

# ***Analysis of dynamic of a virtual compressor***

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## ABSTRACT

Adams simulation software has been successfully utilized to develop a virtual model of reciprocating compressor for refrigeration. The system can be utilized for several purposes: analysis of balancing to minimize vibrations, prevision of force transmitted to the shell, which has an impact on noise emission, force on mechanical components, valve motion prevision. This work presents the development of the supporting system for a domestic hermetic compressor, from the numerical analysis to the experimental test.

## 1.INTRODUCTION

Today the increasing demand of high efficiency, low noise and low cost compressor for domestic refrigerators and the high competitiveness of the market has led producers to perform high level of optimization of the product. A virtual dynamic model of a hermetic compressor has been created to fulfill this target.

Presently the model has been useful to optimize balancing of the crank mechanism reducing low frequency vibrations (about 50 Hz), and variable speed compressors, but the model can be used for many other purposes:

- Analysis of dynamic forces on mechanical components
- Analysis of vibration of the supporting system
- Prediction of power consumption
- Considering flexible shaft also flexional and torsional vibrations can be analyzed
- Analysis of hydrodynamic lubrication
- Start up analysis of the compressor
- Analysis of valves dynamic
- Numerical computation of noise emission throw vibro-acoustic analysis

The model will be connected to a thermo-fluid dynamic package (written in Fortran or C) or a Simulink model to calculate the correct PV cycle in specific conditions or the correct speed vs. torque in variable conditions. This is useful to study variable speed compressors. The paper describes the dynamic model, which uses multi-body [1] and flexible elements. So this approach has less simplification than classical analytical approach. Dynamic of a single piston reciprocating hermetic compressor has been described from several authors. Equation of the motion of the slider crank mechanism can be solved to compute the force applied on the supporting system. The analysis of a rigid body applied on the system permits to evaluate acceleration. Unfortunately this approach has several simplifications and it is generally difficult to know the uncertainty of the result.

The parametric model has been developed with the use of finite element analysis and dynamic analysis. Some components of the compressor, such as supporting springs, the internal discharge pipe, valves have been considered as flexible bodies[2], so that they have a dynamic transfer function which has been

calculated through a spectral analysis performed with Ansys [3]. All other elements has been assumed as rigid, even if it's possible to analyze a system with all flexible bodies. This is useful to analyze vibration sources in a higher range of frequency.

## 2.MECHANICAL MODEL

The main assumption is that only discharge pipe and supporting springs are flexible bodies, so that other elements can be considered as rigid. All rigid body elements have ten characteristic parameters:

- A center of mass (three coordinates)
- A mass
- An inertia (six coordinates)

Coupling of substructures permits to analyze flexible bodies, which have a more complex structure because they have the same ten parameters of a rigid body, but they have several modal frequencies (eigenvalues) and modal shapes (eigenvectors). A modal analysis can be performed to find these parameters. The result of the analysis is generally normalized to the maximum displacement, so the real displacement can be evaluated only if the correct excitation set is known. A mechanical dynamic software can evaluate the excitation from the solution of differential equations of the motion of crank mechanism. A great advantage of substructures is that it's possible to modify independently each component of the assembly. A flexible body has been described as a substructure. A constrained modal analysis of the body has been performed, to evaluate modal frequencies and then power spectral density analysis permits to evaluate participation factor of each mode. It's possible to evaluate experimentally a damping coefficient for each modal frequency. Normal constrained modes have been transformed into normal free-free modes plus a set of interface eigenvector modes[4]. Rigid body modes have a very low frequency (zero) and are normally disabled. The motion of a flexible body can be described as a combination of shape vectors or mode shapes: the position of master nodes of the flexible body is [5]:

$$r_i = x + A(s_i + \Theta_i q)$$

where:

- $x$  defines the position vector in the global reference system to the local reference frame, B
- $A$  is the directions cosines of the orientation between B and the global origin
- $s_i$  is the undeformed location of the  $i^{\text{th}}$  node in B
- $\Theta_i$  define the contribution of the mode shapes to the  $i^{\text{th}}$  node.
- $q$  is the vector of modal amplitude.

Motion can be represented using Euler angles. Generalized coordinates are:

$$\xi = \left\{ \begin{array}{l} X \\ \psi \\ q \end{array} \right\} \quad \text{where:}$$

- $X$  is a vector of coordinates of the local body reference frame
- $\psi$  is the vector of Euler angles of B relative to the global origin.
- $q$  is the vector of modal amplitudes of the  $m$  contributing mode shapes.

