CALCULATION OF RIGID BODY PROPERTIES FROM FRF DATA : PRACTICAL IMPLEMENTATION AND TEST CASES

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ABSTRACT

Accurate estimation of the rigid body properties of a structure is important. These properties are for instance used to determine the appropriate locations and stiffness characteristics for engine mounts and they are often needed in modal substructuring. Traditionally, the rigid body properties are experimentally obtained from a laborious pendulum test. An alternative method, which happens to be easier and more accurate, makes use of FRF data from a specific modal test. The first part of this paper explains the theory and the practical implementation of this method. How many inputs and how many outputs should be used and what is the optimal measurement procedure ? Which options are to be chosen during the estimation of the centre of gravity and the inertia properties ? The second part is about practical test cases, which were investigated. Not only simple objects, which were easily verified, but also real-life structures were examined. Both types of experiments lead to satisfying results and confirmed the validity of the method.

NOMENCLATURE

$\begin{Bmatrix} \ddot{X}_{1_x} \\ \ddot{X}_{1_y} \\ \ddot{X}_{1_z} \\ \ddot{\alpha}_{1_x} \\ \ddot{\alpha}_{1_y} \\ \ddot{\alpha}_{1_z} \end{Bmatrix}$: reference acceleration vector of input 1

$\begin{Bmatrix} F_{1_x} \\ F_{1_y} \\ F_{1_z} \\ M_{1_x} \\ M_{1_y} \\ M_{1_z} \end{Bmatrix}$: reference force vector of input 1

$\{F_1\}$: applied force in input 1

$\{L\}$: vector of total impulse towards reference axis system

$[A]$: matrix of inertia (symmetrical)

$\{\omega\}$: vector of velocity

$\ddot{X}_{1P}$: translational acceleration of node P, caused by input 1

$\ddot{\alpha}$: rotational, angular acceleration

X_P, Y_P, Z_P : co-ordinates of node P

X_1, Y_1, Z_1 : co-ordinates of node corresponding with input 1

m : total mass of the system

$X_{cog}, Y_{cog}, Z_{cog}$: co-ordinates of centre of gravity

I_{xx}, I_{yy}, I_{zz} : moments of inertia towards reference

I_{xy}, I_{yz}, I_{xz} : products of inertia towards reference

I_1, I_2, I_3 : principal moments of inertia

1. INTRODUCTION

Experimental frequency response functions (FRFs) are more and more successfully used in different fields of dynamics and vibrations. In case they are used to derive the inertia properties of a system, globally two types of methods are applied. A first type determines the inertia characteristics using the rigid body mode shapes obtained from test data (Modal Model Method described in reference [1]). A second type starts from the massline, i.e. the FRF inertia restraint of the softly suspended structure. This massline is used in a set of kinematic and dynamic equations, from which the rigid body characteristics (mass, centre of gravity, principal directions and moments of inertia) can be determined (reference [2]). Some of these methods also look for the suspension stiffnesses. Others consider the mass of the system as known (reference [3]). This latest method, a massline method with given mass, will be described and illustrated by application examples.

2. KINEMATIC AND DYNAMIC EQUATIONS

A set of FRFs between excitation DOFs 1,2,... (minimum 2) and response DOFs (minimum 6) in nodes P,Q,... is used in the calculations. A frequency band of these FRFs, which represents the massline, is to be selected between the rigid body modes and the first deformation mode (see figure 1). In stead of the 'Original FRF' values themselves, 'Corrected FRF' values or 'Lower residual' values can be taken. These two alternative methods are dealt with in next chapter. A separate solution of the inertia properties can be determined at each spectral line of the chosen frequency band or a global (least squares) solution can be calculated instead.

2.1. Calculation of the reference acceleration matrix

For all spectral lines of the selected band, for all response nodes P, Q,... and for all inputs 1, 2, ...:

