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Title: Vibration Fatigue Analysis in the Finite Element Environment

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ABSTRACT

Fatigue damage is traditionally determined from time signals of loading, usually in the form of stress or strain. However, there are many design scenarios when the loading, or fatigue damage process, cannot easily be defined using time signals. In these cases the design engineer usually has to use a test based approach to evaluate the fatigue life of his structure or component. Or, alternatively, a frequency based fatigue calculation can be utilised where the loading and response are categorised using Power Spectral Density (PSD) functions.

One very important design problem, which falls into this category, is that of acoustic fatigue. However, there are also many other situations where structures are subjected to a random form of loading such as wing flutter, landing gear runway profiles, engine vibrations and so on. All of these situations can be analysed using new fatigue life estimation techniques now incorporated in MSC/FATIGUE.

The theory of random vibration fatigue has seen a number of important developments over the last fifteen years. The authors have been personally involved in developing new fatigue analysis theories and structural analysis techniques in the frequency domain. More recently this work has focused on the link with Finite Element Analysis (FEA) because of the powerful design opportunities which this creates. The work has found many important practical applications. This paper will provide a state of the art perspective of random vibration fatigue technology. A number of design applications will be presented.

INTRODUCTION

Frequency based techniques for fatigue life prediction are accessible to the designer through a number of stand-alone (non FEA) technical approaches. This technology is also now available within the FEA world as part of the MSC/FATIGUE package distributed by the MacNeal Schwendler Corporation. This paper is intended to provide the required technical background for the practical designer. Where more detailed information is required the reader is referred to the list of technical papers cited at the back of the document. It is necessary to clarify the term ***vibration fatigue*** as the estimation of fatigue life when the stress or strain histories applied to the structure, or component, are random in nature and therefore best specified using statistical information about the process. The same approach can also be described using the terms spectral fatigue analysis or frequency based fatigue techniques.

Nearly all structures or components have traditionally been designed using time based structural and fatigue analysis methods. However, for many complex structures such as offshore platforms, wind turbines, and more generally, any dynamically responsive system, a frequency-based approach for the structural and fatigue analyses is more appropriate. Furthermore, because of the nature of a time based approach the complexity of the stress or strain time histories means that the stress or strain time histories have to be considerably simplified. In addition, the fatigue analysis is only undertaken as a checking procedure at the end of the overall design process. By developing a frequency based fatigue analysis approach the true composition of the random stress or strain responses can be retained within a much optimised fatigue design process. This can yield many advantages, the most important being, (i) an improved understanding of system behaviour, (ii) the capability to fully include the true structural behaviour rather than a potentially inadequate simplified version and (iii) a more computationally efficient fatigue analysis process. MSC/FATIGUE offers these same advantages but in addition, it offers an integrated FEA solution linked to most of the computational solvers such as, for instance, MSC/NASTRAN. This will enable a designer to optimise for fatigue at the preliminary stage of the overall design process in a way, which has not previously been possible.

The fatigue life of structures is usually estimated towards the end of a typical design cycle. The reason for this is that the stress or strain histories used in a fatigue analysis are usually measured from prototypes. In this respect fatigue design can often be considered to be a checking process. If a component or prototype passes such a check it would be highly unlikely that the designer would have the opportunity to reduce material volumes or make other structural cost savings because of the implications this might have on other aspects of the design process. However if the structural and fatigue analyses can be incorporated within an FEA approach at a much earlier point in the design cycle, then the opportunity to design for fatigue becomes feasible. This link with FEA is what makes vibration fatigue tools so powerful.

Alternative descriptions of engineering processes

Most designers, if asked to specify a random loading input, or response output, for a structural system would specify the random time signal shown in figure 1. This process can therefore be described as ***random*** and in the ***time domain***. The process is described as random because, strictly speaking, it can only be determined statistically. A second sample taken for the same process would obviously have different values to the first. There are several alternative ways of specifying the same random process. Fourier analysis allows any random loading history of finite length to be represented using a set of sine wave functions, each having a unique set of values for amplitude, frequency and phase. Such a representation is called ***deterministic*** because the individual sine waves can be determined precisely at any given point in time. It is still time based and so is therefore specified in the time domain.

