MODELLING HELICOPTER ROTOR DYNAMICS WITH ADAMS

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Abstract: this paper describes the results of a feasibility study aiming at the design of an ADAMS-based simulation program for the main rotor of the AGUSTA A109c helicopter. Due to the complexity of the dynamics and aerodynamics of the helicopter rotor (blade flexibility, unsteady aerodynamic loading, non-uniform and time-varying wake) extensive use has been made of ADAMS's features such as FEA modelling, user-written subroutines and linear systems simulation; an *ad hoc* subroutine (CONSUB) for simulation management and control has also been developed.

I. INTRODUCTION

In recent years, a great deal of activity has been devoted in the helicopter community to the research aiming at improving the performance and quality of the existing and future aircraft; in particular, the so-called "*jet smooth*", low vibration helicopter is the target of most manufacturers.
The major source of vibrations in helicopters is the main rotor; therefore a reduction in the vibratory level can be achieved by two different strategies:

1. by modifying the design of existing rotors, after a careful *design optimization* has been performed.

2. by making use of *active control techniques* in order to reduce the effect of rotor vibratory loads on the fuselage and, as a consequence, on the passengers and the on-board equipment.

Both strategies imply a deep understanding of the rotor as a dynamic system and the availability of an advanced simulation tool in order to carry out the required optimization or control system design. The latter strategy (i.e., active control of vibrations) is currently a subject of research at the Politecnico di Milano, in cooperation with the Italian helicopter manufacturer AGUSTA SpA (see, e.g., [1]-[3]); one of the objectives of such a research activity is the development of a suitable simulation program, to be applied in the design and testing of active control systems for helicopter rotors.

As will be made clearer in the following Sections, the multibody approach to the analysis of mechanical systems has been recently recognised as a promising paradigm for the helicopter industry ([4], [5]); this paper will therefore describe the results of a feasibility study aiming at the design of an ADAMS-based simulation program for the dynamics and aerodynamics of the main rotor of the AGUSTA A109c helicopter.

The paper is organised as follows: after a short introduction describing the helicopter rotor and its major dynamic and aerodynamic features (Section II), the first part of the paper (Sections III-VI) will focus on the modelling issues related to the "building blocks" the rotor model is based on, namely:

- the mechanical model of the control system and the hub.
- the structural model of the blades.
- the model of the aerodynamic loads acting on the blades and the associated integration scheme.
- the aerodynamic model of the *rotor induced flow*, i.e., of the wake generated by the motion of the blades with respect to the free air-stream, which is of major importance in the analysis of rotor dynamics.

For each of these building blocks the selected mathematical models will be presented and their assumptions will be discussed.

The second part of the paper (Sections VII-X) will then describe the ADAMS implementation of the aforementioned building blocks (providing the reader with more details about the use of FEA modelling, user-written subroutines and linear systems simulation) and the development of an *ad hoc* subroutine (CONSUB) for simulation management and control.

II. ROTOR DYNAMICS AND AERODYNAMICS

The main rotor of a helicopter has the function of developing the forces and moments required to fly and control the aircraft; in a rotor of the articulated type, as the one this paper deals with, each blade is attached to the rotor's central hub by a set of hinges which allow it to rotate

- out of the rotor disk plane (up and down, *flapping motion*)
- in the disk plane (back and forth, *lagging motion*)
- around its longitudinal axis (*pitching* or *feathering motion*).

The hub rotates at an (almost) constant angular rate Ω (typically Ω ranges from 20 to 50 rad/sec, depending on the particular aircraft), thanks to the torque provided by one or more engines located in the fuselage of the aircraft.
The pilot can control the rotor by varying the pitch angle of each blade by means of an actuator called *swash-plate*; by commanding the blade pitch angle one can indirectly control the amplitude and orientation of the loads the rotor applies to the fuselage, as these are (roughly) proportional to such an angle.

From the modelling point of view, the rotor can be considered as being constituted by four separate subsystems or blocks; this decomposition is represented in Fig. 1, which shows a functional block diagram of a helicopter rotor:

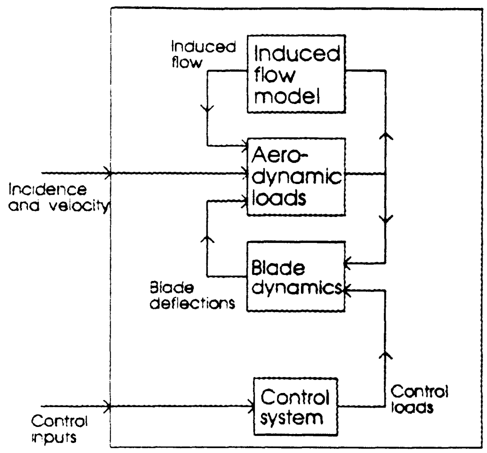

Fig. 1. Block diagram of helicopter rotor dynamics and aerodynamics

The figure puts into evidence the interactions between dynamics and aerodynamics, which is typical of helicopter rotors: the control inputs influence directly the aerodynamic loads acting on the blade, which, in turn, act as forcing inputs for the blade dynamics (both rigid body and flexibility modes) and perturb the distribution of the airflow around the rotor; the loop is closed by taking into account the effects of blade motion (and elastic deformations) and induced flow on the aerodynamic behaviour of the blade's lifting surface.

