

## Distribution of loads in Cycloidal Planetary Gear (CYCLO) including modification of equidistant

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### Abstract

The complex construction of planet wheels in cycloidal planetary gear (Cyclo) practically makes impossible its optimal design. In planetary gear (Cyclo), toothings of planet wheels has shape of equidistant of shortened epicycloid.

Till now, there are applied nominal toothings of wheels in which only one equidistant occurs identical for planet wheel and co-operating wheel. In the paper it is presented original modification of inside cycloidal meshing based on diversification of equidistant of planet wheel toward co-operating wheel.

Two methods and comparisons between analytical and ADAMS model are presented.

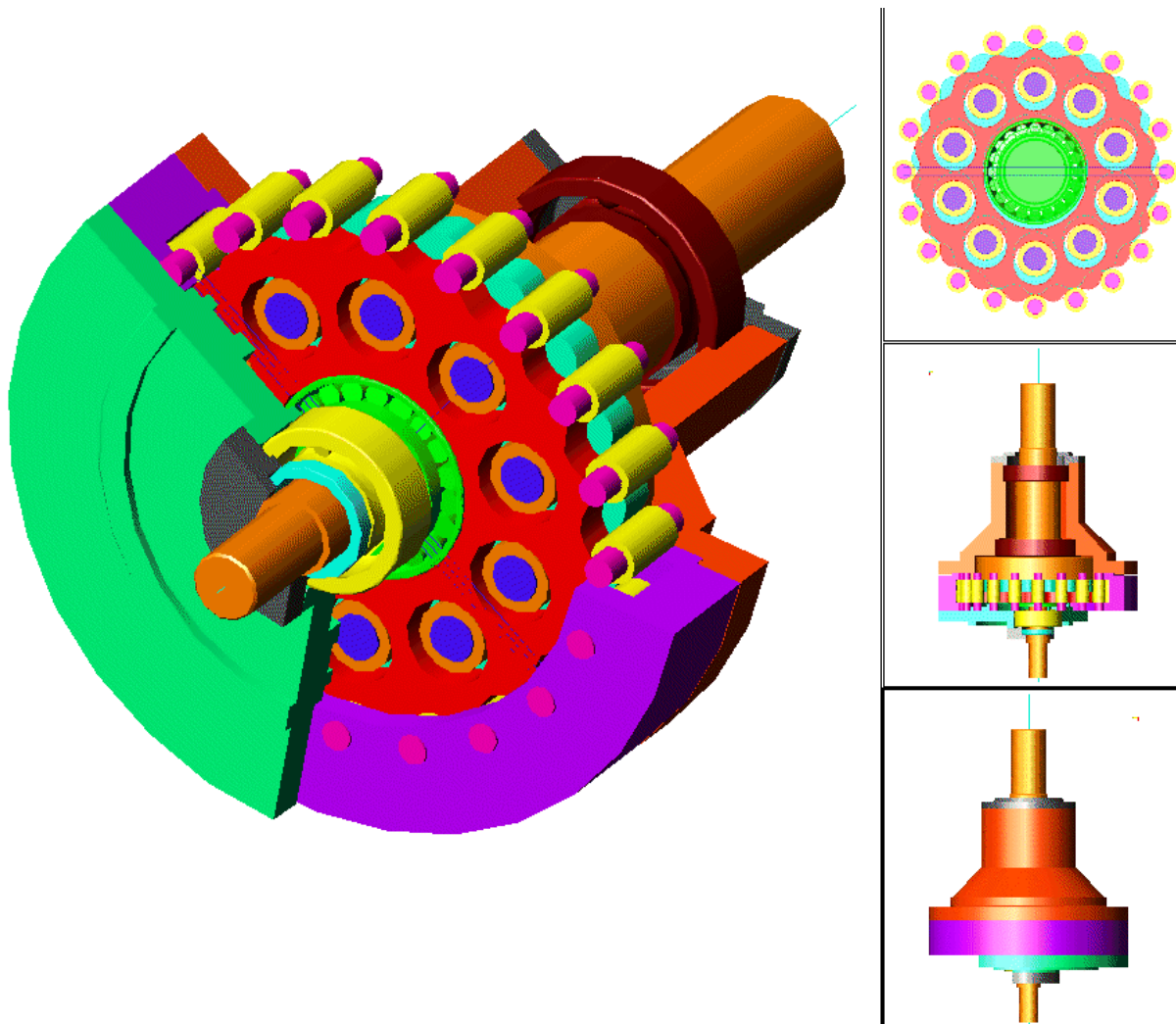


Fig.1: Cycloidal Planetary Gear (CYCLO)

**Introduction**

Currently, in heavy industry, high speed engines are used for many types of machines that requires application high ratio mechanical gears. Relatively, the smallest mechanical gear is the cycloidal planetary gear known as cyclo gear (CPG).

Cycloidal gear implements inside, out of centre meshing to obtain high ratios at one stage, high coefficient of teeth in contact and low dissipation of energy resulting from occurring only roller friction in the gear and coaxiality of shafts [2,3,4].

Figure 2 shows the kinematics scheme. The cyclo gear consist of planetary gear Fig.2 and straight-line mechanism Fig.2b in series connection. Because of that kind of connection we get compact gear with stationary central gear (2), which is meshing with one or two planet wheels (1, 1') driven by the eccentric yoke (3), Fig. 2c. In case of immovable stationary wheel (2), a kinematics ratio is given as follows:

$$i = -\frac{z_1}{z_2 - z_1} = -\frac{z_1}{\Delta z} \tag{1}$$

where:

- $z_1=z_s$  is a number of teeth of planet wheel 1 or 1',
- $z_2=z_k$  is a number of teeth (rolls) of stationary gear 2.