As an extension of Fourier analysis, Fourier transforms allow any process to be represented using a spectral formulation such as a ***Power Spectral Density*** function (PSD). Such a process is described as a function of frequency and is therefore said to be in the ***frequency domain.*** It is still a random specification of the function.

For the vast majority of engineering problems, if you have one form of the above three loading specifications you can quite easily get to one of the two alternative specifications. These transformations rely on the assumption that the process is stationary, random and Gaussian. Fortunately, most engineering processes conform reasonably well to these assumptions.

Coupled with each of these three specifications for the loading are three alternative analysis types. The key question for a designer is therefore which type of structural analysis to use, and subsequently which type of fatigue analysis approach, to use.

Analysis Options

In nearly every industrial sector designers have a choice about which analysis approach to use. For instance, the environmental conditions experienced by aircraft structures can be represented using either a discrete gust approach or a continuous gust spectrum of atmospheric turbulence. The data for discrete gusts is typically obtained from sources such as ESDU data sheets. This data was obtained several decades ago using aircraft which were typically much stiffer than those in service today. Designers therefore have to tune the shape of the gust to ensure that possible dynamic modes in modern slender structures are induced. It is important to note that these measured gusts are not, therefore, independent of the structure from which the measurements were taken, or the structure being designed. In contrast, the continuous gust spectrum of atmospheric turbulence is a characteristic of the atmospheric loading only. This de-coupling of input loading and structural system is typical of a random (PSD) type of analysis where a transfer function is used to represent the structural system. It is also true to say that the gust spectrum is a more accurate representation of the input loading than the derived discrete gust.

Because of these factors it is usually desirable, where possible, to do the structural analysis in the frequency domain using PSD's and transfer functions. However, until recently there was no generally applicable fatigue tool to complete the analysis and so this has significantly restricted the use of the random PSD approach by fatigue designers. However, due to the introduction of the frequency based fatigue tools described in this paper, such approaches can now be seriously considered by the designer.

WHAT IS THE FREQUENCY DOMAIN?

Structural analysis can be carried out in either the time or frequency domains as shown in figure 2. In the time domain the input takes the form of a time history of load (in this case wind speed). The structural response can be derived using a finite element representation coupled with a transient (convolution) solution. In the frequency domain the input is given in the form of a PSD of wind speed and the structure is modelled by a linear transfer function relating input wind speed to the output stress at a particular location in the structure. The output from the model is expressed as a PSD, in this case it is the PSD of stress. Most of the computational time is spent in solving the structural model. In the time domain, the structural model is solved for each time history of input, hence 20 load cases would take 20 times as long to calculate. In the frequency domain the linear transfer function is only calculated once, hence 20 load cases takes little more time to analyse. Obviously, if we are calculating a linear structural model then the structure must behave linearly. Fortunately in most engineering situations this is a reasonable assumption.

The Fourier Transform

The French Mathematician J. Fourier (1768-1830) postulated that any periodic function can be expressed as the summation of a number of sinusoidal waves of varying frequency, amplitude and phase. Each individual sinusoidal wave can be expressed as a spike in the frequency domain and as the number of sine waves increase, the difference in frequencies between them tend to zero, and so the spikes tend to merge into a continuous function. Basically, the frequency domain is another way of representing a time history. Certain information about a random process becomes apparent in a frequency domain plot, which is difficult to see in the time domain. It is easy to flip back and forth between the two domains using the Fourier Transformation and Inverse Fourier Transformation respectively (see figure 3). In this way an Engineer can see both time and frequency domain representations of a signal in the same way as he would flip a graph between log and linear axes to gain a different perspective.

What is a Power Spectral Density (PSD)?