Equations of the motion of the flexible body can be derived from Lagrange's equations:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\xi}} \right) - \frac{\partial L}{\partial \xi} + \frac{\partial F}{\partial \dot{\xi}} + \left[ \frac{\partial \Psi}{\partial \xi} \right]^T \lambda - Q = 0$$

$$\Psi = 0$$

where:

- $L$  is the Lagrangian:  $L=T-V$  where T and V denote kinetic and potential energy.

- $\Theta$  is the energy dissipation function
- $\Psi$  are the constrain equations.
- $\xi$  are the generalized coordinates
- $\lambda$  are the Lagrange multipliers for the constraints

Kinetic energy can be expressed in generalized coordinates  $\xi$  :

$$T = \frac{1}{2} \dot{\xi}^T M(\xi) \dot{\xi}$$

$M(\xi)$  is the mass matrix.

$$M = \begin{bmatrix} M_{tt} & M_{tr} & M_{tm} \\ & M_{rr} & M_{rm} \\ \text{sim} & & M_{mm} \end{bmatrix}$$

Where  $t, r, m$  refer to translational, rotational, and modal coordinates.

$M_{tt} = m I$  where  $m$  is the mass and  $I$  the identity matrix

$$M_{tr} = - \int_V \rho A (s_p + \Phi(P) q) B dV \quad \text{the first term is the center of the mass location multiply by the}$$

mass<sup>1</sup>.  $\Phi(P)$  is the shape function.

$$M_{tm} = \int_V \rho A \Phi(P) dV \quad M_{rr} = \int_V \rho B^T (s_p + \Phi(P) q)^T (s_p + \Phi(P) q) B dV$$

$$M_{rm} = - \int_V \rho B^T (s_p + \Phi(P) q)^T \Phi(P) dV \quad M_{mm} = \int_V \rho \Phi(P)^T \Phi(P) dV$$

Potential energy contains gravitational energy and elastic energy:  $V = \frac{1}{2} \xi^T K \xi + V_g(\xi)$  where

$K$  is the generalized stiffness matrix (only modal coordinates  $q$  participate to elastic energy), and  $V_g$  is the gravitational energy. The damping force is generally dependent on modal velocity  $w$ , according to Rayleigh's dissipation function  $\Theta$ .

$$\Theta = \frac{1}{2} w^T D w \quad \text{where } D \text{ is damping coefficient matrix and } w \text{ is the modal velocity vector.}$$

The asynchronous electric motor has a characteristic speed vs. torque curve, so the speed

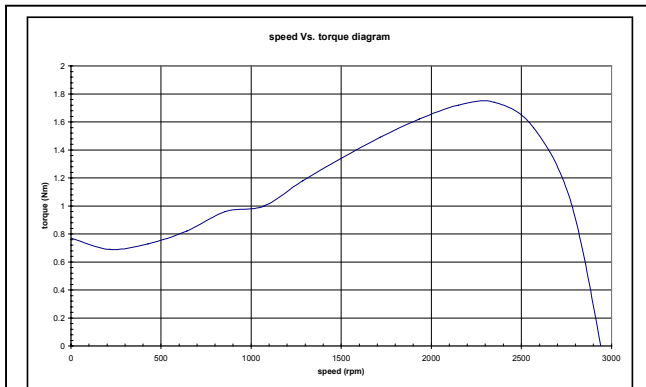


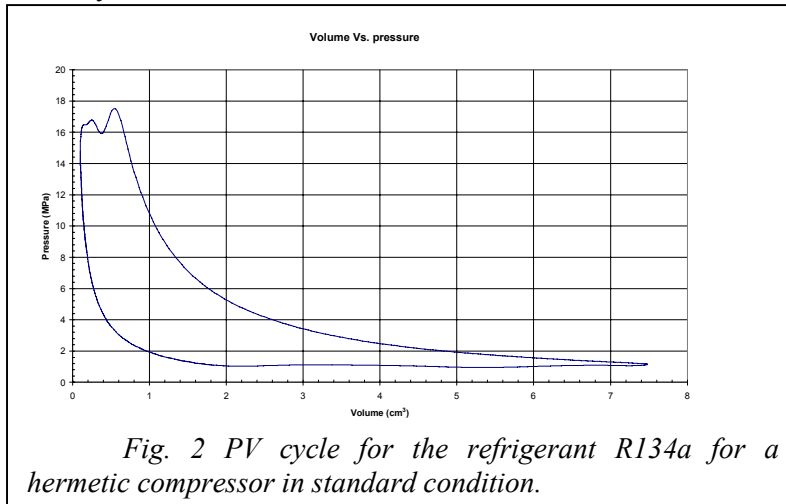
Fig. 1 Motor torque characteristic curve

and the power consumption depends on the load. The angular velocity is about 308 rad/sec, even if this is not constant and brushless motors have also a pulsing torque of double frequency, which has a null mean value. Motor torque characteristic can be determined in regime, if this function is used to study the start up, there is a high degree of uncertainty.

