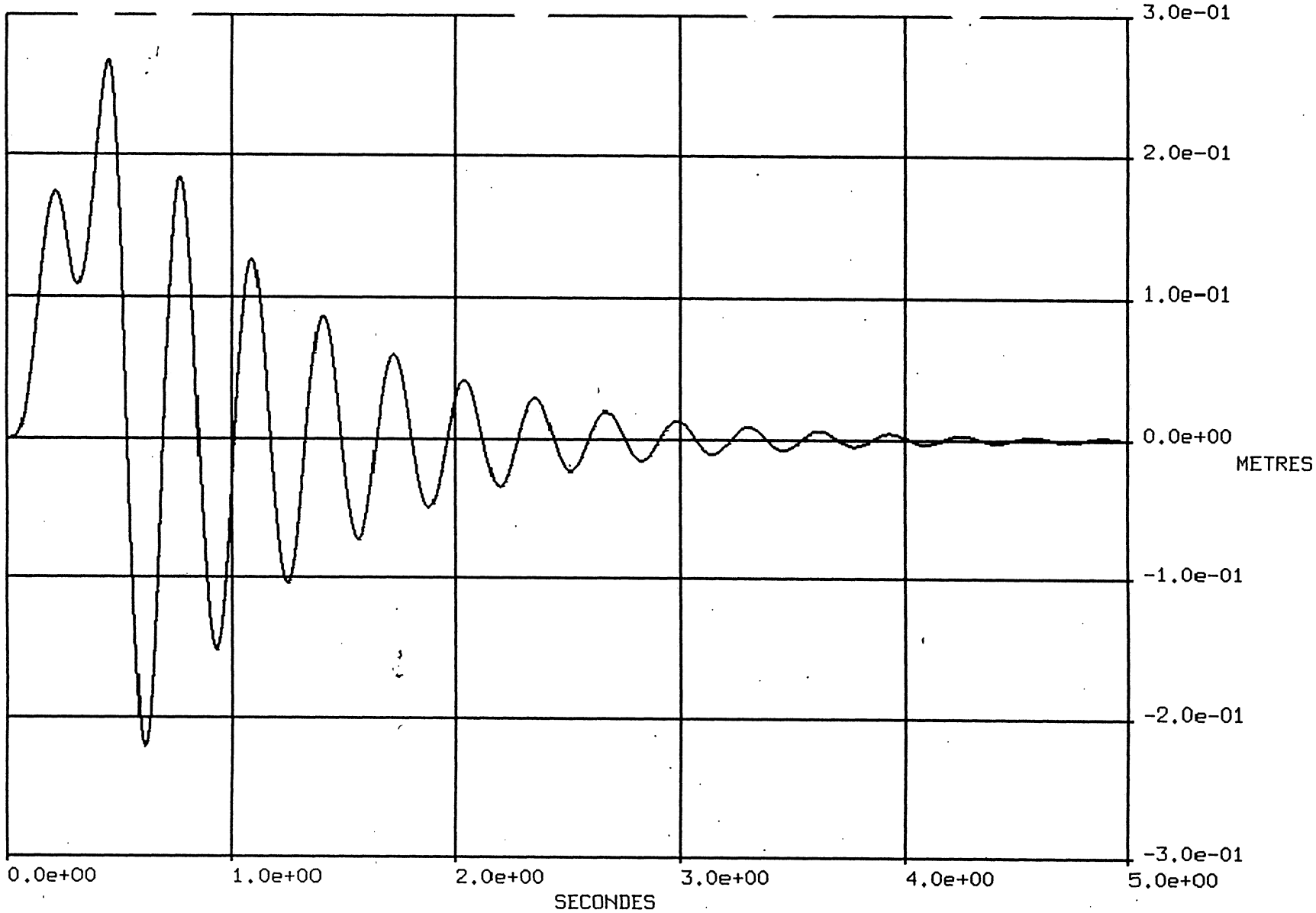
3.0e-01
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SECONDES

