STATE OF STRESS EVALUATION OF STRUCTURAL ELEMENTS BY MODAL SYNTHESIS

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ABSTRACT

In this paper various modalities of determination of stress time histories obtained by means of numerical simulation (MB modelling of mechanical systems combined with FE modelling of their components) are shown. A stress evaluation procedure, implemented by the authors for a particular MBS/FEM environment is shown. Meaningful comparisons are proposed between various procedures of stress calculation, using as reference a simple notched bar, whose finite element model has been validated by means of comparison with experimental result in literature. Analogous and further considerations on damage are developed on a simple articulated mechanical system, analysed within MBS/FEM, using the hypothesis of high cycle fatigue.

INTRODUCTION

In analysing the fatigue life of components belonging to mechanical systems subjected to variable loads in time, the evaluation of damage by means of numerical simulation is increasingly accompanying the traditional means which use experimental tests in laboratory or on field. Figure 1 represents a plot of the phases of a typical evaluation procedure to evaluate damage and thus the fatigue life of a mechanical component. The typical operative procedure is to define the structural loadings, to evaluate the state of stress and strain and finally, to assess damage and to predict life. The experimental approach, in particular, needs the measure of the operative loads, the measure of local strain by means of strain gauges placed in crucial points along the structure, the tests on the material to assess its fatigue behaviour and then, to use the typical estimation algorithms for damage. The numerical assessment procedure differs from the experimental one at least in the first two phases since it uses dynamical models and simulations of the system to define the operative loads and also finite element models of the components to assess the state of stress.
As regards the latter approach, the FE model (FEM) integrated in the multibody model (MBS) allows to quickly obtain useful data to assess damage and also to define important tests to evaluate (accelerated tests) the fatigue life of a mechanical component subjected to unknown loads, which can vary in time. This objective can be achieved by analysing time histories of the components of the local state of stress by Rainflow type counting methods, i.e. to count the closed hysteresis loops which are causes of cumulative damage of the component.
The procedure that is followed in this type of approach needs therefore not only a FE model of the component (honed in order to evaluate the state of stress and in particular, the stress concentration conditions), but also a multibody modelling of the mechanical system, of the knowledge of the material characteristics and of algorythms to analyse the stress/strain history and to evaluate damage.
In this paper in particular, the various modalities to determine the stress time histories obtained by means of MBS model of the mechanical system and of the FE model of the component are analysed. In particular, an evaluation procedure of the state of stress

implemented by the authors for a specific MBS/FEM simulation environment is being illustrated.

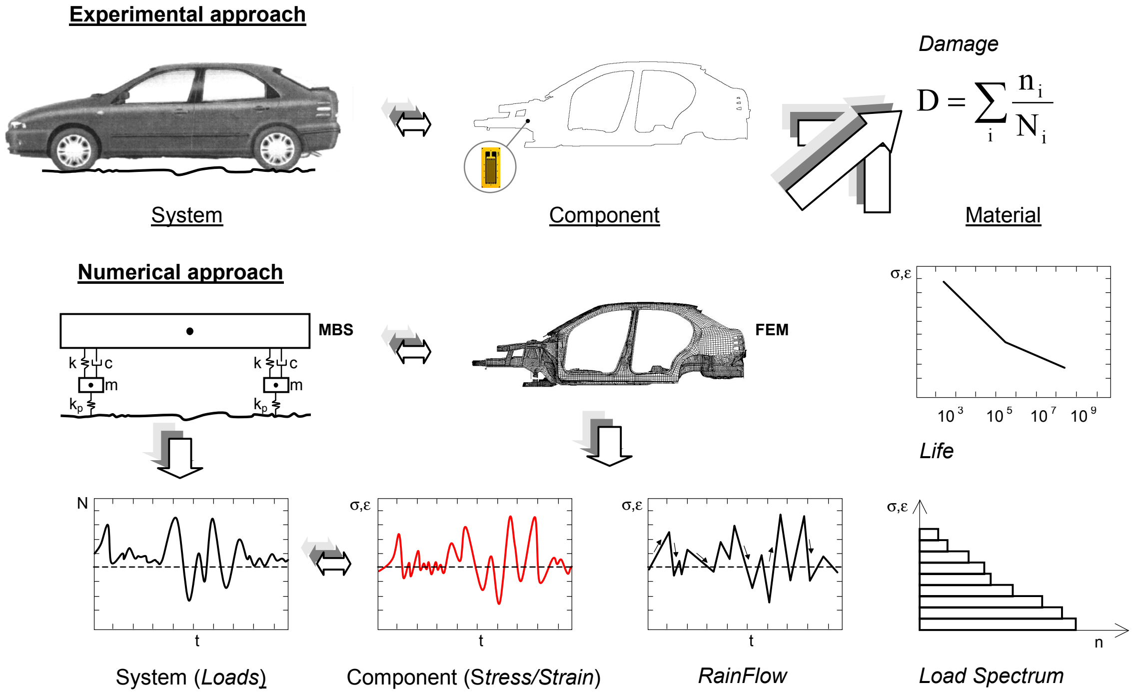


Fig.1 - Plot of a typical assessment procedure of damage

Important comparisons are suggested between the different stress evaluation modalities using, as reference point, a simple notched bar the finite element model which has been assessed comparing with the experimental results found in literature.
Further and analogous considerations have been developed on a simple articulated mechanical system, analysed in the MBS/FEM integrated environment, on which damage assessments were made assuming high cycle fatigue.

EVALUATING BY MEANS OF NUMERICAL SIMULATION OF THE STATE OF STRESS OF MECHANICAL COMPONENTS SUBJECTED TO FATIGUE

In order to develop a hypothetical automatic evaluation process of damage and therefore, of the fatigue life of a mechanical component by means of numerical simulation, in high and low cycle fatigue, it is necessary to know the time histories of stress and /of strain. When assessing damage, the time histories can then be processed by Rainflow counting method which supplies data in terms of matrixes of the closed hysteresis loops and of the residual open ones. Furthermore, data on the material should be available, that is to say, to define Wohler's curve which can be modified with Miner, Haibach or Liu-Zenner's corrections. Correction coefficients need to be introduced so that even the superficial finishing and the dimensional effects can be taken into account; in any case, the residual stresses should be considered which are added on as average stresses along with those induced by the loading histories. Finally, the cumulative damage theory must be defined. Since Rainflow's calculation does not preserve the cycle order, the option naturally remains to be Miner's law.
The "virtual" approach allows not only simulations to be repeated but also prototipation velocity which the "real" approach can not compete with. This does not imply that this methodology is without applicability difficulty nor without problems related to the method. As

mentioned in the introduction, the process needs FE modelling of the component. In order for it to be tuned enough to realistically simulate the state of stress and in particular, the stress concentration conditions, the FE model results usually to be very expensive to use and manage; anyhow, it needs a thorough experimental validation. Furthermore, it is necessary to model the dynamics of the entire mechanical system (MBS) in order to simulate its operations correctly and therefore the state of loadings which will involve the component. For this modelling phase, a full validation of the multibody model is difficult to achieve. For this reason, numerical evaluation of the damage and of the fatigue life of mechanical components by numerical simulations should always be considered in qualitative terms and therefore used mainly in the decisional phase of design choices.
From a methodological point of view, when modelling the component behaviour two main cases emerge: the dynamics of the component is negligible, that is its frequency content is oblivious to the external loads dynamic; the dynamics of the component is fundamental, that is its first frequency falls into the dynamic range of the external actions. A third possibility could also emerge: it is not possible to assess beforehand the relation between input and output dynamics.

Negligible dynamics: "static" approach

Let's consider the case when the dynamics of the component is negligible. The component is introduced in the multibody model of the system as a rigid body and is subjected to a series of external loads $L_1, L_2, \dots L_S$.. The time histories of these ones $L_1(t), L_2(t), \dots L_S(t)$ are available as simulation outputs. In this case, when assuming elastic material behaviour, and afterwards applying the necessary corrections in the case plastic material and of low cycle fatigue, the principle of an overlapping effect is valid so that it is only necessary to solve the elastic problem for each $L_1, L_2, \dots L_S$ load, applied separately but assumed united.
In every p point of the body, the coefficients $c_{ij,k}(p)$ are obtained relating to the k_{th} load which link intensity with the state of stress generated in point p. From a formality point of view, in a generic point of the body, the state of stress generated by the k_{th} load equals:

$$\boldsymbol{\sigma}_{\mathbf{ij},\mathbf{k}}(\mathrm{p}) = \mathbf{c}_{\mathbf{ij},\mathbf{k}}(\mathrm{p}) \tag{1}$$

When the effects are overlapped, the pseudo-stress elastically calculated equals:

$$\boldsymbol{\sigma}_{\mathbf{ij}}(\mathrm{t},\mathrm{p}) = \sum_{\mathbf{k}=1}^{\mathbf{S}} \mathbf{c}_{\mathbf{ij},\mathbf{k}}(\mathrm{p})\, \mathbf{L}_{\mathbf{k}}(\mathrm{t}) \tag{2}$$

Therefore, S elastic problems were solved with a unitary load and a simple linear combination was carried out to attain the state of stress and strain all along the structure.