PSD's are effectively obtained by taking the modulus squared of the FFT. Frequency domain techniques were pioneered by the electronics industry in the early 1940's. The method was employed to investigate the cause of electrical noise in circuits. As power was generally the parameter being measured, it found its way into the title of the spectrum. The mean square amplitude of the sinusoidal waves is used because this was easily obtained by the early analogue circuits of the day, it would be more difficult to derive the amplitude spectra and so this step was never carried out.

Getting a representative time series.

Many design standards give data on random processes in the form of PSDs. This is particularly true of environmental conditions such as wave elevations, wind speeds and earthquake accelerations. Using the vibration fatigue analysis techniques detailed in this paper, it is possible to take these spectra and calculate the anticipated fatigue life of a structure. However, if a time domain analysis was adopted it would be necessary to regenerate a statistically similar time history from the PSD data. Figures 4 and 5 give an engineering perspective of how this might be done. Firstly, if the PSD is split into, say, 40 strips and the area of each strip found, the area of each strip can be used to produce an equivalent sine wave. The amplitude of each equivalent sine wave is equal to the square root of the area times 1.41. This comes from the observation that the rms of a sine wave is equal to its peak height divided by 1.41, and the rms of each strip in the PSD is equal to the square root of its area. In this way an equivalent set of 40 sine waves can be produced. The PSD contains information on the amplitude and frequency content of the sinusoidal waves but does not show the phase relationships. However, it is known that for engineering processes that are Gaussian, the phase angles are randomly distributed. To regenerate a time history from a PSD we must therefore reintroduce the random phase angles. After adding together the 40 sine waves the time history regenerated will not be exactly the same as the original but will be statistically equivalent.

THE CHARACTERISATION OF ENGINEERING PROCESSES USING STATISTICAL MEASURES

Time histories & PSD's

Engineering responses vary in character and figure 6 is useful as a means of characterising these different types of processes in both the time domain and frequency domain.

In figure 6(a) a ***sinusoidal time history*** appears as a single spike on the PSD plot. The spike is centred at the frequency of the sine wave and the area of the spike represents the mean square amplitude of the wave.

In theory this spike should be infinitely tall and infinity narrow for a pure sine wave, however because of the numerical analysis the spike will have a finite width and will therefore have a finite height. Remember, with PSD plots we are interested in the area under the graph and not the height of the graph.

In figure 6(b) a ***narrow band process*** is shown which is built up of sine waves covering only a narrow range of frequencies. A narrow band process is typically recognised in the time history by the amplitude modulation, often referred to as a 'beat' envelope.

In figure 6(c) a ***broad band processes*** is shown which is made up of sine waves over a broad range of frequencies. These are shown in the PSD plot as either a number of separate spikes, as illustrated in the plot, or one wide peak covering many frequencies. This type of process is usually more difficult to identify from the time history but is typically characterised by its positive valleys and negative peaks.

In figure (d) a ***white noise process*** is shown which is a time history built up of sine waves over the whole frequency range.

Expected zeros, peaks and irregularity factor from the time signal.

Random stress or strain time histories can only properly be described using statistical parameters. This is because any sample time history can only be regarded as one sample from an infinite number of possible samples that could occur for the random process. Each time sample will be different. However, as long as the samples are reasonably long then the statistics of each sample should be constant. Two of the most important statistical parameters are the number of so-called zero crossings and peaks in the signal. Figure 7 shows a 1-second piece cut out from a typical wide band signal. E[0] represents the number of (upward) zero crossings, or mean level crossings for a signal with a non-zero mean. E[P] represents the number of peaks in the same sample. These are both specified for a typical 1-second sample. The irregularity factor is defined as the number of upward zero crossings divided by the number of peaks.

$$\gamma = \frac{E[0]}{E[P]}$$

In this particular case the number of zeros is 3 and the number of peaks is 6 so the irregularity factor is equal to 0.5. This number can, theoretically, only fall in the range 0 to 1. For a value of 1 the process must be narrow band as shown in figure 4(b). As the divergence from narrow band increases then the value for the irregularity factor tends towards 0.

