UNILATERAL CONTACT MODELING WITH ADAMS

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SUMMARY

This paper is concerned with the modeling of intermittent unilateral contact conditions for multibody system dynamics. We present a general method that is based on the following basic ingredients: a flexible description of the geometry of the contacting parts based on the NURBS parameterization of curves and surfaces; a minimum distance problem implemented through a set of holonomic constraint conditions that determine at each time instant of the simulation whether the bodies are in contact or not; a general constitutive law that describes the interaction forces between the contacting parts. We implement the numerical procedure in ADAMS, hiding the unnecessary details of the method so as to derive a user friendly interface. We validate the proposed procedure with the help of some application examples.

INTRODUCTION

Many electromechanical products can be modeled with rigid and flexible bodies interconnected with bilateral constraints. In reality, a vast class of systems can only be realistically modeled with the previous ingredients plus the addition of unilateral contact conditions. In particular we refer to two significant categories: circuit breakers and robots. The first, spanning all the range of voltages, are modeled by kinematic chains with variable topology, undergoing very fast motion. The catching or the unlatching of the chain or of its parts is guaranteed by unilateral constraints, and require the modeling of contact/impact phenomena. The second category is represented by mechanisms characterized by open chains undergoing fast motions, with some points following complex paths, often performing highly sophisticated mechanical operations. Robots usually require position and force control, whose success depends on the modeling of phenomena such as friction, backlash and contact.

The modeling of contact/impact processes represents an important topic in advanced multibody simulation, and is crucial for an effective use of this technology in many industrial applications. A numerical procedure with unilateral contact modeling capabilities, needs to address the following problems:

- give a mathematical representation of the geometry of the contacting parts;
- solve the minimal distance problem between the curves or surfaces where contact occurs, in order to evaluate the position of the candidate contact points;
- apply, at the contact points, the interaction forces defined by a constitutive law that is appropriate for the phenomenon being investigated.

In this paper we present a procedure that addresses all the above mentioned areas of concern, with the specific goal of flexibility, generality, robustness and ease of use.

In particular, note that the position of the contact point is generally an unknown of the problem that depends on the dynamics of the mechanism, and cannot be stated a priori. In fact, it is the very motion of the parts that defines the contact points.

Presently, this problem is usually solved in ADAMS for the 2D case by applying the so-called “Dummy Part Technique” (DPT). Using this approach, a fictitious body is connected to each one of the contacting bodies, and it is constrained to slide on the curves that describe the geometry of the parts by means of two PTCV (“point on curve”) constraints. These constraints are so close that the connecting segment approximates the tangent to the curve. Two primitive joints (e.g., two reciprocal INLINE or one INLINE and one PARALLEL AXIS), force the fictitious bodies to lie reciprocally on the normal of the related curves. The technique, although quite effective in simple cases, presents some drawbacks:

- it needs a cumbersome modeling work, which implies a heavier than necessary work-load for the user;
- it does not solve the minimum distance problem in a rigorous way and lacks numerical robustness, which in our experience usually hampers the solution of models of industrial relevance;
- it presents a difficult generalization to the 3D case.

In the following pages, we detail our proposed procedure for avoiding the current limitations of the software in this area.

CONTACT MODELS

The approaches to the modeling of unilateral contact conditions fall into two main categories. The first one considers an impact as an impulsive phenomenon of null duration [6,14,7]. The configuration of the system is "frozen" during the impact, and an appropriate model is used for relating the states of the system immediately before and immediately after the event. There are two alternative flavors to this theory: Netwon's method and Poisson's method. The first relates the relative normal velocities of the contacting bodies through the use of an appropriate restitution coefficient. The second divides the impact in two phases: a first compression phase brings the normal relative velocity of the bodies to zero through the application of an impulse at the contact location. Then, an expansion phase applies an impulse of opposite sign. The magnitude of the second impulse is related to the magnitude of the first one through a restitution coefficient. Although this method has been used with success for multibody contact/impact simulations [11], it is quite clear that it seems most suited to the modeling of events of very short duration. Furthermore, its implementation requires the interruption of the numerical integration of the equations of motion when an impact is detected, followed by the solution of a linear complementarity problem that implies the manipulation of entire descriptive matrices of the whole multibody system prior to the restarting of the integration process. The resulting procedure would not be easily implemented in ADAMS through the standard user-accessible channels.

A second approach models the contact/impact condition as a finite duration event, and tries to describe the time history of the resulting interaction forces for the whole duration of the phenomenon [8,10,3,1]. This is achieved by introducing a suitable phenomenological model for the contact forces, that are usually expressed as functions of the inter-penetration, or "approach", between the contacting bodies. Extension of this methodology to the case of contacts with friction and rolling is straightforward [2], although it will not be detailed here. The computation of the approach at each time instant of the simulation is achieved solving the same set of holonomic constraints that express the minimum distance problem when the bodies are not in contact. As for all the contact models, we have even in this case a complementarity problem: either the sum of the relative distance and the approach is greater than zero, and in this case the contact forces are null, or the same sum is null, and the relative distance is equal and opposite to the approach (inter-penetration) with interaction forces that are not null.

In this work, the second approach is adopted and the contact event is assumed of finite duration. ADAMS already implements a module with the required functionality for modeling the interaction forces: IMPACT. This module allows to describe in very general form arbitrary constitutive laws relating the contact forces with the approach and its time derivatives. The activation of the module is triggered by the detection of inter-penetration between the contacting bodies, as a result of the minimum distance problem.