The following Sections will be devoted to a discussion of the major modelling issues concerning each of the "building blocks" of Fig. 1; the interested reader can find a more detailed presentation of the full mathematical model in [6].

III. CONTROL SYSTEM AND ROTOR HUB

a) Rotor control system

A schematic representation of the rotor control system is given in Fig. 2:

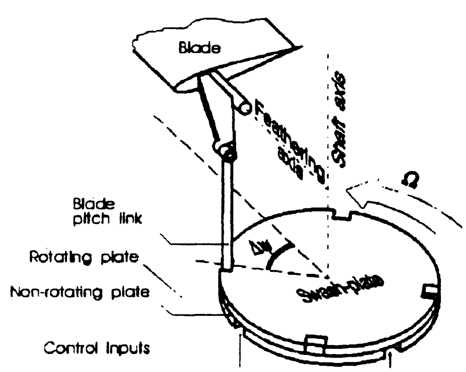

Fig. 2. Rotor blade control system

the pilot can act on the rotor by commanding a vertical displacement (*collective control*) or a longitudinal/lateral tilt (*lateral and longitudinal cyclic* control, respectively) of the non-rotating swash-plate; such displacements are transferred, via the linkage kinematics, to the blade *pitch horn*, as a time-varying pitch command of the form

$$\vartheta(t) = \vartheta_o + \vartheta_{1s}\sin(\Omega t) + \vartheta_{1c}\cos(\Omega t). \tag{1}$$

It is possible to prove (see, e.g., [7]) that by the application of such kind of controls the pilot can influence the motion of the blades and thus modify the spatial orientation of the rotor *thrust*, (T), i.e., of the total aerodynamic force generated by the rotor, as shown in Fig. 3:

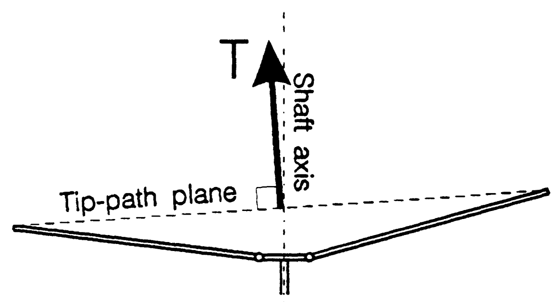

Fig. 3. Rotor thrust and tip-path plane

It is therefore of great importance for a rotor model to accurately describe the kinematics of the control system. This also implies that the model should account in some way for the flexibility of the control system itself (in particular w.r.t. the blade pitch links), as it is subject to large periodic loads coming from the blades. Such a flexibility plays an important role in determining the dynamic characteristics of the rotor; unfortunately, it is extremely difficult to determine an exact analytical value for the control system flexibility, so that it constitutes one of the most critical model parameters, which should be accurately tuned during the model validation phase.

b) Rotor hub

As was mentioned in the previous Section, each blade is attached to the rotor's central hub by a set of hinges, as in Fig. 4.

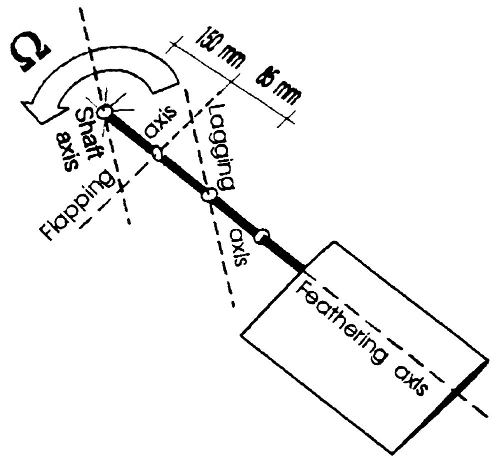

Fig. 4. Rotor blade attachment: hinges arrangement

The presence of hinges instead of a rigid attachment is motivated by stress relief at the root of the blade, while the reason for the hinge offset from the hub centre of rotation is to enable the rotor to transmit moments as well as forces to the fuselage. As a matter of fact, hub moments bring an important contribution to the helicopter's handling qualities: as a matter of fact the flapping hinge offset provides a very good measure of the manoeuvrability of an articulated helicopter.

It is of particular relevance to mention that, while the blade pitching and flapping motions are characterized by a satisfactory level of aerodynamic damping, the blade lagging motion is very lightly damped: this is why each blade attachment is provided with a hydraulic damper, with the function of stabilizing the blade lagging motion (see Fig. 5).

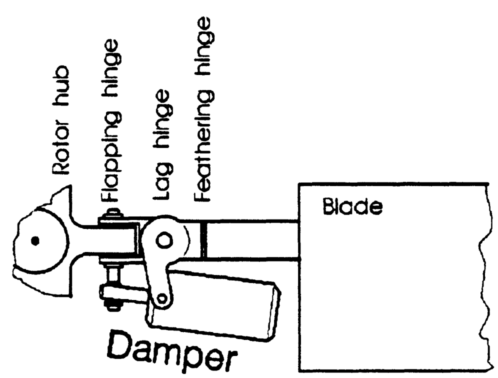

Fig. 5. Blade lag damper

The blade lag damper is in turn modelled by its force-velocity characteristic function, which is usually non-linear, of the form depicted in Fig. 6:

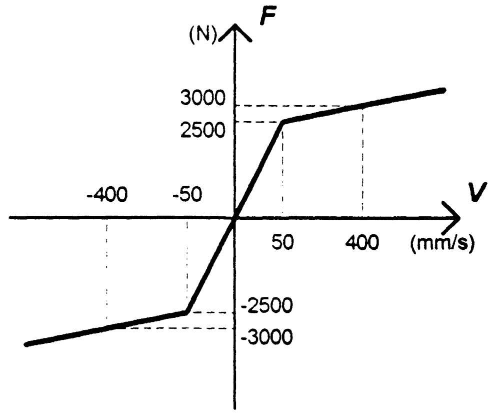

Fig. 6. Lag damper characteristic law

The control system, hub and blade attachments have been modelled in ADAMS as a set of rigid bodies; the swash plate has not been explicitly included in the model, although its addition would not imply any major modification but the inclusion of a few additional parts. The ADAMS implementation of the model for the control system and rotor hub will be presented in detail in Section VII; however it should be clear from the previous description that modelling of the hub and control system is extremely simple in ADAMS, as these components can be naturally described as a set of rigid bodies.

IV. BLADE STRUCTURAL DYNAMICS

The blade is the mechanical part of the rotor which requires the greatest care in the modelling process: rotor blades are very light, slender structural elements (Fig. 7), subject to a distributed and time-varying external loading and characterized by non-uniform structural and inertial properties (see Fig. 8); therefore flexibility (both in bending and torsion) cannot be neglected, especially if one is interested in analyzing the vibratory (i.e., high frequency) response of the system.

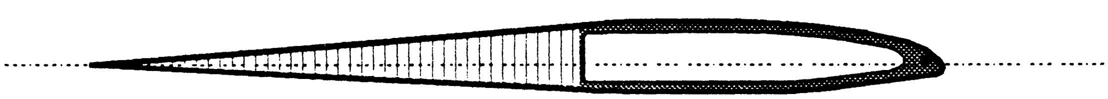

Fig. 7. Typical rotor blade section

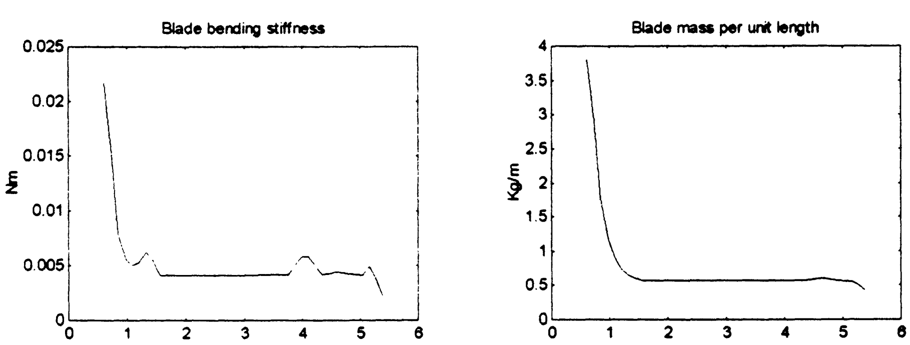

Fig. 8. Typical out-of-plane stiffness and mass distribution for a rotor blade

A full mathematical model for the structural dynamics of the blade is given by a set of three partial differential equations (PDEs), one for in-plane bending, one for out-of-plane bending and one for torsion, with the appropriate boundary conditions for the hub attachment (see, e.g., [8] for a thorough treatment of this subject).

As a first approximation, one could simplify the analysis by assuming that the three PDEs are uncoupled and then resort to the classical results of engineering beam theory for torsion and simple bending (although slightly modified to account for the effects of rotation). This approach, leading to approximate models of the blade, has been followed in the past and is extensively documented in the literature ([7], [9]): unfortunately, due to the complexity of their structure, modern composite rotor blades do not normally satisfy such assumptions, so that it is necessary to consider the structural problem in the general case, for coupled bending and torsional deformations.

Flexibility is handled as usual through discretization, i.e., by representing the solution of the system equations as a set of time-dependent coordinates multiplied by appropriate space functions. The classical approach to the solution of the blade equations, which is still used by many manufacturers, is the modal analysis approach: as is well known, this method is characterized by its mathematical elegance and somehow "natural" derivation, in particular for the case of beam vibrations; however it has a few drawbacks which are of major importance for the case of helicopter rotor dynamics. The modal approach, for example, cannot account for geometric non-linearities (large rotations and deflections). Furthermore, when considering the analysis of a rotating structure, the mode shapes can only be determined w.r.t the nominal value of the angular rate, thus neglecting the (possibly relevant) effect of perturbations of the angular rate due to such causes as gusts or manoeuvres.

In view of these considerations, although new tools for treating modal flexibility in ADAMS have recently become available, ([10]), it has been decided to model the rotor blade by the discrete flexible body approach first introduced by Ryan ([11]) and implemented in the ADAMS/FEA module ([12]). As is well known, this method uses no assumed modal functions, but achieves discretization by substituting the flexible body with a number of rigid sub-bodies connected together by massless compliance elements; each rigid body is characterized in term of concentrated mass and inertia properties, while the compliance elements are described in term of stiffness matrices, which can be determined by a finite element analysis.