Moments from a PSD

Since we are concerned with structural systems analysed in the frequency domain we require a method for extracting the pdf of rainflow ranges directly from the PSD of stress. The characteristics of the PSD, which are used to obtain this information, are the nth moments of the PSD function. The relevant spectral moments are easily computed from a one sided PSD G(f) in units of Hertz using the following expression.

$$m_n = \int_0^{\infty} f^n \, G(f) \; df$$

The n^{th} moment of area of the PSD (m_n) is calculated by dividing the curve into small stripes as shown in figure 8. The n^{th} moment of area of the strip is given by the area of the strip multiplied by the frequency raised to the power *n*. The n^{th} moment of area of the PSD is then found by summing the moments of all the strips. In theory all possible moments are required to fully characterise the original process. However, in practice we find that m_0, m_1, m_2 and m_4 are sufficient to compute all of the information required for the subsequent fatigue analysis.

Expected zeros, peaks and irregularity factor from a PSD.

The first serious effort at providing a solution for estimating fatigue damage from PSDs was undertaken by SO Rice in 1954 [22]. He developed the very important relationships for the number of *upward mean crossings per second (E[0])* and *peaks per second (E[P])* in a random signal expressed solely in terms of their spectral moments m_n.

$$E[0] = \sqrt{\frac{m_2}{m_0}} \qquad E[P] = \sqrt{\frac{m_4}{m_2}} \qquad \gamma = \frac{E[0]}{E[P]} = \sqrt{\frac{m_2^2}{m_0 m_4}}$$

FATIGUE LIFE ESTIMATION FROM PSD's

We shall now consider the theory behind vibration fatigue analysis in the frequency domain. Before introducing the concepts needed to estimate fatigue damage in the frequency domain it is useful to set out a parallel approach in the time domain. The approach highlighted is that of a traditional S-N (Stress-Life) approach. This section is therefore split into two parts; the first discusses the theory of fatigue analysis in the time domain, and the second discusses the parallel approach in the frequency domain.

Time Domain Fatigue Life Estimation - General Procedure

The starting point for any fatigue analysis is the response of a structure or component, which is usually expressed as a stress or strain time history. The structural model highlighted here could be a laboratory component, test track prototype or an FEA model. If the response time history was made up of constant amplitude stress or strain cycles then the fatigue design could be accomplished by referring to a typical S-N diagram. However, because real signals rarely conform to this ideal constant amplitude situation, an empirical approach is used for calculating the damage caused by stress signals of variable amplitude. Despite its limitations, the Palmgren-Miner rule is generally used for this purpose. This linear relationship assumes that the damage caused by parts of a stress signal with a particular range can be calculated and accumulated to the total damage separately from that caused by other amplitudes. A ratio is calculated for each stress range, equal to the number of actual cycles at a particular stress range n divided by the allowable number of cycles to failure at that stress N (obtained from the S-N curve). Failure is assumed to occur when the sum of these ratios, for all stress ranges, equals 1.0.

If the response time history is irregular with time, as shown in the plot, then rainflow cycle counting is widely used to decompose the irregular time history into equivalent sets of block loading. The numbers of cycles in each block are usually recorded in a stress range histogram. This can then be used in the Palmgren Miner calculation. Matsuishi and Endo first introduced the concept of rainflow ranges to the scientific community over twenty years ago. An example of the way rainflow ranges are extracted from a time signal is given in [3]. Figure 9 highlights this process.

The Frequency Domain Model

In the frequency domain a transfer function would first be computed for the structural model. This is completely independent of the input loading and is a fundamental characteristic of the system, or model. The PSD response caused by any PSD of input loading is then obtained by multiplying the transfer function by the input loading PSD. Further response PSD's caused by additional PSD's of input loading can then be calculated with a trivial amount of computing time. An essential requirement of a structural analysis in the frequency domain is that it results in a PSD, which is equivalent to the time history obtained using the transient approach. The rest of the design process is then concerned with using the new vibration fatigue tools to compute fatigue life directly from these PSD's of stress. These new tools either estimate rainflow histograms (or pdf's), or fatigue life directly. Figure 9 is intended to show that the time and frequency domain processes are actually very similar. The only differences being the structural analysis approach used (time or frequency domain) and the fact that a fatigue modeller is required to transform from a PSD of stress to the rainflow cycle histogram. In this context the vibration fatigue modeller can be envisaged as just another form of rainflow cycle counting. Details of all the methods can be found in the references. Here, just the three most common will be described.