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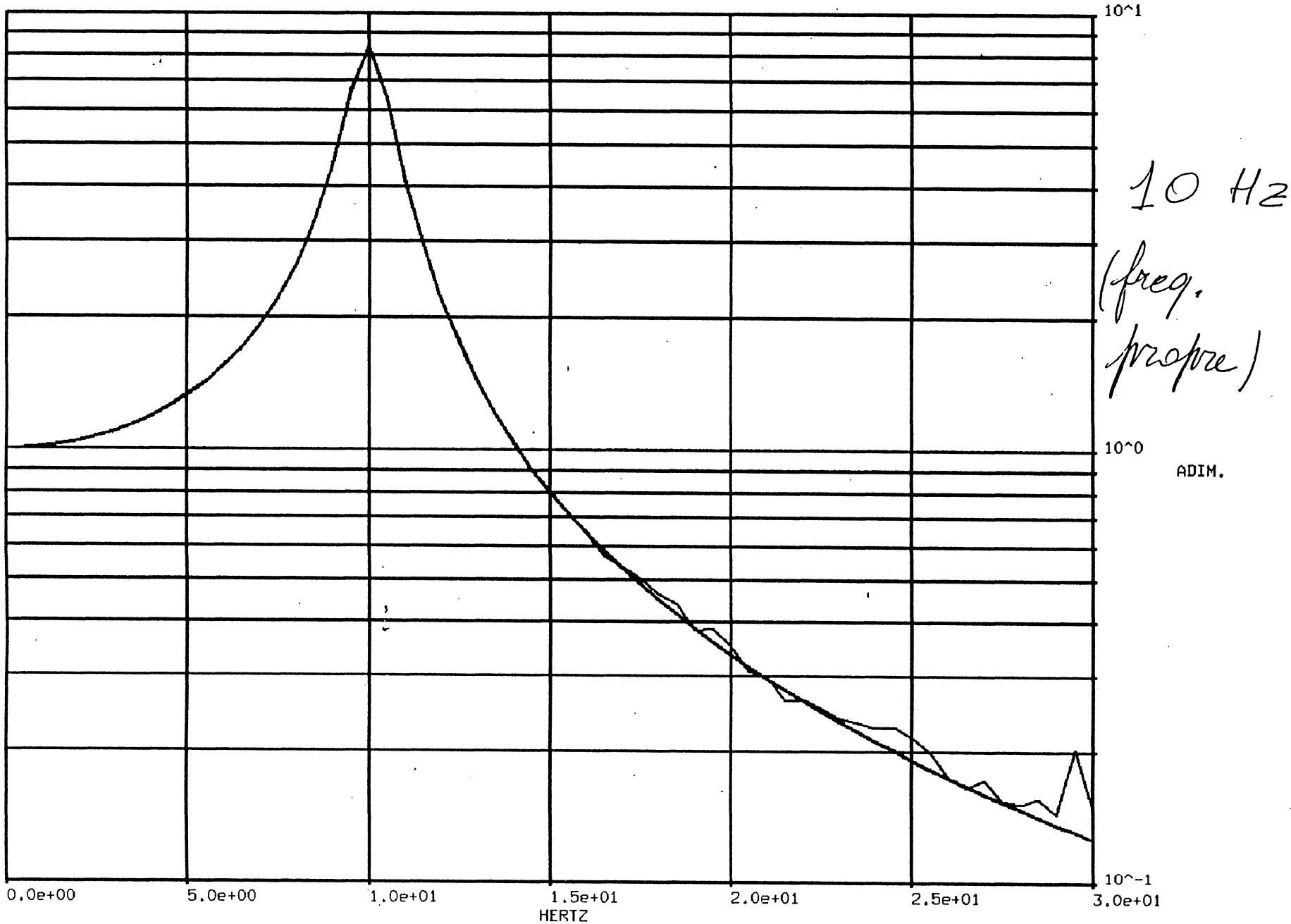
REPONSE ADAMS