PARAMETRIZATION OF CONTACT CURVES AND SURFACES

Following the methodology just outlined, we have seen that we need to provide a mathematical description of the geometry of the contacting parts. This will allow the solution of the minimum distance problem, that in turn will trigger the activation of the interaction forces between the contacting parts. Therefore, we need some kind of parameterization of the geometric entities of the form:

- 2D case x-y: $\xi \rightarrow (x(\xi), y(\xi))$,
- 3D case x-y-z: $\xi, \eta \rightarrow (x(\xi, \eta), y(\xi, \eta), z(\xi, \eta))$,

where x, y and z are the point coordinates in the global reference frame, while ξ and η are parametric coordinates.
It is clear that the parameterization adopted should be as general and flexible as possible, in order to allow the greatest range of possible applications. One parameterization that is broadly used for its generality and flexibility is based on Non Uniform Rational B-Splines (NURBS) [4,13], often used for CAD solid modeling. Based on this representation, it is possible to evaluate all the geometric attributes of the entities, for example tangents, normals and point locations. These geometric attributes will then be the ingredients of the minimum distance problem that is solved at each time instant of the simulation.
We have implemented a NURBS-based parameterization of curves and surfaces in a library of C routines. This library is linked with the rest of the solver, and provides the needed geometric functionalities to the "user defined subroutines" implementing the minimal distance problem.
From the user's viewpoint, the geometric description proceeds through the positioning of a suitable number of control points, following the NURBS conventions. Graphic display of the geometry is automatically obtained by using some of the ADAMS graphic tools. Note that there is no coding required by the user, that will simply use the modeling functionality of the provided routines for describing the particular geometry being analyzed. The details of the whole process are completely hidden to the user, that operates through a proper ADAMS macro.

THE PROBLEM OF MINIMAL DISTANCE

Letting q be the minimal distance between two entities, we say that they are in contact if

$$q \leq 0. \qquad (1)$$

Hence, at any integration time step, the distance q has to be evaluated and a contact force F must to be applied as

$$F=\delta(q)\ F(a, a', a'', \ldots), \qquad (2)$$

where $a=-q$ is the approach between the entities and $\delta(q)$ is a Kronecker switching function, null for positive q. To decide on the application of the impact force, we have to evaluate the minimal distance between the entities. Let us formalize the problem that has to be solved at any time step to find the point eligible for contact in both the 2D and 3D cases [11,12].

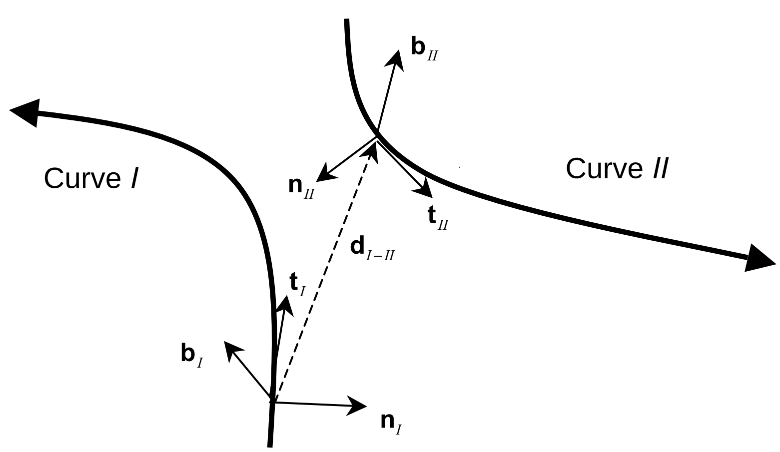

Figure 1 – Geometry of contacting curves.

2D CASE

The regular curves $C_i \subset R^2$ ($i = I, II$) are defined through the mapping $\boldsymbol{u}_i$: $A_i \mapsto R^2$ as

$$C_i := \{\ \boldsymbol{u} \in R^2 \quad : \boldsymbol{u} \in \boldsymbol{u}_i\ (\xi_i) \quad \forall\ \xi_i \in A_i \subset R\},$$

where ξ_i is the curvilinear coordinate of the curve and $\boldsymbol{u}_i \in C^1(A_i)$. The points eligible for contact are then obtained by solving the nonlinear system:

$$\mathbf{n_I}(\xi_I) \cdot \mathbf{t_{II}}(\xi_{II}) = 0, \qquad (3)$$
$$\mathbf{d_{I\text{-}II}}(\xi_I, \xi_{II}) \cdot \mathbf{t_{II}}(\xi_{II}) = 0, \quad (4)$$

where, in accordance with Figure 1, **t**, **n** and **b** are the vectors representing respectively the tangent, normal and bi-normal to the curves. Equation (3) forces the normal vector on C_I to be perpendicular to the tangent vector on C_{II}, while equation (4) forces the distance vector **d** to be perpendicular to the tangent vector on C_{II}. The nonlinear system is solved by two values ξ_I and ξ_{II} corresponding to the minimal distance points on the curves.

