Prediction of steering efforts during stationary or slow rolling parking maneuvers.

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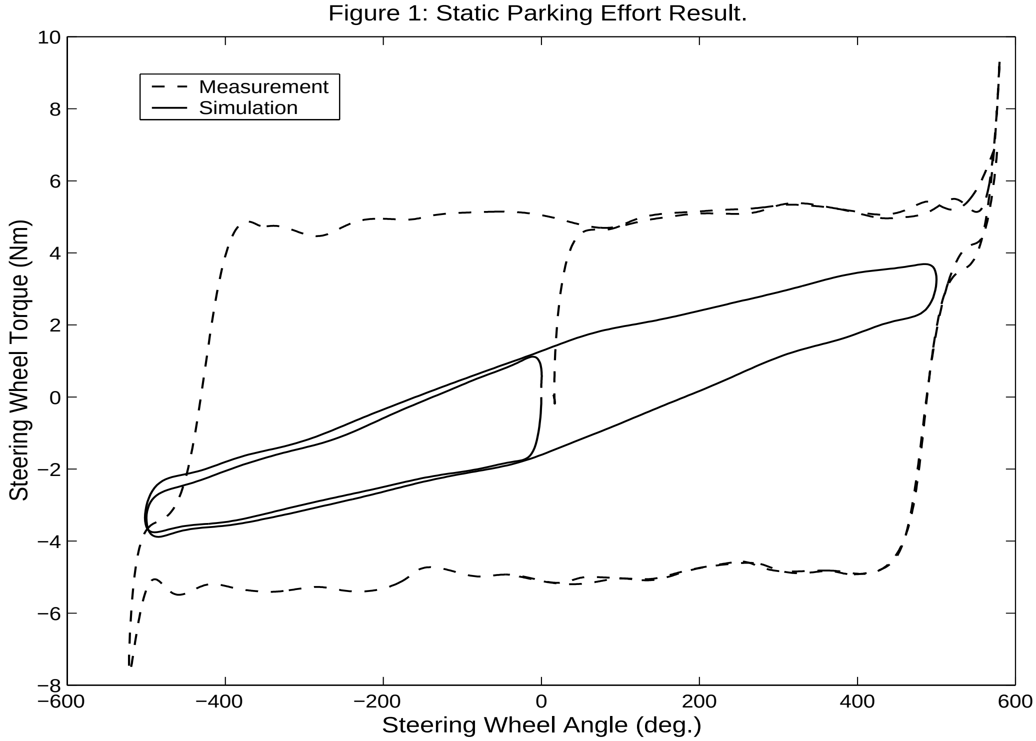
ABSTRACT

Existing tire models for Vehicle Dynamics analysis only generate forces and moments under the influence of the slip velocity of the tire contact patch. This assumption is valid for most driving maneuvers we want to simulate. If we, however, want to predict the forces and moments of a tire under stationary conditions, for example in the case of a parking maneuver, these models will not predict the correct forces due to the absence of tire slip. This report describes the development of a new empirical model which predicts the correct forces and moments under these stationary conditions. This new model has a smooth transition to the dynamic model and all effects are negligible at higher speeds.

Introduction:

Their is a need from some Adams users for a tiremodel extension to be able to describe the correct torques (Mz) during low speed parking maneuvers. The current Adams-tire model only generates forces and moment under the influence of a velocity of the tread relative to the ground (slip). If a vehicle is stationary, there will be no slip and the existing tire models will predict to low steering efforts (figure 1).

Figure 1: Static Parking Effort Result.



To improve the result of the simulation at low or no speed parking, the moment due to yawing of the tire plane needs to be added to the dynamic tires forces and moments.

Some additional requirements for the model extension were:

- No modification of the Adams data set needed to activate this option.
- Smooth transition from stationary to rolling.
- No effect at higher speeds (>10 MPH).
- Tire manufacturers should be able to measure the parameters for this model.

This paper will describe the theory behind this parking force tire model, the testing carried out for model validation and the results of the Adams simulations.

Model Theory.

When I started, there was not much experimental data available to develop a theory for this parking model. Data was only available on two tire constructions. When more test data became available, some of the equations described below where slightly modified. This is described later in this report.

An example of the torque generated by a non-rolling tire due to a steer (yaw) motion relative to the ground can be seen in figure 2.

Looking at the initial part of the curve we can see that this part is linear. This area could be modeled as an torsional spring between the tire and the ground.

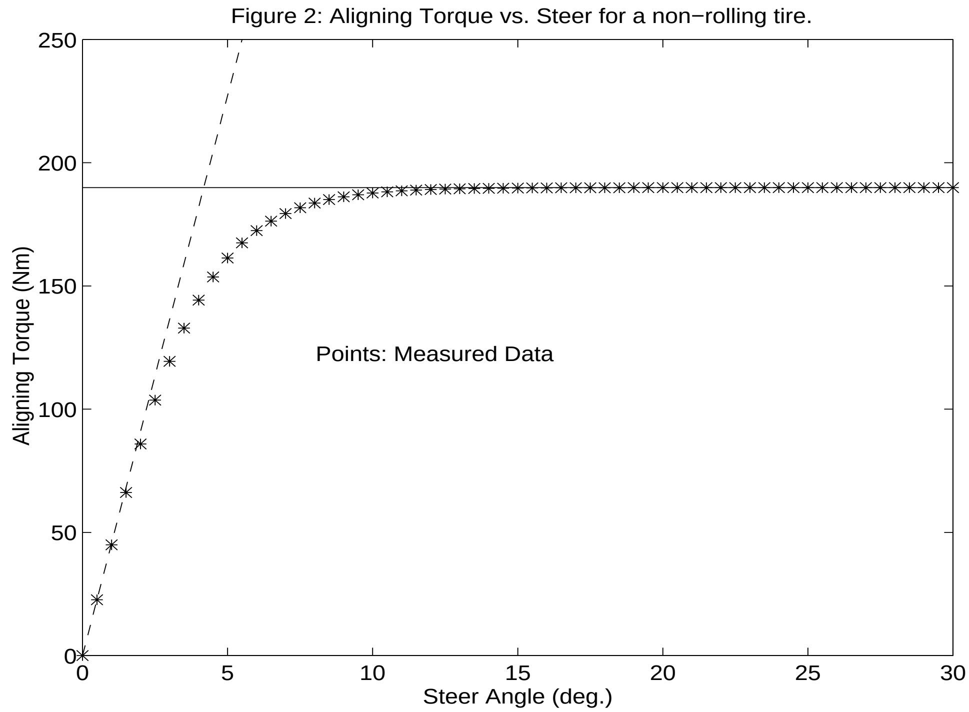
$$M_z = K_y . \Psi \qquad \text{eq. 1}$$

with:

M_z: Aligning Torque.

K_ψ: Torsional stiffness of the tire.

Ψ: Yaw angle of the wheel plane.



This equation generates the dashed line in figure 2. With this equations the tire would continue to build up torque if it continued to rotate. In reality, it will reach a point where the tire contact patch can not generate more torque and it will start sliding. Looking at the measured data, it is a good approximation to hold the torque constant if the maximum torque is reached. The effect can be described by:

$$\begin{aligned} M_z &= K_\psi . \Psi & \quad if \quad & \Psi < \Psi_{\max} \\ M_z &= M_{z\max} & \quad if \quad & \Psi \geq \Psi_{\max} \\ \Psi_{\max} &= M_{z\max} / K_\psi \end{aligned} \qquad \text{eq. 2}$$

with:

$M_{z\max}$: Maximum Torque that can be generated by the tire.

This model is already quite a good approximation of the torque generated but is not usable for simulation for the following reasons:

- This model has a discontinuous derivative in the transition point between sticking and sliding. This would cause problems with the variable step integrators like the ones used by Adams.

- This model will not show any hysterises, e.g. returning the steering would follow the same torque path. Hysterises is one of the important characteristics which show up in steering wheel torque measurements during parking maneuvers.