The starting point for the hybrid FE/MSA modelling process is therefore a finite element model of the flexible element to be discretized. A NASTRAN model of the rotor blade has been provided to us by Agusta: in this model, the blade is discretized using 42 BEAM elements to characterize its stiffness properties, while an identical number of concentrated masses (both translational and rotational) has been used to approximate the distributed inertial properties. This representation, however, is far too detailed to be directly imported in ADAMS, as this would imply the generation of 42 new parts for each blade in the model (i.e., a total of 4*42=168 parts, being the A109c a four-bladed helicopter).

In order to simplify the FE model, an intermediate step of stiffness condensation and mass lumping has been necessary: a more detailed account of this procedure, as well as of the validation and implementation of the simplified structural model will be given in Section VIII.

V. AERODYNAMIC LOADS

As was briefly mentioned in the previous Section, rotor blades are subject to a distributed, time-varying loading, which is mainly of aerodynamic origin; this Section will describe the mathematical model which has been used for the calculation of the aerodynamic loads acting on each blade element; Section IX will provide more details on the implementation of this model in the form of ADAMS user written subroutines as well as on the integration scheme which has been adopted in order to determine the total forces and moments acting on each blade.

The model is based on classical *lifting line theory*; the major assumption of such a theory is that the aerodynamic loads (lift, drag and pitching moment) acting on the generic blade (wing) section are determined only by the local airflow, which, in turn, is due to:

- the overall motion of the aircraft
- the rotation of the blade
- the rigid body motion and the elastic deformations of the blade
- the rotor induced flow

By summing the contributions of each of these velocities in the local reference frame of a given, generic, blade section, one can then determine the aerodynamic incidence α (see Fig. 9):

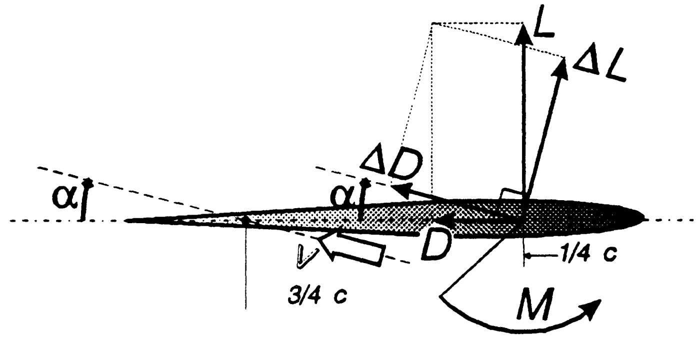

Fig. 9. Determination of section forces and moments (direct flow)

In order to determine the section loads, the classical expressions for their steady components can be used:

lift: $$\Delta L = \tfrac{1}{2}\,\rho\, c\, v^2\, C_l \tag{2}$$

drag: $\Delta D = \frac{1}{2} \rho c v^2 C_d$ (3)

pitching moment: $M = \frac{1}{2} \rho c^2 v^2 C_t$ (4)

where ρ is the air density, c is the blade chord, v is the modulus of the resultant section velocity and C_l, C_d, C_t are non-dimensional coefficients representing the airfoil aerodynamic properties. Such coefficients are, in turn, non-linear functions of α and v; they are obtained by linear interpolation over three look-up tables based on wind-tunnel test data.

Unlike the case of fixed wing aircraft, in helicopter rotors it is not uncommon for a blade section to meet *reverse flow* conditions, i.e., situations when the chordwise component of the local velocity is negative; in such a case, (Fig. 10), one needs to modify the above given expressions, in order to account for the change of sign in the aerodynamic loads. Furthermore, while lifting line theory requires the direct flow local velocities to be evaluated at three quarters of the blade section chord, reverse flow conditions require the value of the local velocities at the first quarter chord of the section ([7]); this effect has been accounted for in the model.

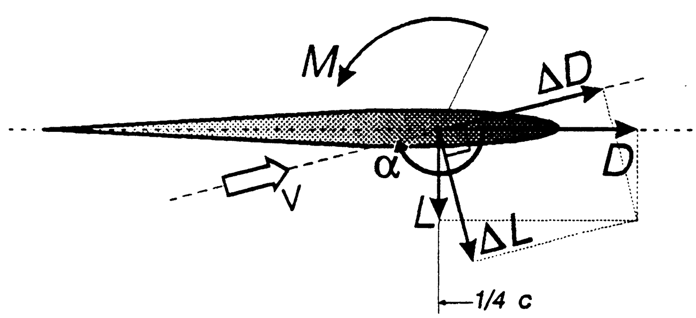

Fig. 10. Determination of section forces and moments (reverse flow)

Remark

Expression (4) for the section pitching moment needs further work before it can be applied in simulations. This is because such an expression provides only the mean (steady) value of the pitching moment and does not account for the pitch rate of the section, which is nonzero due to the periodicity of the pilot commands (see (1)). As the dependence of the pitching moment on the section pitch rate is the *only* source of damping for the blade's pitching dynamics, it is mandatory to modify expression (4) in order to account for this effect. This can be achieved by resorting to unsteady airfoil theory ([7]) and retaining in the expression for the pitching moment the relevant pitch damping term; the section moment then becomes:

$$M = \frac{1}{2} \rho c^2 v^2 C_t - \frac{1}{16} \pi \rho c^3 v \omega_{sx} \quad (5)$$

where ω_{sx} is the pitching angular rate.□

VI. ROTOR INDUCED FLOW

It has been mentioned in the previous Section that the induced flow plays an important role in the determination of the section aerodynamic incidence and, as a consequence, of the section aerodynamic loads. Nowadays a large variety of induced flow models is available in the helicopter literature (see, e.g., [13]); two representative models have been included in the simulation program: the *Glauert* model and the *Peters-He* model. The former is a classical, simple, quasi-steady representation of the rotor inflow, while the latter is a recently developed ([14], [15]) dynamic model, much more sophisticated but also more expensive from the computational point of view. The choice between the two models is left to the user, according to his needs and his requirements for accuracy and simulation time.