$$\begin{bmatrix} \ddot{X}_{1P_x} & \ddot{X}_{2P_x} & \cdots \\ \ddot{X}_{1P_y} & \ddot{X}_{2P_y} & \cdots \\ \ddot{X}_{1P_z} & \ddot{X}_{2P_z} & \cdots \\ \ddot{X}_{1Q_x} & \ddot{X}_{2Q_x} & \cdots \\ \ddot{X}_{1Q_y} & \ddot{X}_{2Q_y} & \cdots \\ \ddot{X}_{1Q_z} & \ddot{X}_{2Q_z} & \cdots \\ \vdots & \vdots & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_P & -Y_P \\ 0 & 1 & 0 & -Z_P & 0 & X_P \\ 0 & 0 & 1 & Y_P & -X_P & 0 \\ 1 & 0 & 0 & 0 & Z_Q & -Y_Q \\ 0 & 1 & 0 & -Z_Q & 0 & X_Q \\ 0 & 0 & 1 & Y_Q & -X_Q & 0 \\ & & & \vdots & & \end{bmatrix} \begin{bmatrix} \ddot{x}_{1_x} & \ddot{x}_{2_x} & \cdots \\ \ddot{x}_{1_y} & \ddot{x}_{2_y} & \cdots \\ \ddot{x}_{1_z} & \ddot{x}_{2_z} & \cdots \\ \ddot{\alpha}_{1_x} & \ddot{\alpha}_{2_x} & \cdots \\ \ddot{\alpha}_{1_y} & \ddot{\alpha}_{2_y} & \cdots \\ \ddot{\alpha}_{1_z} & \ddot{\alpha}_{2_z} & \cdots \end{bmatrix} \quad (1)$$

This over-determined system of equations (number of output DOFs is higher than or equals 6) is solved for each spectral line in a least square sense. In this way at each spectral line, the reference acceleration matrix is found. Further, a general solution of the reference acceleration matrix over the total frequency band is calculated by solving in a least squares sense the global set of equations containing all outputs and all spectral lines.

2.2. Calculation of the reference force matrix

For all inputs 1, 2, ...:

$$\begin{Bmatrix} F_{1_x} \\ F_{1_y} \\ F_{1_z} \\ M_{1_x} \\ M_{1_y} \\ M_{1_z} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -Z_1 & Y_1 \\ Z_1 & 0 & -X_1 \\ -Y_1 & X_1 & 0 \end{bmatrix} \{F_1\} \quad (2)$$

The left side of this equation describes the equivalent force vector in the reference node of input 1.

2.3. Calculation of co-ordinates of centre of gravity and moments and products of inertia

For each input and for each spectral line:

$$\begin{Bmatrix} F_x - m\ddot{X}_x \\ F_y - m\ddot{X}_y \\ F_z - m\ddot{X}_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} =$$

$$\begin{bmatrix} 0 & -m\ddot{\alpha}_z & m\ddot{\alpha}_y & 0 & 0 & 0 & 0 & 0 & 0 \\ m\ddot{\alpha}_z & 0 & -m\ddot{\alpha}_x & 0 & 0 & 0 & 0 & 0 & 0 \\ -m\ddot{\alpha}_y & m\ddot{\alpha}_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & F_z & -F_y & \ddot{\alpha}_x & 0 & 0 & -\ddot{\alpha}_y & 0 & -\ddot{\alpha}_z \\ -F_z & 0 & F_x & 0 & \ddot{\alpha}_y & 0 & -\ddot{\alpha}_x & -\ddot{\alpha}_z & 0 \\ F_y & -F_x & 0 & 0 & 0 & \ddot{\alpha}_z & 0 & -\ddot{\alpha}_y & -\ddot{\alpha}_x \end{bmatrix} \begin{Bmatrix} X_{cog} \\ Y_{cog} \\ Z_{cog} \\ I_{xx} \\ I_{yy} \\ I_{zz} \\ I_{xy} \\ I_{yz} \\ I_{xz} \end{Bmatrix} \quad (3)$$

These sets of equations (for each excitation) can be put together and the coordinates of the centre of gravity and the inertia moments and products can be solved in a least squares way.
This solution can happen in two steps. First, the co-ordinates of the centre of gravity can be solved from the first three equations per excitation. Afterwards, these values can be filled in the last equations to solve the inertia moments and products.

Step 1
For each input and for each spectral line and for each input over the total band:

$$\begin{Bmatrix} F_x - m\ddot{X}_x \\ F_y - m\ddot{X}_y \\ F_z - m\ddot{X}_z \end{Bmatrix} = \begin{bmatrix} 0 & -m\ddot{\alpha}_z & m\ddot{\alpha}_y \\ m\ddot{\alpha}_z & 0 & -m\ddot{\alpha}_x \\ -m\ddot{\alpha}_y & m\ddot{\alpha}_x & 0 \end{bmatrix} \begin{Bmatrix} X_{cog} \\ Y_{cog} \\ Z_{cog} \end{Bmatrix} \quad (4)$$