In considering a way to achieve this hysterises effect, an idea arose to assume that one can never reach a larger torsional deflection of the tire then Ψ_{defm}. This value Ψ_{defm} is determined by dividing the maximum achievable torque ($M_{z\max}$) by the torsional stiffness (K_ψ) of the tire. This means if you steer a tire to a larger angle then Ψ_{defm} the tire contact patch will start slipping and the deflection stays constant. Returning the steering will directly reduce the tire deflection and the tire will reach zero torsional deflection before the original starting position. Assuming the tire is symmetric, this behavior can be described with the following equations:

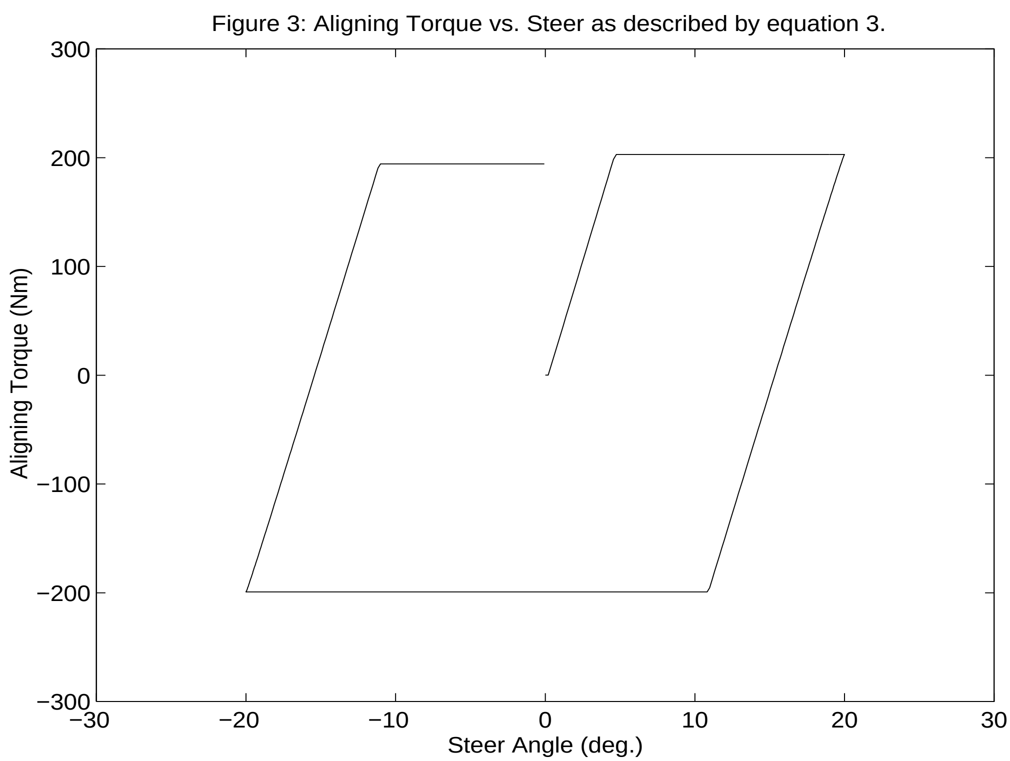
$$
\begin{aligned}
&\dot{\Psi}_{def} = \dot{\Psi} && \text{if} \quad \left|\Psi_{def}\right| < \Psi_{defm} \;\text{ or }\; \mathrm{sign}(\Psi_{def}) \neq sign(\dot{\Psi}) \\
&\dot{\Psi}_{def} = 0 && \text{if} \quad \left|\Psi_{def}\right| = \Psi_{defm} \;\text{ and }\; \mathrm{sign}(\Psi_{def}) = sign(\dot{\Psi}) \\
&\Psi_{defm} = M_{z\max} / K_{\psi} \\
&\Psi_{def} = \int_0^t \dot{\Psi}_{def} \cdot \partial t \\
&M_z = K_{\psi} \cdot \Psi_{def}
\end{aligned}
\qquad \text{eq. 3}
$$

with:

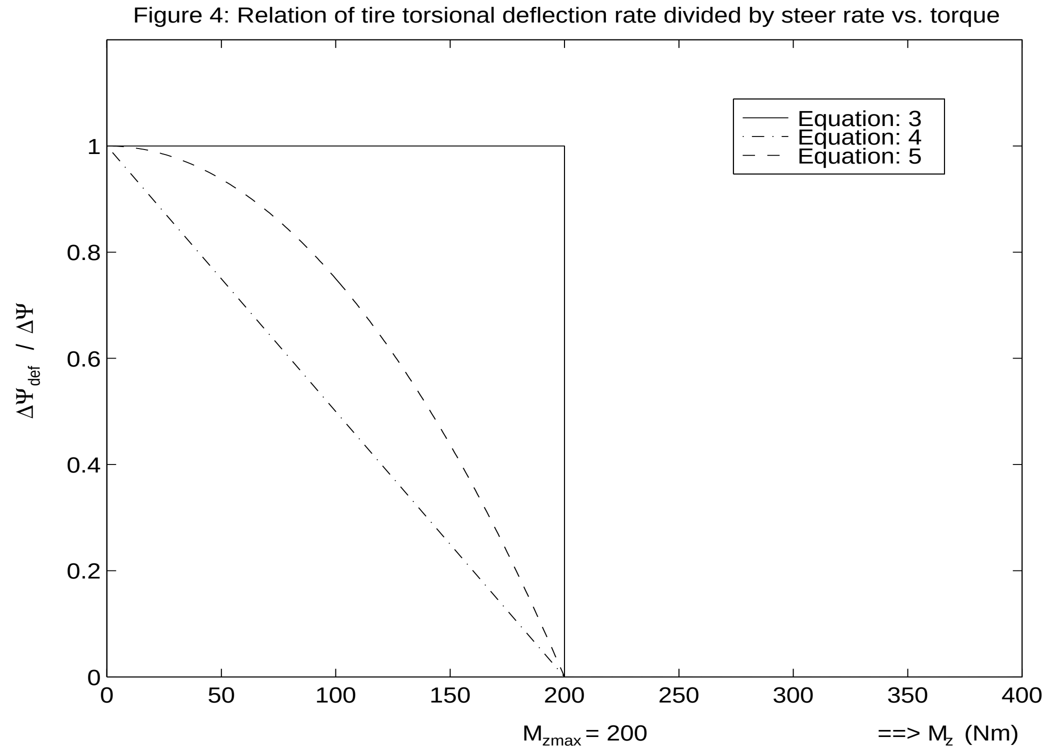
Ψ_{def}: Torsional deflection of the tire.

Ψ_{defm}: Maximum possible deflection of the tire.

A plot of this characteristic can be seen in figure 3, which now shows a hysterises effect.



The next problem was to solve the discontinues derivative at Ψ_{defm}. Figure 4 shows a plot of the relation between Ψ and Ψ_{def} versus torque as described by equation 3. If we would now describe this relation by a linear function (dash-dotted line), instead of a step, we solve the problem with the discontinuity.



This will change the first two lines in equation 3 to:

$$\begin{aligned}
\dot{\Psi}_{def} &= \left(1 - \left|\frac{M_z}{M_{z\max}}\right|\right) \cdot \dot{\Psi} && \text{if } \operatorname{sign}(\Psi_{\mathrm{def}}) = sign(\dot{\Psi}) \\
\dot{\Psi}_{def} &= \dot{\Psi} && \text{if } \operatorname{sign}(\Psi_{\mathrm{def}}) \neq sign(\dot{\Psi})
\end{aligned} \qquad \text{eq 4.}$$

To achieve a good fit of the measured data available, the following model extensions were decided on.

$$\begin{aligned}
\dot{\Psi}_{def} &= \left(1 - \left(\left|\frac{M_z}{M_{z\max}}\right|\right)^{c_0}\right) \cdot \dot{\Psi} && \text{if } \operatorname{sign}(\Psi_{\mathrm{def}}) = sign(\dot{\Psi}) \\
M_{z\max} &= f(F_z) = a_2 \cdot F_z^{\ 2} + a_1 \cdot F_z \\
K_\psi &= f(F_z) = b_2 \cdot F_z^{\ 2} + b_1 \cdot F_z
\end{aligned} \qquad \text{eq. 5}$$