When a high ratio is required within the limits of  $i=11:87$  - a difference  $\Delta z = 1$ .

Outline of planet wheel is an equidistant of shortened epicycloid abbreviation ESE [1,2,3]. Central gear 2 consists of set of rolls. Torque from the planet wheels is transmitted by the bolts and disk of straight-line mechanism 4 Fig.2.

The main element of the cyclo gear connecting others elements is the planet wheel 1. For balancing body forces and lowering of meshing forces two identical planet wheels 1, 1' are applied, with reverse angle between them equal  $\pi$ . Planet wheel (1 or 1') is a flat disk which perimeter has a shape of equidistant of shortened epicycloid. In the centre of the wheel there is a big round hole for high effort roller bearing of eccentric and round about it smaller holes for bolts of straight-line mechanism.

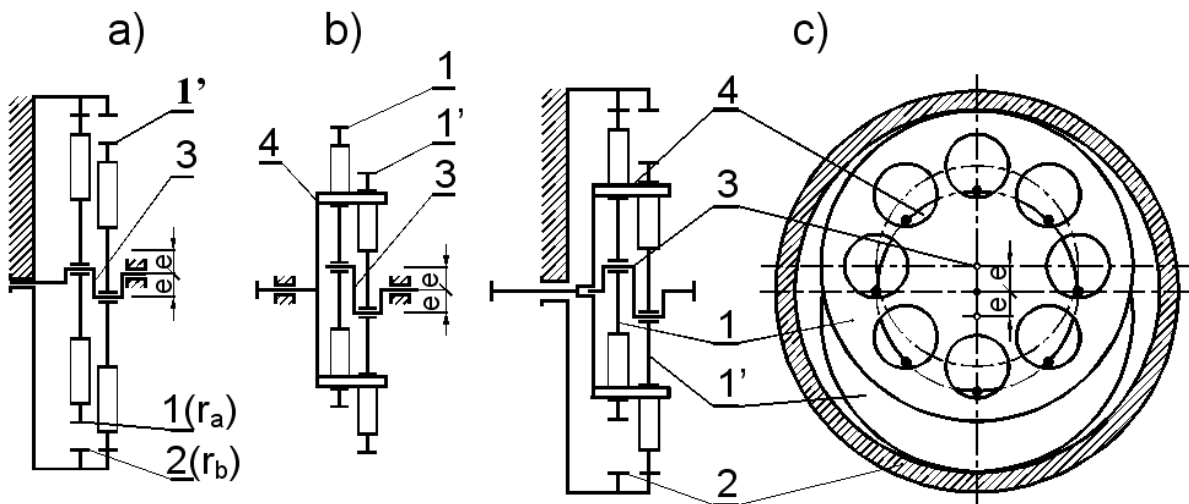


Fig 2. Kinematics scheme of planetary cycloidal gear

Inside toothing is realised by two planet wheels and immovable central wheel, Fig.2. Planet wheel or wheels have inside teeth and curvilinear shape of equidistant of shortened epicycloid in abbreviation ESE [2]. Till now it is applied unmodified inside cycloidal meshing which is based on occurring one and the only one ideal ESE as well for planet wheel as for cooperating wheel. That kind of meshing can be called **nominal** and its characteristic feature is absence or casual configuration of clearances between teeth being the reason of discontinuity or even stopping of revolving movement.

Parameters of equidistant:

- Module pitch of epicycloid 
$$M = \frac{a}{b} \quad (2)$$

- coefficients of shortening of epicycloid 
$$m = \frac{e}{b} = \frac{ez_K}{r} \quad (3)$$

where

e – eccentric of gear

a – radius of constant wheel of epicycloid (Fig.2 – 1 (ra))

b – radius of rolling wheel (Fig.2 – 2 (rb))

r = a + b – sum of radius ‘a’ and ‘b’

In this paper is shown differences between models of CYKLO with nominal and modified equidistant’s parameters applied as numerical method in higher level approximates calculation model to conditions in real gear. To visualize differences there is also the analytical method as well as numerical one presented and obtained results has been compared with each other.

### Modification of inside cycloidal meshing

Basic idea of modification of inside cycloidal meshing is diversifying the equidistant of planet and co-operating wheel. It will occur two different equidistant, Fig.4:

Nominal equidistant SQT characterised by known parameters r, e, q, m representing stationary co-operating wheel (set of rolls)

Corrected equidistant S<sub>1</sub>Q<sub>1</sub>T<sub>1</sub> with unknown parameters r<sub>k</sub>, e<sub>k</sub>, q<sub>k</sub>, m<sub>k</sub>

Corrected equidistant will cause clearances between teeth {Δr<sub>i</sub>} and pitch play δ. When equidistants are co-linear then initial configuration of clearances between teeth occur {Δr<sub>oi</sub>} enabling assembling of the elements of the gear, Fig.3 and 4. After revolution of planet wheel with angle δ modified equidistant will be tangent to nominal in point S and clearances between teeth would create other asymmetric configuration {Δr<sub>i</sub>} with infinitely low clearances in active part of toothing and higher clearances in idle part of toothing, Fig.3.

