Realization of an interaction model between flexible structures and piezoelements in multibody modeling

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ABSTRACT

In this paper a general procedure useful in multibody design software for the synthesis of a reduced dynamic model for the description of flexible structure coupled with piezoelectric sensors and actuators is shown. Using a interaction modeling procedure (using theoretical and FEM approach) built by authors in precedent papers, modifying and developing an existing modal synthesis method, using by commercial software ADAMS, a new mode component synthesis methodology was build. This new procedure allows to simulate any system, containing flexible parts coupled with piezoelectric materials, in linear and non linear behavior, under any mechanical and electrical boundary condition both in the frequency and time domain. This chance isn't allowed both by theoretical-analytical and FEM approach unless heavy and long computational loads.

1. INTRODUCTION

The analysis of complex systems dynamic behavior can't leave out of consideration the use of mechanical system simulation softwares that allow to study their behavior both in kinetic and dynamic terms, in any working conditions. Moreover, the necessity to consider bodies flexibility in a constant growing number of applications, in order both to evaluate deformations and stresses associated to system motion (time dependent) and to design efficient position and/or vibration control systems taking into account system flexibility, causes the necessity to combine MSS softwares with FEM softwares functionality.

At present, dynamic simulation software houses are developing or have already developed transfer procedures for systems flexible behavior by retrieving necessary information from structural analysis commercial softwares. This paper is set in this research scope.

Considering the constantly growing utilization of piezoelectric materials such as ceramics PZT and films PVDF in flexible structures vibration control, in particular in aerospace applications in which a smaller structural influence of the control system, both in terms of mass and stiffness, is requested, the principal aim of this work has been to transfer into the flexible structure ADAMS modal synthesis method (in particular ANSYS/ADAMS procedure) an interaction model built by the authors in precedent papers [11,12] for the description of flexible structure coupled with piezoelectric sensors and actuators.

In this way, this work introduces the possibility to model and simulate action of piezoelectric actuators and sensors in multibody modeling and in rigid and flexible bodies simulation (in linear and non linear system behavior conditions), thus allowing to design, with those elements, position and vibration control systems in co-simulation environments in which multibody software allows to easily receive algorithms coming from dedicated sofwares (MATLAB and MATRIXX).

2. MODAL SYNTHESIS

The ADAMS multibody code analyses rigid systems motion by integration of the motion equations written in Euler-Lagrange form.

So, the study of flexible body are developed according to this formulation. To consider a flexible body a "modal" approach is followed. Deformations are evaluated by a discrete displacement function obtained by the multiple modal matrix [Φ] with generalized (modal) co-ordinates q, identifying body position X by coupling elastic deformations δ with body general motion, considered rigid, x:

$$\{X\}=\{x\}\cdot\{\delta\} \quad with \quad \{\delta\}=[\Phi]\cdot\{q\} \tag{1}$$

Euler-Lagrange equations assume a form described by (2), where L represents the lagrangian, F the dissipation energy, Ψ the constraint equations, λ the Lagrange constraint multipliers, ξ the generalized co-ordinates, q_k the flexible body modal co-ordinates (n) and m the constraints number.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}_i}\right) - \frac{\partial L}{\partial \xi_i} + \frac{\partial F}{\partial \dot{\xi}_i} + \sum_{j=1}^{m}\left[\frac{\partial \Psi_j}{\partial \xi_i}\right]^T \lambda_j - Q_i = 0$$

$$\Psi_j = 0 \qquad \text{with} \quad i = 1,\dots,n+6 \quad j = 1,\dots,m \qquad \text{e} \qquad \xi = \begin{Bmatrix} x \\ y \\ z \\ \psi \\ \vartheta \\ \varphi \\ q_k \end{Bmatrix} \tag{2}$$

2.1 Mode Component Synthesis: Craig and Bampton method (*C&B*)

In ADAMS a modal synthesis method, based on Craig and Bampton one [2,3,4,5], is used. This method allows to reduce generalized co-ordinates number and to obtain greater freedom in boundary conditions definition on boundary points (constraint and/or force application ones), describing completely local flexibility effects. Flexible structural component motion, with N degree of freedom and with defined boundary points (or interface ones), is described by combination of P normal modes (*normal modes*) and S constraint modes (*static correction modes*). The firsts are obtained by modal analysis considering boundary degree of freedom fixed, instead the seconds are S static deflections obtained by imposing a unitary displacement at interface degree of freedom one at a time and taking others fixed.