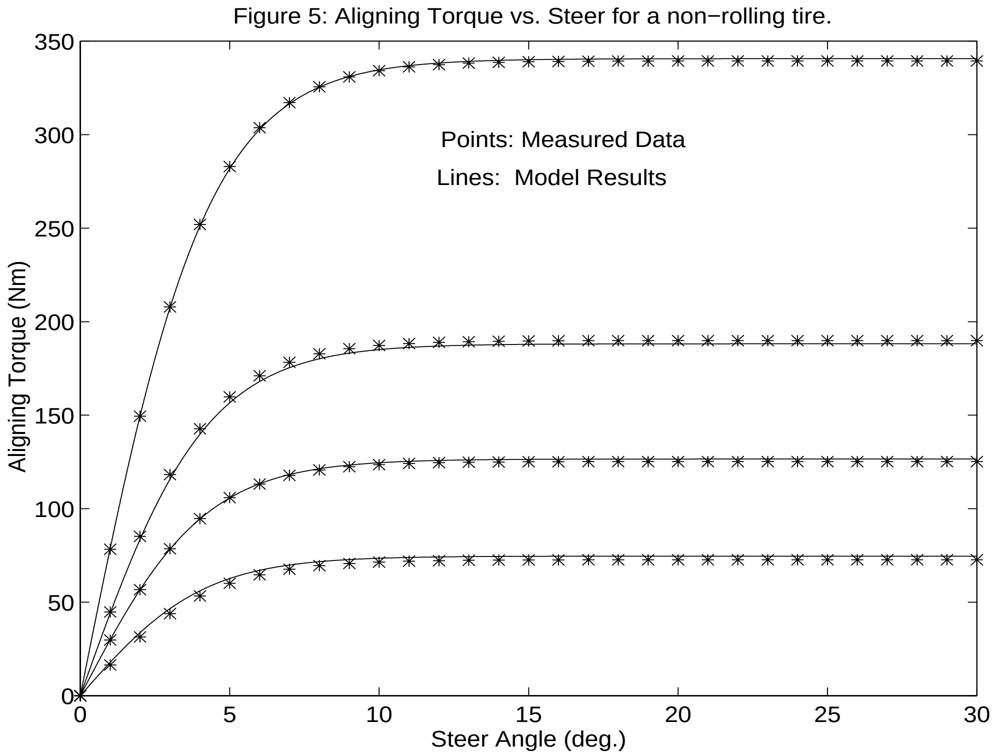
with:

Fz: Vertical load on the tire.

$a_1,\ a_2\ ,\ b_1,\ b_2,\ c_0$: Model parameters.

For this tire, a quadratic relation for $M_{z\max}$ and K_ψ versus load and also a quadratic relation between $\dot{\Psi}_{def}$ and $\dot{\Psi}$ fits the data well. When more experimental data becomes available, these relations can be improved if necessary.

Figure 5 shows the correlation between the model and the measured data using the following parameter values: $a_2 = 6.245$, $a_1 = 31.263$, $b_2 = 1.374$, $b_1 = 7.867$ and $c_0 = 2.0$



The model shows the correct characteristics for a non-rolling tire. It is known that the influence of this torque has to get negligible with speed. It is also known from experience that if the steering system of a stationary vehicle is wound up and then driven forward for a few meters and stopped that there is no torque remaining in the steering system. To describe these two effects with this model, the following theory was developed based on well known tire behavior.

From the literature [Lit. 1] the behavior of a rolling tire under transient slip angle input conditions is known. For example, the influence of these transient effects on the lateral force can be described with the following differential equation.

$$\tau \cdot \frac{\partial F_y}{\partial t} + F_y = F_{yss} \qquad \text{eq. 6}$$

Which can be written as:

$$F_y = \left(1 - e^{-t/\tau}\right) \cdot F_{yss} \qquad \text{eq. 7}$$

with:

- τ: Time constant.
- F_y: Tire Lateral Force
- F_{yss}: Steady State Lateral Force.

Thus a tire under transient input will reach 67% of it's steady state value after a time span τ. Investigations have shown that the value τ is a function of the tire velocity and can be described by $\tau = X_{rel} / (\omega \cdot r)$. The value X_{rel} is the relaxation length. A tire will have reached 67% of its steady state value after it has rolled over a distance X_{rel}.

This same theory is used for the static parking model, when we make the assumption that the tire will lose 67% of its torsional deflection after it has rolled over a distance X_{rel}. This can be described with the following additions to equation 4.

$$\begin{aligned} \dot{\Psi}_{def2} &= -\frac{1}{\tau} \cdot \Psi_{def} \\ \tau &= X_{rel} / (\omega \cdot r) \\ \Psi_{def} &= \int_0^t \left(\dot{\Psi}_{def} + \dot{\Psi}_{def2} \right) \cdot \partial t \end{aligned} \qquad \text{eq. 8}$$

with:

Xrel: Tire relaxation length.

ω: Tire rotational velocity.

r: Tire rolling radius.

These equations describe the desired behavior: e.g. after winding up the steering and driving three times X_{rel} distance and stopping, 99% of the torque is dissipated. At higher speed torsional deflection is dissipated more quickly than it builds up and the additional torque generated by this model is negligible.

Test Program:

The measurements were carried out on the "Flat-Plank" tire test machine of Delft University of Technology (Figure 6). This machine was developed in the past as a so called "Glass-Plate" tire tester, which was used to look at the tire footprint under different operating conditions. The glass plate can travel in longitudinal direction under the tire with a velocity of 0.0475 meters per second. The glass plate is no longer used, and has been replaced by a steel plate which enables greater travel (8 meters) and can be coated to give more realistic frictional properties. For testing, "3M Safety Walk - 80 Grid" was used on the steel plate This material is used throughout the tire industry as a standard surface for tire force and moment testing.

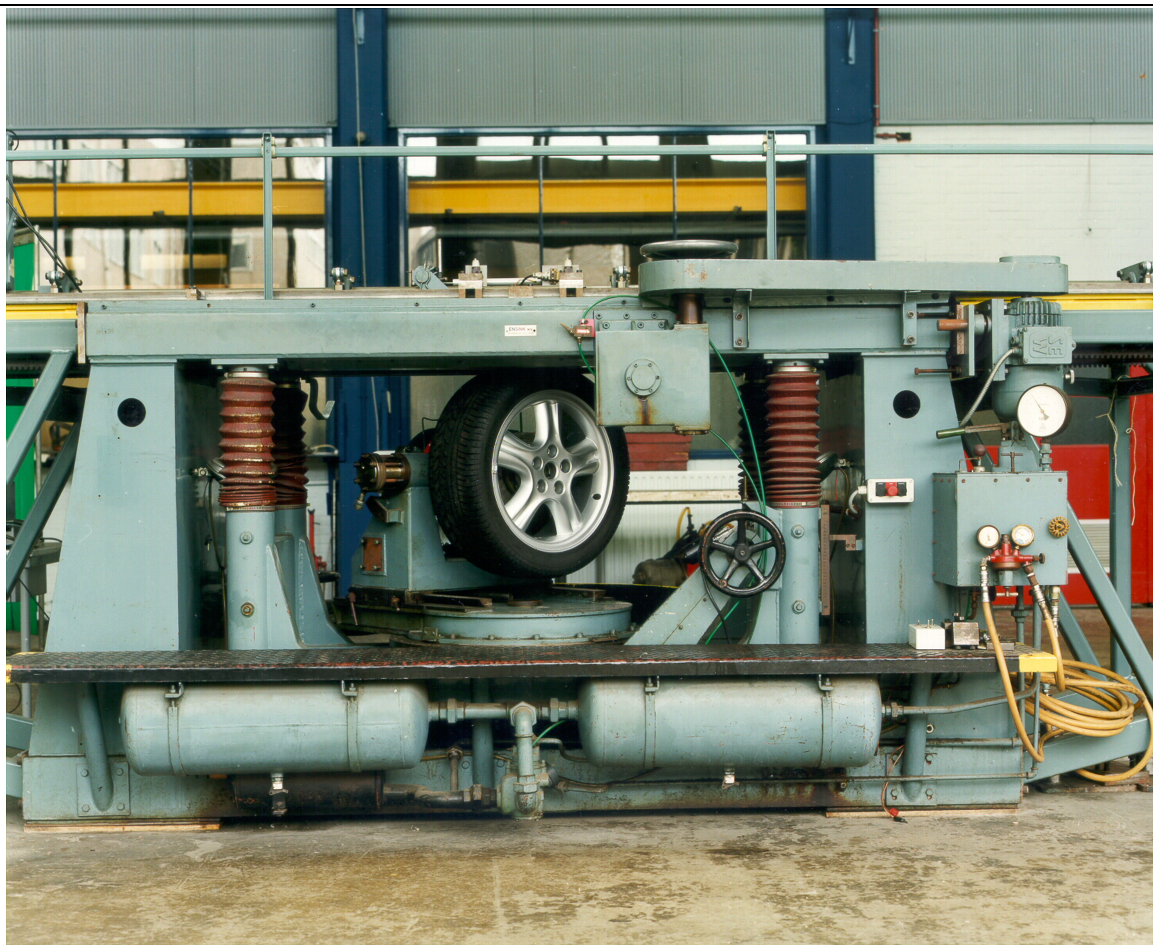


Figure 6: "Flat-Plank" Tire tester.

Photographer: H. Kempers.