A measurement of distribution of loads in active part of toothing can be factor of contact (or pseudo-contact) designating sets of clearances lower than assumed criteria value Δr:

$$\varepsilon = \frac{|\alpha_2 - \alpha_1|}{\alpha_{pc}} \cong \frac{|\alpha_{2p} - \alpha_{1p}|}{\alpha_{pc}} \quad (4)$$

where:

α<sub>2</sub>, α<sub>1</sub> - generating angles of nominal ESE, in extreme points 1 and 2 at the contact arc;

α<sub>2p</sub>, α<sub>1p</sub> - angles of position of extreme points of contact arc 1 and 2, Fig. 4;

α<sub>pc</sub> - angular pitch of toothing in gear.

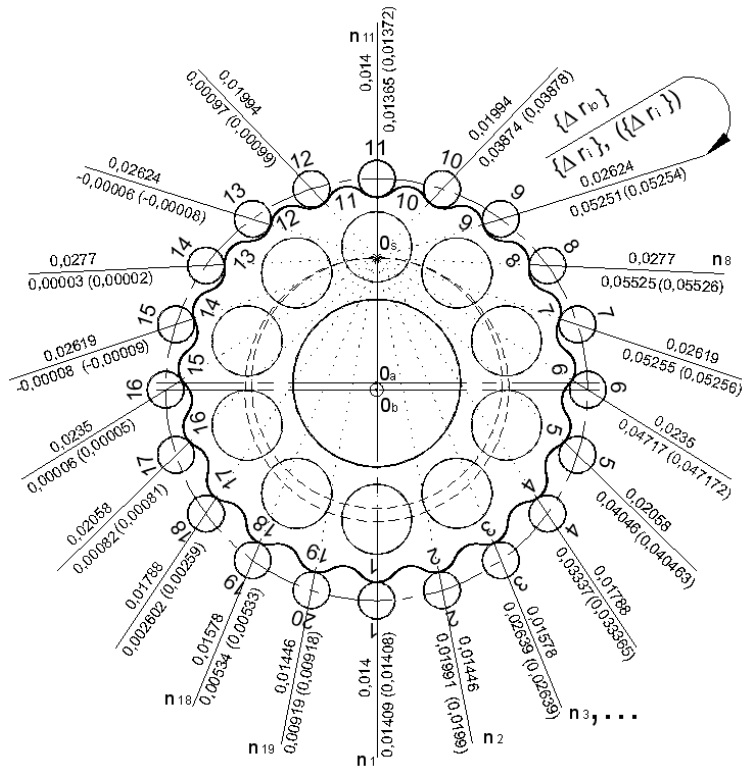


Fig. 3. The inside cycloidal meshing in planetary gear with configuration of clearances between teeth modified planet wheel for  $\epsilon = \epsilon_{\max} = 0,45$  (90 % teeth in active contact)

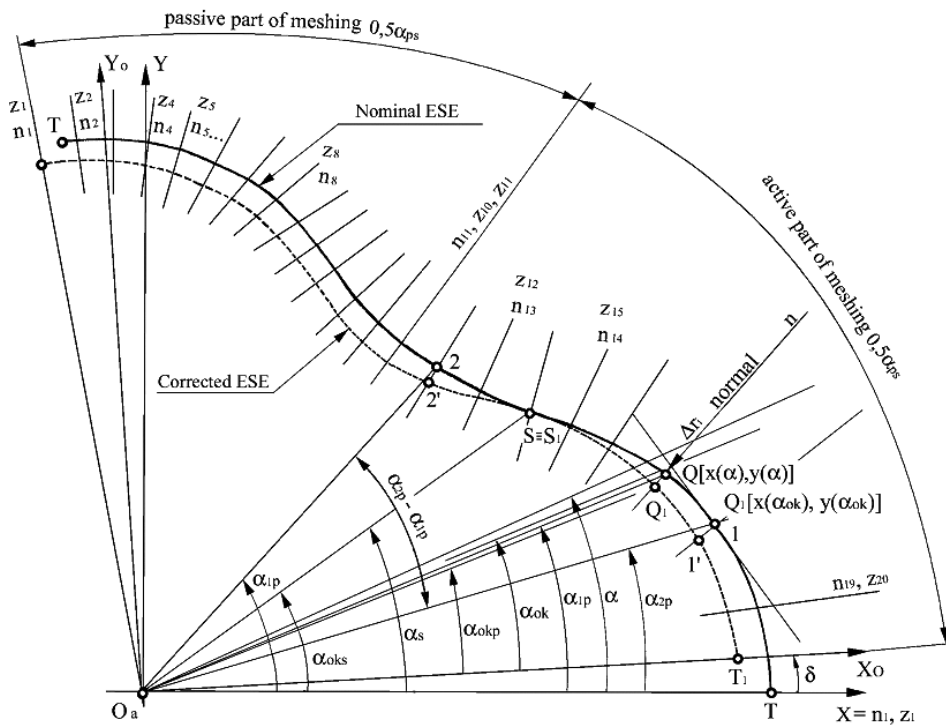


Fig. 4. Distances of equidistant  $\{\Delta r_i\}$  and contact arc  $|\alpha_{2p} - \alpha_{1p}|$  after revolution of planet wheel with angle  $\delta > 0$

## Mathematical model of modification of toothing with implementation of diversification of equidistant

The goal of modification is modelling of clearances (distances) between teeth by means of correction of equidistant. For known parameters of nominal equidistant  $r, e, q, m$  in gear with ratio  $i = z_s = z_k - 1$  it is searched corrected equidistant fulfilling criteria of optimisation :

- sufficiently high factor of contact  $\epsilon$ ,
- critical values of clearance between teeth  $\Delta r, \Delta r_o$ ,
- sufficiently low angle of pitch play  $\delta \leq \delta_{\min}$ ,
- equal values of equidistant eccentrics  $e = e_k$ .