a) The Glauert model

This model prescribes an algebraic relationship between the rotor thrust and the mean value of the induced velocity; such a relationship is only valid in the case of vertical flight, but it has been heuristically generalized to the more common situation of forward flight. In this case, the distribution of the induced velocity is assumed to be linear over the rotor disk, i.e., the induced velocity v_i is expressed in the following form:

$$v_i(x,\psi) = v_{io}\left(1 + Kx\cos(\psi)\right) \tag{6}$$

where x and ψ are respectively the radial and azimuthal coordinates over the rotor disk and K is a coefficient which depends on the considered flight condition.

The mean value of the induced velocity (v_{io}) is obtained by solving the following algebraic equation:

$$v_{io}^{4} + 2sen(\alpha)Vv_{io}^{3} + V^{2}v_{io}^{2} - U_T^{4} = 0 \tag{7}$$

where α is the rotor disk incidence (see Fig. 11), V is the free stream velocity and $U_T^{2} = \dfrac{T}{4\pi R\rho}$.

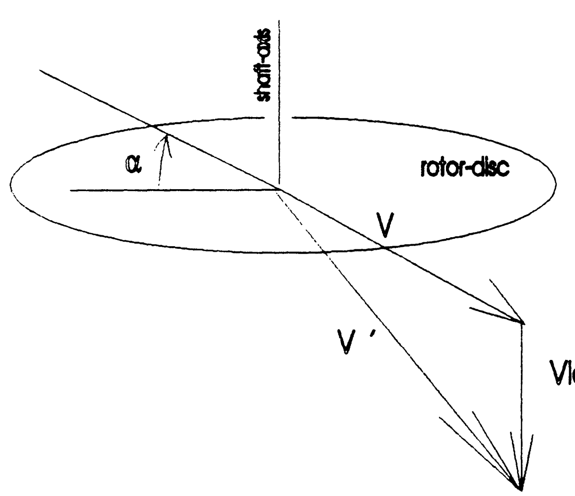

Fig. 11. Rotor induced velocity: Glauert model

This model combines a great simplicity with a good accuracy for low-frequency applications (i.e., rotor performance studies), but it is not adequate to the analysis of high frequency characteristics of rotor response. For this kind of application, a more accurate model is required.

Remark

Strictly speaking, the quantities involved in this model are values averaged over one rotor revolution; from the implementation point of view, this time-averaging could be obtained by low-pass filtering the instantaneous values of T. However, this filtering would only change the high frequency behaviour of the model, which is anyway beyond the scope of the model itself. This choice significantly simplifies the implementation, which only requires a variable (to store the value of v_{io} to be used in aerodynamic load computations) and an iterative (Newton-Raphson) procedure, performed by the VARSUB subroutine to solve the 4th order algebraic equation (7).□

b) The Peters-He model

This model is based on a straightforward application of mass and momentum conservation laws to the airflow across the rotor surface: as in Section IV, the problem is that of solving a set of PDEs, with the associated boundary conditions ([14]).

Under a few restrictive assumptions it can be proved that the PDEs associated to the aforementioned conservation laws can be solved over the rotor disk by a modal expansion approach: the (unknown) distribution of induced velocity over the disk is expanded in a Fourier series over the rotor azimuth ψ and in series of Legendre polynomials over the radial coordinate.

Therefore, a finite set of time-dependent modal coefficients ($\{\alpha_j^r\}, \{\beta_j^r\}$, j=1,.., n and r=1,.., m, with n, m selected by the user) is sufficient for the determination of the velocity distribution.

The modal coefficients can be determined as the state of a linear dynamic system of the form:

$$\begin{aligned} \{\dot{\alpha}_j^r\} &= -[M]^{-1}[L^C]^{-1}\{\alpha_j^r\} + \frac{1}{2}[M]^{-1}\{\tau_n^{mc}\} \\ \{\dot{\beta}_j^r\} &= -[M]^{-1}[L^S]^{-1}\{\beta_j^r\} + \frac{1}{2}[M]^{-1}\{\tau_n^{ms}\} \end{aligned} \tag{8}$$

where $[M], [L^S], [L^C]$ are matrices which depend on the considered flight condition and $\{\tau_n^{mc}\}, \{\tau_n^{ms}\}$ are integrals of the thrust distribution over the rotor disk, weighted with the assumed expansion mode shapes.

The induced velocity distribution can be reconstituted by weighted sums of the modal coefficients $\{\alpha_j^r\}, \{\beta_j^r\}$ with the respective disk mode shapes:

$$w(\nu,\psi,t)=\sum_{\substack{r=0,1\ldots\\ j=r+1,r+3\ldots}}^{\infty}\phi_j^{\,r}(\nu)\left[\alpha_j^{\,r}(t)\cos(m\psi)+\beta_j^{\,r}(t)\,\mathrm{sen}(m\psi)\right] \tag{9}$$

Unlike the Glauert model, the Peters-He model does not prescribe a fixed induced velocity distribution and is therefore much more accurate in describing the complexity of the rotor wake induced flow; however this increased accuracy is to be paid in terms of computational effort, as will be discussed in Section X.