Using a degree of freedom partition, between internal and external dofs, flexible body motion equation becomes:

$$\begin{bmatrix} m^{BB} & 0 \\ 0 & m^{II} \end{bmatrix}\begin{Bmatrix} \ddot{x}^B \\ \ddot{x}^I \end{Bmatrix} + \begin{bmatrix} k^{BB} & k^{BI} \\ k^{IB} & k^{II} \end{bmatrix}\begin{Bmatrix} x^B \\ x^I \end{Bmatrix} = \begin{Bmatrix} f^B \\ f^I \end{Bmatrix} \tag{3}$$

with I internal dofs (equal to $R = N - S$) and B boundary ones (equal to S).

$$\text{(a)} \begin{bmatrix} k^{BB} & k^{BI} \\ k^{IB} & k^{II} \end{bmatrix}\begin{Bmatrix} x^B \\ x^I \end{Bmatrix} = \begin{Bmatrix} f^B \\ f^I \end{Bmatrix} \qquad \text{(b)} \left[\phi^C\right] = -\left[k^{II}\right]^{-1}\left[k^{IB}\right] \tag{4}$$

$$\text{(a)} \left[-\omega^2\left[m^{II}\right] + \left[k^{II}\right]\right]\cdot\left\{\phi^I\right\} \qquad \text{(b)} \left[\phi^N\right] = \left[\left\{\phi^I\right\}_1, \dots, \left\{\phi^I\right\}_P\right] \tag{5}$$

From equation (3), considering *static correction modes* and *normal modes* definitions (4a e 5a) matrices $[\phi^C]$ (SxS) of constraints modes (4b) and $[\phi^N]$ (RxR) of normal modes (5b) are obtained. Of this last matrix, only a sub-group P ($< R$) modes is considered with the aim to reduce computational load, relating to maximum frequency range to analyze.

By approximating physical co-ordinates x with a group of modes and realizing relative co-ordinates change (6) from equation (3) equation (7) is obtained in which p represents the new co-ordinate system and mass and stiffness matrices are called generalized.

$$\{x\} = \begin{Bmatrix} x^B \\ x^I \end{Bmatrix} = \begin{bmatrix} [I] & [0] \\ [\phi^C] & [\phi^N] \end{bmatrix}\begin{Bmatrix} p^B \\ p^I \end{Bmatrix} = [\Phi]\{p\} \tag{6}$$

$$[\overline{M}]\{\ddot{p}\} + [\overline{K}]\{p\} = \begin{bmatrix} \overline{m}^{BB} & \overline{m}^{BN} \\ \overline{m}^{NB} & \overline{m}^{NN} \end{bmatrix}\begin{Bmatrix} \ddot{p}^B \\ \ddot{p}^I \end{Bmatrix} + \begin{bmatrix} \overline{k}^{BB} & 0 \\ 0 & \overline{k}^{NN} \end{bmatrix}\begin{Bmatrix} p^B \\ p^I \end{Bmatrix} = \begin{Bmatrix} \overline{f}^B \\ \overline{f}^I \end{Bmatrix} = \{\overline{f}\} \tag{7}$$

It's important to highlight the problem dimension reduction in terms of total dofs: from N they become (P+S).

In ADAMS, moreover to simplify flexible body analysis an ortho-normalization of the reduced system described by equation (7) is realized, compared to original *C&B* method. This last phase produce a diagonal model, that is to say a modal matrix, flexible body free-free behavior image, but what's more elastic contribution of boundary points deformability. This last one isn't more unequivocally identifiable through static correction modes, but it is connectible to high frequency modes generated by ortho-normalization.

Initial modal and static information are obtained by FEM structural analysis of flexible body and then handled by an automatic transfer procedure that creates a binary file (*modal neutral file*) readable by multibody software and that can be generated by all principal commercial structural analysis codes (ANSYS, NASTRAN, ABAQUS, I-DEAS). Subsequently the multibody code has the task to connect flexible bodies to other parts writing necessary compatibility equations.

2.2 Changes realized in ANSYS/ADAMS modal synthesis procedure

From an operative point of view, in this modal synthesis procedure, compared to the original method (*C&B*), the user chooses boundary nodes (*Hard Point*), considering all their dofs as boundary ones and assuming the other dofs, relative to the remaining nodes, as internal ones.

This mode of action limits the potentiality of the method.

First of all, it has to consider all the node and all the relative dofs; this causes a great computational load also in those cases in which some nodes or/and some dofs could be not considered (i.e. rotational dofs in a membrane structure).

Moreover, in those cases in which constraints, realized in multibody model doesn't fix all hard points dofs (i.e. spherical joint) modal matrix itself is over-dimensioned (the constraint modes number is greater when all the hard points dofs are considered).

If this means, from a numerical point of view, greater time analysis and matrices over-dimensioning, it is possible, from a theoretical one, that it introduces an evaluation error located in *C&B* method characteristics. Both the authors of method (*R.R. Craig e M.M.C. Bampton*) and some experiences developed on simple structures in this paper suggest to consider a number of boundary dofs strictly necessary to constraint requirements, avoiding, when possible, to let boundary dofs unconstrained and, therefore, to consider those high frequency modes generated by ortho-normalization.

In this research, we tried to make ADAMS modal synthesis method more general and, consequently, more versatile. What pointed out previously constitutes the motivation for the changes introduced; changes made by utilization of new Fortran routines and ANSYS/ADAMS macros defined by the authors (*Modified Mode Component Synthesis*).

An ANSYS procedure has been written to realize a set of analyses necessary to obtain useful data for creation of the transfer file (*mnf*); this macro allows user to interactively select, besides normal modes number to be considered, boundary dofs (*hard dofs*), nodes subset and internal dofs, freeing him from only hard point selection.