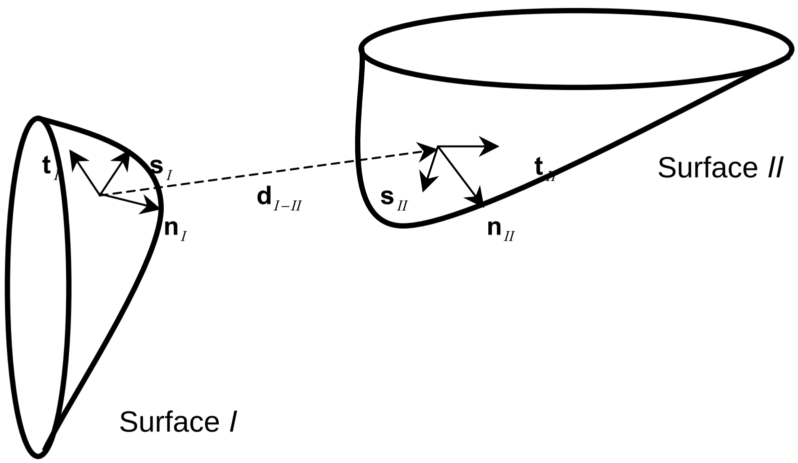

Figure 2 – Geometry of contacting surfaces.

3D CASE

The regular surfaces $S_i \subset R^3$ ($i = I, II$) are defined through the mapping $\boldsymbol{u}_i$: $B_i \mapsto R^3$ as

$$S_i := \{\ \boldsymbol{u} \in R^3 \quad : \boldsymbol{u} \in \boldsymbol{u}_i\ (\xi_i,\eta_i) \quad \forall\ (\xi_i,\eta_i) \in B_i \subset R^2\},$$

where ξ_i and η_i are the curvilinear coordinates of the surfaces and $\boldsymbol{u}_i \in C^1(B_i)$. The points eligible for contact are obtained by solving the following nonlinear system of equations

$$\mathbf{n_I}(\xi_I, \eta_I) \cdot \mathbf{s_{II}}(\xi_{II}, \eta_{II}) = 0, \qquad (5)$$
$$\mathbf{d_{I\text{-}II}}(\xi_I, \eta_I, \xi_{II}, \eta_{II}) \cdot \mathbf{s_{II}}(\xi_{II}, \eta_{II}) = 0, \quad (6)$$
$$\mathbf{n_I}(\xi_I, \eta_I) \cdot \mathbf{t_{II}}(\xi_{II}, \eta_{II}) = 0, \qquad (7)$$
$$\mathbf{d_{I\text{-}II}}(\xi_I, \eta_I, \xi_{II}, \eta_{II}) \cdot \mathbf{t_{II}}(\xi_{II}, \eta_{II}) = 0. \quad (8)$$

In accordance with Figure 2, **t** and **s** are two orthogonal R^3 vectors lying on the tangent planes and **n** is the R^3 vector normal to the surfaces. Equations (5) and (7) force the normal vector on S_I to be perpendicular to the tangent vectors on S_{II}; equations (6) and (8) force the distance vector to be perpendicular to the tangent vectors on S_{II}. The nonlinear system is solved by the values ξ_I, η_I, ξ_{II}, η_{II} corresponding to the minimal distance points on the surfaces.

Once the minimal distance point have been evaluated, in both the 2D and 3D case, the minimal distance value q is then given by:

$$q = \mathbf{d}_{I-II} \cdot \mathbf{n}_I .$$

ADAMS IMPLEMENTATION

The minimum distance problem needs to be slightly reformulated in order to be implemented in ADAMS using the functionalities provided by the "user defined subroutines". In fact, ADAMS requires to specify a-priori the point of application of forces. This problem can be solved by introducing massless dummy parts. Clearly, these additional parts must be of null mass in order not to modify the dynamics of the system. The dummy parts are constrained to move on the curves or surfaces that describe the geometry of the contacting bodies, with axes x and y belonging to the tangent plane and axis z aligned with the local normal. The minimum distance problem is then defined using the axis system associated with the dummy parts. Furthermore, the interaction forces are applied to the same parts [9].

In contrast with the DPT approach, each dummy part is in this case explicitly constrained to a curve or surface through a set of rigorous kinematic conditions. These constraint relations are implemented using appropriate UCON STATEMENTS and a UCOSUB SUBROUTINE [15]. Equations (1,2) and (3–6) univocally determine the parametric coordinates ξ_I and ξ_{II} (ξ_I, η_I, ξ_{II}, η_{II} in the 3D case), which are the only remaining degrees of freedom for the dummy parts. The constraint conditions are completely general, in the sense that they do not depend on the particular parameterization of the geometry adopted.

A total number of 14 scalar constraint equations is written for the 2D case, that determine the 6 degrees of freedom of each dummy part plus the ξ_I and ξ_{II} parametric coordinates. For the 3D case, we have 16 scalar constraints, corresponding to the 6 degrees of freedom of each dummy part plus the ξ_I, η_I, ξ_{II}, η_{II} parametric coordinates.