Relevant dynamics: "modal" approach

In the case that the dynamic behaviour of the component is important, in compliance with the main multibody codes approach to analysing the system's motion, the introduction of bodies with elastic properties occurs by integrating a system of equations of motion expressed by Eulero-Lagrange. The approach used to describe a flexible body is the "modal" one. Deformations are estimated with a discrete displacements function obtained by multiplying the modal matrix $\mathbf{\Phi}$ with the generalised coordinates (modal) $\mathbf{q}$ (3), and by finding the position of the body $\mathbf{B}$ by coupling the small elastic deformations $\boldsymbol{\delta}$ with the general motion of the body considered as a rigid one $\mathbf{b}$ (fig.2).
Eulero-Lagrange's equations for a flexible object are expressed by (4) where L is Lagrangian, F is dissipation energy, Ψ the constrain equations, λ the constraints multipliers of Lagrange, ξ the generalised coordinates ,q_κ the modal coordinates of the flexible body equal to n and m the number of constraints.

$$\mathbf{B} = \mathbf{b} + \boldsymbol{\delta} \quad \text{con} \quad \boldsymbol{\delta} = \boldsymbol{\Phi} \cdot \mathbf{q} \tag{3}$$

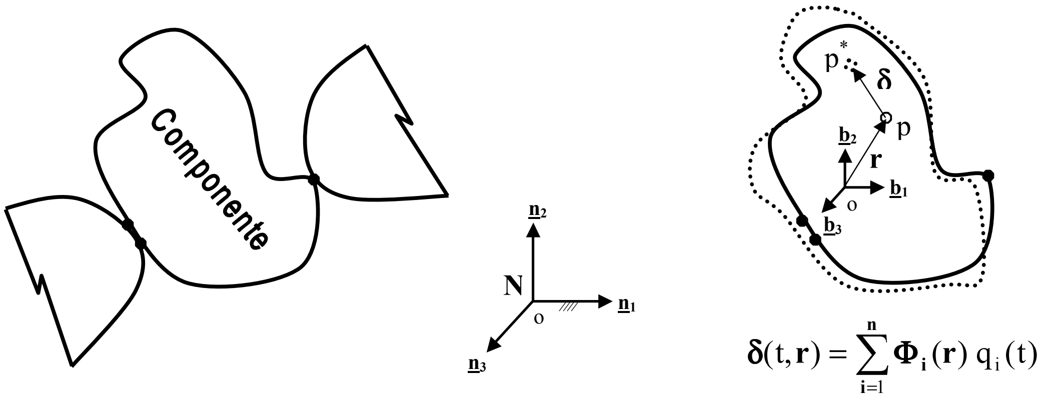


Fig.2 - Deformation state of the flexible component

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}_i}\right) - \frac{\partial L}{\partial \xi_i} + \frac{\partial F}{\partial \dot{\xi}_i} + \sum_{j=1}^{m}\left[\frac{\partial \Psi_j}{\partial \xi_i}\right]^T \lambda_j - Q_i = 0 \quad \text{con} \quad \Psi_j = 0 \tag{4}$$

$$\text{con } i = 1,\ldots,n+6, \quad j = 1,\ldots,m \quad \text{e} \quad \xi = [x \quad y \quad z \quad \psi \quad \vartheta \quad \varphi \quad q_k]$$

The method above expressed is difficult due to the choose of the best sets of generalised modal coordinates, i.e. of the modal forms to consider in modelling the behaviour of the flexible body.
So, by means of MBS simulation, the displacement of the generic point of the model can be expressed by modal overlap:

$$\boldsymbol{\delta}(t) = \sum_{\mathbf{i}=1}^{\mathbf{n}} \boldsymbol{\Phi}_{\mathbf{i}}\ q_i(t) \tag{5}$$

and, similarly, if the corresponding modal forms expressed in terms of stress $\boldsymbol{\Phi}_{\mathbf{k}}^{\boldsymbol{\sigma}}$ are known, the tensor of the state of stress can be attained in every point:

$$\boldsymbol{\sigma}_{\mathbf{ij}}(t) = \sum_{\mathbf{k}=1}^{\mathbf{n}} \boldsymbol{\Phi}_{\mathbf{k}}^{\boldsymbol{\sigma}}\ q_k(t) \tag{6}$$

Therefore it can be said that even when the dynamic contribution of the component is present, the stresses are in all points linear combinations according to known coefficients of quantities which depend on time and which in this case are not the loads but the modal coordinates.

Unknown dynamics: "MCS modal" approach (Mode **C**omponent **S**ynthesis**)**

It is not always possible to assess the importance of the dynamics of the component in determining its state of stress and strain. In some cases its flexibility is not easy to assess and therefore neither its modal contents inside the assembled system; in other cases it is difficult to assess the contents of loads frequencies to which it is subjected.
This paper indicates what mistakes can be made when assessing the state of stress of the component by choosing incorrectly between the two approaches described above. In particular, an evaluation procedure was developed which used a modal synthesis method

known in literature allowing to attain contemporarily a good modelling not only of the static behaviour but also of its dynamics doesn't affect the calculation of the entire numerical procedure in a considerable way.
Various modal synthesis methods exist in literature such as Craig-Bampton [12,13,14], MacNeal-Rubin [13,14] and Benfield-Hruda [13] which are all based on a certain selection of the generalised modal coordinates that is of the modal forms to consider when modelling the behaviour of the flexible body. In particular, a method of modal synthesis has been developed based on *Craig and Bampton*'s. On the one hand, this method allows to reduce to a minimum the number of generalised coordinates and on the other, to allow more freedom in defining the boundary conditions of the boundary points (constraints or applying forces points) by completely describing the effects of local flexibility. The motion of the flexible structural component, characterised by N degree of freedom and with fixed boundary points (or interface) is described by the combination of P *normal modes* and by S *static correction modes*. The former are the results of a modal analysis of the body, considering its boundary dofs as being fixed, whereas the latter are static deformed S obtained by imposing a unitary displacement for each degree of freedom of the interface and keeping all the others fixed.
The modal transformation which characterises this model is described in (7) in which the physical coordinates $\mathbf{x}$ are approximised with a modes summation $\overline{\mathbf{\Phi}}$ and $\mathbf{p}$ represents the new coordinate system; I indicates the degrees of freedom of the internal points (equal to R=N-S) and B the boundary dofs (equal to S). The $\mathbf{\Phi}^{C}$ (SxS) represents the static correction modes and matrix $\mathbf{\Phi}^{N}$ (RxR) the normal modes. Of this last matrix only P modes (< R) is considered of the latter.

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^{B} \\ \mathbf{x}^{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{\Phi}^{C} & \mathbf{\Phi}^{N} \end{bmatrix} \begin{bmatrix} \mathbf{p}^{B} \\ \mathbf{p}^{I} \end{bmatrix} = \overline{\mathbf{\Phi}}\,\mathbf{p} \tag{7}$$

$$\overline{\mathbf{M}}\,\ddot{\mathbf{p}} + \overline{\mathbf{K}}\,\mathbf{p} = \begin{bmatrix} \overline{\mathbf{m}}^{BB} & \overline{\mathbf{m}}^{BN} \\ \overline{\mathbf{m}}^{NB} & \overline{\mathbf{m}}^{NN} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{p}}^{B} \\ \ddot{\mathbf{p}}^{I} \end{bmatrix} + \begin{bmatrix} \overline{\mathbf{k}}^{BB} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{k}}^{NN} \end{bmatrix} \begin{bmatrix} \mathbf{p}^{B} \\ \mathbf{p}^{I} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{f}}^{B} \\ \overline{\mathbf{f}}^{I} \end{bmatrix} = \overline{\mathbf{f}} \tag{8}$$