Step 2

For each input and for each spectral line and for each input over the total band:

$$\begin{Bmatrix} M_x - Y_{cog}F_z + Z_{cog}F_y \\ M_y + X_{cog}F_z - Z_{cog}F_x \\ M_z - X_{cog}F_y + Y_{cog}F_x \end{Bmatrix} = \begin{bmatrix} \dot{\omega}_x & 0 & 0 & -\dot{\omega}_y & 0 & -\dot{\omega}_z \\ 0 & \dot{\omega}_y & 0 & -\dot{\omega}_x & -\dot{\omega}_z & 0 \\ 0 & 0 & \dot{\omega}_z & 0 & -\dot{\omega}_y & -\dot{\omega}_x \end{bmatrix} \begin{Bmatrix} I_{xx} \\ I_{yy} \\ I_{zz} \\ I_{xy} \\ I_{yz} \\ I_{xz} \end{Bmatrix} \qquad (5)$$

At each spectral line, these over-determined sets of equations (number of excitations larger than or equal to 2) are solved in a least square sense. Also a global solution for these rigid body properties over the total band can be found from the global acceleration matrix over the total frequency band (equation (1)).

2.4. Calculation of principal axis of inertia and principal moments of inertia

In general:

$$\{L\} = [A]\{\omega\}$$

$$\begin{Bmatrix} L_x \\ L_y \\ L_z \end{Bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \qquad (6)$$

This is an eigenvalue problem, where the eigenvalues are the principal moments of inertia and the eigenvectors the directions of principal axis of inertia.

Eigenvalues : I_1, I_2, I_3: 3 principal moments of inertia

Eigenvectors: $\{e_1\}$, $\{e_2\}$, $\{e_3\}$: directions of the 3 principal axes of inertia

3. MASSLINE METHODS

In theory, only the high frequency values of the FRFs of rigid body structures (without deformation modes) are massline values, which can be used in the former calculations. In practice, these values can be derived from the measured FRFs in three ways:

- **Original FRFs**
  When rigid body modes and deformation modes are sufficiently spaced, the amplitude values (with sign of real part) of the original measured FRFs in the frequency band between the last rigid body mode and the first deformation mode can be used.
- **Corrected FRFs**
  When the spacing between rigid body modes and deformation modes is not sufficient, the FRFs have to be corrected. In this case the influence of the first deformation modes, if significant, can be subtracted from the original FRFs:
  - The first set of deformation modes is estimated from the FRFs, using common modal estimators, e.g. Least Squares Complex Exponential (LSCE) and Least Squares Frequency Domain (LSFD), see reference [4].
  - FRFs are synthesized with this set of deformation modes in the frequency band of interest (between rigid body and first deformation mode), without using lower residual terms.
  - These synthesized FRFs are subtracted from the measured FRFs and the amplitude values (with sign of real part) of these corrected FRFs are used in calculations
- **Lower residuals**
  In case accurate measured FRFs are not available in the frequency range directly above the rigid body modes, lower residual terms can be used.
  - The first set of deformation modes is estimated from the FRFs, using common modal estimators, e.g. LSCE and LSFD, with lower and upper residual terms. About the use of residual terms during estimation: see reference [4].
  - The lower residual terms (each excitation in the data set leads to one lower and one upper residual term) represent the influence of the modes below the deformation modes, therefore they represent the rigid body modes.

4. MEASUREMENT SET-UP

- **Measurement locations**
  In theory 2 excitations and 6 responses are needed for calculations. Practical tests show that best results are obtained with at least **6 excitations** (e.g. 2 nodes in 3 directions) and **12 responses**. The locations are regularly spread over the structure to improve the spatial observability.
- **Suspension**
  A very **soft suspension** is used, which allows rigid body movements in all directions, translational and rotational.
- **Hardware**
  A front-end with a **high dynamic range** is needed, e.g. DIFA SCADAS II 16 bits ADC. Because of the high number of excitations, **impact (hammer) testing** is preferable. The frequency range of interest is very low: a **soft rubber hammer tip** is to be used. The **frequency resolution** has to be taken very **high** (small frequency step, wide time band). Obviously the **calibration** (e.g. ratio calibration) of the force cells and the accelerometers has to be taken care of very carefully.

5. ACADEMIC CASE : BEAM

First, calculations are tested on a beam structure. The inertia properties of this structure can easily be calculated from the dimensions and the mass density of the beam.

A MSC NASTRAN F.E. model of this beam is available, which contains the 6 rigid body modes and 4 deformation modes. These modes are used to synthesize FRFs in 24 response DOFs (8 nodes, each in 3 directions) and in 12 excitation DOFs (4 nodes, each in 3 directions), see figure 1. The synthesized FRFs, together with the total mass of the beam are used in the rigid body calculations. The three mass line methods are applied, see table 1. All estimated results are very close to the theoretical results: because the rigid body modes and the first deformation modes are well separated, also the 'Original FRF' method leads to good results.