Remark

Although more accurate, the Peters-He inflow model is only valid in the forward flight condition, since its first principle equations assume that the induced velocity is small when compared with the free-stream velocity (this is necessary to ensure the linearity of the model equations and therefore allow their modal expansion (8)). For this reason, the model is not directly applicable in the hovering case; it is anyway possible, as suggested in [14], to use it as a perturbation model in the neighbourhood of the average value of induced velocity (usually significantly high in hovering flight) as provided by the Glauert model.□

VII. CONTROL SYSTEM AND ROTOR HUB: MODEL IMPLEMENTATION

a) ADAMS implementation of the control system

As was described in Section III, the *N* blade pitch angles (*N* being the number of blades) clearly provide a set of degrees of freedom which could be used to influence the rotor response; although only three degrees of freedom are actually used by the pilot to control the rotor (see (1)), many automatic control algorithms might require the use of the complete set of DOFs, as could be achieved by the use of a hydraulic actuator on each blade.

In Section III it was suggested that the links and the swash plate could be implemented as ADAMS parts, but this option was considered unnecessarily complicated, for the following reasons:

- it is possible to derive a simple yet sufficiently accurate analytical relation between the three control variables (collective pitch, longitudinal and lateral cyclic pitch) and the lower end position of each blade pitch link in the following terms:

  $$\Delta z = b\,\vartheta(t),$$

  where b is a geometry-dependent constant and θ is the blade pitch command.

- as was mentioned in Section III, the flexibility of the control system is an important part of the model. However most of the elastic properties of the control system are not easily available; therefore it is necessary to introduce an approximate description of its structural characteristics, i.e., to use an equivalent elasticity, as will be described shortly.

- one of the goals of the simulation program is to provide a framework to test active control algorithms for vibration reduction: to this purpose, a modelling scheme which leaves available all N degrees of freedom is by far preferable, due to its more general applicability.

The implemented model simulates the action of each link on the corresponding blade by means of an SFORCE statement, linearly dependent on the distance d(t) between two markers which represent the ends of the pitch link. However, due to the absence of the swash plate, the actual length of the link is given by

$$l(t) = d(t) - b\vartheta(t) = \varepsilon(t) + l_0 \qquad (10)$$

where ε(t) represents the elastic deformation of the pitch link. It is then straightforward to introduce in the SFORCE expression the linear dependence on ε(t) with a suitable elastic constant K_{el}.

This simple scheme makes it possible to :

- include in the model the main structural characteristics of the control system.

- properly describe the position of the upper end of the pitch link, which will depend on the full blade motion, thus automatically taking into account the important effects of flapping and lagging on the actual pitch angle.

- implement the pitch control θ of each blade as an ADAMS variable, thus leaving available all the N degrees of freedom (although they can, of course, include a term depending on the collective and cyclic pitch commands, as if the swash plate were present).

It should finally be noted that in the case of explicit modelling of the swash plate the implementation would become more involved: as a matter of fact, if one wants to impose a motion to be computed on line by a MOTSUB subroutine (as is the case for control algorithms), the subroutine should also provide the first and second derivatives of the displacements. On the contrary, the use of variable ε(t) also simplifies the introduction of a damping term in the SFORCE, by means of ADAMS's DIFF statement; such a damping term has proved indispensable, mainly for numerical reasons.

b) Stability and trimming

Although the implementation of the control system is relatively short in the ADAMS model file, it is not surprising that it requires a special care due to its physical relevance.

Specifically, two problems must be solved.

The first one is related to the choice of the elastic constant K_{el}, as well as of the damping coefficient: in fact these parameters greatly influence the blade's torsional dynamics as regards respectively the first mode of vibration and its stability.

A simplified analytical model for coupled flapping and pitching motion of the blade (see [9]) has allowed the study of the collocation of its poles, in order to eventually set

K_{el} and the damping coefficient to values ensuring both stability and matching of the experimental value for the first torsional frequency. Some fine tuning has then been performed on the more comprehensive simulation program.

The second problem, namely trimming, is that of finding such values of the three pilot commands as to ensure the desired constant speed, steady state, forward flight condition. In the simulation program, the fuselage is absent, so that the flight condition is represented by a marker and a variable, describing, respectively, the direction and the amplitude of the velocity vector v; therefore the trimming procedure aims at setting the triple $\vartheta_0, \vartheta_{1c}, \vartheta_{1s}$ and v to consistent values, in order to obtain realistic simulation results, particularly for the blade periodic motion. Also in this case, a simplified (low frequency) model of the rotor was used, in order to predict and to invert the dependency

$$\underline{v} = \underline{f}(\underline{\vartheta}).$$

Finally, the user of the simulation program is left the possibility to choose the desired flight condition; as a matter of fact, the ADAMS model file contains only the statements which are independent of the selected flight condition, as well as of all the other user-defined parameters (like, e.g., the order of the modal expansion for the induced velocity distribution); the program takes care of updating the ADAMS model file according to the user's needs (see also Section X); this feature has been implemented by the CONSUB subroutine.