Using temporary matrices, pointers and some Fortran subroutines for output reading, an ascii file set can be obtained; these are handled by a Fortran program (*MMNF*) and by a series of subroutines that realize model modal synthesis (*C&B*), its ortho-normalization and writing of the transfer file. Also, it has to be highlighted the user possibility to select interactively or by external ascii file the normal modes to be considered into modal synthesis after their FEM creation, to the aim to not consider in synthesized model possible "local modes" or influenced by a wrong meshing.

Realizing this synthesis procedure with two phases, using ascii text files, penalizes the procedure itself in terms of analysis time but, during method set-up, it allows an easier control and tests of the analysis phases.

A series of analyses were realized on various finite elements models, that, as shown in the paragraphs below, prove correctness of changes, as well as the hypotheses on which these changes are based.

3. FLEXIBLE STRUCTURE COUPLED WITH PIEZOELECTRIC ELEMENTS: MODAL SYSNTHESIS METHOD

Finite element modeling of a flexible structure coupled with piezoelectric transducers is quite easy, considering that there are finite elements that simulate their behavior. However, it's very difficult, using this modeling, to realize control strategies synthesis and optimization and to simulate output time histories caused by external actions, considering relative computational loads.

To make this interaction manageable in a easy way and to allow vibration and position control systems synthesis, in two precedent papers [11,12] the authors built a general procedure which is useful in modal synthesis of a generic flexible structure equipped with piezoelectric actuators and sensors. This procedure uses a finite element model of the structure with piezoelectric elements on it. Realizing simple and little onerous structural analyses (modal and static analyses) it's possible to synthesize a reduced model of whatever complex structures, immediately utilizable in state space representation and, so, easily useful both in simulation phase and in control design one.

In this work these results have been extended to multibody modeling. In the next paragraph, a modal synthesis procedure developed by the authors and transferred into ADAMS Mode Component Synthesis is shown (*Modified Mode Component Synthesis*).

3.1 Piezoelectric elements in the *Modified Mode Component Synthesis*

The dynamic behavior of a structure equipped with piezoelectric J sensors e W actuators can be written in the form below:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F_e\}+\sum_i \{F_V\}_i \tag{8}$$

where $[M]$ e $[K]$ are FEM model mass and stiffness matrix, $\{x\}$ displacements, $\{F_e\}$ external forces and each $\{F_V\}_i$ is the applied action realized by i-th piezo-actuator reduced to nodal loads. Neglecting damping and writing equation (8) using *C&B* approach:

$$\begin{bmatrix} m^{BB} & 0 \\ 0 & m^{II} \end{bmatrix}\begin{Bmatrix} \ddot{x}^B \\ \ddot{x}^I \end{Bmatrix}+\begin{bmatrix} k^{BB} & k^{BI} \\ k^{IB} & k^{II} \end{bmatrix}\begin{Bmatrix} x^B \\ x^I \end{Bmatrix}=\begin{Bmatrix} F_e^{\ B} \\ F_e^{\ I} \end{Bmatrix}+\sum_i \begin{Bmatrix} F_V^{\ B} \\ F_V^{\ I} \end{Bmatrix} \tag{9}$$

It can be verified that piezoelectric actuators presence doesn't modify normal and static correction modes definition (4b e 5b).

Moreover, after changing co-ordinates (6) equation (8) becomes:

$$[\overline{M}]\{\ddot{p}\}+[\overline{K}]\{p\}=[\Phi]^T\{F_e\}+[\Phi]^T\sum_i \{F_V\}_i \tag{10}$$

the $\{F_V\}_i$ forces, applied by the i-th piezo-actuator can be related with the relative applied voltage V_{Ai} with the relation below:

$$\{F_V\}_i=\{\alpha\}_i\cdot V_{Ai} \tag{11}$$

where the array $\{\alpha\}_i$ can be evaluated, thanks to linearity conditions, by imagining to decompose effects into those which are due to external loads and those which are due to piezoelectric actuators and performing a static analysis applying an unitary voltage at the i-th piezo-actuator:

$$\begin{gathered}[K]\cdot\{x\}_i=\{\alpha\}_i\cdot V_{Ai}=\{F_V\}_i \\ V_{Ai}=1 \qquad \{\alpha\}_i=[K]\cdot\{x_s\}_i\end{gathered} \tag{12}$$

Equation (10) becomes:

$$[\overline{M}]\{\ddot{p}\}+[\overline{K}]\{p\}=\left[[\Phi]^T \quad [\Phi]^T\{\alpha\}_i\right]\begin{Bmatrix}\{F_e\} \\ V_{Ai}\end{Bmatrix} \tag{13}$$

$$[\overline{M}]\{\ddot{p}\}+[\overline{K}]\{p\}=\left[[\Phi]^T \quad \{\varphi_A\}_i^T\right]\begin{Bmatrix}\{F_e\} \\ V_{Ai}\end{Bmatrix}=[\hat{\Phi}]\{\hat{F}\}$$

$$[\hat{\Phi}]=\begin{bmatrix}[\Phi] \\ \{\varphi_A\}_i\end{bmatrix},\quad \{\varphi_A\}_i^T=[\Phi]^T\{\alpha\}_i,\quad \{\hat{F}\}=\begin{bmatrix}\{F_e\} \\ V_{Ai}\end{bmatrix}$$

It can be observed that fictitious dofs $\{\varphi_A\}_i$, one for each actuator, are added to the modal matrix $[\Phi]$. Although there is no physic correspondence, it can be stated that, for each mode, the $\{\varphi_A\}_i$ component value represents the actuator capacity to excite that mode.