The motion equation of the flexible body can be expressed as in (8), where the mass and stiffness matrixes are generalised.
For a simpler management of the flexible body, compared to the *C&B* 's original model, an orthonormalisation of the reduced system is carried out in the multibody simulation codes. This last phase yields a diagonal model, that is a $\widetilde{\mathbf{\Phi}}$ modal matrix image of the free-free behaviour of the flexible body, but with an elastic contribution of its deformability in the boundary points. (5) and (6) thus are still valid where the modal matrix in terms of displacement is represented by $\widetilde{\mathbf{\Phi}}$ in terms of stress $\widetilde{\mathbf{\Phi}}^{\boldsymbol{\sigma}}$ is attainable by applying the transformation used to orthonormalise the displacement modes at the matrix of the stress modes $\overline{\mathbf{\Phi}}^{\boldsymbol{\sigma}}$ corresponding to $\overline{\mathbf{\Phi}}$.
This modal modeling of the component yields, as the results illustrated in following paragraphs will show, how the elastic behaviour in static conditions is faithfully simulated; of course, this is guaranteed by a correct choice and determination of the static correction modes. As far as static conditions are concerned, it has been demonstrated how this approach is correct even when only using the static correction modes as modal base.
The above stated also demonstrates how this method is from a computational point of view equivalent to the "static" one. In fact, in both cases it is necessary to perform the FE model of the component; for the "static" approach the elastic analysis of the model has to be performed by solving many static analysis (S) with a unitary load for each external load; for the "MCS" approach, the static correction modes are needed, which means that many static analysis (S) have to be performed for unitary displacement for each boundary degree of freedom. In both cases therefore, there is the same computational burden except for the

procedure of image generation of the component for the multibody environment; finally it will only be necessary to have the MBS simulation results, in terms of loads in one case and in terms of lagrangian coordinates for the other, in order to carry out a simple linear combination to have the state of stress and strain in each point of the structure.
Therefore, the strenght of this approach is in the possibility to faithfully simulate not only the static conditions of the mechanical component but also its dynamical ones in terms of displacement and strain/stress by simply considering a modal model of the flexible body with a number of freedom degrees equal to $(P+S)$ obtained from P normal modes and S static modes.
In cases where it is difficult to assess the dynamical contribution of the component this type of approach allows to simulate the behaviour with a relatively higher computational burden (modal analysis to obtain P normal modes).
Another consideration should be made for this aspect of modelling. Theoretically, it is definitely more correct to model the component inside the dynamic model as a flexible part and not as a rigid one. This allows, as will be shown in this paper, to have a loads distribution inside the structure which allows for the elasticity of the single flexible components. Of course this does not mean that the evaluation of the state of stress cannot be carried out anyhow with a "static" approach that is considering the load histories and the $c_{ij,k}$ coefficients.
When considering the calculation burden, this method appears to need not only the modal model of the component, even if only with a modal base composed of sole static correction modes, but also to carry out a set of static analyses to achieve $c_{ij,k}$ coefficients: the modal approach would directly yield the correct evaluation of the loads condition and the state of stress.
It must be pointed out that the above described modalities are directly applicable to the study of high cycle fatigue by solely supplying the state of stress in the elastic field. Obviously, as regards low cycle fatigue the elastic tensor of pseudo stress must be modified in an elasticplastic tensor; as far as the procedure is concerned, this can be attained for instance, by Jiang’s transformation or by more approximative approaches such as Neuber’s law or Ramberg-Osgood’s formula: these are not however the topics of this paper.

Stress Recovery Procedure

Basing on what we have already shown we implemented a procedure to calculate stress which, starting from Adams MBS model of the system and Ansys FE model of component, lets to compute automatically the stress state of the whole system (or a part of it) using “static” and/or “Modal MCS”.
Analyzing the system it allows to identify the points where forces are applied and get their time histories. Furthermore it is possible to get modal coordinates time histories of modal coordinates, both orthonormalized and un-orthonormalized (split in original normal constrained modes and static correction modes). It is possible to export modal shapes of a part or of the whole model. Lastly it is possible to compute, directly within the software, a modal superposition of modes, in terms of displacement, and export time histories of the whole model or a part of it. Every output file can be created in binary or ascii format.
On the FEM side, starting from the MBS output, stress recovery is automatically available:

1. by a pseudo-static, motion driven analysis, imposing displacements on the model;
2. performing the S unit force static analysis on free component (IRLF) and using force time histories as combination factors;
3. by modal superposition, first applying mode shapes as motion driven static analysis to the model (partial or full) and then combining them with orthonormal modal coordinates or using mode shapes contained into the FEM result file got from the modal neutral file generation with de-orthonormaized modal coordinates (on full model).

With the aim of comparing the different methods shown, we also implemented a damage evaluation procedure which, based on critical plane theory and using Rainflow hysteresis counting algorithm, got load spectra at critical points of the flexible component letting to evaluate, chosen the fracture criterium and material properties, the damage, i.e. using the Miner damage cumulation law.

MBS (Adams)

Time Domain Analyses
$\mathbf{L_k(t)}$ Time Histories
$\mathbf{q_k(t)}$ Time Histories (Ortho)
$\mathbf{q_k(t)}$ Time Histories (deOrtho)
Modal Model
$\mathbf{\Phi_k}$ Full or **Reduced** Model Modes (Ortho)
Modal Linear Superposition (Ortho)
$\mathbf{u_{ij}}$ **(t)** Full or **Reduced** Model Disp. Time History

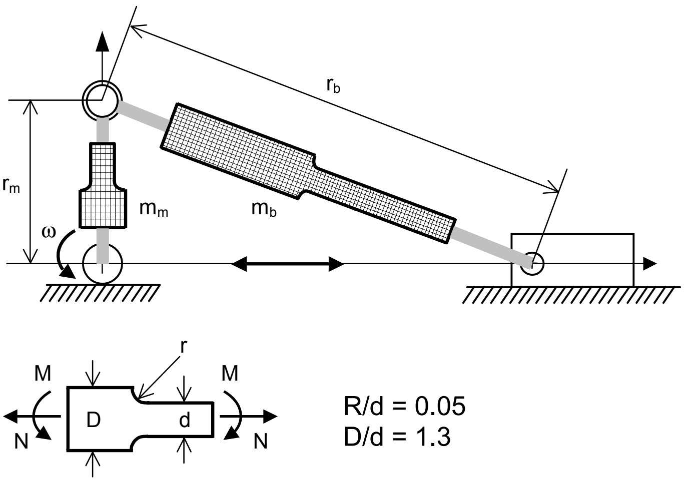
FEM (Ansys)

Time domain Analyses
on Full or Reduced Model
SR (by applied disp. $\mathbf{u_{ij}}$ **(t)**)
Static Linear superposition
on Full Model
Static analises ($\mathbf{L_k = 1}$)
SR by superposition ($\mathbf{L_k(t)}$ from MBS)
Modal Linear superposition (Ortho)
on Full or Reduced Model
Modes Recovery from MBS (by applied Disp. $\mathbf{\Phi_k}$)
Model SR by superposition (Ortho $\mathbf{q_k(t)}$ from MBS)
Modal Linear superposition (deOrtho)
on Full Model
SR by superposition (deOrtho $\mathbf{q_k(t)}$ from MBS)

Fig.3 – Stress Recovery toolkit capabilities

RESULTS

The methodology set up in a MBS/FEM simulation environment (Adams/Ansys) was analysed and verified on a simple in plane mechanism. The system is a slider-crank (fig.3) with no physical comparison with real slider-cranks, but nevertheless suitable in this context to point out each time the methodological problems in reconstructing the state of stress and in assessing damage. In MBS simulations the connecting rod and the crank are considered to be alternating rigid and flexible; the state of stress was set up in time by the "static" approach and the "MCS modal" described previously. Two different FE models were carried out for the connecting rod and the crank that would yield not only an extremely rigid crank and therefore, a presumed negligible dynamic contribution but also a connecting rod with, vice versa, extremely sensitive dynamics. For both components, the presence of notch with equal dimensions was considered (fig.3).



R/d = 0.05
D/d = 1.3

Fig.3 – Slider-crank sketch

Modelling the flexible component

First of all, a FE mesh was defined which would allow an optimal correspondence between the values of the stress concentration factors found in literature and

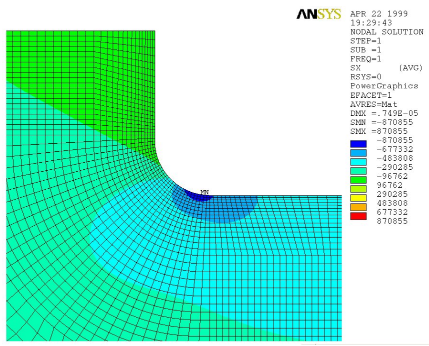


Fig.4 - FE Model detail of notch area

those attained numerically (fig.4).

Table 1 shows the results in terms of concentration factors as concerns the elastic static analysis performed in FE environment and by the "static" reconstruction and the "MCS modal" approaches. As regards the "MCS modal" reconstruction and the “static” one obtained from the loads assessed on a flexible model of the component, a modal model based on a modal base composed of only static modes was used. It is therefore obvious that there is perfect agreement among the results. The dynamic behaviour of the two components was then analysed and therefore the ability to reconstruct the dynamics in stress by the methods described previously. The comparison was carried out by using an FE harmonic analysis of the two models, considering a bending state of stress and an axial one separately. As far as the procedure of reconstructing in the multibody code is concerned homologous transient dynamical analyses were adopted by using a random loads as input (with frequency constant up to 200Hz) and by processing afterwards the results with classical experimental signal analysis procedures so that a comparison was possible in the frequency field. As regards the connecting rod, the flexible model was performed with a modal base composed of, not only by the same family of static correction modes as before but also by ten normal ones. The results show how the “MCS modal” reconstruction and the “static” one on the rigid and flexible model, coincide in static conditions (0HZ frequency); they remarkably differ however in the flexible model case (fig.5).