The following test program was carried out:

<u>General Conditions:</u>

Tire:	General Tire Ameri*G4S
Size:	P205/65R15
Rim:	15x6
Inflation:	33 Psi
Sample Rate:	50 Hz
Measured Channels:	1. Longitudinal Force (Fx) in N. 2. Lateral Force (Fy) in N. 3. Vertical Load (Fz) in N. 4. Overturning Moment (Mx) in Nm. 5. Aligning Torque (Mz) in Nm. 6. Steer Angle in deg. 7. Ground Surface traveled distance in m.

<u>Test Procedure 1:</u>

A static (non-rolling) tire was steered to very large angles. This test was performed at four different vertical loads.

Vertical Loads:	1, 3, 5 and 7 kN.
Steer angle:	0 ==> 20 ==> -20 ==> 20 deg.
Steer Angle Rate:	≈ 1 deg/sec. (varied slightly between the different vertical loads.)

<u>Test procedure 2:</u>

A static (non-rolling) tire was steered from zero degrees to a certain angle and held at this angle. After the desired angle was reached, the tire was rolled forward for approximately two meters.

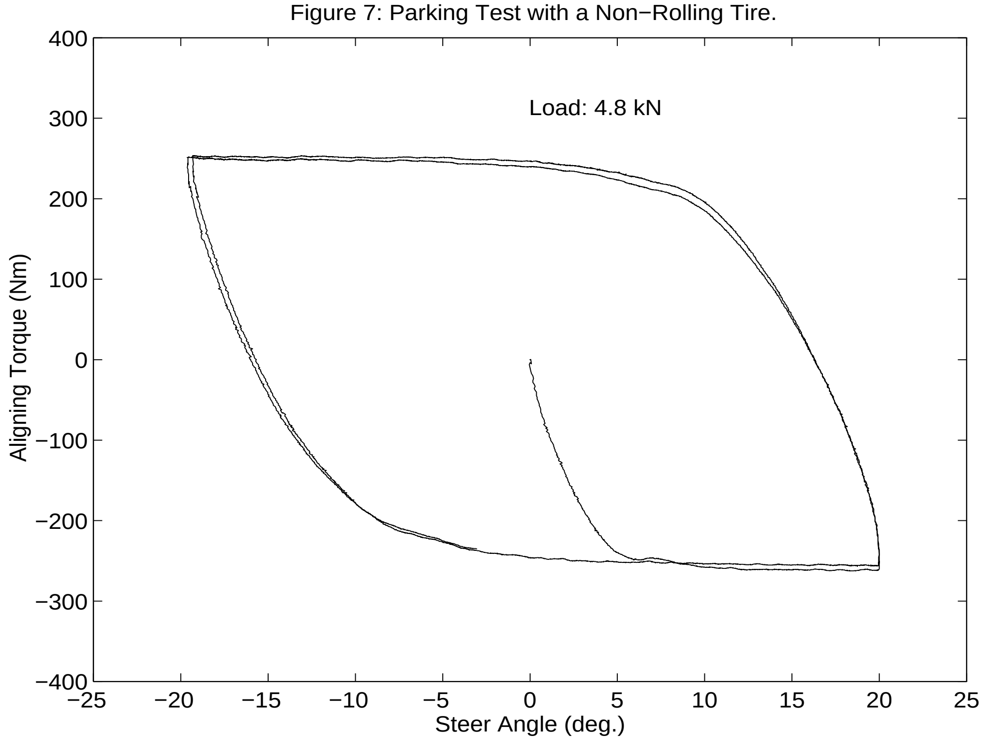
Vertical Loads:	3 and 5 kN.
Steer angle:	0, 1, 2, 4, 8, 12 and 20 deg.
Ground Speed:	0.0475 m/s

Results and Analysis of Test procedure 1:

Figure 7 shows the example of an output from the first test procedure.

Looking at this data, it can be seen that the stiffness (slope) of the initial part is much higher than the stiffness of the consecutive loops thereafter. There is nothing in the current version of the model that enables us to be described both slopes correctly with one set of parameters.

Figure 7: Parking Test with a Non−Rolling Tire.



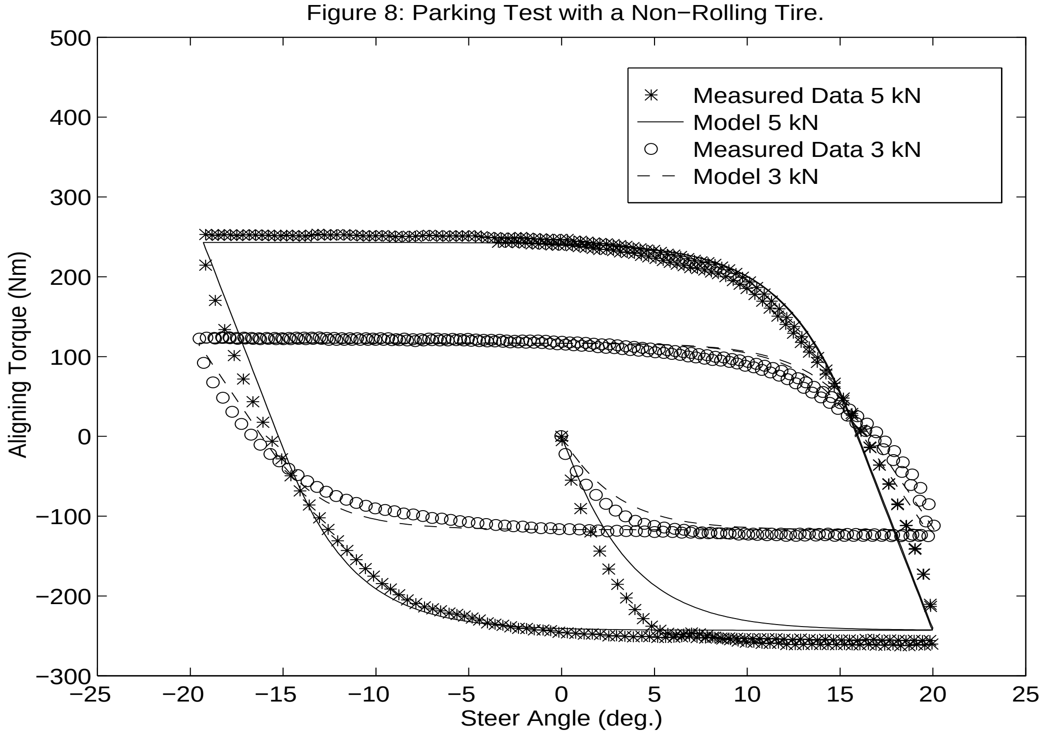
A model which describes both the initial slope and the following loops is difficult to achieve. The current model does a good job of describing the major effects (max. torque and hysteresis) of static tire behavior. The good correlation between the simulation results of this new tire model and the instrumented vehicle data of a park maneuver test confirmed that there is no need to more accurately describe this phenomena . These correlation results will be discussed later in this paper.

The next task was to derive model parameters from the measured data. Optimizing the model parameters gave the following result:

$$a_2 = 4.987 \text{ , } \mathrm{a}_1 = 26.003 \text{ , } b_2 = 0.308 \text{ , } b_1 = 10.898 \text{ and } c_0 = 1.189$$

The result from this optimization can be seen in figure 8.

Figure 8: Parking Test with a Non−Rolling Tire.