After introduction stationary and movable co-ordinate system, Fig. 4:

- co-ordinates of nominal equidistant in stationary co-ordinate system Oxy

$$x(\alpha) = r \cdot \cos(\alpha) + e \cdot \cos(z_k \cdot \alpha) - q \cdot \cos \left\{ \alpha + \arctg \left[ \frac{\sin(z_s \cdot \alpha)}{m^{-1} + \cos(z_s \cdot \alpha)} \right] \right\} \quad (5)$$

$$y(\alpha) = r \cdot \sin(\alpha) + e \cdot \sin(z_k \cdot \alpha) - q \cdot \sin \left\{ \alpha + \arctg \left[ \frac{\sin(z_s \cdot \alpha)}{m^{-1} + \cos(z_s \cdot \alpha)} \right] \right\} \quad (6)$$

- co-ordinates of corrected equidistant in movable co-ordinate system O<sub>x<sub>o</sub></sub>y<sub>o</sub>

$$x_{ok}(\alpha_{ok}, r_k, q_k) = r_k \cdot \cos(\alpha_{ok}) + e_k \cdot \cos(z_k \cdot \alpha_{ok}) - q_k \cdot \cos \left\{ \alpha_{ok} + \arctg \left[ \frac{\sin(z_s \cdot \alpha_{ok})}{\frac{r_k}{e_k \cdot z_k} + \cos(z_s \cdot \alpha_{ok})} \right] \right\} \quad (7)$$

$$y_{ok}(\alpha_{ok}, r_k, q_k) = r_k \cdot \sin(\alpha_{ok}) + e_k \cdot \sin(z_k \cdot \alpha_{ok}) - q_k \cdot \sin \left\{ \alpha_{ok} + \arctg \left[ \frac{\sin(z_s \cdot \alpha_{ok})}{\frac{r_k}{e_k \cdot z_k} + \cos(z_s \cdot \alpha_{ok})} \right] \right\} \quad (8)$$

- co-ordinates of corrected equidistant in stationary co-ordinate system Oxy

$$x_k(\alpha_{ok}, r_k, q_k, \delta) = x_{ok}(\alpha_{ok}, r_k, q_k) \cdot \cos(\delta) - y_{ok}(\alpha_{ok}, r_k, q_k) \cdot \sin(\delta) \quad (9)$$

$$y_k(\alpha_{ok}, r_k, q_k, \delta) = x_{ok}(\alpha_{ok}, r_k, q_k) \cdot \sin(\delta) + y_{ok}(\alpha_{ok}, r_k, q_k) \cdot \cos(\delta) \quad (10)$$

where:

- $r, e, q, m, z_s, z_k$  – known parameters of nominal equidistant [2],
- $r_k, q_k, m_k$  – sought parameters of corrected equidistant,
- $\alpha, \alpha_{ok}, \delta, e_k = e$  – sought additional parameters, connected with modification, Fig.4
- $m, m_k$  - coefficients of shortening of epicycloid.

Modification of meshing requires searching of values of 9 variables:

parameters of corrected equidistant  $r_k, q_k, m_k$

angle of revolution  $\delta$ , (compensating pitch play),

additional parameters, describing position angles  $\alpha_1, \alpha_2, \alpha_{ok1}, \alpha_{ok2}$  of extreme points 1, 1', 2, 2' on contact arc and position angles  $\alpha_s, \alpha_{oks}$  of tangent point S, Fig.4

Configuration of clearances can be determined from set of distances of two points positioned on the equidistant and normal line to nominal equidistant, Fig.4. Unknown parameters of corrected equidistant and additional parameters of modification resulting from optimisation criteria can be calculated from the following set of equations:

$$y_k(\alpha_{ok1}, r_k, q_k, \delta) + \frac{1}{y'(\alpha_1)} \cdot x_k(\alpha_{ok1}, r_k, q_k, \delta) - \left[ y(\alpha_1) + \frac{1}{y'(\alpha_1)} \cdot x(\alpha_1) \right] = 0 \quad (11)$$

$$y_k(\alpha_{ok2}, r_k, q_k, \delta) + \frac{1}{y'(\alpha_2)} \cdot x_k(\alpha_{ok2}, r_k, q_k, \delta) - \left[ y(\alpha_2) + \frac{1}{y'(\alpha_2)} \cdot x(\alpha_2) \right] = 0 \quad (12)$$

$$\left\{ [x(\alpha_1) - x_k(\alpha_{ok1}, r_k, q_k, \delta)]^2 + [y(\alpha_1) - y_k(\alpha_{ok1}, r_k, q_k, \delta)]^2 \right\}^{0,5} = \Delta r \quad (13)$$

$$\left\{ [x(\alpha_2) - x_k(\alpha_{ok2}, r_k, q_k, \delta)]^2 + [y(\alpha_2) - y_k(\alpha_{ok2}, r_k, q_k, \delta)]^2 \right\}^{0,5} = \Delta r \quad (14)$$