Moreover, taking into account the output voltage at the i-th piezoelectric sensor V_{Si} as ulterior fictitious dof, considered as a displacement degree of freedom, V_{Si} itself can be related with modal parameters p through the co-ordinates transformation described by (14) in which x_{Si} represents fictitious displacement relative to i-th sensor and $\{\varphi_S\}_i$ represents relative influence coefficients. There is a perfect formal analogy between V_{Si}/p relation and x/p (*x physical co-ordinates, p modal co-ordinates*).

$$V_{Si}=\{\varphi_S\}_i\{p\}, \qquad V_{Si}=x_{Si} \tag{14}$$

So, as for physical co-ordinates (6), it can be stated that piezo-sensor output voltage is defined as combination of the constraint and the normal modes. For a single sensor it can be written:

$$x_S=V_S=\{\varphi_S\}\{p\} \qquad \begin{gathered}\{\varphi_S\}=\{\varphi^C{}_{S1}\ldots\varphi^C{}_{SS} \quad \varphi^N{}_{S1}\ldots\varphi^N{}_{SP}\} \\ \{\varphi^C{}_S\}=\{\varphi^C{}_{S1}\ldots\varphi^C{}_{SS}\} \\ \{\varphi^N{}_S\}=\{\varphi^N{}_{S1}\ldots\varphi^N{}_{SP}\}\end{gathered} \tag{15}$$

In equations (15) $\{\varphi^C{}_S\}$ represents the constraint modes array relative to the piezofilm voltage, with dimension 1xS, that can be evaluated by measuring the sensor voltage for each constraint mode; and $\{\varphi^N{}_S\}$ represents the normal mode array relative to the piezofilm voltage, with dimension 1xP, by measuring sensor voltage for each normal mode. The single component of the modal array $\{\varphi^N{}_S\}$ represents the sensor ability to be excited from the single mode.

So, the modal matrix and the related array of forces become:

$$[\tilde{\Phi}]=\begin{bmatrix}[\Phi] \\ \{\varphi_A\}_i \\ \{\varphi_S\}_i\end{bmatrix}, \qquad \{\tilde{F}\}=\begin{bmatrix}\{F_e\} \\ V_{Ai} \\ 0\end{bmatrix} \tag{16}$$

It was hereby demonstrated that the introduction of the fictitious dofs doesn't modify generalized stiffness and mass properties and it does not alter its modal behavior either.

Therefore, through subsequent model ortho-normalization, a system reduced modal model is obtained, which is characterized by a number of lagrangian co-ordinates which is equal to $(S+P)$

and with the number of nodes increased by $(J+W)$.
It's important to highlight that the introduction of piezo-elements into flexible body doesn't change the number of lagrangian co-ordinates.
Some changes were realized to *MMNF* program and to ANSYS macros to transfer these theoretical results into multibody simulation environment. For each piezo-actuator a static analysis is performed imposing an applied unit voltage in order to obtain a relative displacement array x_S . Moreover, besides displacement and rotation values, the piezo-sensor output voltage is evaluated for each normal mode and for each static correction one. In MMNF new subroutines were built to evaluate the arrays $\{\varphi_A\}_i$ and $\{\varphi_S\}_i$, suitably partitioned, and, therefore, the new modal matrix $\left[\bar{\Phi}\right]$.
Due to the introduced analogy between physical and "piezo-element" co-ordinates, we needs to associate fictitious nodes (marker) to fictitious dofs (fig.3); from structural point of view, these nodes aren't linked in terms of stiffness and mass to the flexible structure; they are introduced with zero mass (to preserve system inertial properties), arbitrary position (user defined) and with only one dof in a direction of the flexible body local co-ordinate system. Each piezo-actuator fictitious node has the task to transmit the load condition that applied voltage would generate; this action is realized by applying a force on it along the enabled direction, with an amplitude which is equal to the applied voltage. Instead, each piezo-sensor fictitious node has the task to give as result the output voltage that would be produced by any external load condition through evaluation of the node displacement along its enabled local direction.
As concerns theoretical content, previous explanation is complete but, to correctly use this new flexible body, a further change has to be realized in the program, that is in the modal reduced model. In fact, the force application, besides generating deformed shape, which is equivalent to that realized by the applied voltage, loads the system with an external load that voltage doesn't really realize. In order to equilibrate this external action, two fictitious nodes were created for each piezo-actuator (fig.3) with previous characteristics, lying along the fictitious external load direction; a further zero modal array ($\{\varphi_A\}_i$) is associated to one of these; this increases the modal matrix and loads vector dimensions; this modification changes only nodes number increased at the end by $(J+2W)$. On these two nodes two forces are applied with the same amplitude and opposite sign along local enabled direction that realize piezo-equivalent load condition, canceling external global action.