Results and Analysis of Test procedure 2:

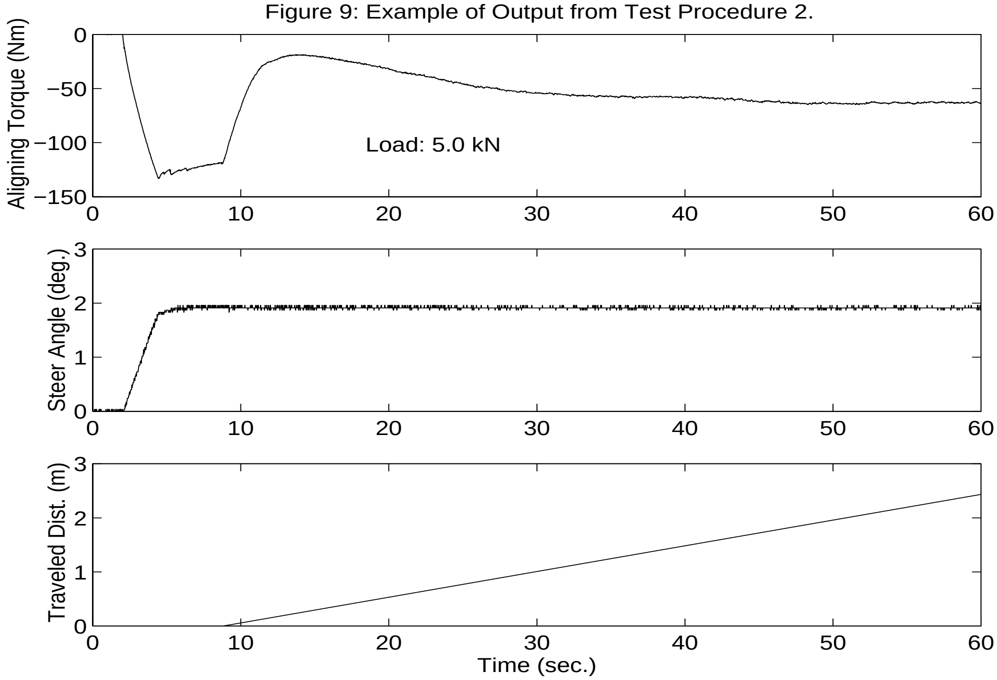
In the second test the non-rolling tire was steered to a desired angle. When this angle was reached the tire started rolling. After the tire rolled over a certain distance, the lateral force (Fy) and aligning torque (Mz) will settle at a “steady state” value. This test is often performed to measure the relaxation length of the tire [Lit. 1]. The relaxation length of the tire is used to describe tire behavior under transient slip conditions. The theory developed for the static parking model assumed that the tire would lose the static deflection (parking torque) at the same rate as it build up its normal dynamic aligning torque caused by slip. This means the same relaxation length can be used for both effects. No experimental data was available when the parking model was developed to test this theory.

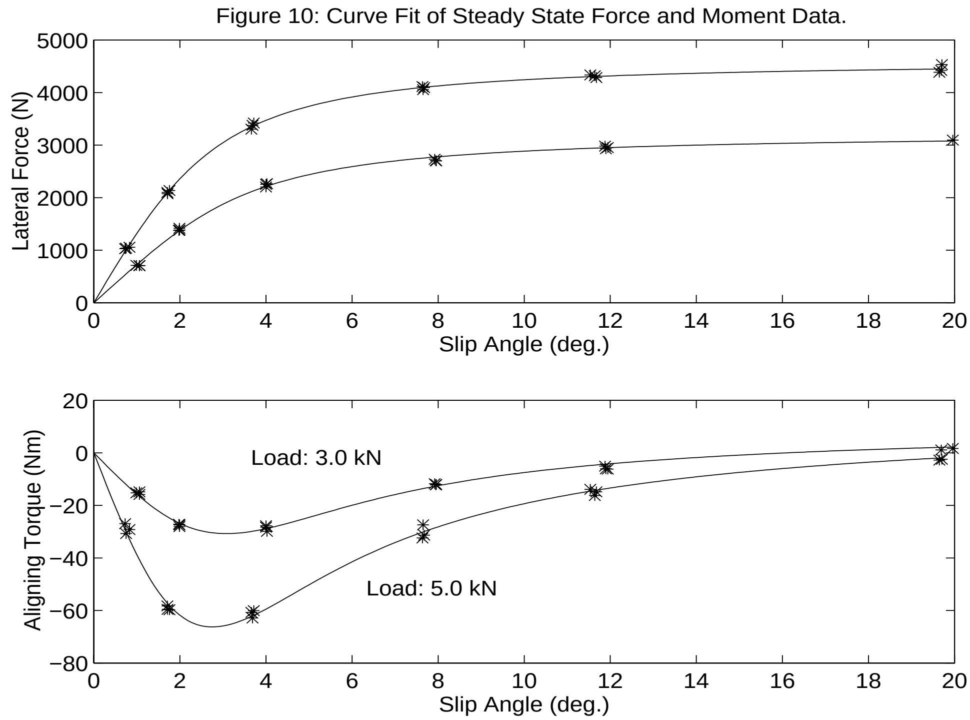
Figure 9 shows an example of the results of this test. Looking at this plot, another effect which is not included in the parking model is noticed. In the small time period between the angle build up and the start of the ground plane motion, the torque decreases. This effect occurs at all steered angles. Because of this effect, the data will be analyzed from the point where the ground plane motion starts.

The test data was analyzed following several different steps:

1) The steady state values of lateral force (Fy) and Aligning torque (Mz) were plotted versus slip angle. The so called “Magic Formula” parameters [Lit. 2] can be fitted through this data to get a continuous relationship between Fy and slip ($F_y = f(\alpha)$) and Mz and slip ($M_z = f(\alpha)$). The results of these fits are shown in Figure 10.

Figure 9: Example of Output from Test Procedure 2.





2) With the function for Fy versus slip known, the value for the relaxation length can be optimized by minimizing the error between the measured and calculated Fy.

$$\alpha = \alpha_{ss} \cdot \left(1 - e^{-\tau}\right)$$
$$\tau = x / X_{rel}$$
$$F_{ycal} = f(\alpha)$$

with:

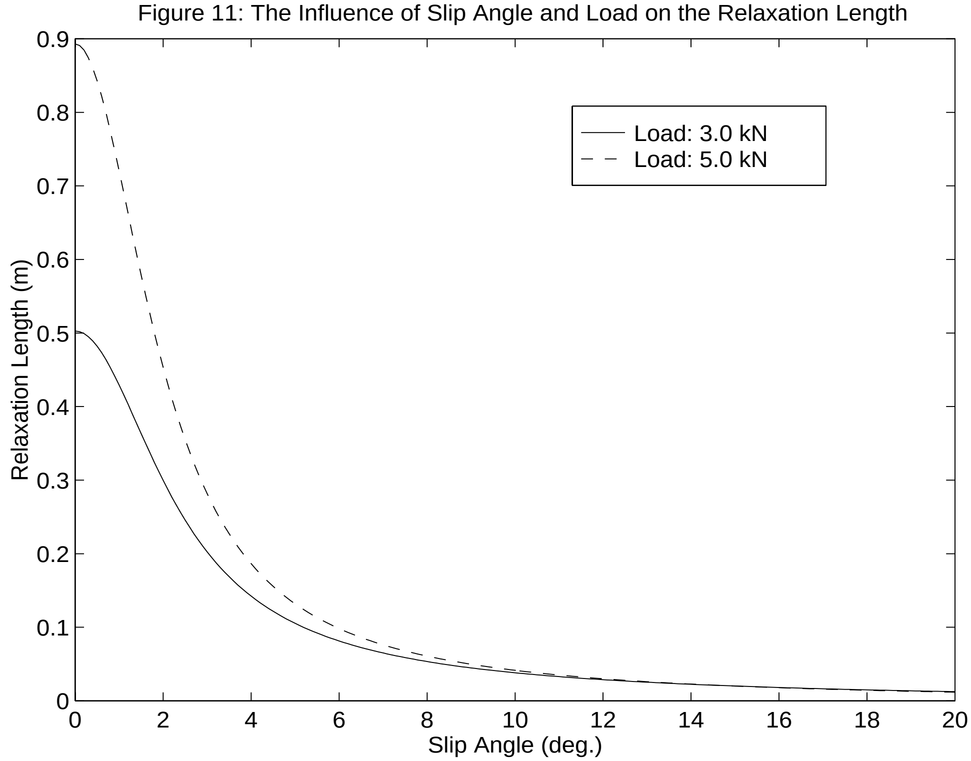
α_{ss}: Steady State Slipangle (deg.)

α: Slip Angle (deg.)

x: Travelled Distance (m)

X_{rel}: Relaxation Length (m)

Figure 11 shows the values for the relaxation length versus slip angle for the two measured vertical loads. The influence of slip angle and load on the relaxation length is what we expected and is known from literature [Lit. 1].



3) The last step was to check if the relaxation length found for the lateral force can be used to describe the decrease of the static parking moment while rolling. The total aligning torque can be described as:

$$\alpha = \alpha_{ss} \cdot \left(1 - e^{-\tau}\right)$$
$$\tau = x / X_{rel}$$
$$M_{zdynl} = f(\alpha)$$
$$M_{zsta} = M_{z0} . e^{-\tau}$$

with:

M_{zdyn}: Aligning Torque due to tire slip (Nm).

$M_{zsta:}$: AligningTorque due to tire torssional deflection (Nm).

M_{z0}: Initial Static Parking Torque (Nm).