In this case, it is obvious how by only considering the modal modelling it is possible to correctly simulate the component behaviour in terms of stress both in static and dynamic conditions. As far as the crank is concerned, the extreme rigidity and therefore, the high frequencies which characterise its modes allow the rigid modelling and the consequent "static" reconstruction to yield correct results.

Table 1 – Stress Concentration Factors

Analysis	Bending Moment	Axial Force
Theory [1]	2.225	2.450
FEM	2.243	2.588
“Static” approach	2.243	2.588
“MCS” approach [2]	2.243	2.588

[1] Experimental data [1]

[2] Obtained only using the static correction modes

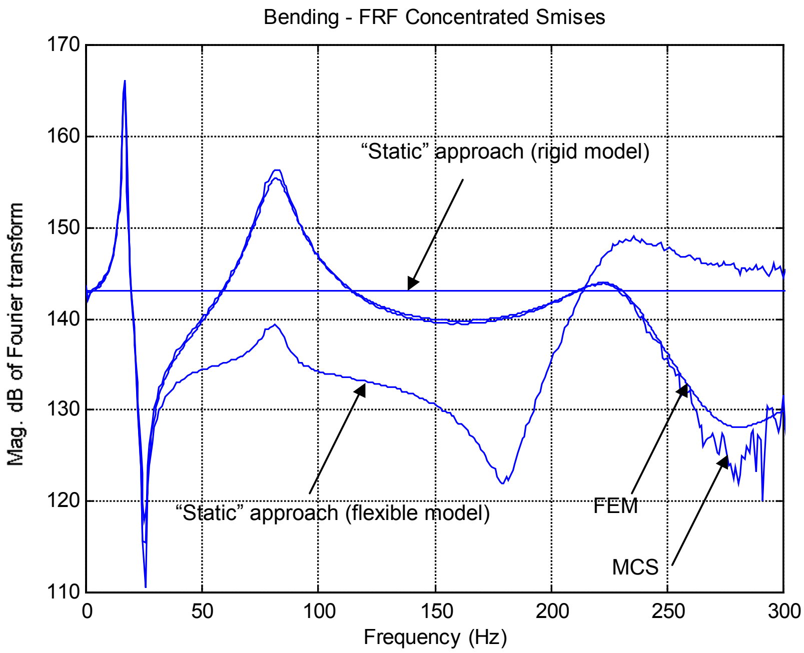


Fig.5 – Stress / Bending Moment FRF

Modelling the mechanical system

From system modelling point of view, It should be pointed out that when simulating complex mechanical systems (i.e. cars) and reconstructing the state of stress and damage of their flexible components, considered separately with high rigidity, (i.e. suspension arm), that is with natural frequencies beyond the field in exam, the current trend, confirmed by a vast bibliographical research, is to use the “static” approach with rigid modelling of the component.

In order to verify the correctness of this approach and especially the correctness of the rigid modelling of the component, some comparisons were made among the several stress recovery modalities from the transient analyses carried out on the kinematism with constant rotational speed of the connecting rod. The rotational frequency was largely lower to crank first free-free natural one. The system was characterised by perfect internal constraints, except for the connecting rod/crank node modelled with clearance and Hertz contact.

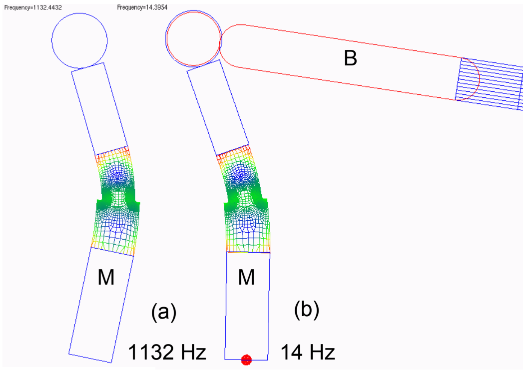


Fig.6 - (a) Crank first mode (free-free) (b) System first mode (M flexible, B rigid)

The simulations carried out in this work show also how it is surely more correct to always model the component, inside the dynamical model, as a flexible part of the component rather than as a rigid one. In fact, in this way, it is possible to consider the elastic behaviour variations of the system as function of the elasticity introduced by the single flexible components. In our testing system, the crank is a component which singularly shows high rigidity (fig.6a). Its elasticity, introduced by a "MCS modal" model only by static corrections modes, is enough to determine a noticeable variation in the behaviour of the system both in terms of natural frequencies (fig6) and then, in terms of loads, stress and damage. In fact, by comparing two system models, the former which assumes the two components as rigid and the latter which considers the flexible connecting rod it can be seen how there are remarkable behavioural differences. When analysing the section of time histories relative to 10 slider-crank cycles, for the two models, in the notch, have been evaluated: the critical plane, the stress time history relative to this plane (fig7), the rainflow matrix of this stress history (fig8), the load spectrum (fig9), and in the assumption of Miner's damage cumulation law, by using Miner's curve modified by Haibach, the damage and the component's life have been evaluated. These comparisons have demonstrated the previous statements, supplying a life of $8.6 \cdot 10^5$ repetitions for model with rigid crank, and $2.4 \cdot 10^5$ for the model with flexible one.
In the case of the latter model (flexible component with high rigidity) the use of the "static" approach results to be equivalent to the "MCS modal" as far as the choice of reconstruction modality of the state of stress is concerned.
As regards the connecting rod, the model with both components being rigid, and the model

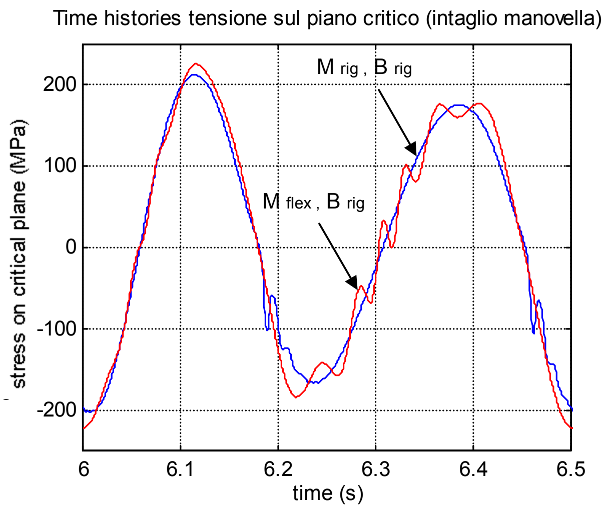


Fig.7 – Stress time histories zoom on notch critical plane.

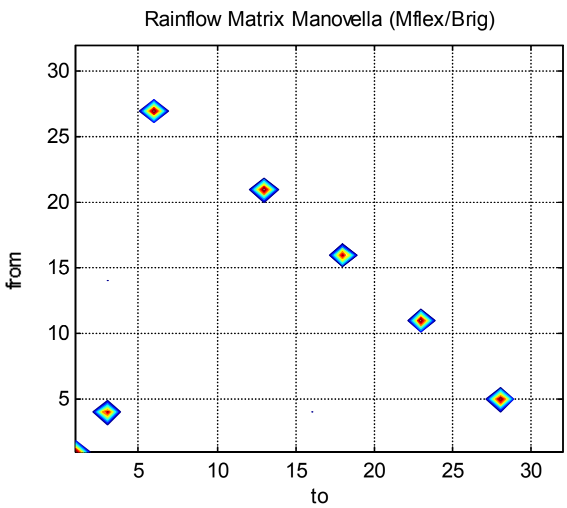


Fig.8 - Rainflow matrix of stress on notch critical plane

for a flexible connecting rod were considered, the latter being modelled by static modes and 15 normal modes.
Furthermore, the excitation low frequency content, mostly represented by the rotational frequencies of the crank, shows for this component also, assumed to be highly flexible that the "static" reconstruction is similar enough to the one carried out by "MCS modal" modelling. It is also obvious that, as shown in the analyses performed on the single components (fig 5), when in presence of excitation with a frequency content distributed in a significant range, the "MCS modal" model is the only analysis modality which correctly reconstructs the component behaviour in terms of strain and stress.