Narrow band solution

JS Bendat (1964) presented the theoretical basis for the so-called ***Narrow Band*** solution. This expression was defined solely in terms of the spectral moments up to m_4. However, the fact that this solution was suitable only for a specific class of response conditions was an unhelpful limitation for the practical engineer. The narrow band formula is given below.

$$E[D] = {}_{i}\ \frac{n_i}{N(S_i)} = \frac{S_t}{k}\ S^b . p(S) dS = \frac{E[P].T}{k}\ S^b .[\frac{S}{4m_0} e^{\frac{-S^2}{8m_0}}] dS$$

This was obtained by substituting the Rayleigh pdf for $p(S)$ and noting that S_t is equal to $E[P].T$, where *T* is the life of the structure in seconds. *k* and *m* are material parameters from the S-N curve.

This was the first frequency domain method for predicting fatigue damage from PSD's and it assumes that the pdf of peaks is equal to the pdf of stress amplitudes. The narrow band solution was then obtained by substituting the Rayleigh pdf of peaks with the pdf of stress ranges. Figure 10 explains why the narrow band solution is so conservative for wide band cases? The overhead shows two time histories. The narrow band history is made up by summing two independent sine waves at relatively close frequencies, while the wide band history uses two sine waves with relatively widely spaced frequencies. Narrow banded time histories are characterised by the frequency modulation known as the beat effect. Wide band processes are characterised by the presents of positive troughs and negative peaks and these are clearly seen in the overhead as a sinusoidal ripple superimposed on a larger, dominant sine wave. The problem with the narrow band solution is that positive troughs and negative peaks are ignored and all positive peaks are matched with corresponding troughs of similar magnitude regardless of whether they actually form stress cycles. To illustrate why the narrow band solution becomes conservative with wide band histories, take every peak (and trough) and make a cycle with it by joining it to an imaginary trough (peak) at an equal distance the other side of the mean level. This is shown in figure 10. It is easy to see that the resultant stress signal contains far more high stress range cycles than were present in the original signal. This is the reason why the narrow band solution is so conservative.

Empirical correction factors (Tunna, Wirsching, Hancock, Kam and Dover)

Many expressions have been proposed to correct this conservatism. Most were developed with reference to offshore platform design where interest in the techniques has existed for many years. In general, they were produced by generating sample time histories from PSD's using Inverse Fourier Transform techniques, from which a conventional rainflow cycle count was then obtained. The solutions of **Wirsching** et al (1990)**, Chaudhury and Dover** (1885), **Tunna** (1986), **Hancock** (Kam and Dover, 1988), and **Kam and Dover** (1988) were all derived using this approach. They are all expressed in terms of the spectral moments up to m_4 **.**

Steinberg Solution

The approach of Steinberg leads to a very simple solution based on the assumption that no stress cycles occur with ranges greater than 6 rms values. The distribution of stress ranges is then arbitrarily specified to follow a Gaussian distribution. This defines the stress range cycles to occur with the following probability.
68.3% time at 2rms
27.1% time at 4rms
4.3% time at 6rms.

Dirlik's empirical solution for rainflow ranges

Most of the above solutions assume that the pdf of rainflow ranges is the factor that controls fatigue life. The best approach is therefore to estimate this directly from the PSD without using the narrow band approach as a starting point. Both empirical and theoretical expressions have been produced in this way. Dirlik (1985) has produced an empirical closed form expression for the pdf of rainflow ranges, which was obtained using extensive computer simulations to model the signals using the Monte Carlo technique. Dirlik's solution is given below.