$$x(\alpha_s) = x_k(\alpha_{oks}, r_k, q_k, \delta) \quad (15)$$

$$y(\alpha_s) = y_k(\alpha_{oks}, r_k, q_k, \delta) \quad (16)$$

$$\frac{y(\alpha_s)}{x(\alpha_s)} = \frac{y_k(\alpha_{oks}, r_k, q_k, \delta)}{x_k(\alpha_{oks}, r_k, q_k, \delta)} \quad (17)$$

$$\arctg \frac{y(\alpha_s)}{x(\alpha_s)} - \arctg \frac{y_{ok}(\alpha_{oks}, r_k, q_k)}{x_{ok}(\alpha_{oks}, r_k, q_k)} = \delta \quad (18)$$

$$\left[ x(\alpha_s)^2 + y(\alpha_s)^2 \right]^{0,5} = \left[ x_k(\alpha_{oks}, r_k, q_k, \delta)^2 + y_k(\alpha_{oks}, r_k, q_k, \delta)^2 \right]^{0,5} \quad (19)$$

$$\left[ \frac{\sqrt{1+m^2}-1}{1-\sqrt{1-m^2}} \cdot (q_k - q) + \frac{e \cdot z_k}{m} \right] = r_k \quad (20)$$

$$\frac{|\alpha_2 - \alpha_1|}{\alpha_{pc}} = \varepsilon \quad (21)$$

Equations (11), (12) are for normal lines transiting through points 1, 1' and 2, 2'. Equation (13), (14) represent distances of equidistants between these points. Equation (15)-(19) describe conditions of tangency of equidistants in basic system Oxy. Equation (20) results from including corrected equidistant in nominal one and (21) connects sought angles  $\alpha_1, \alpha_2$  with coefficient of contact  $\varepsilon$ .

System of equation (11) ÷ (21) is solved by iterative method of Levenberg-Marguardt which is a variation of gradient method in MathCAD 6.0 software. Precision of calculation (error vector) was set on level  $10^{-13}$ . As the result of solving system of equations for assumed value  $\varepsilon_j$  we can get 9 sought values, which can be components of vector of meshing modification:

$$z_j(\varepsilon_j) = \begin{bmatrix} z_{1j} \\ z_{2j} \\ z_{3j} \\ \vdots \\ z_{9j} \end{bmatrix} = \begin{bmatrix} (\alpha_1)_j \\ (\alpha_{ok1})_j \\ (\alpha_2)_j \\ \vdots \\ \delta \end{bmatrix} \quad \text{for } j = 1, 2, \dots, m \quad (22)$$

For m-criterial values of vector of contact ratio:

$$\varepsilon = [0,24; 0,27; 0,30; 0,33; 0,36; 0,39; 0,42; 0,45] \quad ,$$

we get matrix of modifications (matrix of possible solutions):

$$\mathbf{A} = [\mathbf{z}_1(\varepsilon_1), \mathbf{z}_2(\varepsilon_2), \dots, \mathbf{z}_m(\varepsilon_m)] \quad (23)$$

Exemplary matrix A for cycloidal gear with ratio  $i = z_s = 19$  is shown in Table 1.

Elements of matrix has been calculated for the following nominal equidistant:

$r = 96\text{mm}$ ;  $e = 3\text{mm}$ ,  $q = 8,5\text{mm}$ ;  $z_k = z_s + 1 = 20$ ;  $m = 0,625$ ;  $\alpha_{pc} = 18,947368^\circ$

and criteria of optimisation:  $\Delta r = 0,01\text{m}$ ;  $\max \Delta r_{io} = 0,03\text{mm}$ ;  $\delta_{\min} = 0,05^\circ$

Table 1: Matrix of optimal solutions for cycloidal gear with ratio  $|i| = 19$ :

Coefficient of contact $\varepsilon_j$										
$\varepsilon_1=0,24$	$\varepsilon_2=0,27$	$\varepsilon_3=0,30$	$\varepsilon_4=0,33$	$\varepsilon_5=0,36$	$\varepsilon_6=0,39$	$\varepsilon_7=0,42$	$\varepsilon_8=0,45$			
Number of teeth in contact										
4 (5)	5	5 (6)	6	6 (7)	7	8	8 (9)			
A =	3,919643	3,484203	3,044094	2,598228	2,147577	1,692691	1,234555	0,774210	$\alpha_1 [^\circ]$	Parameters of modification and correction of meshing
	3,783628	3,391341	2,974621	2,544749	2,104391	1,657213	1,204858	0,748404	$\alpha_{ok1} [^\circ]$	
	8,467011	8,599992	8,728305	8,850859	8,968629	9,082165	9,192550	9,300526	$\alpha_2 [^\circ]$	
	8,416554	8,567535	8,705395	8,834097	8,955678	9,071909	9,184118	9,293456	$\alpha_{ok2} [^\circ]$	
	5,842463	5,659194	5,928649	5,898093	6,329706	6,299692	5,382737	6,184599	$\alpha_s [^\circ]$	
	5,850020	5,658225	5,933204	5,900872	6,334117	6,302904	5,380887	6,186348	$\alpha_{oks} [^\circ]$	
	96,78121	96,44742	96,28635	96,19335	96,13864	96,10282	96,07875	96,06247	$r_k [\text{mm}]$	
	9,456107	9,047590	8,850465	8,736635	8,669677	8,625841	8,596389	8,576465	$q_k [\text{mm}]$	
	0,345288	0,197962	0,126963	0,085755	0,061649	0,045725	0,034897	0,027774	$\delta [^\circ]$	
	0,619955	0,622101	0,623141	0,623744	0,624099	0,624331	0,624488	0,624594	$m [-]$	