<sup>1</sup>  $\Phi(P) q$  is the deformation of a point P of a flexible body ( $\Phi(P)$  is the shape function,  $s_p$  is the undeformed position of the point P).

For variable speed compressor there are many characteristic curves varying the frequency of the current. Pressure load on the system has been computed through thermo-fluid dynamic software for the specific compressor. So the analysis isn't fully coupled, but can give a good result in stationary conditions.

The pressure has been applied to piston, but also suction and discharge valves have  $\Delta P$  applied to evaluate their dynamic.



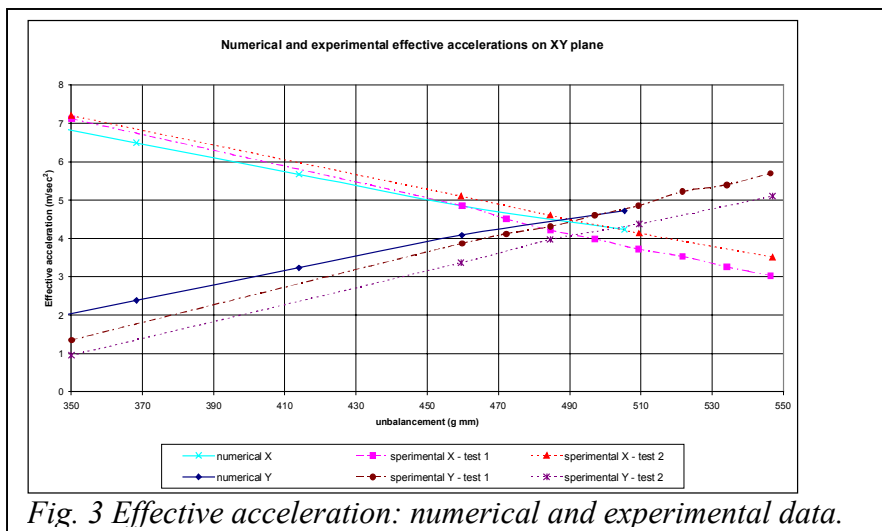
The model also considers friction of the sliding parts of the system. Friction model torque has been computed from the theory of hydrodynamic lubrication of bearings. The steady incompressible Reynolds equation has been employed with Sommerfeld boundary conditions.

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{w}{2} \frac{\partial h}{\partial x}$$

where:

- $h$  is the film thickness (normally this is  $h = h_{\min} + h_{\text{term}} \cos(\varphi) + x \sin(\varphi)$  where  $h_{\text{ter}}$  is due to thermal expansion).
  - $w$  is the speed
- Then hydrodynamic friction force can be evaluated on shear stress.

### 3. NUMERICAL AND EXPERIMENTAL RESULTS



The first application of the virtual compressor model has been the optimization of balancing. It is possible to optimize the magnitude of a goal function in the frequency domain, for example the goal function at the fundamental frequency (50 Hz). A domestic hermetic compressor<sup>2</sup> has been analyzed varying the counterbalance weight. It is possible to analyze the variation of the effective acceleration in some

reference points. Defined as coordinate system on this plane  $x$  as the piston motion direction and  $y$  direction orthogonal to  $x$ , and defined as origin the center of the crankshaft, maximum effective

<sup>2</sup> This compressor uses R134a and it has a cooling capacity of 170 kcal/h

acceleration on  $x$  and  $y$  direction have been computed and measured. This value corresponds to the effective value of the FFT magnitude diagram at the current frequency (about 50 Hz).