In Figure 12 is clearly visible that using the same relaxation length for both effects does not reduce the Static Moment fast enough. Instead of using the same number for the dynamic and static portions a separate number for the static portion was optimized. This fit gives an excellent correlation between the measured data and the model (Fig. 13). The relaxation length for the static portion is independent of load and slip angle (Fig. 14). A fixed value for the static relaxation length (0.05 m) gives very good results.

Figure 12: Aligning Torque Response. (Load: 3.0 kN, Slip Angle: 4 deg.)

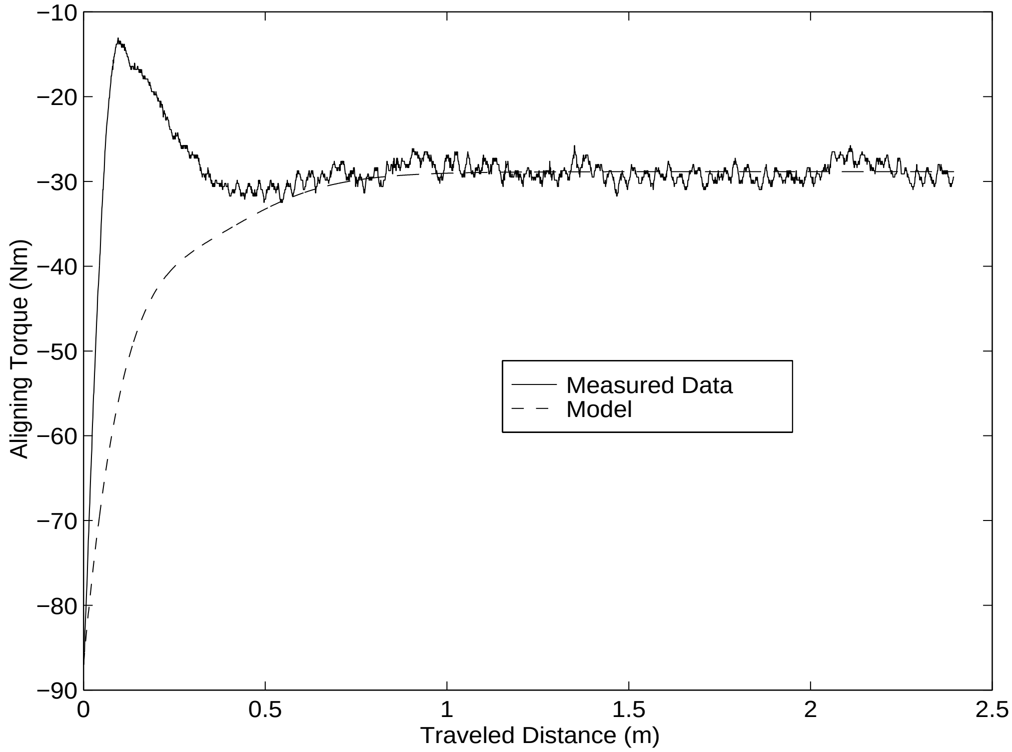


Figure 13: Aligning Torque Response. (Load: 3.0 kN, Slip Angle: 4 deg.)

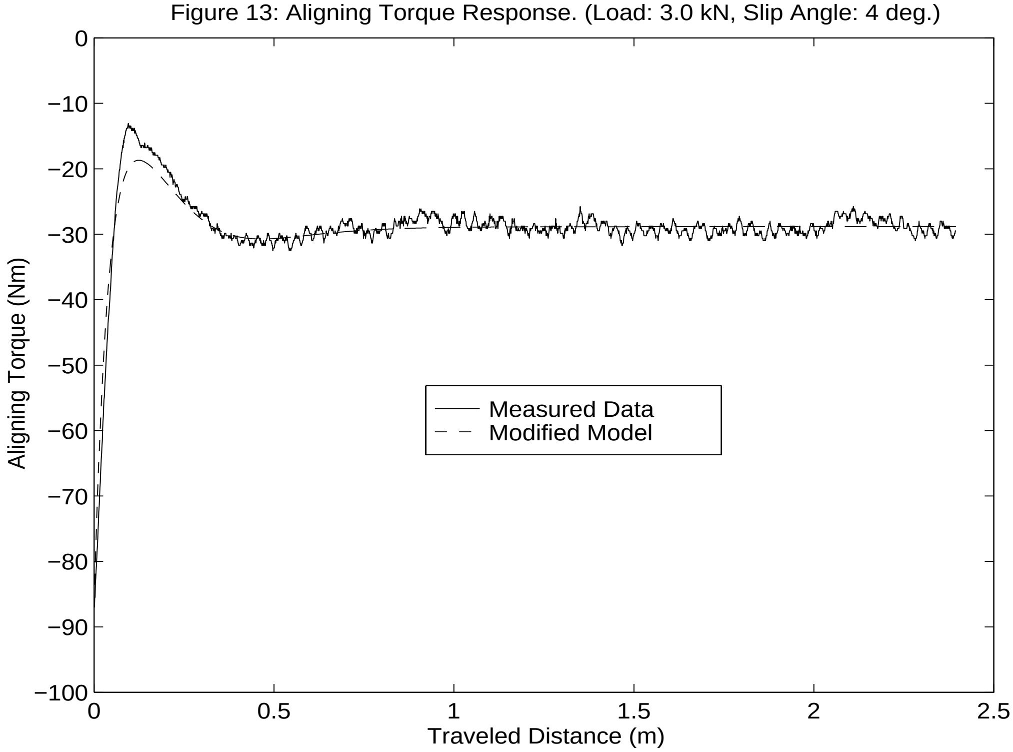
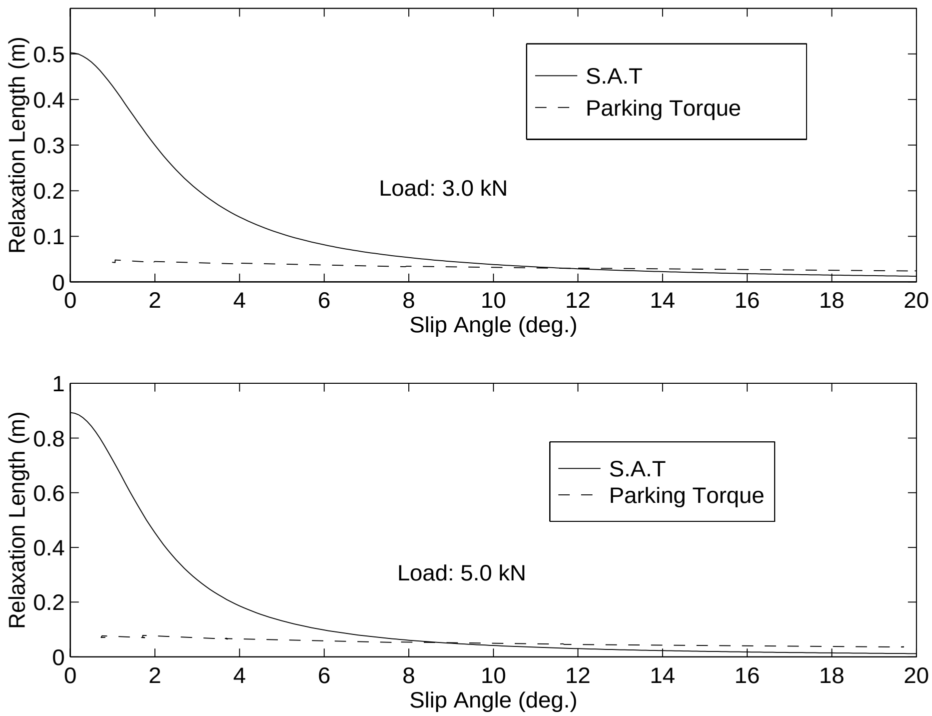