### Distribution of loads acting on planet wheels

Dependence between torques in CPG is [ 2,6,7 ] :

$$M_1 = 2M_c = M_h \cdot |i| \cdot \eta \quad (24)$$

$$M_h = R \cdot e \cdot \cos \alpha_R \quad (25)$$

$$M_1 - M_2 + M_h = 0 \quad (26)$$

where:

- $M_1 = 2M_c$  – torque, arising in planet wheels 1 i 1',
- $M_2$  – torque giving load on interacting central wheel 2,
- $M_h$  – input torque (driving) on eccentric shaft (yoke shaft),
- $i, \eta$  – kinematics ratio and efficiency of gear;  $\eta \cong 1$ ,
- $R$  – eccentric reaction force,
- $\alpha_R$  – incline angle of force R,
- $e$  – eccentric of gear,  $e = O_a O_b$ .

Torque  $M_c$  arises in planet wheel as a consequence of loading the gear of drive torque  $M_h$  and can be calculated from that wheels assuming that load is distributed equally on both wheels. Relation (27) results from balance of moments in planetary gear with positive base ratio  $i_0 > 0$  acc. to [ 7 ] and CPG is that kind of gear. So yoke's shaft is differential shaft and central wheel's shaft is aggregating shaft.

Torques acting on planet wheel produces in CPG three unknown load distributions, as follows:

load distribution in meshing ,distribution of forces  $P_i$  between teeth,

load distribution ( $Q_j$ ), acting on bolts of straight-line mechanism

load distribution of eccentric  $R$  on  $Q_{ri}$  loading roller elements (rolls) in bearing hole.

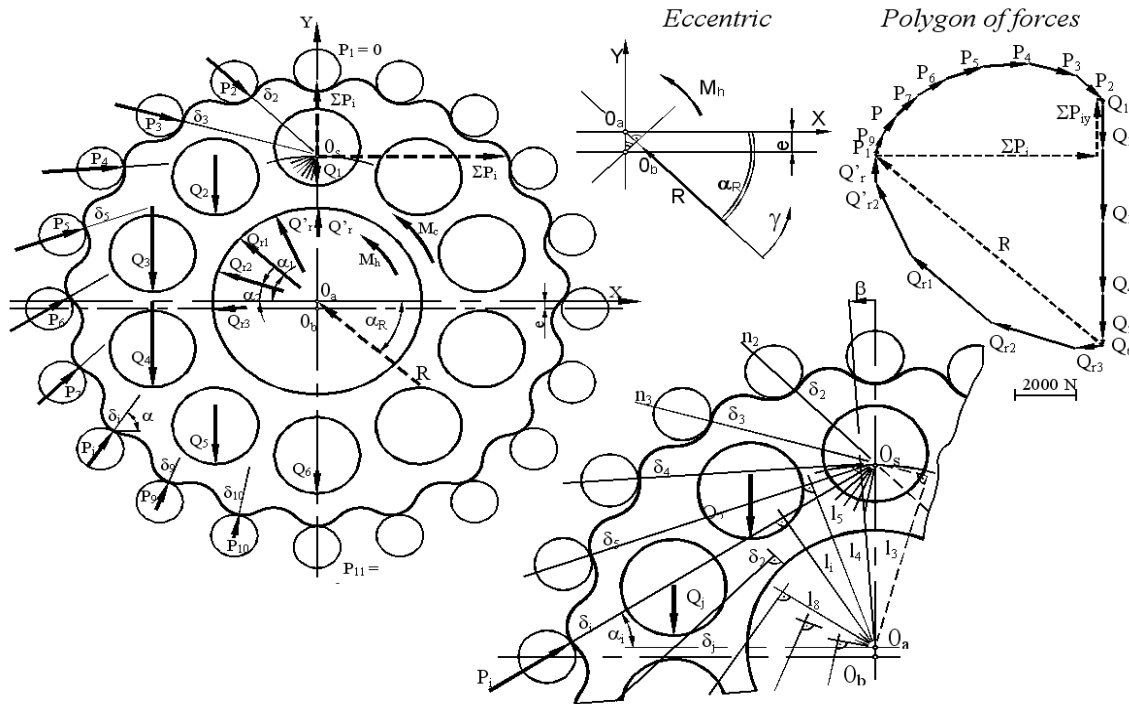


Fig.5 System of forces, strains and moments and rule of balancing forces acting on planet wheel

Figure 5 shows how to balance the forces acting on the planet wheel 1 or 1'. Forces between teeth  $P_i$  and forces  $Q_j$  are function of displacements  $\delta_i$  and  $\delta_j$  which arise in points of application of forces. And forces  $Q_{ri}$  depending on resolving of force  $R$  are the function of geometrical features of roller bearing and mainly depend on radial clearance. [3].

To calculate forces between teeth  $P_i$  and reaction forces  $Q_j$  there is applied analytical method, making use of simplifying assumptions [ 5,6,7 ].