VIII. BLADE STRUCTURE: MODEL IMPLEMENTATION

As already mentioned in Section IV, the blade structural properties are described by a NASTRAN finite element model, and must be included in the program in order to insure that the model will reproduce at least the vibratory modes of the blade whose natural frequencies fall in the range of interest. Moreover, for the case of a helicopter rotor blade, it is particularly important to consider such non linear effects as (in order of importance):

- the presence of a relevant centrifugal force, having a stiffening effect on blade bending
- the blade rotation around its feathering axis (see the equations of motion of the blade as derived in [8])
- the presence of Coriolis forces.

Under this respect, the multibody approach implemented in ADAMS is particularly attractive, allowing at the same time:

- to represent the structural properties of the blade in a way which is very similar to the original FE formulation, the only difference being the implicit constraint on the inertial matrix (which must be equivalent to a set of concentrated masses)

- to eliminate any a priori assumption (notably constant angular velocity Ω) in the generation of the dynamic equations of motion, thus preserving the non linear effects in full generality.

The second point presents the obvious drawback of an increased computational load, if compared to FE; in order to compensate for this, the possibility to proceed to a superelement reduction of the reference NASTRAN model was analyzed, with the aim of reducing the number of degrees of freedom to be represented in the ADAMS model. Since, as already stated, the reduced order model must still be able to predict accurately the first modes of vibration, the following steps have been taken for its determination (see, e.g., [12] and [16]):

- determine one possible choice of the internal nodes, and compute the corresponding stiffness condensation (using NASTRAN, see [17])

- determine the lumped mass representation corresponding to a suitable re-grouping of the concentrated masses (both described by CONM2 cards); this has been done by importing in MATLAB the data contained in the GRID and CONM2 cards present in the NASTRAN data deck and using MATLAB functions for mass condensation developed to this purpose

- compute the reduced order model eigenvalues and compare them with the ones of the reference model, using a boundary condition resembling the one the blade is subjected to in the actual full rotor model.

The first two steps correspond to the determination of a coarser-grain discrete model by assembling the basic components of the reference FE model, and they are therefore partially subjective. However, provided that the extensions and inertias of the condensed elements are chosen as similar as possible, the only really influential parameter is the number of elements,.
Moreover, the third point provides an objective evaluation criterion for the performance of the reduced order model.

Of course, these models must be eventually implemented in ADAMS, hence the choice of the lumped mass method for the inertia matrix condensation; it should also be remembered that the motion of each rigid body in the ADAMS model will be defined by the DOFs of one of the master nodes, therefore the inertia condensation should provide centers of mass as close as possible to such nodes (in order to be physically significant).
In the ADAMS model, the flexible blades have been represented by nine rigid bodies, for a total of 54 DOFs in each blade; as the data in Table 1 clearly indicate, this choice sacrificed very little accuracy in the frequency band of interest (that is, at least up to the first torsion-dominated mode, indicated with T1 in the table), allowing at the same time considerable computational savings. Current work is aiming at a further simplification of the blade structural model.

mode	NASTRAN MODEL (42 elements)	ADAMS MODEL (9 parts)	rel. err. (%)
F1	6.097	6.063	0.57
F2	19.90	19.53	1.85
L1	36.74	36.51	0.64
F3	38.21	37.43	2.04
T1	58.71	58.52	0.34
	67.57	64.48	4.57
	99.77	94.00	5.78
	113.8	111.3	2.23
	124.3	120.4	3.12

Table 1. Natural frequencies (hz) for the first nine modes of a hinged, non-rotating blade.

Since this reduced order model was created with NASTRAN, it has been almost straightforward to convert it to an ADAMS model, by using the modules NASUNI and UNIADM, although version 7.0 of such modules does not properly handle BEAM cards, so that some manual intervention turned out to be necessary.
As a result, each blade is constituted by nine rigid bodies, whose main characteristics are their inertia, their center of mass markers and the master node markers; every couple of adjacent bodies reciprocally interact by means of an NFORCE statement (attached to the corresponding master nodes), which accounts for the elastic properties of the blade.

Remark
A rigid blade model is also supplied in the simulation program; this is only to take advantage of the inherently higher simulation speed, for the cases when high frequency (4Ω and multiples) accuracy is not required. This model is by far more efficient from the computational point of view and proves very useful for first order analysis.□

IX. AERODYNAMIC LOADS: MODEL IMPLEMENTATION

In this Section, the computation and the application of the aerodynamic forces will be explained; the expressions used to determine the loads per unit length have already been discussed in Section V; the value of the induced velocity will be here assumed as known wherever necessary.
Three main points may be identified:

- discretization of the distributed aerodynamic loads

- representation of the relevant geometric and aerodynamic properties of the blade in the ADAMS model

- numerical integration of the spanwise loading distribution

The first point deals with the necessity of identifying a finite number of significant characteristics of the load distribution to be used as the model inputs: clearly, they must be supplied by the method used to discretize the equation of motion. In our case the most accurate choice would be the one suggested by the finite element analysis, i.e., a weighted integral of the section loads to be applied to the corresponding master node as an equivalent concentrated load.
Unfortunately this approach presents two difficulties:

- it is not very well suited to the ADAMS MSA approach, since the weight functions depend on more information than the one contained in the NFORCES

- it increases the computational load, due to the higher degree of interaction between the parts composing the blade, since each integration interval extends on more than one part

So it was considered more reasonable to be coherent with the rigid body decomposition scheme, which ADAMS would anyway use to compute the inertial forces; this means that the usual resultant/moment decomposition has been employed (equivalently stated, the weight functions are constant/linear in the integration interval).