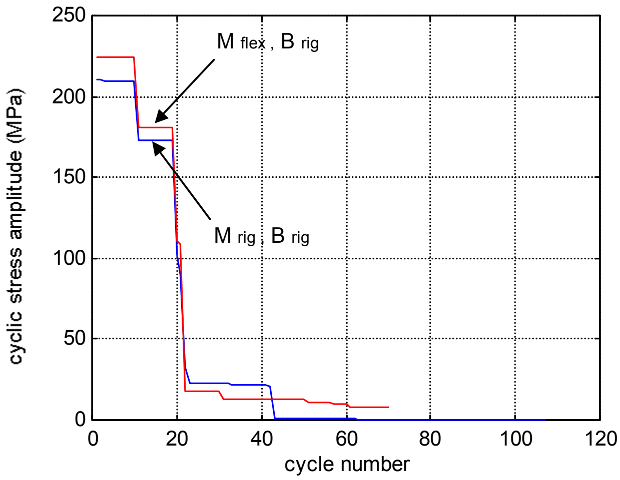


Fig.9 – Weighted load spectra

CONCLUSIONS

This work has shown how a correct evaluation of the mechanical component damage, even with high rigidity, belonging to articulated and complex mechanical systems, needs its flexible modelling.
Future developments in research can finalise the criteria for the analysis and the synthesis of load and stress histories triggered by numerical simulations and/or experimental acquisitions, and of algorithms to automatically evaluate damage.

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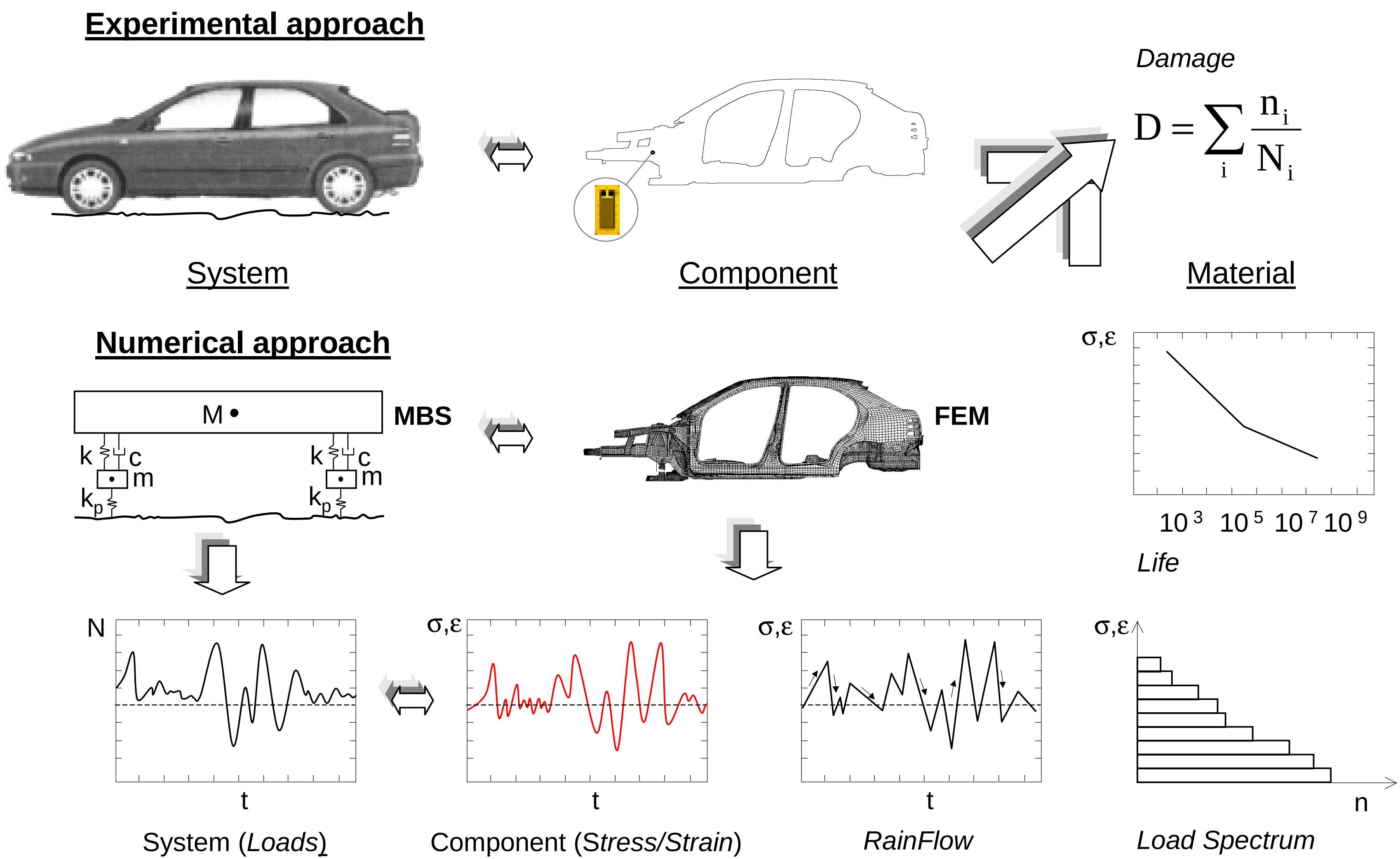
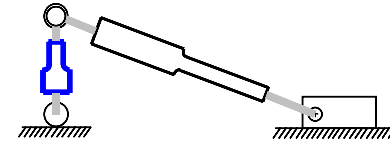
Mechanical Dynamics

Damage evaluation procedure

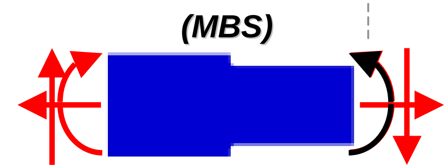


Stress Recovery

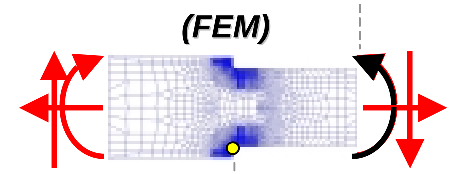
*Neglegible dynamics: **"static" approach***