### Analytical method for determining loads in cycloidal gear

- Following assumptions in analytical method has been done, Fig.5 :
- loads are distributed equally on both planet wheels and each transmits torque  $M_c$  being half value of output moment  $M_1$
- load is transmitted only by one (active) side of planet wheel and directions of meshing force action make pencil of lines with common starting point in roller point of meshing  $O_s$  ;



- displacements  $\delta_i$  in place of acting of meshing forces  $P_i$  result from slight angular displacement  $\beta$  of planet wheel as rigid plate coming from shifts of rolls of stationary wheel and local mutual strain of rolls and teeth;
- load from planet wheel is transmitted onto one (active) side of straight line mechanism and directions of action of forces  $Q_j$  are parallel towards line  $O_a O_b$  (of eccentric) ;
- displacements  $\delta_i$  in point of action of forces  $Q_j$  result from slight angular displacement  $\Delta\phi$  of straight line mechanism's disk and are created by deflection of bolts and mutual strains of bolts and holes of planet wheel ;
- eccentric reaction force  $R$  is a concentrated force and is distributed into components  $Q_{ri}$  and results from the value of input torque  $M_h$  and conditions of equilibrium.
- Method of calculation of meshing forces  $P_i$  and bolts reaction forces  $Q_j$  analytically is described in references [ 5, 6 ].

### Method of calculations and applied numerical models

In ADAMS has been created model of cycloidal planetary gear (Cyclo) with planet wheel and cooperating elements (bolts and rolls). The geometry of interacting elements is simple (circles with different radius) and it is no problem to create the geometrical model of them. The more complicated is the shape of external edge of the planet wheel. It is described by parametric equations of equidistant [ 2, 6 ] :

$$\left. \begin{aligned} x_{ees} &= r \cdot \cos \alpha + e \cdot \cos(z_k \cdot \alpha) - q \cdot \cos(\alpha + \gamma) \\ y_{ees} &= r \cdot \sin \alpha + e \cdot \sin(z_k \cdot \alpha) - q \cdot \sin(\alpha + \gamma) \end{aligned} \right\} \quad (27)$$

where:

$\alpha$  – generating angle of equidistant,

$r, e, q, z_k$  – parameters of meshing, Fig.2;

$\gamma$  – overtaking angle, depending on coefficient of shortening equidistant  $m$  and gear ratio.

In the ADAMS model of Cyclo, was implemented two different kind of equidistant – with nominal and corrected parameters – as profile of teeth of planetary wheel. The profile of teeth was implemented as generation of the curve points (equidistant) on the basis of parametric equations (5,6) with given tolerance every  $0,05^0$  with parameters of nominal equidistant:

$r = 96,0\text{mm}; e = 3\text{mm}, q = 8,5\text{mm}; m = 0,625; z_k = z_s + 1 = 20, ; \alpha_{pc} = 18,947368^\circ$   
and corrected equidistant : given in Table 1 for  $\varepsilon = 0.27, 0.39$  and  $0.45$

Values of forces in both of methods has been calculated for given size of cycloidal gear as in Fig.5 with ratio  $i = 19$ , power  $N = 6.4[\text{kW}]$  and rotational speed  $n_h = 750 [\text{rpm}]$ . For this gear  $M_1 = 2M_c = 880 [\text{Nm}]$ , and force  $R = 10,3 [\text{kN}]$ .

Results from analytical calculation was precise presented in [2,3].

The max values of  $P_i = 1711 [\text{N}]$  and  $Q_i = 3776 [\text{N}]$

In numerical method, the calculated forces are in max level about for  $P_i = 2200[\text{N}]$ , and  $Q_i = 3800[\text{N}]$  (Fig.7). (this is only an example for modified equidistant with parameters where  $\varepsilon = 0.27$  (Table 1))