More precisely, we have in the 2D case:

$$\mathbf{g}_{3_D} \cdot \mathbf{t} = 0, \qquad (9)$$

$$\mathbf{g}_{2_D} \cdot \mathbf{t} = 0, \qquad (10)$$

$$\mathbf{g}_{3_D} \cdot \mathbf{b} = 0, \qquad (11)$$

$$\mathbf{n}_I \cdot \mathbf{t}_{II} = 0, \qquad (12)$$

$$\mathbf{d}_{I-II} \cdot \mathbf{t}_{II} = 0, \qquad (13)$$

$$\mathbf{x}_D = \mathbf{x}_{DP} + \mathbf{x}_P , \qquad (14)$$

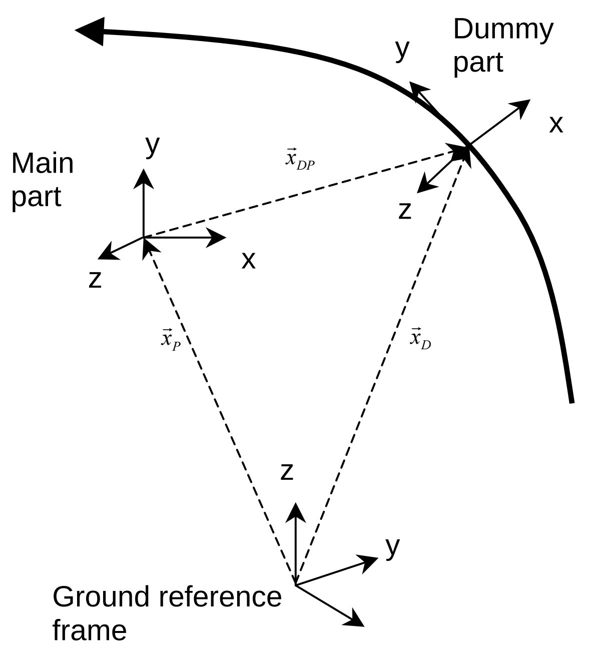

Figure 2 – Main and dummy part geometry.

where $\mathbf{g}_{iD}$ (i=1,3) are the dummy part unit vectors, and

- equation (9) states that the z axis of a dummy part must be orthogonal to the curve tangent;
- equation (10) states that the y axis of a dummy part must be orthogonal to the curve tangent;
- equation (11) states that the z axis of a dummy part must be orthogonal to the vector normal to the plane of the associated curve;
- equation (12) states that the normal to curve *I* must be orthogonal to the tangent to curve *II;*
- equation (13) states that the relative distance between curve *I* and *II* must be orthogonal to the tangent to curve *II*;

- equation (14) translates the point on curve constraint for a dummy part and its associated curve.

Equations (9-11) must be repeated for both curve *I* and *II*.
For the 3D case we have the same equations (9,10) and (12-14), plus the additional conditions:

$$\mathbf{g}_{3_D} \cdot \mathbf{s} = 0, \qquad (15)$$
$$\mathbf{n}_I \cdot \mathbf{s}_{II} = 0, \qquad (16)$$
$$\mathbf{d}_{I-II} \cdot \mathbf{s}_{II} = 0, \qquad (17)$$

where equation (15-17) are homologous to equations (9), (12), and (13), respectively, and impose the conditions for the second tangent to the associated surface. Equation (15) must be repeated for both surface *I* and surface *II*.
In the previous expressions, all vector quantities are represented in the ground reference frame.
Since constraint equations can only be written in ADAMS in terms of state variables, it is necessary to introduce two (four in the 3D case) additional massless parts, called “trackers” in the following, associated with the parametric coordinates. The trackers are constrained to the ground by translational joints, in order to have one single degree of freedom left, for example the displacement in the z direction. The value assumed by this displacement at each time instant represents the value of a parametric coordinate along a curve or surface, this way identifying the contact point location.
Having introduced all the constraint conditions just discussed, we have that all the massless parts are completely determined: the dummy parts are constrained to slide on the curves or surfaces so as to be located at each time instant at minimum distance, while the tracker locations correspond to the values of the parametric coordinates at the candidate contact points. It is now possible, based on the solution of the minimum distance problem, to trigger the application of a contact force, modeled using the IMPACT module.
From the user-interface point of view, this procedure can be implemented in a very effective manner, hiding the details of the implementation and therefore easing the model preparation and verification. In particular, the user must only identify the pairs of curves or surfaces that will be interested by a contact condition, and must specify the constitutive contact force law parameters. A macro automatically generates all the dummy parts, trackers and constraint equations, without user intervention.

NUMERICAL EXAMPLES

In the following, we analyze some mechanical systems characterized by the presence of unilateral contact conditions with the purpose of validating our procedure and showing its basic capabilities.

The Bouncing Cams Problem

This problem is concerned with the simulation of two interacting cams, depicted in Figure 4. The purpose here is to show that arbitrary shapes can be easily modeled, and to give a brief description of the user interface developed.
Body 2 is grounded, while body 1 is free. Under the effect of gravity, body 1 falls and impacts on body 2. Although extremely simple, this problem has all the typical features that characterize contact conditions: unilateral constraint, time varying contact points, stiffness of the equations.
The user defines a contact curve selecting the option *“nurbs_for_impact”* in the Command Navigator, as shown in Figure 5. This will open a

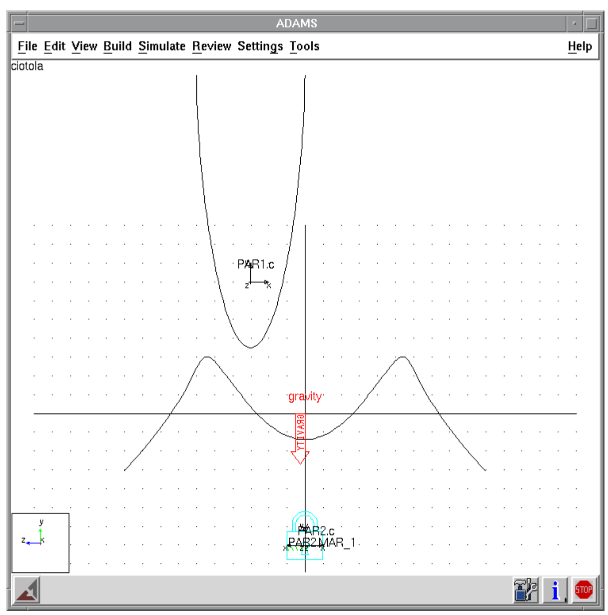

Figure 4 – The bouncing cams problem.