4. RESULTS

To verify this new modal synthesis procedure a FEM model of a plate coupled with one piezo-sensor and one piezo-actuator was considered;

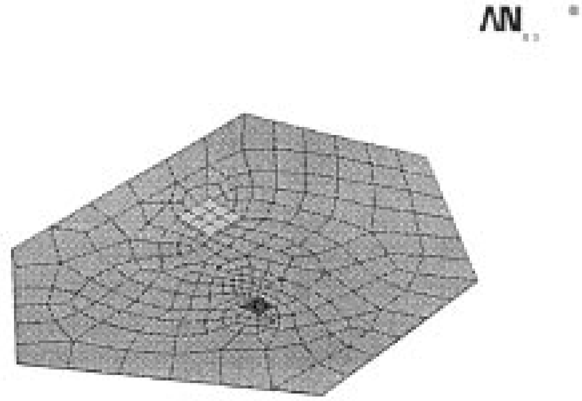

Fig.1 - ANSYS model of a plate equipped with a piezo-sensor (PVDF) and a piezo-actuator (PZT)

the plate model was built using ANSYS Shell elements with four nodes and six dof per node.

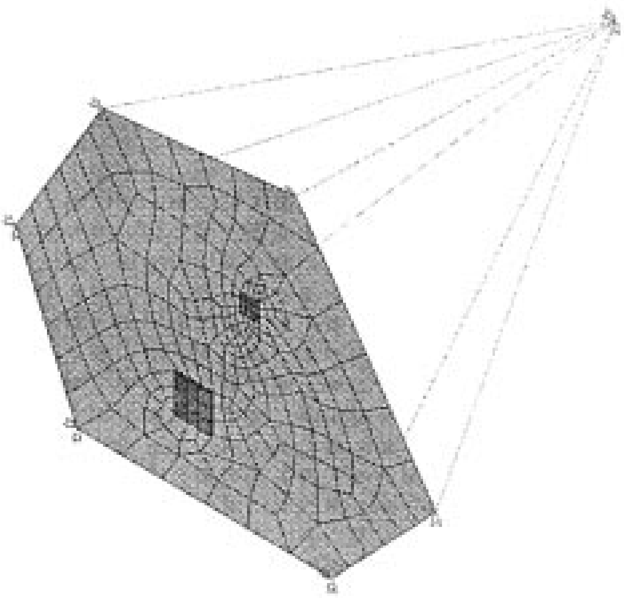

Fig.2 - *"Shaker"* ANSYS model

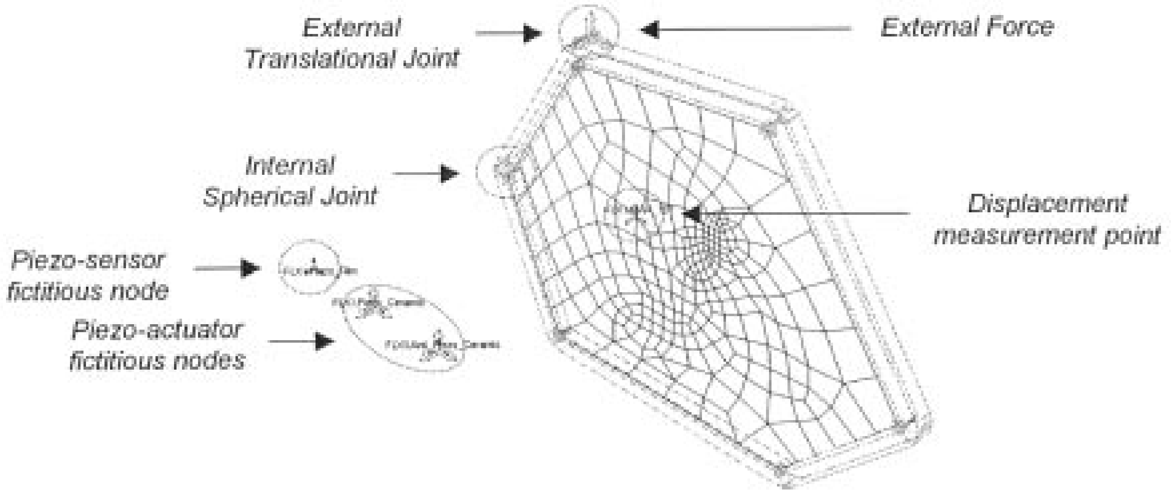

Fig.3 - *"Shaker"* ADAMS model

Only locally, in correspondence of the piezo, 3D elements with three dof per node were used. To modeling the piezo, a particular ANSYS element dedicated to these applications was considered. The electric-mechanic piezo-material characteristics, that are requested in matrix form, were obtained using a generalized formulation of classical elastic and electrical relations [11,12]. Therefore, both in ANSYS (fig.2) and in ADAMS (fig.3) a model was built, realized with