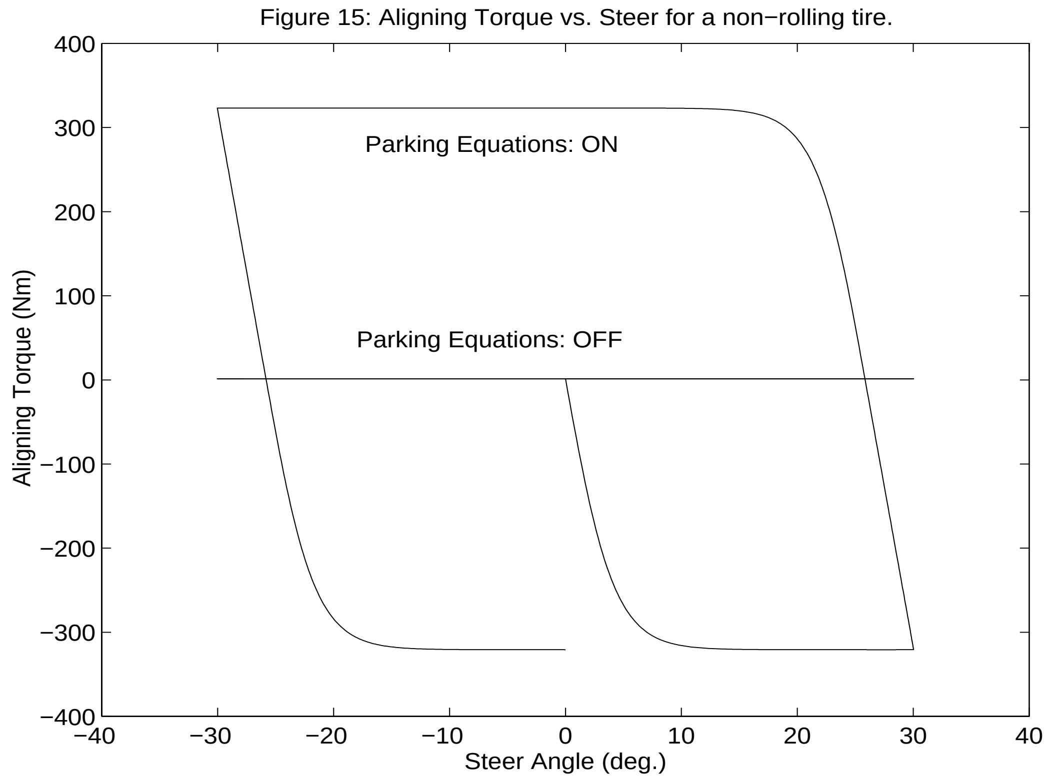
Figure 14: Relaxation lengths for Self Aligning Torque & Parking Torque



Adams Simulation Results.

The simulations were performed with two different Adams models. The first one being a model of a simple one wheel tire tester. This model has only two degrees of freedom: forward velocity and tire rotation. Motions are imposed to the model for steer, camber and to load the tire. An additional moment can be applied to the spindle to simulate drive/brake torque. This model is always used to test any modification made to the tiremodel because it is able simulate a tire testing machine. The second model is a full Adams model of a Ford Mondeo. For this vehicle we also had steering wheel torque and angle data available measured during static and slow-rolling park maneuver tests.

The first simulation was performed with the tire tester model. In this simulation there was no forward or rotational velocity and the tire was steered to +/- 20 degrees. This simulation was performed with the new park option on and off. Figure 15 shows the result of the torque vs. steered angle. It can seen that the torque shows the desired characteristics.



The next simulation performed is similar to the first one, but there is now a forward velocity (10 m/s). The results from this simulation (Figure 16) show that at this speed there is just a small influence from the additional parking torque.

The results from last simulation carried out with this model are displayed in figure 17. In this simulation the tire was steered to +20 deg., rolled forward for ten meters and stopped again. This figure shows that the transition from static to dynamic aligning torque has the desired characteristics.

Figure 16: Aligning Torque vs. Slip Angle

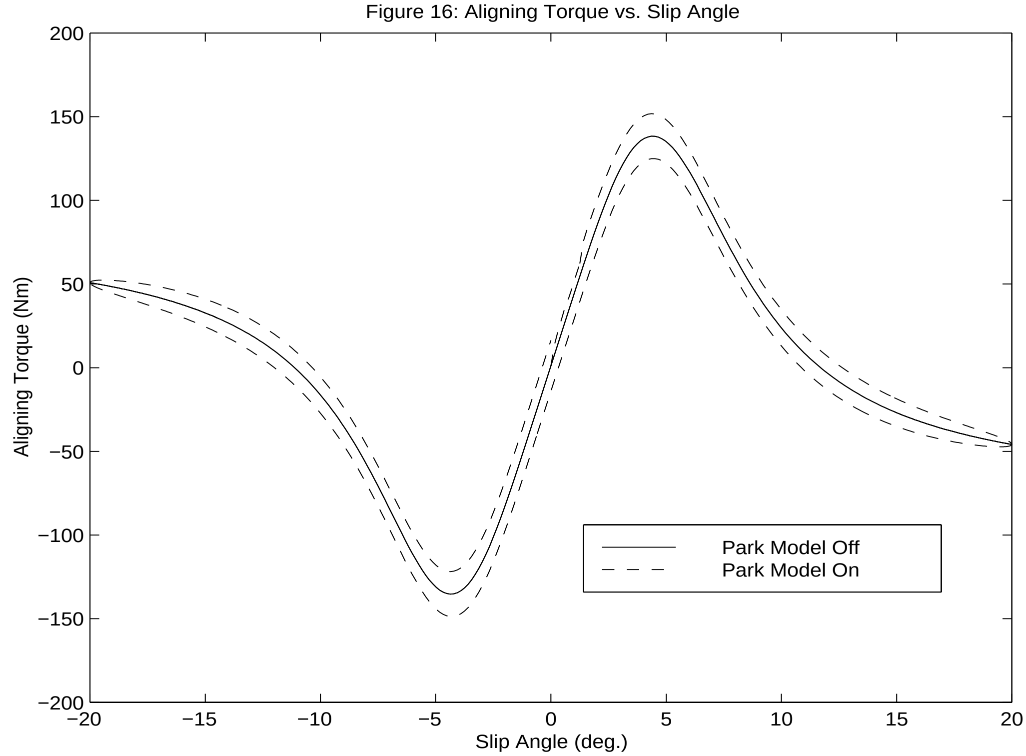
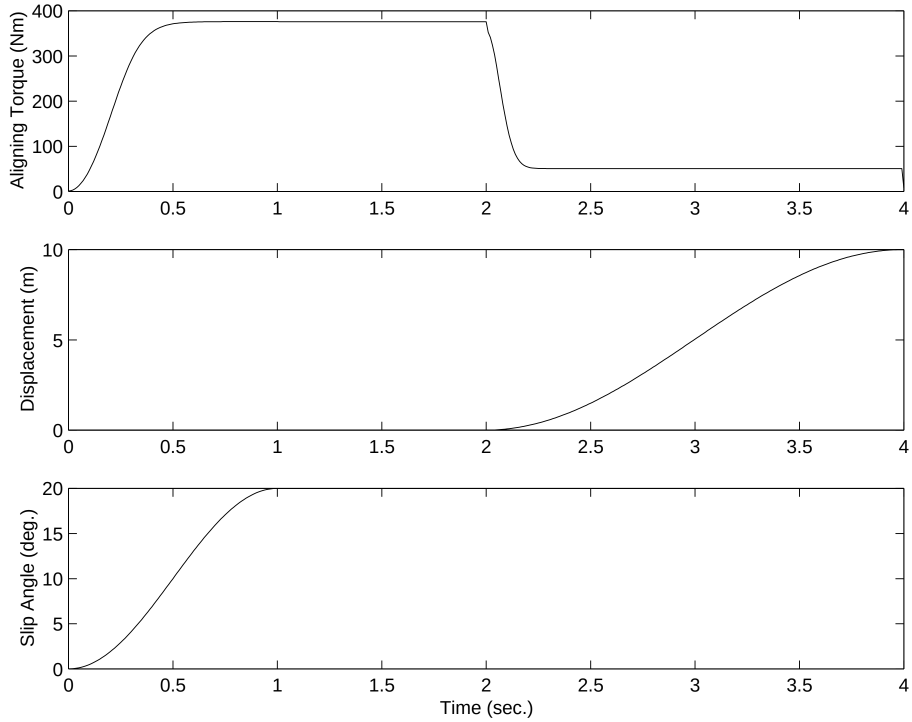


Figure 17: Behavior of the new tire model under the transition from static to rolling.



The next simulation we performed with a full vehicle model of a Ford Mondeo. With this model we performed our standard test procedure for static and slow rolling (8 KPH) parking efforts. Figure 18 and 19 show the results of a simulation with the new tire model together with actual measured vehicle data. In this figure it can be seen that the new model has an excellent correlation with the measured data. The large spikes at the end of the measured data are due to the mechanical stops in the steering system which do not exist in the simulation model.

Figure 18: Static Parking Effort Result.