new window that allows to input all the parameters that are required for the curve definition. Using this window, the user specifies the body associated with the curve, a unique identifier for the curve itself, the number of NURBS control points, the polynomial degree of the representation, a flag that specifies whether the curve is rational or not, the control points, and the number of curve segments. This last information is required for the curve visualization using the ADAMS graphic tools.

Once the curves have been generated, the user can now proceed to the definition of the unilateral constraint. Selecting the option *"impact_generator"* from the Command Navigator will open the window shown in Figure 6. The user must now specify the following parameters: a unique identifier for the unilateral constraint, the bodies and the identifiers of the curves involved, the values of the parametric coordinates that approximately solve the minimum distance problem in the initial configuration. This value will be used as initial starting guess by a Newton procedure that will identify the solution of the same problem at time t=0. The contact/impact definition is completed by inserting the parameters required by the IMPACT module in the last rows of the window.

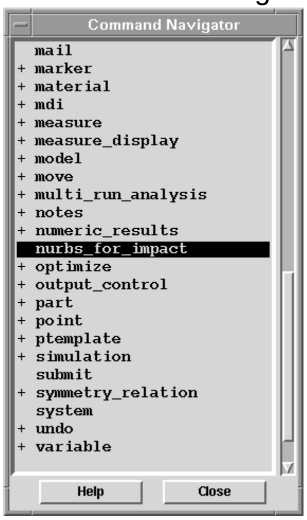

Figure 5 – Command Navigator window.

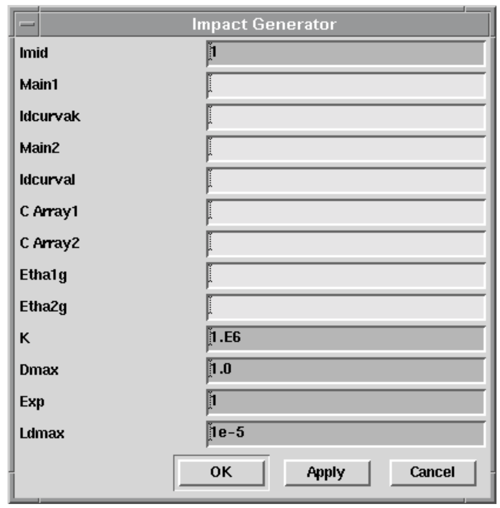

Figure 6 – "Impact Generator" window.

Clicking on OK will automatically generate all the ingredients necessary for the analysis: dummy parts, trackers and UCON STATEMENTS, as shown in Figure 7.

Selecting *"Verify Model"*, we obtain a check of the state of the systems, verifying the presence of all the components required for the simulation, as shown in Figure 8.

It is now possible to start the simulation in the usual manner. In Figure 9 we give a sequence of images representing the system configuration at various time instants of the simulation. The candidate contact points at each time instant are identified by a cross symbol. The two points on the left of the grounded body are the trackers associated with the

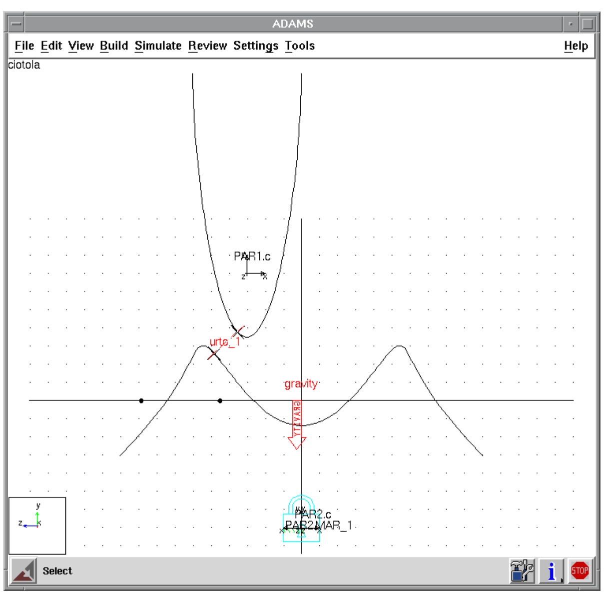

Figure 7 – Cams with virtual parts.