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2(m_0)^{1/2}}$$

Where x_m, D_1, D_2, D_3, Q and R are all functions of $m_0, m_1, m_2,$ and m_4 .Z is a normalised variable equal to $\frac{S}{2(m_0)^{1/2}}$.

Bishop's theoretical solution for rainflow ranges

Dirlik's empirical formula for the pdf of rainflow ranges has been shown to be far superior, in terms of accuracy, than the previously available correction factors. However, the need for certification of the technique before its use meant that theoretical verification was required. This was achieved by **Bishop** (1988) when a theoretical solution for predicting rainflow ranges from the moments of the PSD was produced. A detailed description of this method is given in Bishop and Sherratt, [3].

OBTAINING THE STRESS PSD'S USING MSC/FATIGUE

Before techniques such as the Dirlik approach can be applied the PSD's of stress response have to be computed for each point of interest on the FEA model.

Standard techniques inside MSC/NASTRAN can be utilised to compute the transfer functions for these points. However, before a fatigue life calculation can be performed these results have to be transformed into a relevant stress system, such as principal stress. MSC/FATIGUE now utilises a state of the art technique for rotating these stress transfer functions into such axis systems. Once these transfer functions are obtained the standard approach of MSC/NASTRAN, for multiple load application points, is used to estimate the output stress PSD's. These calculations are performed inside MSC/FATIGUE and a flexible system of input PSD load application is allowed. An overview of the procedure is given in figure 11.

EXAMPLES OF FEA BASED VIBRATION FATIGUE ANALYSIS

In order to assess the FEA based vibration fatigue approach a number of comparison calculations have been performed on the FEA model shown in figure 12. This is a bracket, which is fully fixed at the position of the round hole. Three loading time histories were applied, at the end of the bracket, in the horizontal vertical and twist directions. Figures 13(a), (b) and (c) show the applied time histories, PSD's and cross PSD's respectively. The cross PSD functions quite clearly show some correlation between load input signals.

Two separate comparisons have been made. Firstly, a static analysis comparison has been undertaken between the results from a conventional pseudo static analysis and the results from a PSD based analysis. With the pseudo static approach the results caused by each load application point are linearly superimposed at each node of interest. For both methods principal stresses were used.

In order to properly simulate a static situation the mass of the bracket was set sufficiently low to ensure that the first natural frequency was well above the maximum loading frequency. The first natural frequency was approximately 60Hz, with the highest frequency of loading being approximately 50Hz. Figures 14 and 15 show good agreement between the two approaches for fatigue life. The red area shows the position of the shortest life and the white areas the longest fatigue lives. The PSD and time history results for the most critical node are given in figures 16 and 17. The full set of comparison results for this node is given below.

Static model results in seconds		
	Static	**Vibration**
Vertical	6.6E5	2.5E6
Horizontal	9.3E8	3.2E9
Twist	1.0E9	9.3E7
All together	4.0E4	5.1E3

The second comparison analysis was undertaken using a dynamic example. The same FE model was used but this time the mass was set so that several modes occurred in the loading frequency range. Mode 1 was at approximately 6Hz. Figure 18 shows mode 6, which occurred at 46Hz. For this comparison a transient dynamic analysis was undertaken using MSC/NASTRAN. The stress outputs from this analysis were then analysed using MSC/FATIGUE. Figures 19 and 20 show the output from a critical node and figures 21 and 22 show the fatigue life contour plots for all nodes. Again, there is excellent agreement between the two approaches. . Once again, the red area shows the position of the shortest life and the white areas the longest fatigue lives. The full set of comparison results for this node is given below.

Dynamic model results in seconds		
	Dynamic	**Vibration**
Vertical	145	38
Horizontal	1.9E9	9.8E8
Twist	3.7E7	3.8E5
All together	0.7	0.5

SUMMARY

MSC/FATIGUE has been shown to be an accurate and versatile tool for performing vibration fatigue calculations on FEA models. Comparison results for both a static and dynamic model have shown excellent agreement.