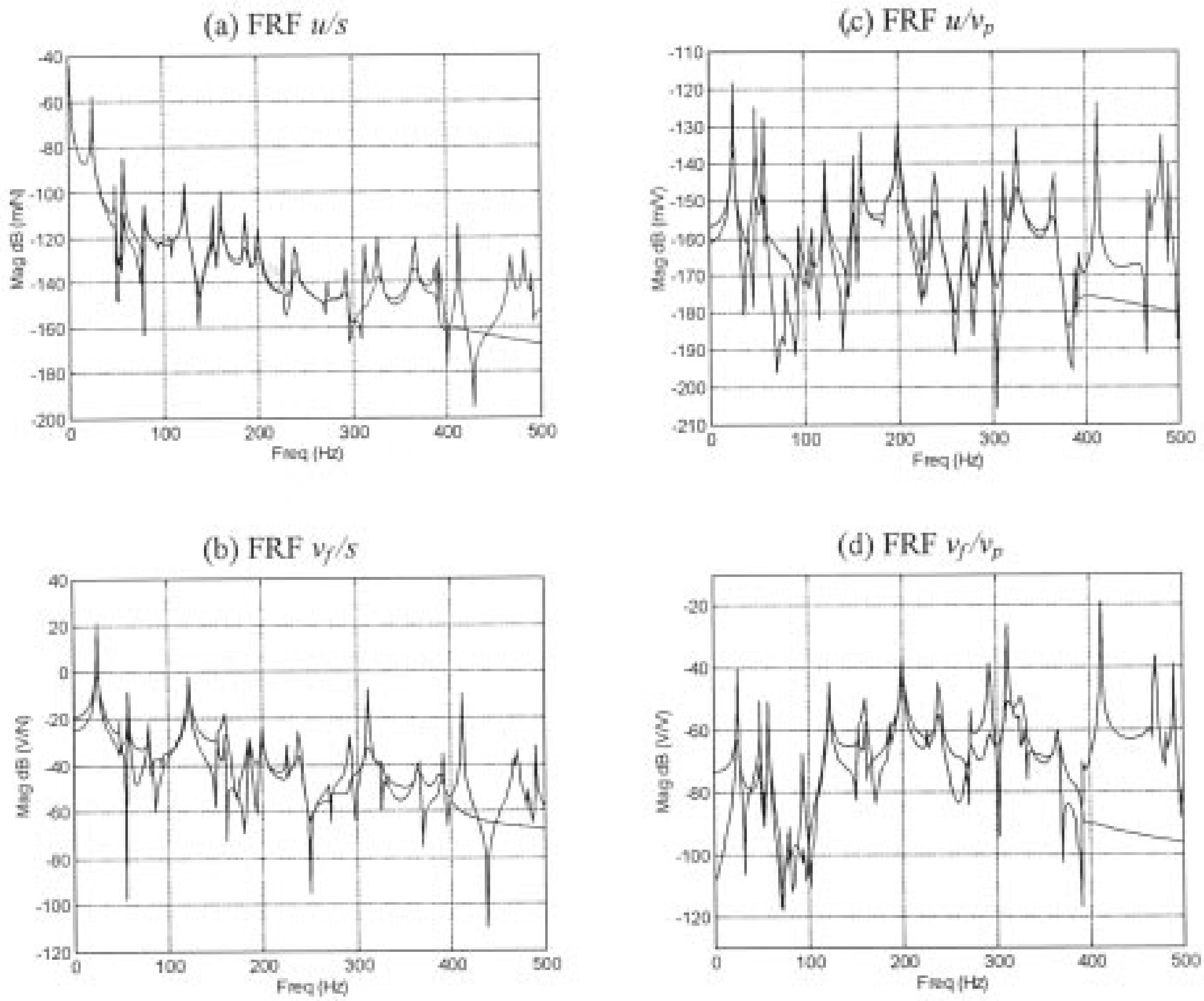

Fig.4 - Comparison between ANSYS (*Harmonic Analysis* with plate complete model) and ADAMS (*ADAMS/Linear Analysis* with reduced plate model) FRFs, obtained on "*shaker*" model.

a rigid system linked to the flexible plate; this model allows to excite the plate through the motion generated by an external force (*"shaker"*). The Plate is constrained to the *"shaker"* with spherical joint in its six vertexes. The system is free along *"shaker"* operating direction (perpendicular to plate surface) allowing to realize non linear analyses too, with large displacements.

For *Modal Neutral File* creation, only nodes on plate surface were considered, thus reducing the *Modal Neutral File* dimensions. Moreover, for constraint nodes only three displacements dofs were enabled that are necessary in order to use spherical joint in multybody model. For other nodes only the vertical displacement dof (perpendicular to plate surface) was enabled.

At last, the static correction modes considered are eighteen (they were thirty-six) with an automatic ulterior reduction of *Modal Neutral File* dimensions; moreover, FEM matrices dimensions, in ANSYS analyses, were reduced to dimensions six times less than original ones.

A comparison between homologous ADAMS and ANSYS modal analyses results had been realized. Good results had been obtained both in terms of mode shapes than in terms of natural frequencies. ADAMS model used was the reduced one, instead ANSYS model was a complete model considering all nodal dofs.

