A Note On ADAMS Functions $DZ(i,j,k)$ and $VZ(i,j,k)$

Sudhakar Medepalli and **Brett Harris**
Consulting Services
Mechanical Dynamics Inc.
Tel: 313 913 2551
email: smede@adams.com

In this article we discuss the relation between the two ADAMS functions $DZ(i,j,k)$ and $VZ(i,j,k)$ and by extension between DX and VX, and DY and VY respectively. We also illustrate the importance of reference frames while deriving the time derivatives of various vector quantities. Finally we conclude the note with an example where the results derived here are useful. We first develop some notation.

Notation

The following notation is used throughout this article.

- ${}^{A}\bar{\omega}^{B}$ stands for the angular velocity of the reference frame B with respect to reference frame A. In general a left superscript stands for a reference frame relative to which a quantity is measured or an operation is performed.
- O stands for the inertial reference frame (Ground).
- If i is the ID of a marker, I is assumed to be the corresponding reference frame with unit vectors $(\hat{\mathbf{x}}_i, \hat{\mathbf{y}}_i, \hat{\mathbf{z}}_i)$ respectively.

Some Useful Formulae

The following standard results of vector algebra and kinematics are used throughout this short paper.

Given the vector quantities $\bar{\mathbf{a}}$, $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$, and the reference frames A, B and C, we have:

- $$\bar{\mathbf{a}} \times \bar{\mathbf{b}} = -\bar{\mathbf{b}} \times \bar{\mathbf{a}} \tag{1}$$
- $$\bar{\mathbf{a}} \cdot [\bar{\mathbf{b}} \times \bar{\mathbf{c}}] \;=\; \bar{\mathbf{b}} \cdot [\bar{\mathbf{c}} \times \bar{\mathbf{a}}] \;=\; \bar{\mathbf{c}} \cdot [\bar{\mathbf{a}} \times \bar{\mathbf{b}}] \tag{2}$$
- $${}^{A}\bar{\omega}^{B} = {}^{A}\bar{\omega}^{C} + {}^{C}\bar{\omega}^{B} \tag{3}$$

- The time derivative of a vector quantity depends on the reference frame in which the differentiation is performed. In fact

$$\frac{{}^A d}{dt}\,\bar{\mathbf{a}} = \frac{{}^B d}{dt}\,\bar{\mathbf{a}} + {}^A\bar{\omega}^B \times \bar{\mathbf{a}} \tag{4}$$

  On the other hand, the time derivative of a scalar quantity is independent of the reference frame.

Time Derivative of $DZ(i,j,k)$

With the above preliminaries, let us derive the time derivative of the ADAMS function $DZ(i,j,k)$. By definition

$$\begin{aligned} DZ(i,j,k) &= [\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j] \cdot \hat{\mathbf{z}}_k && (5) \\ VZ(i,j,k) &= [\bar{\mathbf{v}}_i - \bar{\mathbf{v}}_j] \cdot \hat{\mathbf{z}}_k && (6) \end{aligned}$$

where $\bar{\mathbf{r}}_i$ is the position vector of the i marker (expressed in some convenient reference frame) and $\bar{\mathbf{v}}_i$ is the velocity of the origin of the i marker with respect to the inertial reference frame. Let us choose some arbitrary reference frame A to obtain the time derivative of the scalar DZ. Using Eq.(5) we get

$$\begin{aligned} \frac{d}{dt}DZ(i,j,k) &= \frac{d}{dt}\left[(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \hat{\mathbf{z}}_k\right] \\ &= \left[\frac{{}^A d}{dt}(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)\right] \cdot \hat{\mathbf{z}}_k + (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \left[\frac{{}^A d}{dt}\,\hat{\mathbf{z}}_k\right] && (7) \end{aligned}$$

Using Eq.(4), we can express these derivatives with respect to the inertial reference frame O as

$$\begin{aligned} \frac{d}{dt}DZ(i,j,k) &= \left[\frac{{}^O d}{dt}(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) + {}^A\bar{\omega}^O \times (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)\right] \cdot \hat{\mathbf{z}}_k \\ &+ (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \left[\frac{{}^O d}{dt}\,\hat{\mathbf{z}}_k + {}^A\bar{\omega}^O \times \hat{\mathbf{z}}_k\right] && (8) \end{aligned}$$

Note that since $\hat{\mathbf{z}}_k$ is a unit vector of K,

$$\frac{{}^K d}{dt}\,\hat{\mathbf{z}}_k = 0 \tag{9}$$

and application of Eq.(4) with $A = O$ and $B = K$ gives

$$\frac{{}^O d}{dt}\,\hat{\mathbf{z}}_k = {}^O\bar{\omega}^K \times \hat{\mathbf{z}}_k \tag{10}$$

Using the triple product relation given in Eq.(2) and the property of vector cross products given in Eq.(1) we see that the terms containing ${}^{A}\bar{\omega}^{O}$ in Eq.(8) cancel out. Hence we get

$$\frac{d}{dt}DZ(i,j,k) = \left[\frac{{}^{O}d}{dt}(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)\right] \cdot \hat{\mathbf{z}}_k + (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \left[{}^{O}\bar{\omega}^{K} \times \hat{\mathbf{z}}_k\right] \tag{11}$$

The RHSs of Eqs.(7) and (11) clearly show that the time derivative of a scalar (DZ in this case) is invariant with respect to the reference frame in which it is obtained. Comparing the first term on the RHS of Eq.(11) to Eq.(6) we note that they are the same. Thus,

$$\boxed{\tfrac{d}{dt}DZ(i,j,k) = VZ(i,j,k) + (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \left[{}^{O}\bar{\omega}^{K} \times \hat{\mathbf{z}}_k\right]} \tag{12}$$

The above equation clearly shows that the derivative of $DZ(i,j,k)$ is in general not equal to $VZ(i,j,k)$. They are equal when the second term on the RHS of Eq.(12) is zero, which happens under one of the following circumstances.