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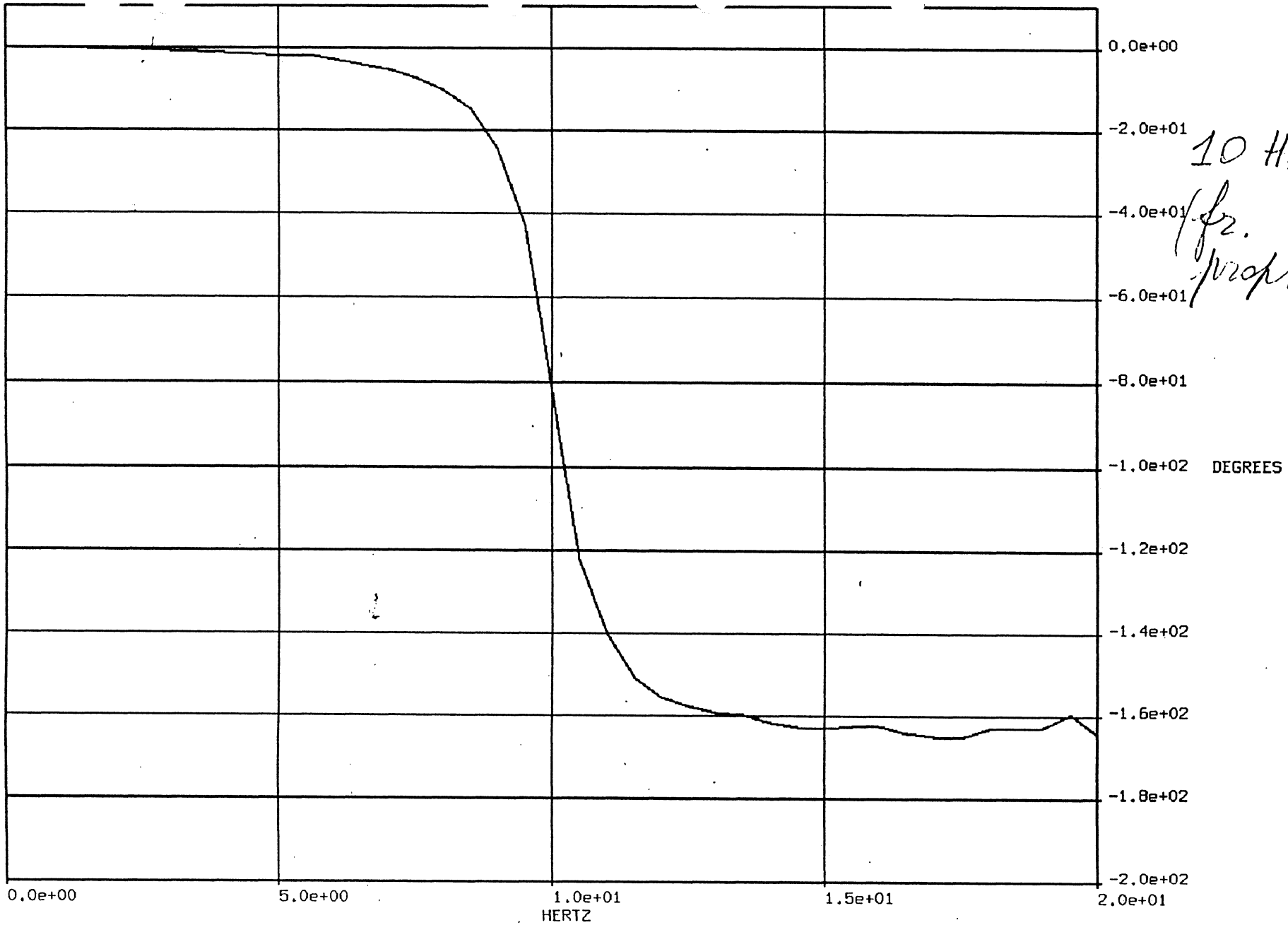

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