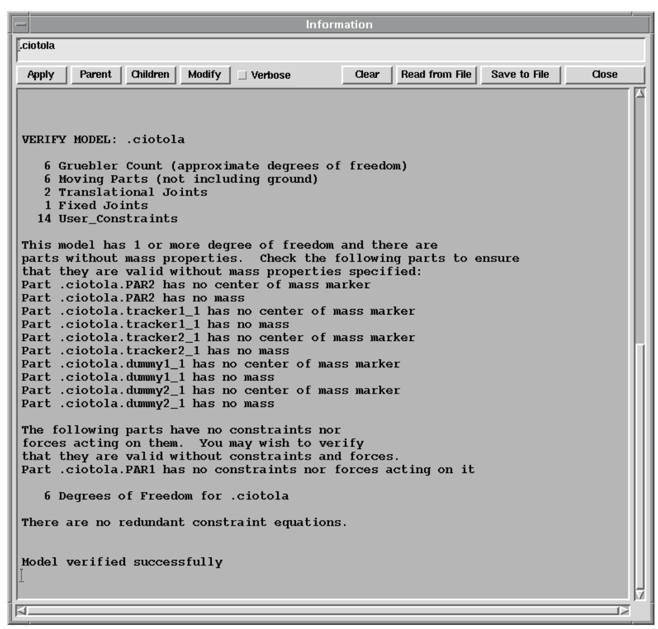

Figure 8 – "Verify model" window.

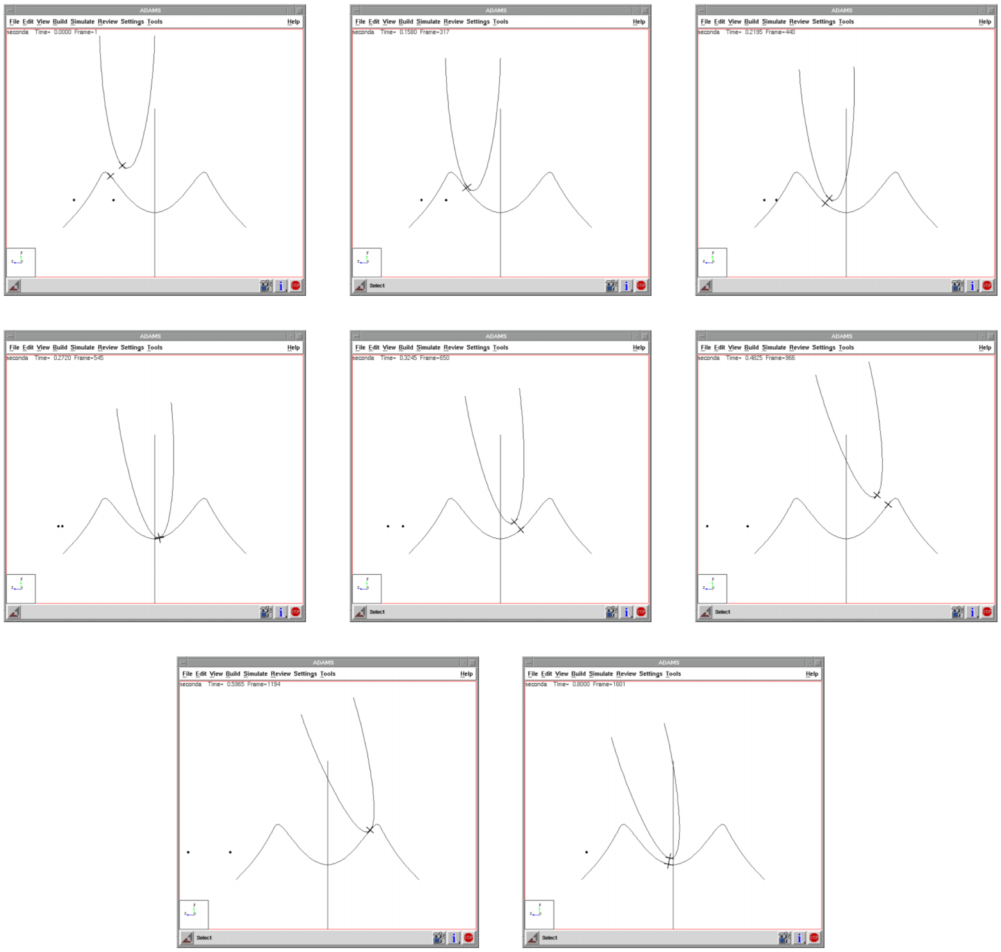

Figure 9 – Snapshots of the system during the analysis.

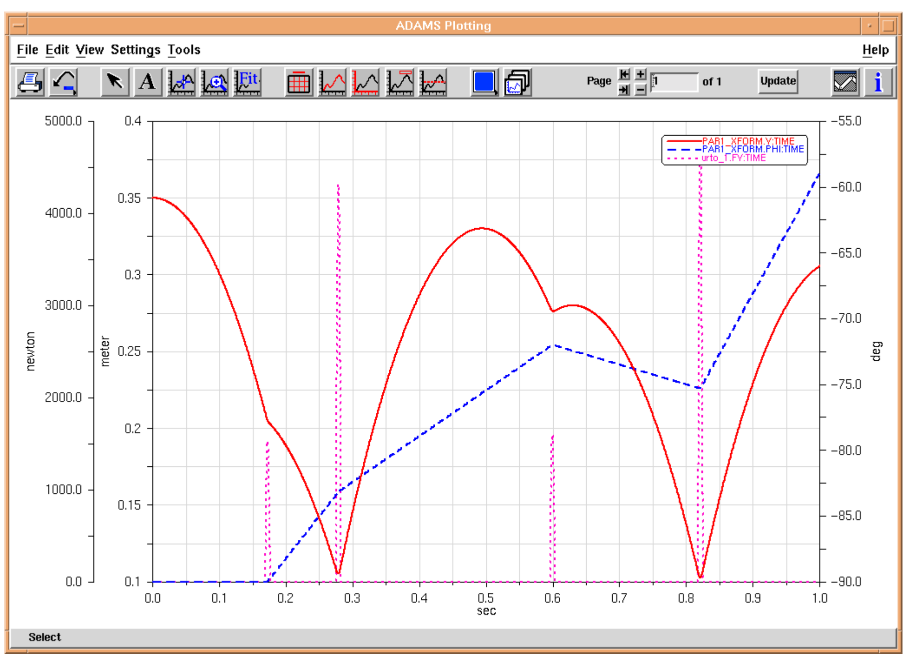

Figure 10 – Results window.

parametric coordinates of the contacting curves. Note how their positions change during the simulation, to account for the time varying candidate contact points.