$L_k(t)$

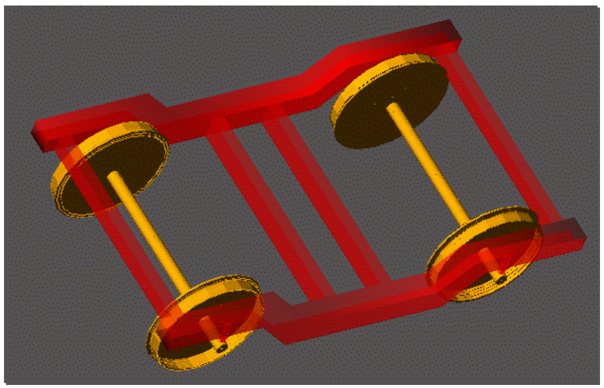


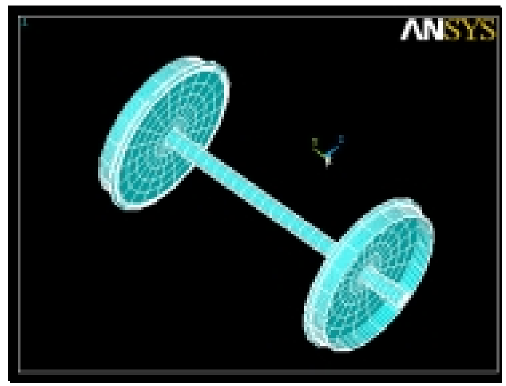
$L_k = 1$



$c_{ij,k}(p)$

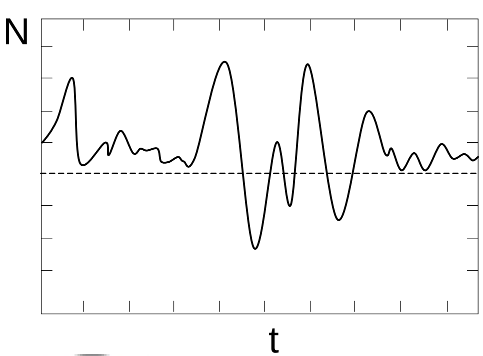
System	Component

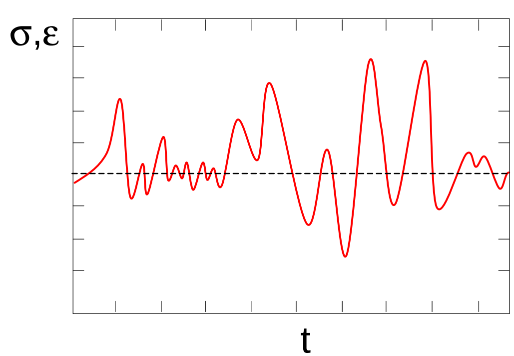




Loads

Stress / Strain



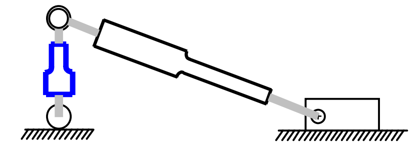


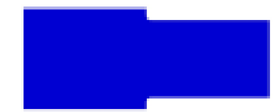
$$\sigma_{ij,k}(p) = c_{ij,k}(p)$$

$$\sigma_{ij}(t,p) = \sum_{k=1}^{S} c_{ij,k}(p)\, L_k(t)$$

Stress Recovery

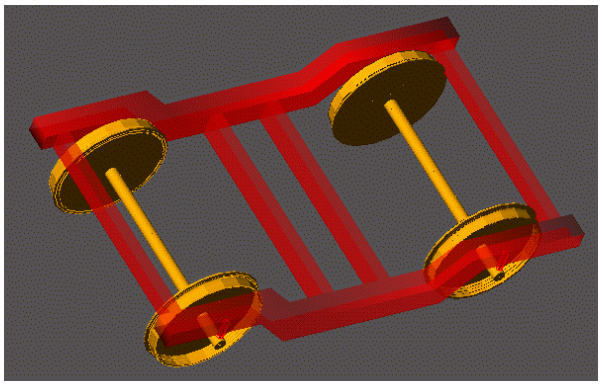
*Relevant dynamics: **"modal" approach***

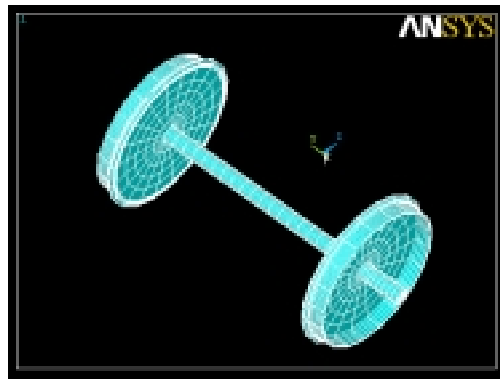


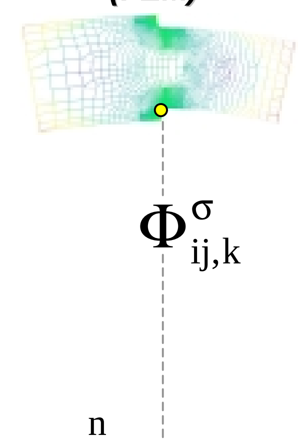


System
(with flexible components)

Component







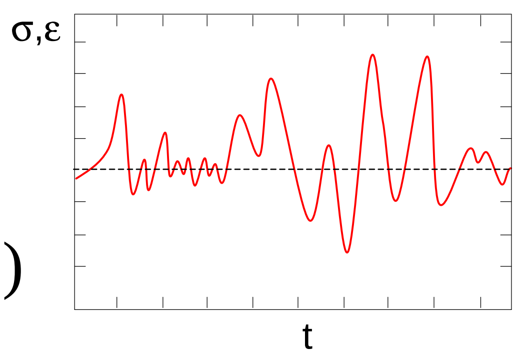
Lagrange Coordinates

Stress / Strain

$$B=b+\delta$$

$$\delta=\Phi\cdot q$$

$$\delta(t,r)=\sum_{i=1}^{n}\Phi_i(r)\,q_i(t)$$

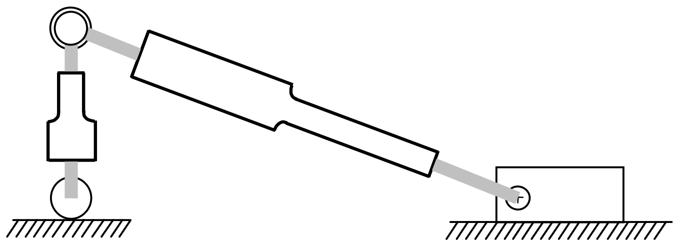


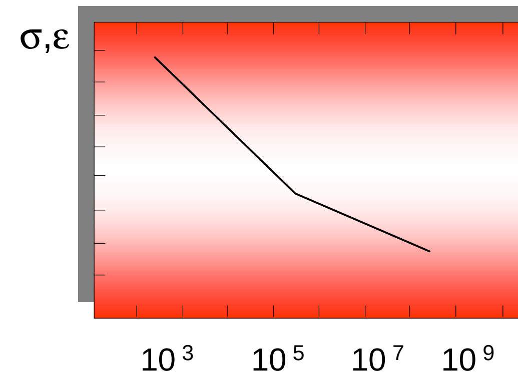
$$\sigma_{ij}(t,p)=\sum_{k=1}^{n}\Phi^{\sigma}_{ij,k}\,q_k(t)$$

Damage Evaluation

Material

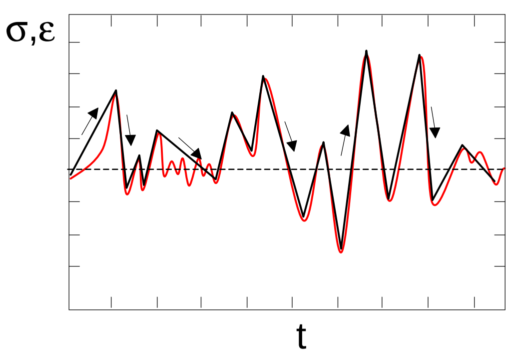
(MBS) with Flexible bodies



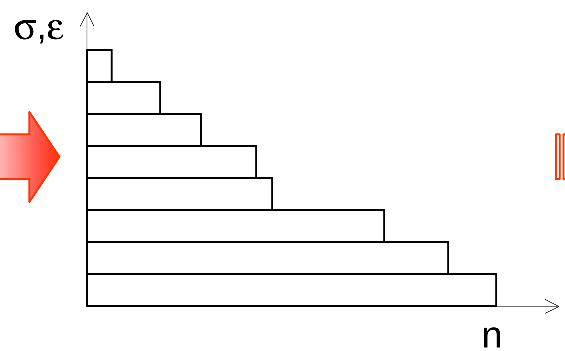


Life

RainFlow



Spectra

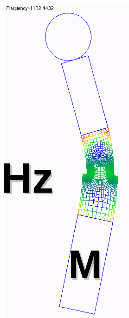


$$D = \sum_i \frac{n_i}{N_i}$$

Damage

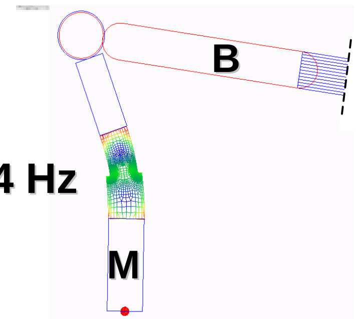
Why use Flexible Approach ?