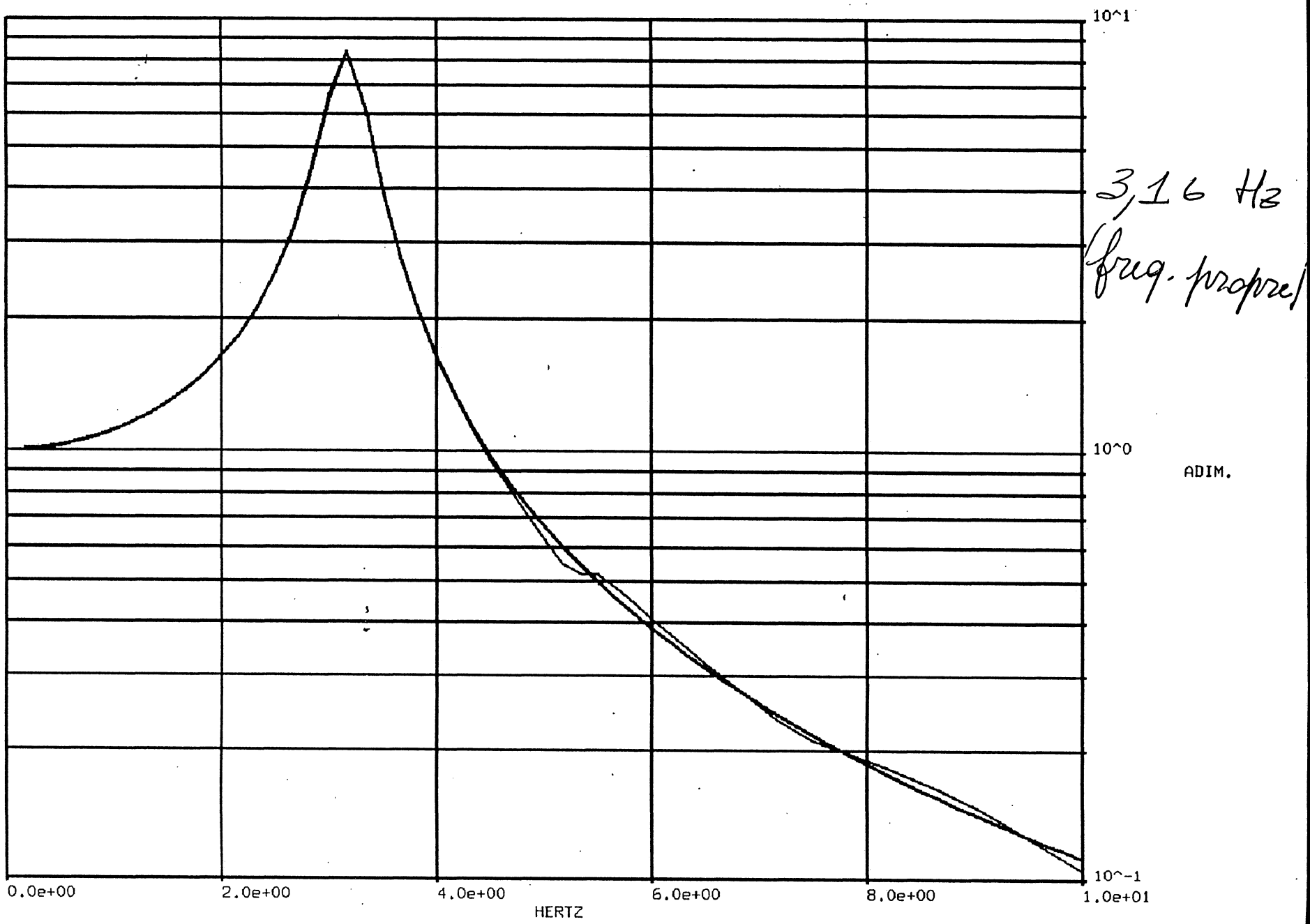

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