Figure 10 gives a time history plot of some of the computed quantities. The solid line represents the vertical position of the falling body, the dashed line represents the angular position of the same body, while the dotted line gives the time history of the vertical component of the contact force.

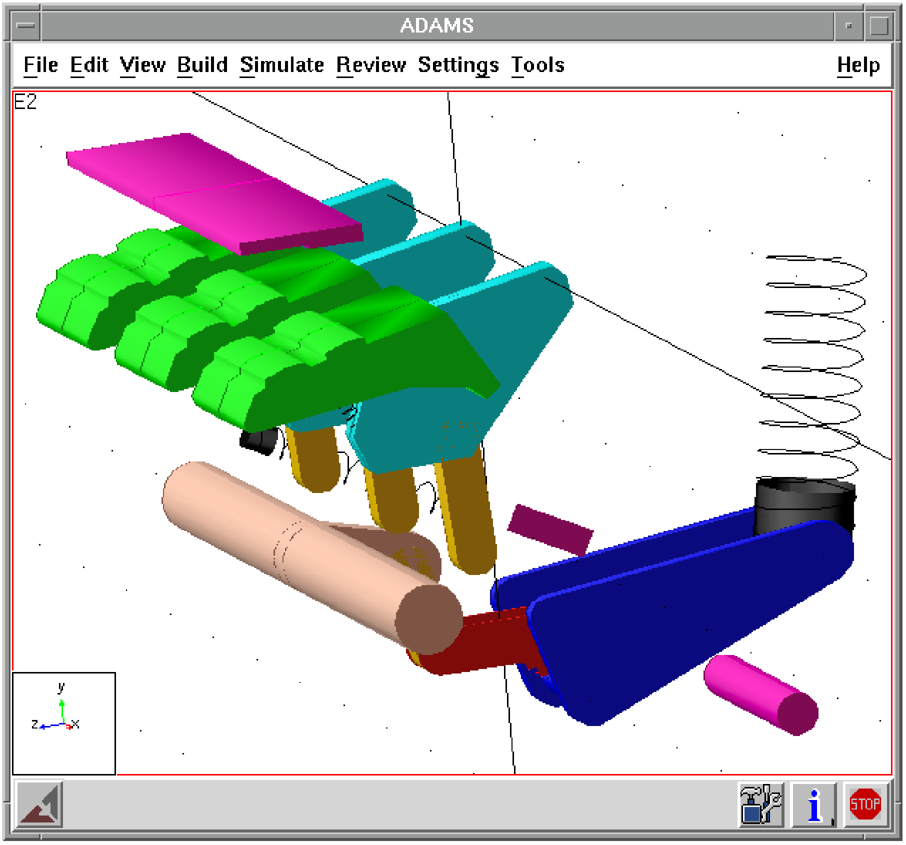

Figure 11 - The circuit breaker problem.

Circuit Breaker

This problem is concerned with the simulation of the closing operation of a circuit breaker. The system is composed of several rigid components and its configuration is depicted in Figure 11. The input energy is provided by a closing spring, that is responsible for both the quick activation time of the mechanism and for the recharging of the opening spring. Ten unilateral contact conditions are present in this example, that exhibits a very complex behavior with short duration impacts as well as prolonged contacts, together with considerable interaction forces.

Figure 12 gives a few snapshots of the system configuration during the simulation. Figure 13 presents a plot of the time history of the contact forces in the various unilateral constraints. A detailed description of the functionality of the system is beyond the scopes of the present work, however we can notice the typical behavior that is expected in the contact/impact analysis of these mechanisms, i.e. large interaction forces characterized by very sharp gradients that account for the changes in system topology. Figure 14 is a zoom of the same quantities in the last part of the simulation. Close inspection of the plot shows multiple repeated contacts followed by separation of the bodies, or sticking conditions when the bodies remain together for prolonged periods of time.

CONCLUSIONS

We have developed a procedure that allows the implementation in ADAMS of general unilateral constraint conditions between curves or surfaces of arbitrary geometry. Our approach achieves this goal through the use of user-accessible features of the code, in particular “user defined subroutines” and “macros”. The basic ingredients of the proposed procedure are the following:

- extremely general and flexible parameterization of the geometry of contact through the use of NURBS;
- rigorous implementation without any ad-hoc simplification of the constraint equations that define the minimum distance problem;