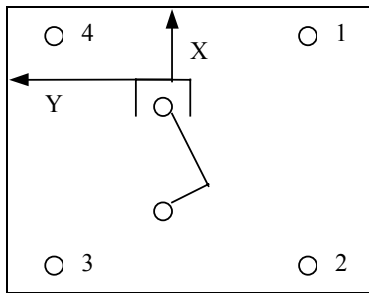


Fig. 4 reference system

Numerical analysis has been performed to find the best counterbalance weight; even if it is always required an experimental validation of the results and the experimental determination of the goal function. Another validation of the model has been the measure of effective acceleration of the top of the four supporting springs. These accelerations have been measured with a particular setup[6] and they have been calculated numerically. It is possible to notice in tables 1-2 that there is a good correlation.

Table 1 Effective acceleration (measured values)

	X		Y		Z	
	Amplitude (m/sec <sup>2</sup> )	Phase (deg)	Amplitude (m/sec <sup>2</sup> )	Phase (deg)	Amplitude (m/sec <sup>2</sup> )	Phase (deg)
Spring 1	0.5	0	0.6	-140	1.3	-130
Spring 2	0.5	0	0.6	106	1.1	-50
Spring 3	0.7	-80	0.5	130	1.3	50
Spring 4	0.7	-80	0.7	-130	1.1	130

Table 2 Effective acceleration (computed values)

	X		Y		Z	
	Amplitude (m/sec <sup>2</sup> )	Phase (deg)	Amplitude (m/sec <sup>2</sup> )	Phase (deg)	Amplitude (m/sec <sup>2</sup> )	Phase (deg)
Spring 1	0.4	0	0.6	-130	1.2	-120
Spring 2	0.4	0	0.5	110	1.2	-60
Spring 3	0.6	-70	0.5	125	1.2	55
Spring 4	0.6	-70	0.6	-120	1.1	110

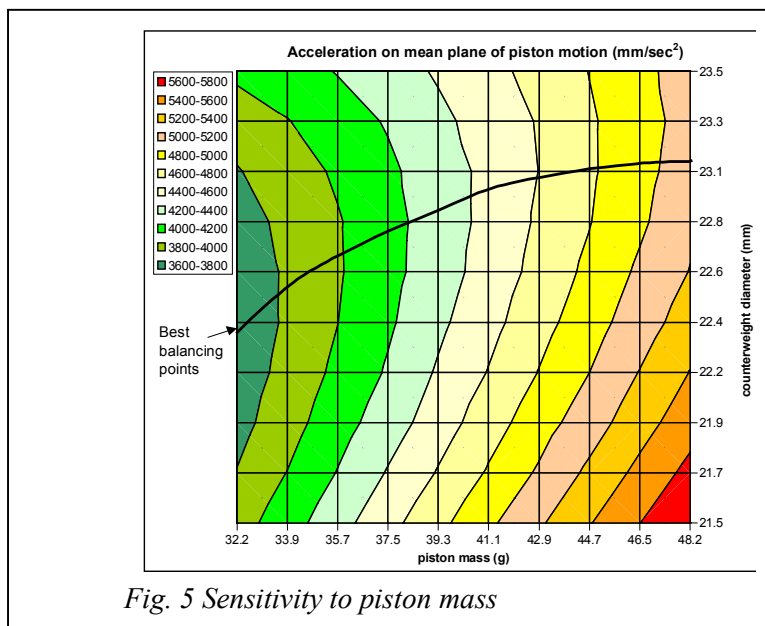


Fig. 5 Sensitivity to piston mass

The sensitivity to various parameters such as piston mass, inertia of rotating elements can be studied. An improvement of this optimization now under development is the reduction of 2<sup>nd</sup> and 3<sup>rd</sup> harmonic.

The model can calculate FFT diagram of acceleration and the force transmitted to the shell. This information can be useful for a vibro-acoustic analysis of the system for the evaluation of emitted noise. It is possible to notice in fig. 5 that there is a good correlation for low frequency vibration (under 200 Hz). For higher frequency it is important to consider other flexible elements. This will be

done in future enhancements of the model.