In order to compute these equivalent load values the following information is therefore needed:

- a reference frame for each section, in order to:

    1. decompose vectors

    2. attach the resulting forces (GFORCE statement)

    3. identify the blade chord direction

- the airfoil tables (N23012)

- the velocity vector v

- radial coordinate and chord of each blade section

In terms of the underlying ADAMS model, the description of an integration interval has required the introduction of a set of markers, named "aero_bnn", and the associated GFORCE statements, sharing a common id number *bnn*, *b* being the blade index and *nn*

the segment index. Two vectors of constants ("chord" and "abscissas") were also necessary, to supply the blade chord values and radial abscissas computed at the boundary points of the integration interval.
It follows from the above that in each blade part one or more of these intervals need to be defined, so that no interval will be shared by more than one part. As a matter of fact, to increase the number of integrals for each part is only useful to locally refine the numerical integration (explained below), and was not considered necessary in the (nine part) flexible blade model; this option was anyway used for the rigid blade model to increase the integration accuracy near the blade tip.

The computation is then performed by a GFOSUB subroutine, relying on ADAMS utility subroutines to obtain the necessary velocities and angular displacements relative to the velocity vector v at the points of interest, in order to compute aerodynamic loads (see Section IV); the aerodynamic coefficients account for static stall and compressibility effects due to Mach number changes; the original, experimental, look-up tables, supplied by AGUSTA, have been spline-interpolated in order to increase the number of available points, thus reducing run-time (linear) interpolation inaccuracies.

For faster convergence, the numerical integration over each interval is based on the well-known Gaussian formulas; their accuracy can be controlled by the user in terms of the number of points to be used (such a parameter can be selected in the preliminary phase of the simulation, handled by the CONSUB subroutine).

X. ROTOR INDUCED FLOW: MODEL IMPLEMENTATION

The implementation of the Peters-He inflow model requires, at each integration step (and depending on the selected values of parameters m and n, see Section VI):

- the determination of the $\{\tau_n^{mc}\}, \{\tau_n^{ms}\}$ integrals of the rotor thrust distribution
- the integration of the set (8) of linear differential equations
- the reconstitution (by modal summation) of the induced velocity distribution

The second point is easily implemented in ADAMS using the LSE statement, taking advantage of the linearity of the ODEs; the dynamic matrix of the linear system must be computed during the preliminary phase (by the CONSUB procedure) on the basis of the selected flight condition, along with the reference steady state values $\overline{T}$ and $\overline{v}$ for the rotor thrust and mean induced velocity (obtained using the Glauert model, see the Remark in Section VI).
As regards the third point, it is left to the GFOSUB procedure to select the points of interest for the calculation of v_{io} (as required by the Gaussian formulas, see Section IX) and to perform the series summation using the LSE state variables as expansion coefficients, during the computation of the aerodynamic loads.

The efficient computation of the input vector $\{\tau_n^{mc}\}, \{\tau_n^{ms}\}$ has on the contrary required particular attention: the main obstacle is given by the fact that each

component of the input vector depends on the whole distribution of the aerodynamic load over the rotor disk; such a distribution, in its turn, depends on almost the entire mechanical state of the rotor (see Section IX).
Since ADAMS implicit numerical integration techniques require the computation of the jacobian matrix, such a wide dependence will require to repeat the determination of the components of the input vector for a significant number of times.
This is clearly unavoidable, as the computation of the jacobian matrix is the very heart of the stiff integrators; however one can take advantage of the fact that the input components differ only in the weighting functions, while they share the most demanding part of the computation, which is that of determining the load distribution.
Unfortunately, the implementation of this scheme is not quite straightforward using the LSE statement, given the fact that the solver will expect the VARSUB subroutine to compute one input component at a time, while it would be more natural to compute the whole input vector in a single call.
This problem has been worked around by allocating a static array where the whole input vector is stored following its computation at the very first call, to be read at each subsequent call. There are two potential shortcomings in this solution:

- this approach relies heavily on an assumption on the order followed by the solver for the computation of the jacobian matrix; this order is not documented and might be modified in future versions

- a large amount of memory must be allocated in order to store the results, because the above mentioned computation is performed row-wise instead of column-wise, that is two successive calls to the VARSUB procedure will differ in the rotor disk state rather than in the input component to be computed

In spite of these potential problems, the overall computational savings have been found to be so significant to make the adoption of the described scheme extremely useful.

XI. CONCLUDING REMARKS

A feasibility study has been performed in order to design an ADAMS-based simulation program for the main rotor dynamics and aerodynamics of the AGUSTA A109c helicopter.
The complexity of the phenomena under investigation (rotor blade flexibility, unsteady aerodynamic loading, non-uniform and time-varying wake) have required extensive use of ADAMS's features such as FEA modelling, user-written subroutines and linear systems simulation and the development of an *ad hoc* subroutine (CONSUB) for simulation management and control.
Current work aims at incresing the computational efficiency of the simulation code; the correlation of the model outputs with experimental flight test data will also be performed during 1995.

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