The FRFs of this beam structure were measured in 24 response DOFs and 8 excitation DOFs. The estimated results, also very close to the theoretical results, are shown in table 2.

6. REAL-LIFE CASE : FRAME

Next, a more realistic frame structure is investigated. An ANSYS F.E. model with 2272 elements and 2347 nodes is available. The inertia properties found with this model will be the reference results for the experimental rigid body analysis.

Using the appropriate test set-up, 408 FRFs are measured in 34 response DOFs and 12 excitation DOFs. The F.E. model and the measurement locations are shown in figure 2. The frame structure is excited in the 4 corner nodes (in the 3 directions).

Although the 'Original FRF' method leads to results close to the F.E. results, in this case the 'Corrected FRF' and the 'Lower residual' massline methods are superior: see table 3.

Starting from the found inertia properties, the rigid body modes can be re-synthesized (method described in reference [5]). Figures 3, 4, and 5 show the 3 synthesized rotational rigid body modes.

7. CONCLUSION

Deriving rigid body properties from modal FRF data can only be successful when a number of conditions are met:

First, the <u>test conditions</u>: sufficient measurement locations on the right place, a soft suspension, and a high measurement accuracy in the lower frequency range.

Second, the <u>correct massline values</u> are to be determined: sometimes the original FRF values can be taken, more often corrected values are preferable.

At last, the kinematic and dynamic equations have to be solved in a <u>least squares</u> way over the inputs, over the outputs and over a number of spectral lines, while the total mass is considered to be known.

When all these conditions are fulfilled, accurate rigid body properties can be found, also for real-life structures.

8. ACKNOWLEDGEMENTS

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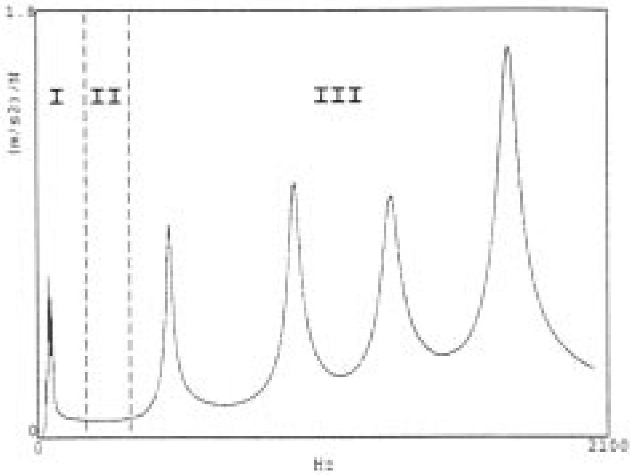

Figure 1 : synthesised FRF :
I : Rigid body modes
II : Massline
III : Deformation mode

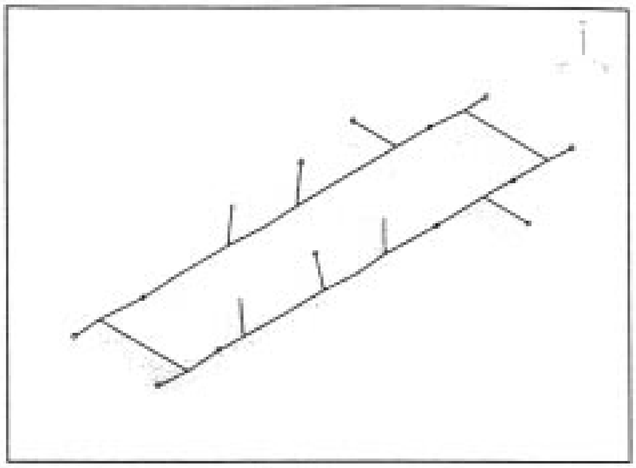

Figure 2 : FRAME structure : FE model and measurement location.

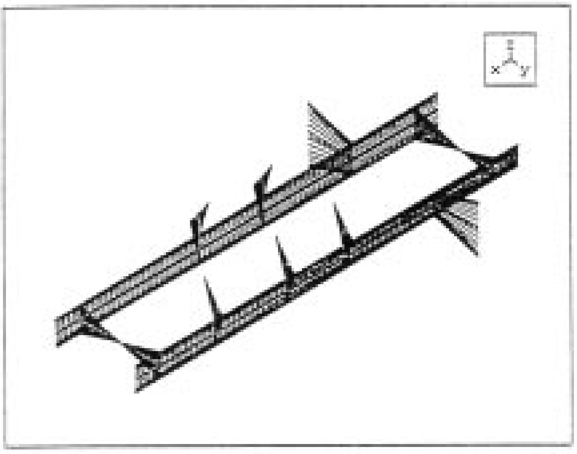

Figure 3 : FRAME structure : first rotational rigid body mode.