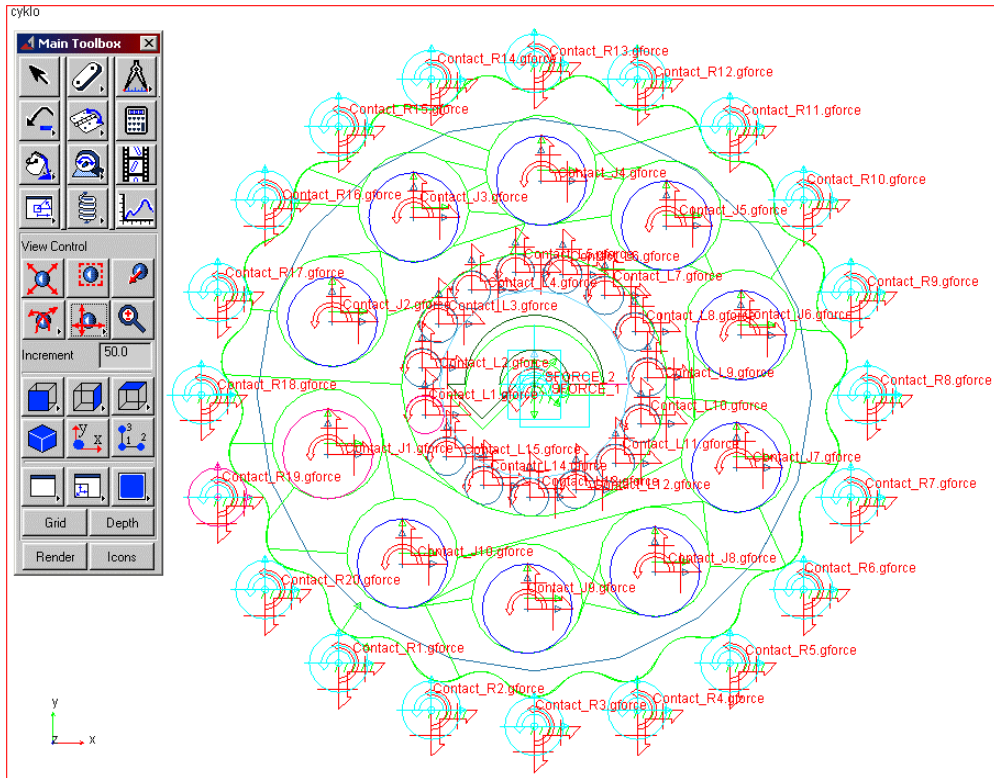


Fig.6. Model of Cycloidal Planetary Gear (CYCLO) built in ADAMS

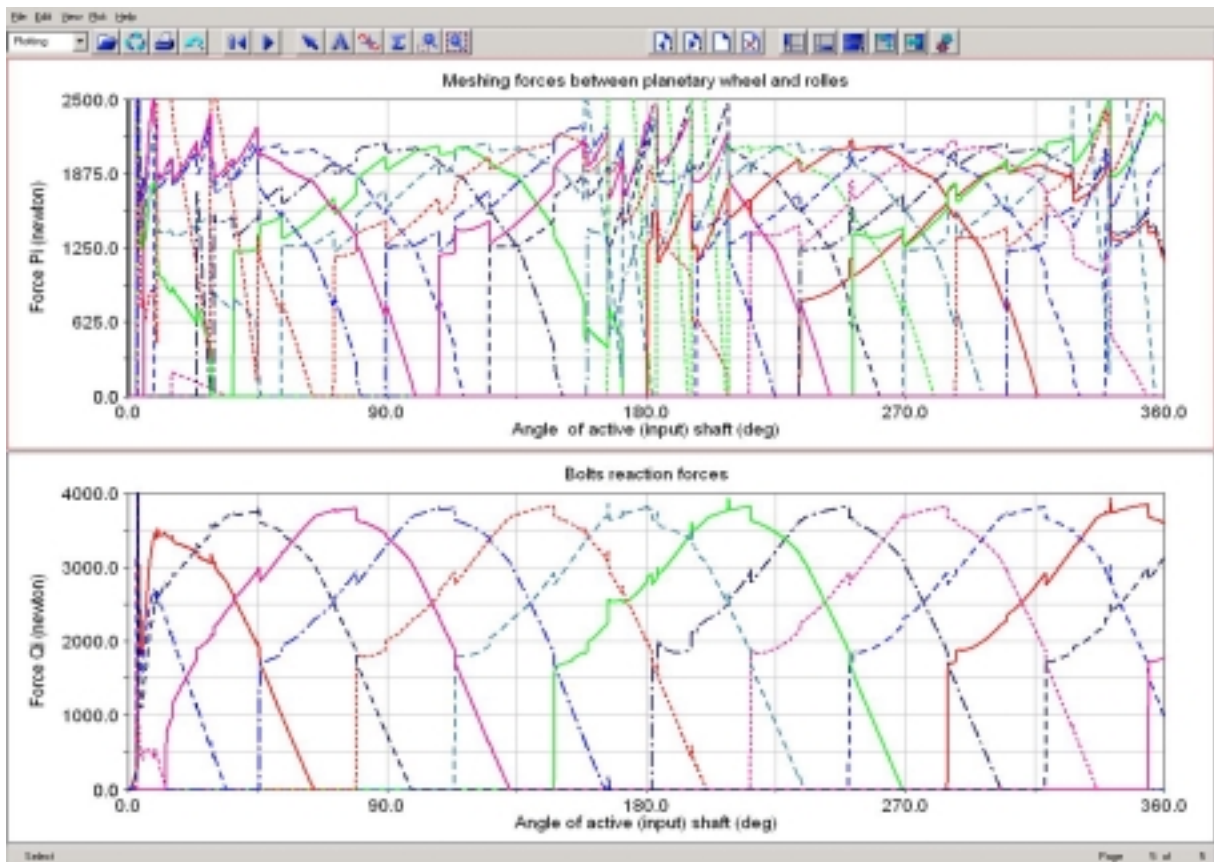


Fig.7. An examples of nonfiltered traces of meshing forces between planetary wheel and stationary wheel and reactions between bolts and planetary wheel. ( $\epsilon = 0.27$ )

## Conclusions:

Presented results concern cycloidal gear with nominal and modified meshing with transmission ratio  $|i|=19$ . It should be also executed calculations of entire range of applied ratios  $|i|=11\div 89$  and basing on them to try to generalise distribution of forces of meshing forces  $P_i$  and reaction forces  $Q_j$

The basic aspect of modification of meshing in cycloidal planetary gear is diversification of equidistant.

Matrix of modification A (Table 1) presents example field of solutions of inside cycloidal meshing for assumed criteria of optimisation. Matrix A enables visualisation of clearances  $\{\Delta r_i\}$  assigned to individual teeth that enables and makes easier selection of efficient parameters of correction of planet wheels.

Calculated parameters of corrected equidistant  $r_k, e_k, q_k, m_k$  can be used while producing toothing that will assure co-operation from 50 ÷ 90% of teeth of planet wheel in its active part of meshing.

Distribution of meshing forces  $P_i$  and reaction forces  $Q_j$  calculated in ADAMS have a little different traces comparing it with distributions determined analytically. It results from omitting in analytical method inertia of moving parts of gear.

This model was modeled in ADAMS as a rigid body, so to get more details about behavior of this kind of gear, it should be also modeled as a flexible bodies.

To make this results more authentic, also was built a real model of CYCLO gear which is actual in testing stage and this work will be continue and develop.

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