Subsequently, the harmonic response of the structure had been considered; the exciting action was realized with the motion generated by external force (*s*) and with piezo-ceramic applied voltage (*vp*). Frequency response functions in terms of displacement (*u*) and in terms of piezo-

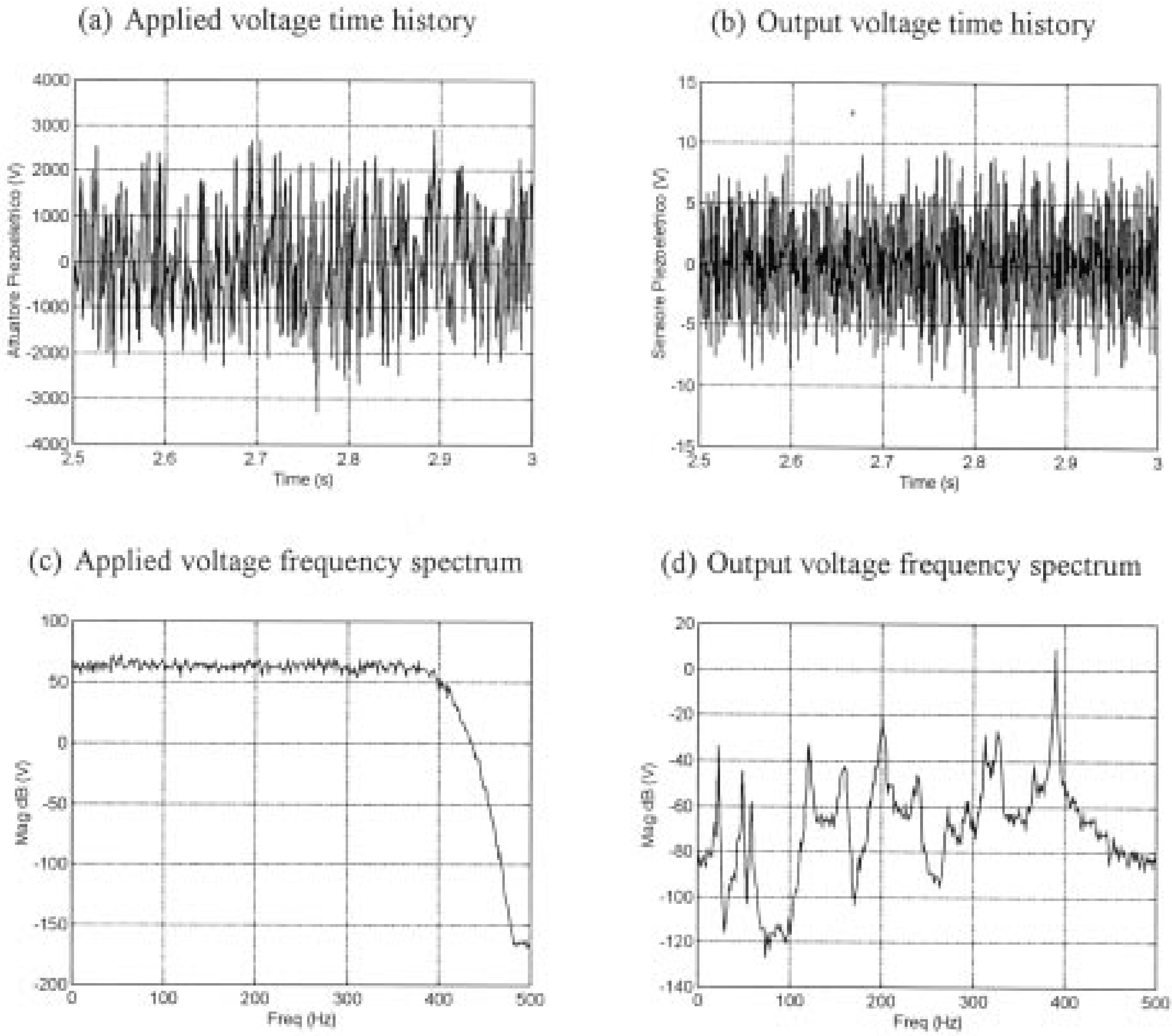

Fig.5 - Some result plots of ADAMS Transient Analysis on "shaker" model using a random applied voltage as excitation.

sensor voltage (v_f), obtained by a series of ANSYS FEM analyses on the complete model, were compared with those obtained in ADAMS on the reduced one (fig.4).

ANSYS FRFs had been obtained through an harmonic analysis using mass and stiffness complete matrices (*full analysis*). ADAMS FRFs, instead, had been obtained both through linearized model in state space form (using ADAMS/Linear analysis and, then, elaborated in MATLAB software) (fig.4), and through ADAMS/transient analyses (fig. 6).

As concerns input type considered, for these last FRFs a random function had been used (fig. 5a) with a frequency content quite constant (fig.5b) from 0 to 400 Hz. Frequency spectra and relative FRFs had been obtained on complete input and output time histories (fig. 5a and 5c). To this aim a MATLAB FRF estimation procedure was realized.; it is based on classical methods used in experimental environment (fig. 5b, 5d e 6).

All analyses prove that new synthesis method is good. ADAMS analyses results show a good correspondence with ANSYS ones in modal terms; this confirms that considering fictitious nodes associated to piezo-elements doesn't influence model modal behavior. Moreover, this correspondence is verified in terms of structure/piezo-elements interaction modeling too.