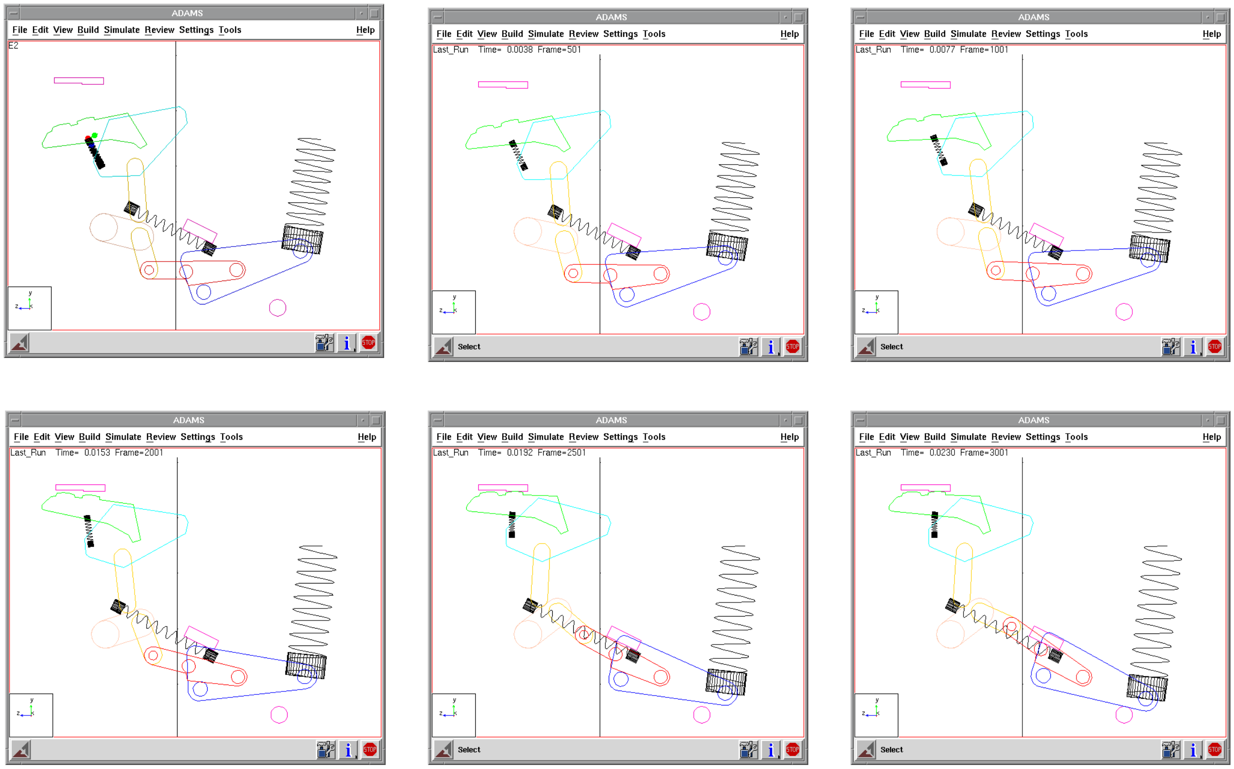

Figure 12 – Snapshots of the system configuration during the simulation.

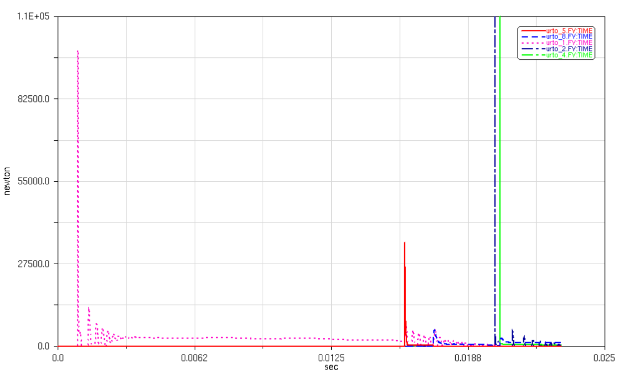

Figure 13 – Time history of the contact forces.

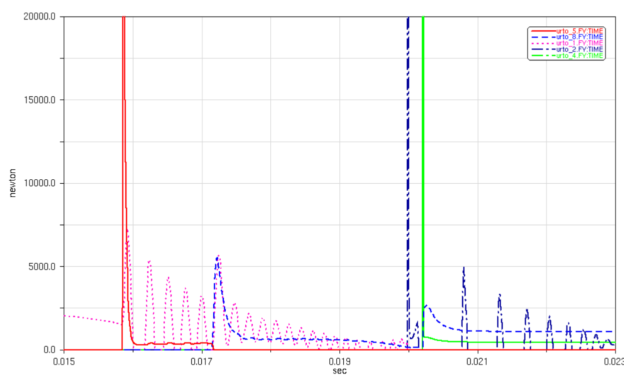

Figure 14 – Zoom for the time history of the contact forces in the last part of the simulation.

- general and flexible modeling of the constitutive laws for the contact forces through the ADAMS IMPACT module;
- straightforward use and full integration with the program, achieved by complete hiding of the details of the implementation to the user.

The approach offers the following advantages:

- rigorous treatment of the kinematic problem of minimum distance, which implies greater robustness and reliability of the simulation procedure;
- generality and flexibility, both in the modeling of the geometry and in the description of the interaction forces exchanged by the contacting bodies, which in turn guarantee a wide range of applicability of the method;
- ease of use of the procedure, with seamless integration with the rest of the code.

This way, we have extended the modeling capabilities of ADAMS to all those situations when the presence of unilateral contact conditions plays an important role.
We have tested and validated our methodology with the help of some numerical experiments.

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