1. ${}^{O}\bar{\omega}^{K} = 0$. This in turn is true if $K = O$, i.e., the reference marker is on ground; or if the reference frame K is translating (not necessarily at constant velocity) without rotating with respect to the ground.

2. $\bar{\mathbf{r}}_i = \bar{\mathbf{r}}_j$, i.e., the i and the j markers are coincident.

3. $\left[{}^{O}\bar{\omega}^{K} \times \hat{\mathbf{z}}_k\right] = 0$ This is true if the reference frame K is rotating about its own z-axis.

4. $(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \left[{}^{O}\bar{\omega}^{K} \times \hat{\mathbf{z}}_k\right] = 0$. This occurs if we can write

$$(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) = \alpha\, \hat{\mathbf{z}}_k + \beta\, \frac{{}^{O}\bar{\omega}^{K}}{|{}^{O}\bar{\omega}^{K}|} \tag{13}$$

   where α and β are any two real numbers and $|{}^{O}\bar{\omega}^{K}|$ is the magnitude of the vector ${}^{O}\bar{\omega}^{K}$. In other words $(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)$ lies in the plane containing $\hat{\mathbf{z}}_k$ and ${}^{O}\bar{\omega}^{K}$.

Example Application

Now we illustrate an application of the above result through an example. The ADAMS function IMPACT has the syntax

$$IMPACT(x, \dot{x}, x_1, k, e, c_{max}, d)\ , \tag{14}$$

where x is the penetration distance and $\dot{x}$ is the corresponding speed and the remaining variables specify various parameters. Most of the time we use $DZ(i,j,k)$ for x

and $VZ(i,j,k)$ for $\dot{x}$ and we have seen that this may lead to incorrect results. Thus, if we want to use VZ for speed, we must make sure that one of the conditions specified above holds true.

1. Note that in the IMPACT expression, if we use $DZ(i,j,k)$ with the reference marker k equal to either i or j, the first condition stated above may not be valid since ${}^{O}\bar{\omega}^{I}$ or ${}^{O}\bar{\omega}^{J}$ may not be zero.

2. The second condition can be satisfied by placing the i and the j markers on two dummy parts (if necessary), one on each colliding part, and constraining the dummy parts in such a way that they always coincide with the point of impact.

3. As already mentioned, the third condition is true if the reference frame K is rotating about its own z axis, which may not be true in general.

4. The fourth condition can be satisfied by aligning the z-axis of the reference marker along the line connecting the two points coming under contact.

If none of the above four conditions are satisfied, we must compute the actual speed $\dot{x}$ as given by Eq.(12). In ADAMS we can implement it as follows. Using Eqs.(1) and (2) we can rewrite Eq.(12) as

$$\frac{d}{dt}DZ(i,j,k) = VZ(i,j,k) - \left[{}^{O}\bar{\omega}^{K} \times (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)\right] \cdot \hat{\mathbf{z}}_k \tag{15}$$

Note that if we express the vector quantities ${}^{O}\bar{\omega}^{K}$ and $(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)$ in terms of the unit vectors $(\hat{\mathbf{x}}_k, \hat{\mathbf{y}}_k, \hat{\mathbf{z}}_k)$ of reference frame K, we just need to evaluate the $\hat{\mathbf{z}}_k$ component of the cross product term in Eq.(15) since we are dotting it with $\hat{\mathbf{z}}_k$. We can write this as

$$\left[{}^{O}\bar{\omega}^{K} \times (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)\right] \cdot \hat{\mathbf{z}}_k = \left[{}^{O}\bar{\omega}^{K} \cdot \hat{\mathbf{x}}_k\right] * \left[(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \hat{\mathbf{y}}_k\right] - \left[{}^{O}\bar{\omega}^{K} \cdot \hat{\mathbf{y}}_k\right] * \left[(\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j) \cdot \hat{\mathbf{x}}_k\right] \tag{16}$$

In terms of ADAMS functions this can be written as

$$\left[{}^{O}\bar{\omega}^{K} \times (\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j)\right] \cdot \hat{\mathbf{z}}_k = WX(k,o,k) * DY(i,j,k) - WY(k,o,k) * DX(i,j,k) \tag{17}$$

Using Eqs.(15) and (17) we get

$$\frac{d}{dt}DZ(i,j,k) = VZ(i,j,k) - WX(k,o,k) * DY(i,j,k) + WY(k,o,k) * DX(i,j,k) \tag{18}$$

Conclusions

Through this short article we illustrated the relation between the ADAMS functions DZ and VZ. We also pointed out the significance of reference frames while finding the time derivatives of vectors. Finally, note that the same type of results apply to DX and VX, and DY and VY respectively. We can summarize the results as follows.

$$\boxed{\frac{d}{dt}DX(i,j,k) = VX(i,j,k) - WY(k,o,k) * DZ(i,j,k) + WZ(k,o,k) * DY(i,j,k)}$$

$$\boxed{\frac{d}{dt}DY(i,j,k) = VY(i,j,k) - WZ(k,o,k) * DX(i,j,k) + WX(k,o,k) * DZ(i,j,k)}$$

$$\boxed{\frac{d}{dt}DZ(i,j,k) = VZ(i,j,k) - WX(k,o,k) * DY(i,j,k) + WY(k,o,k) * DX(i,j,k)}$$

References

[1] *ADAMS Reference Manual*, Mechanical Dynamics Inc., Ann Arbor, 1992.

[2] Kane T.R. and Levinson D.A., *Dynamics* , McGraw-Hill, New York, 1985.