1132 Hz



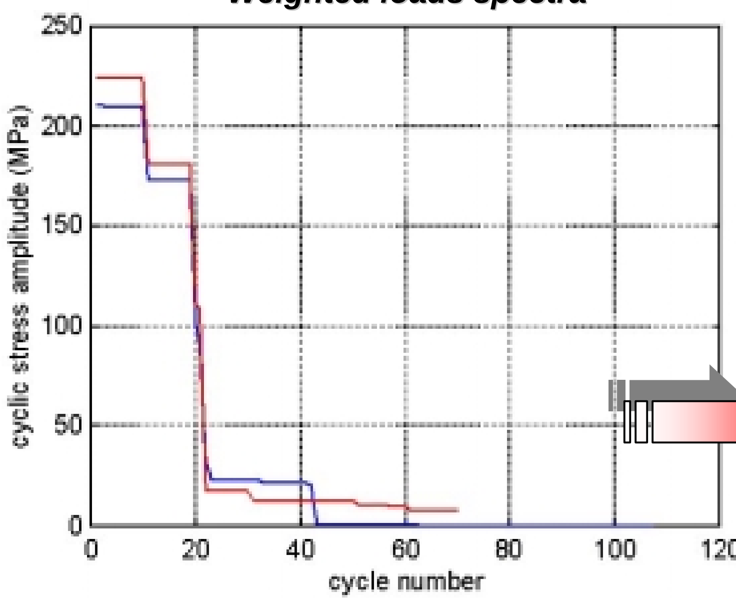
First natural mode of free-free crank

14 Hz

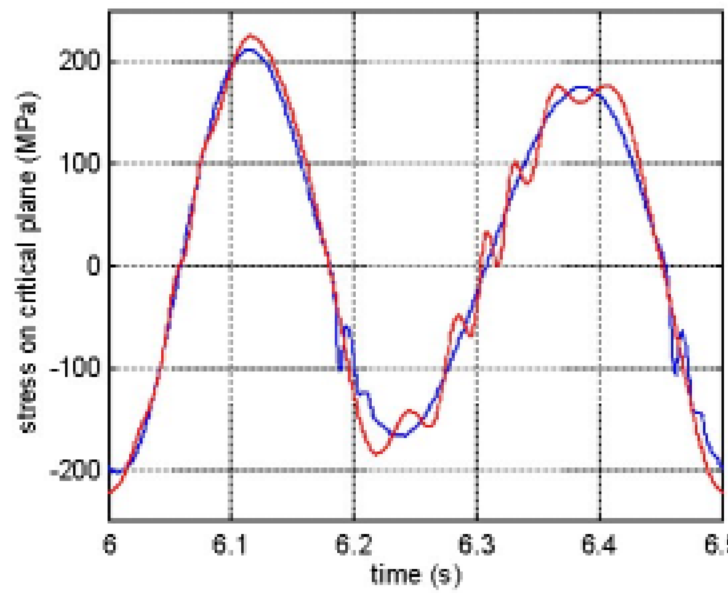


First natural mode of system in a particular configuration

Weighted loads spectra



Stress Time histories on notch critical plane

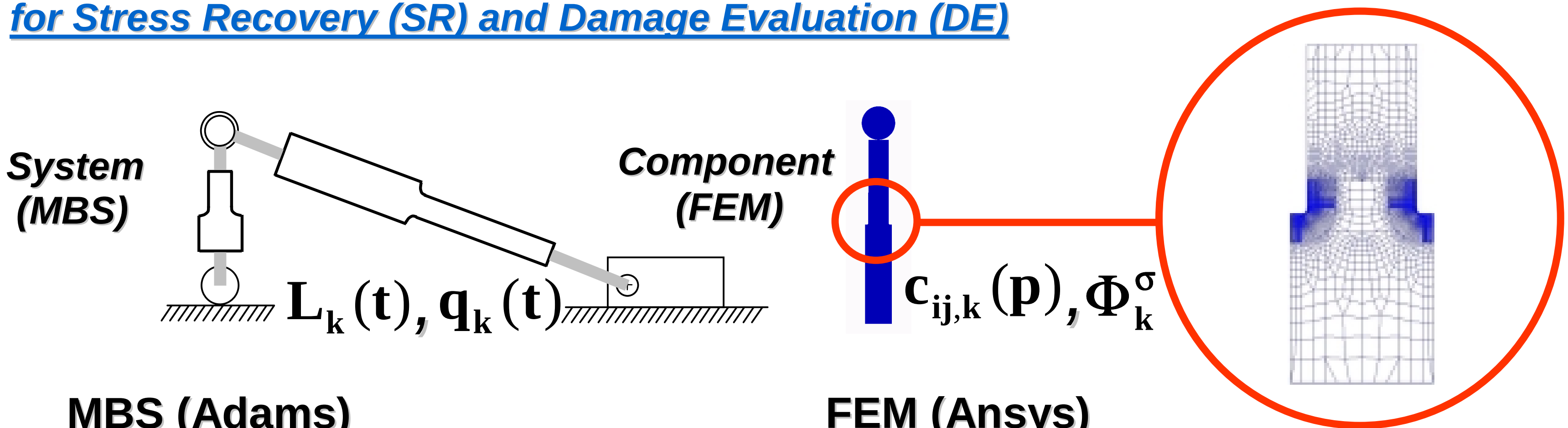


Rigid crank: life of $\underline{8.6 \cdot 10^5}$ cycles

Flexible crank: life of $\underline{2.4 \cdot 10^5}$ cycles

ADAMS & ANSYS Integrated Tools

for Stress Recovery (SR) and Damage Evaluation (DE)



MBS (Adams)

Automatic Procedure

Time Domain Analyses
L_k(t) Time Histories
q_k(t) Time Histories (Ortho)
q_k(t) Time Histories (deOrtho)

Modal Model
Φ_k Full or **Reduced** Model Modes (Ortho)

Modal Linear Superposition (Ortho)
u_{ij} (t) Full or **Reduced** Model Disp. Time History

FEM (Ansys)

Automatic Procedure

Time domain Analyses on Full or Reduced Model
SR (by applied disp. **u_{ij} (t)**)

Static Linear superposition on Full Model
Static analises (**L_k= 1)**
SR by superposition (**L_k(t)** from MBS)

Modal Linear superposition (Ortho) on Full or Reduced Model
Modes Recovery from MBS (by applied Disp. **Φ_k**)
Model SR by superposition (Ortho **q_k(t)** from MBS)

Modal Linear superposition (deOrtho) on Full Model
SR by superposition (deOrtho **q_k(t)** from MBS)

ADAMS & ANSYS Integrated Tools
for Stress Recovery (SR) and Damage Evaluation (DE)

MBS (Adams) GUI

Time Domain Analyses

L_k(t) Time Histories

q_k(t) Time Histories (Ortho)

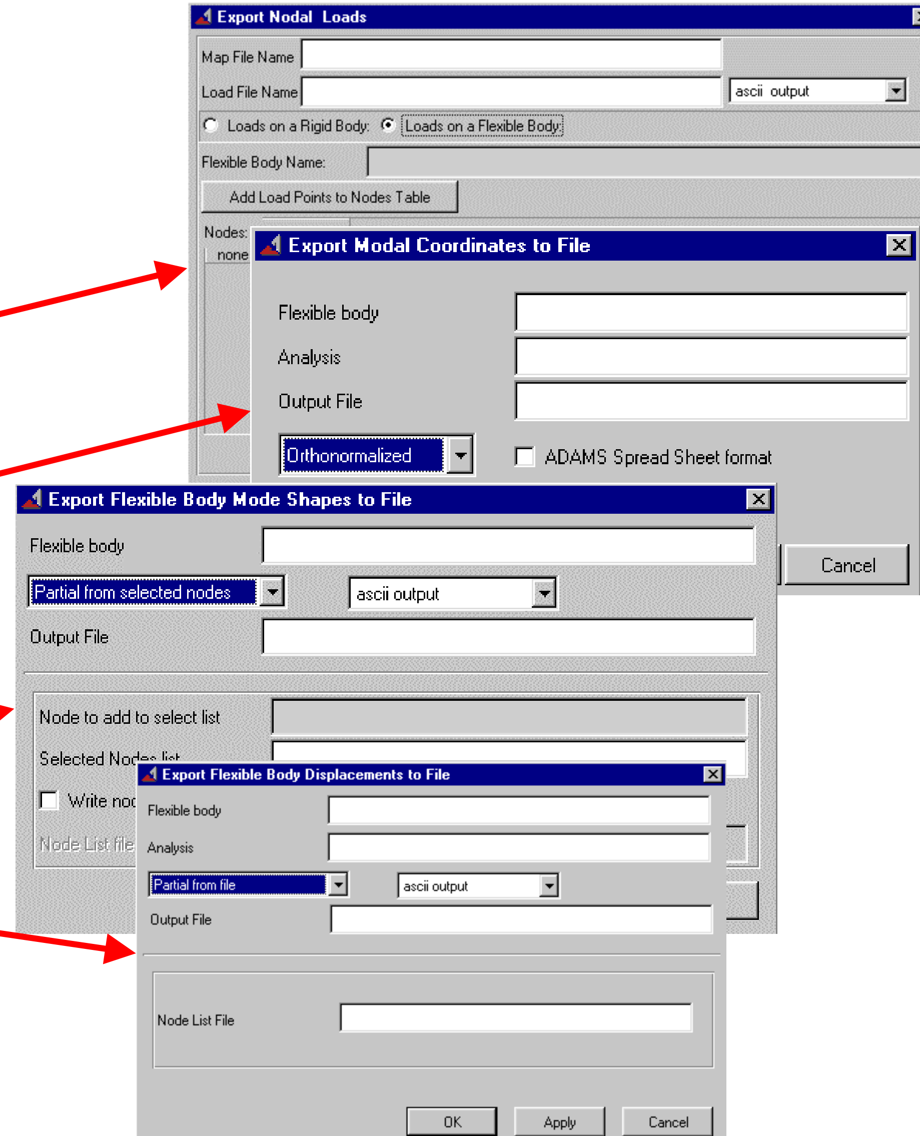
q_k(t) Time Histories (deOrtho)

Modal Model

Φ_k Full or **Reduced** Model Modes (Ortho)

Modal Linear Superposition (Ortho)

u_{ij} (t) Full or **Reduced** Model Disp. Time History



Mechanical Dynamics

FEM (Ansys)GUI

Time domain Analyses

on Full or Reduced Model

SR (by applied disp. $\mathbf{u_{ij}}$ **(t)**)

Static Linear superposition

on Full Model

Static analises ($\mathbf{L_k}$**= 1)**

SR by superposition ($\mathbf{L_k(t)}$ from MBS)