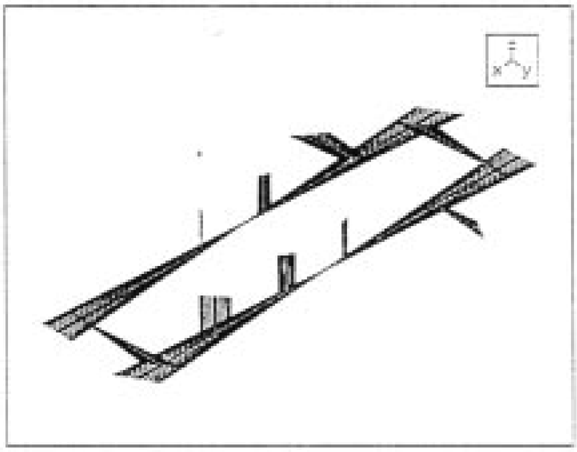

Figure 4 : FRAME structure : second rotational rigid body mode.

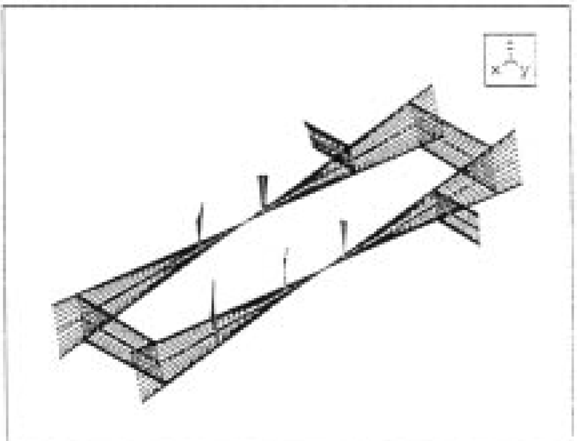

Figure 5 : FRAME structure : third rotational rigid body mode.

Property	Reference (calculated)	Unchanged FRF (32-101 Hz)	Corrected FRF (32-101 Hz)	Lower residual
Mass (kg)	15.61	15.61	15.61	15.61
Centroid (m)				
Xc	0.323	0.322	0.322	0.323
Yc	0.0195	0.0195	0.0195	0.0195
Zc	0.0395	0.0407	0.0395	0.0389
Moment (kgm^2)				
Ixx	0.0101	0.0118	0.0121	0.0121
Iyy	0.549	0.548	0.546	0.541
Izz	0.543	0.552	0.553	0.545

Table 1

Academic case: BEAM

(synthesized FRFs)

Property	Reference (calculated)	Unchanged FRF (32-101 Hz)	Corrected FRF (32-101 Hz)	Lower residual
Mass (kg)	15.61	15.61	15.61	15.61
Centroid (m)				
Xc	0.323	0.322	0.322	0.320
Yc	0.0195	0.0198	0.0197	0.0205
Zc	0.0395	0.0401	0.0399	0.0373
Moment (kgm^2)				
Ixx	0.0101	0.0106	0.0107	0.0114
Iyy	0.549	0.556	0.557	0.529
Izz	0.543	0.548	0.548	0.512

Table 2

Academic case: BEAM

(measured FRFs)

Property	Reference (ANSYS)	Unchanged FRF (25-35 Hz)	Corrected FRF (25-35 Hz)	Lower residual
Mass (kg)	9.81	9.81	9.81	9.81
Centroid (m)				
Xc	0.431	0.426	0.426	0.425
Yc	0.0818	0.0893	0.0867	0.0888
Zc	-0.00211	0.00248	0.00533	0.00239
Moment (kgm^2)				
Ixx	0.0969	0.0875	0.0746	0.0887
Iyy	0.760	0.745	0.750	0.740
Izz	0.843	0.862	0.849	0.857

Table 3

Real-life case : FRAME

Property	Reference (ANSYS)	Unchanged FRF (4.5-10.2 Hz)	Corrected FRF (4.5-10.2 Hz)	Lower residual
Mass (kg)		52.50	52.50	52.50
Centroid (m)				
Xc		0.700	0.696	0.695
Yc		0.0341	-0.00510	-0.00190
Zc		0.193	0.182	0.182
Moment (kgm^2)				
Ixx		15.2	17.9	18.0
Iyy		2.41	2.48	2.50
Izz		18.6	22.4	22.7
Principle Moment				
(kgm^2)		21.9	22.9	23.1
I_1		11.9	17.4	17.7
I_2		2.39	2.40	2.42
I_3				

Table 4

Real-life case: REARAXLE