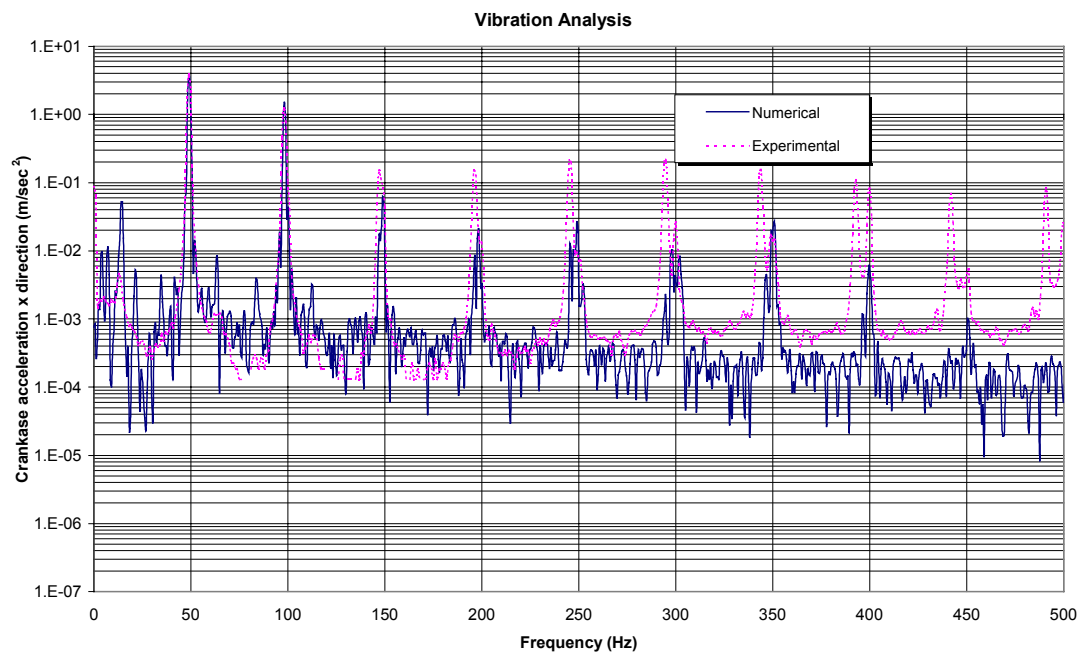


Fig. 6 FFT diagram of acceleration of crankcase

The model has been applied also to evaluate suction and discharge valve dynamic. The dynamic of suction valve can be evaluated considering also impact of flexible elements with valve plate. This effect can be considered also for discharge valve, but in this case there are some models where two flexible bodies may have a contact. This is a very difficult task to achieve.

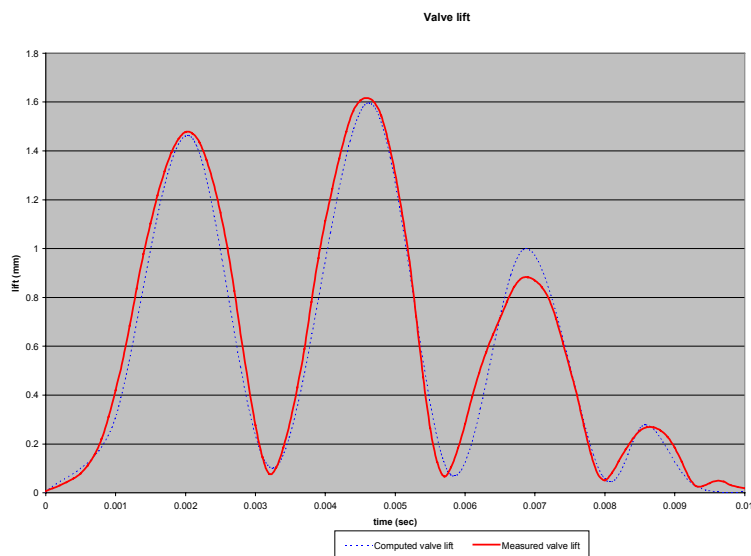


Fig. 7 Suction valve dynamic

### 3.CONCLUSIONS

The first applications of Adams model have a good agreement with experimental results, so this approach can give an answer to many questions. Similar results for a single model could be achieved with an experimental approach, but a virtual model remarkably improves design capabilities. Further work is under progress to improve and validate the virtual model of compressor.

### 4.REFERENCES

- [1]Craig, R.R. and Bampton, C.C.,*Coupling of substructures for Dynamic Analyses*, AIAA journal vol.6 no.7 pp. 1313-1319
- [2]*Adams flex* guide Mechanical Dynamics rev. 10.0
- [3]*Ansys theory reference*, 8<sup>th</sup> Edition SAS IP, Inc.
- [4]Mark Baker, *Component Mode Synthesis Method for Test-Based, Rigidly connected flexible components* Journal of spacecraft vol. 23 No.3
- [5]J. T. Spanos, W. S. Tsuha *Selection of component modes for flexible multibody simulation* AIAA journal vol.14 no.2 pp. 278-286
- [6] “*Running Mode Manual 3.5B*” LMS Leuven, Belgium