Modal Linear superposition (Ortho)

on Full or Reduced Model

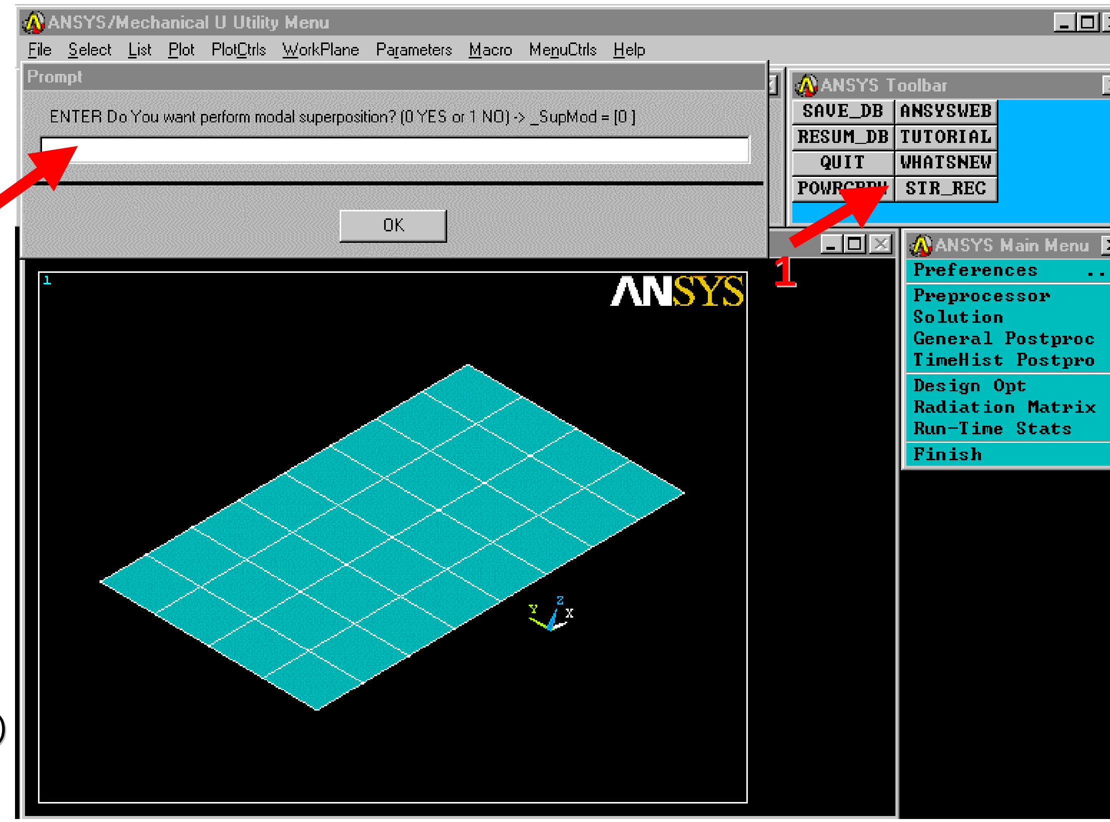
Modes Recovery from MBS (by applied Disp. $\mathbf{\Phi_k}$)

Model SR by superposition (Ortho $\mathbf{q_k(t)}$ from MBS)

Modal Linear superposition (deOrtho)

on Full Model

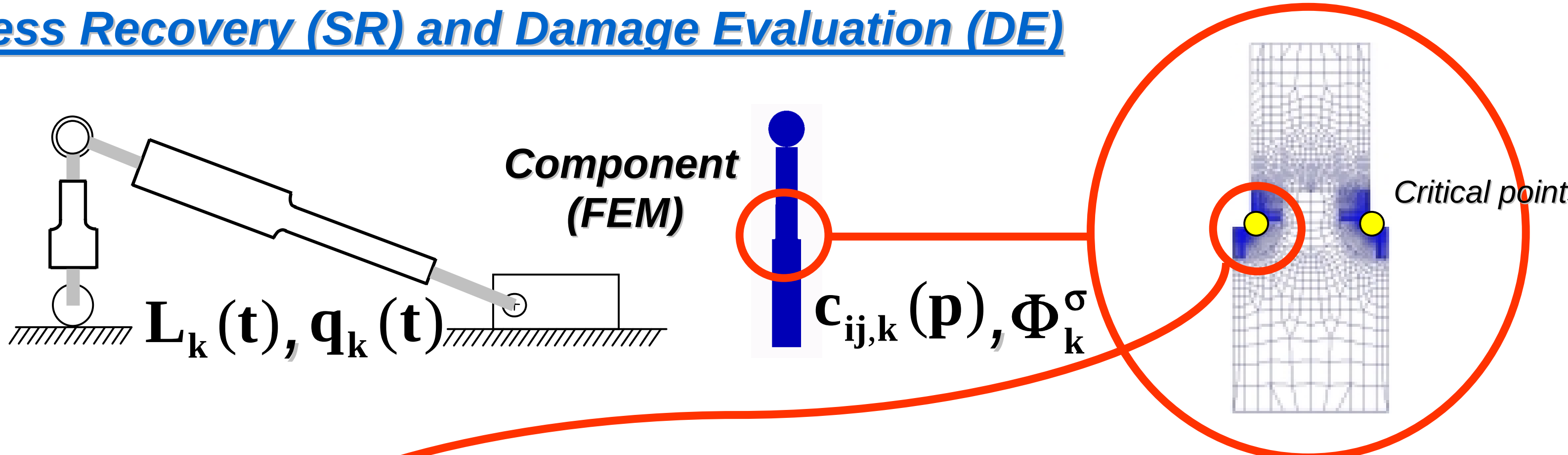
SR by superposition (deOrtho $\mathbf{q_k(t)}$ from MBS)

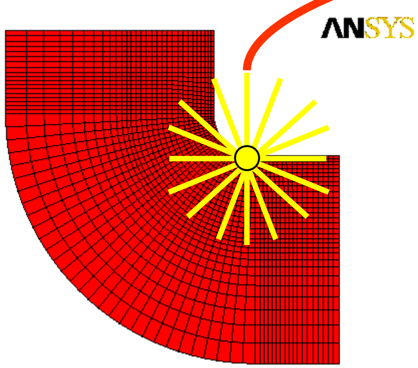


ADAMS & ANSYS Integrated Tools
for Stress Recovery (SR) and Damage Evaluation (DE)

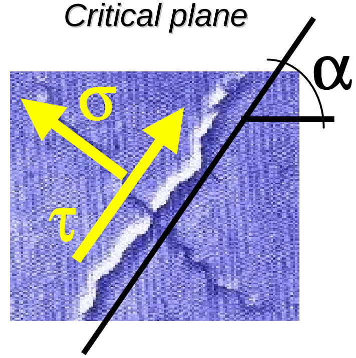
System (MBS)

Component (FEM)





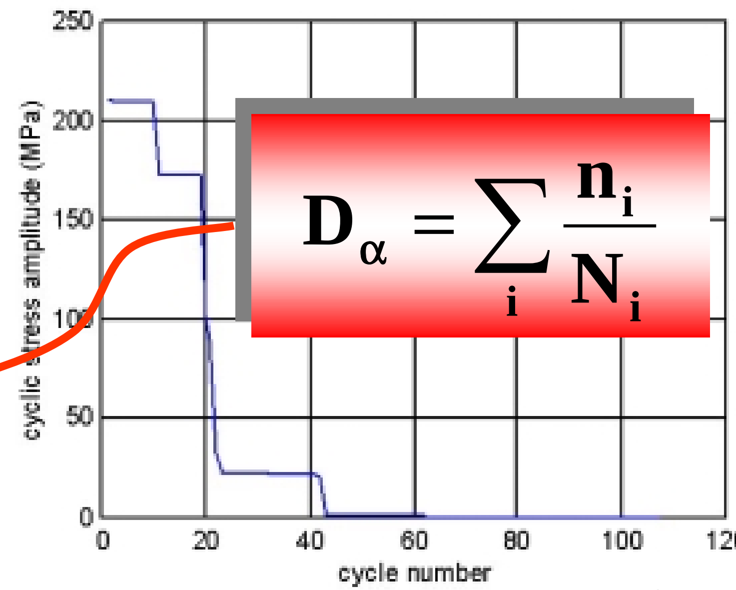
Critical plane



Stress TH on critical plane

Rain Flow Matrix of Stress TH

Weighted Load spectrum



Material

Stress Life — Wöhler; Miner m., Haibach, Liu Zenner; Miner

Strain Life — Ramberg-Osgood; Neuber; Manson-Coffin; Smith-Watson-Topper

Università degli Studi di Perugia - Dipartimento di Ingegneria Industriale

Mechanical Dynamics