It's important to highlight that this new synthesis procedure allows to easily obtain structural response of flexible body and of piezo-elements; this is true in large displacement conditions too, and in presence of a generic external time variant action; this is more evident if its computational load is compared with homologous FEM or theoretical analyses.

5. CONCLUSIONS

A general and easy synthesis method that allows to introduce piezo-elements action in multibody

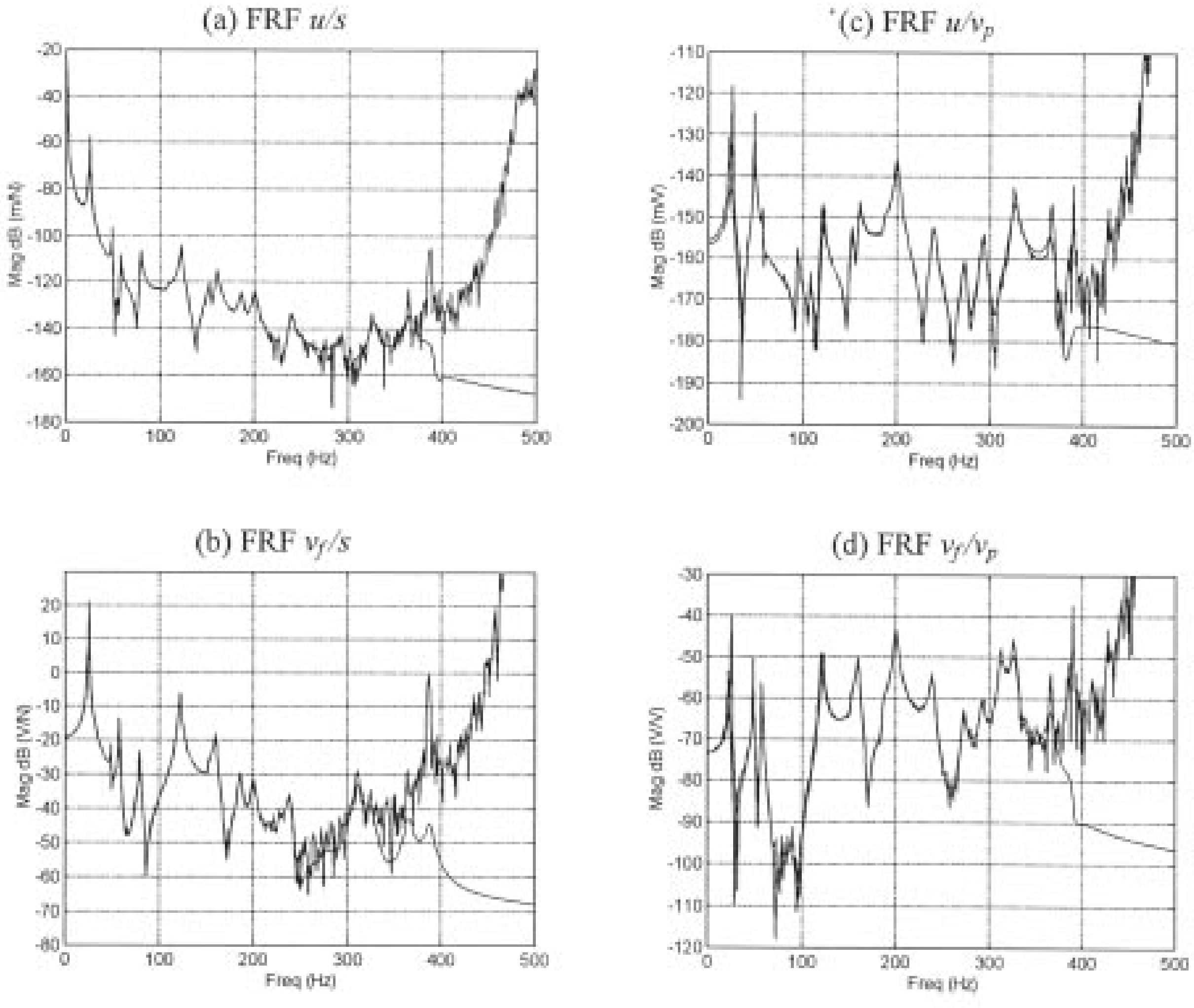

Fig.6 - Comparison between ADAMS/Linear and ADAMS/Transient FRFs

modeling was built; to this aim a flexible body modal synthesis method had been successfully modified and developed. This procedure allows to transfer structure elastic behavior from FEM structural code (ANSYS) into MSS code (ADAMS).

This new synthesis procedure allows to easily obtain structural response of flexible body and of piezo-elements; this is true in large displacement conditions too, and in presence of a generic external time variant action; this is more evident if its computational charge is compared with homologous FEM or theoretical analyses. Moreover, it allows to design, with piezoelectric sensors and actuators, position and vibration control systems in co-simulation environments in which multibody software allows to easily receive algorithms coming from dedicated softwares (MATLAB and MATRIXX).

This will be next step of the research illustrated in this paper.

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