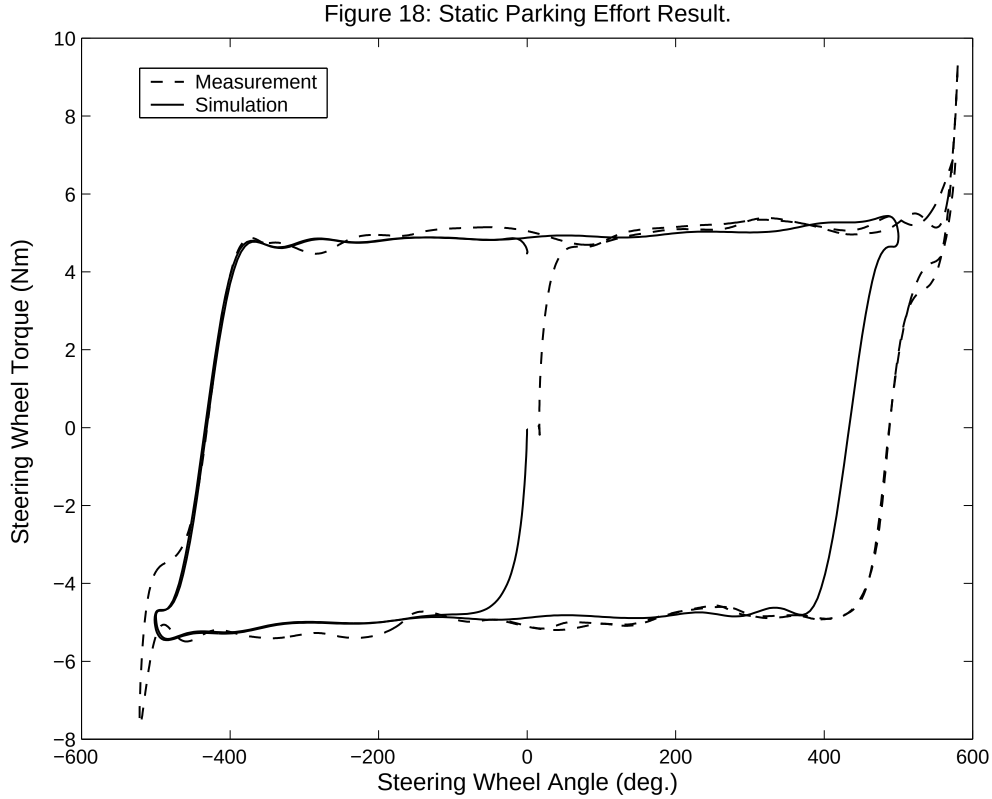
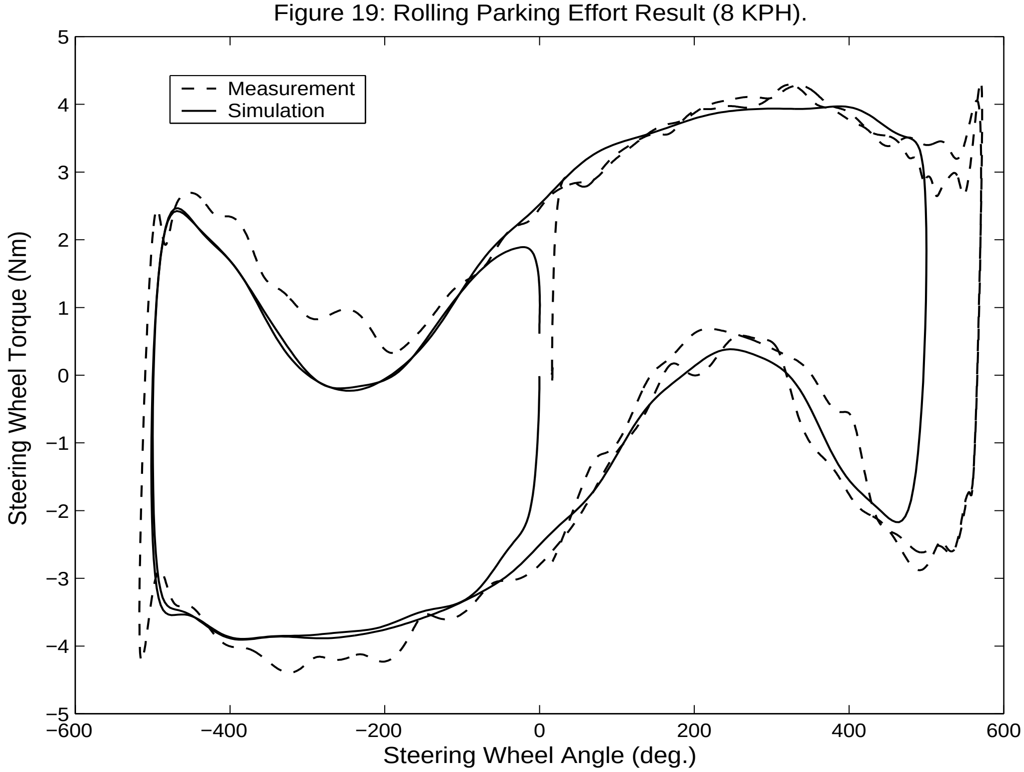


Figure 19: Rolling Parking Effort Result (8 KPH).



From these measurement we normally we derive the following characteristic numbers to describe the steering system

Static test:

Parameter:	Measurement:	Old Tire Model:	New tire Model (with static Mz):
Steering Torque near center. CW (Nm)	4.97	1.22	4.88
Steering Torque near center. CCW (Nm)	4.98	1.46	4.88
Steering Non-uniformity. CW (%)	94.5	10.9	98.6
Steering Non-uniformity. CCW (%)	94.9	3.1	98.5
Maximum Steering Torque. (Nm)	5.49	3.69	5.32

Rolling test (8KPH):

Parameter:	Measurement:	Old Tire Model:	New tire Model (with static Mz):
Steering Torque near center. CW (Nm)	2.39	1.86	2.34
Steering Torque near center. CCW (Nm)	2.32	1.80	2.28
Maximum Steering Torque. (Nm)	4.39	3.16	3.96

Conclusions.

The current model describes well the most important properties of a tire under parking and low speed maneuvers and has a smooth transition from the non-rolling to rolling condition of the tire.
The measurements showed that the initial steering away from straight ahead has a different stiffness then the following steering inputs. The physical properties of the tire that cause this phenomena are not yet completely understood and will be investigated in more detail. The investigation will also seek to determine if there is a large change in the parking characteristics between new and worn tires.

The results from full vehicle simulations with a Ford Mondeo vehicle model show a very good correlation with the measured data.

Literature References:

[1] H.B. Pacejka and T. Takahashi, ‘Pure slip characteristics of tyres on flat and on undulated road surfaces’. AVEC ‘92, Yokohama, Sept. 1992.

[2] E. Bakker , H.B. Pacejka and L. lidner, ‘A new tyre model with an application in vehicle dynamics studies. SAE 890087.

Appendix A: Equations for Parking Model.

$$\dot{\Psi}_{def} = \left(1 - \left(\left|\frac{M_z}{M_{z\max}}\right|\right)^{c0}\right) \cdot \dot{\Psi} \qquad \text{if } \operatorname{sign}(\Psi_{\mathrm{def}}) = sign(\dot{\Psi})$$

$$\dot{\Psi}_{def} = \dot{\Psi} \qquad \text{if } \operatorname{sign}(\Psi_{\mathrm{def}}) \neq sign(\dot{\Psi})$$

$$\Psi_{defm} = M_{z\max} / K_{\psi}$$

$$M_{z\max} = f(F_z) = a_2 \cdot F_z^{\ 2} + a_1 \cdot F_z$$

$$K_{\psi} = f(F_z) = b_2 \cdot F_z^{\ 2} + b_1 \cdot F_z$$

$$\dot{\Psi}_{def\,2} = -\frac{1}{\tau} \cdot \Psi_{def}$$

$$\tau = X_{rel} / (\omega \cdot r)$$

$$\Psi_{def} = \int_0^t \left(\dot{\Psi}_{def} + \dot{\Psi}_{def\,2}\right) \cdot \partial t$$

$$M_z = K_{\psi} \cdot \Psi_{def}$$

with:

M_z:	Aligning Torque.
K_{ψ}:	Torsional stiffness of the tire.
Ψ:	Yaw angle of the wheel plane.
$M_{z\max}$:	Maximum Torque that can be generated by the tire.
Ψ_{def}:	Torsional deflection of the tire.
Ψ_{defm}:	Maximum possible deflection of the tire.
$Xrel$:	Tire relaxation length.
ω:	Tire rotational velocity.
r:	Tire rolling radius.
F_z:	Vertical load on the tire.
a_1, a_2, b_1, b_2, c_0:	Model parameters.