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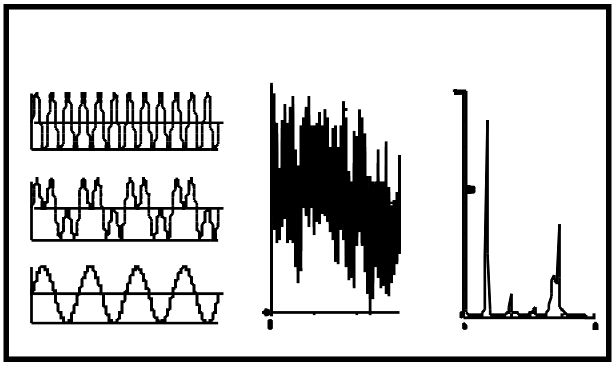

Figure 1. Deterministic or Random?

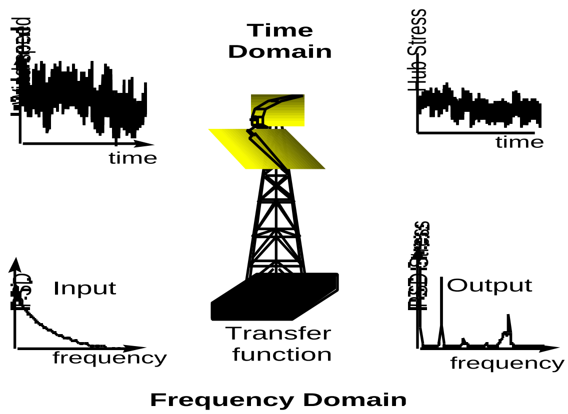

Figure 2. Time or Frequency domain analysis

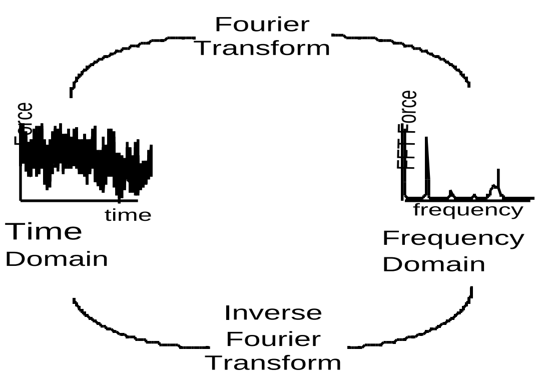

Figure 3. The IFFT and FFT circle

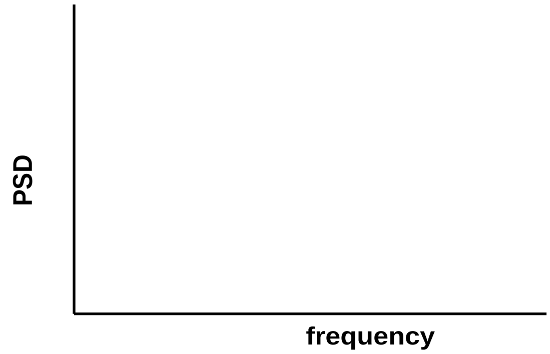

Figure 4. Splitting a PSD up into 40 components.

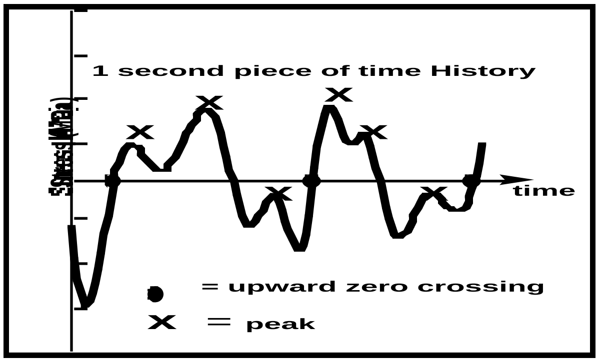

Figure 7.An example of counting zero crossing and peak rates

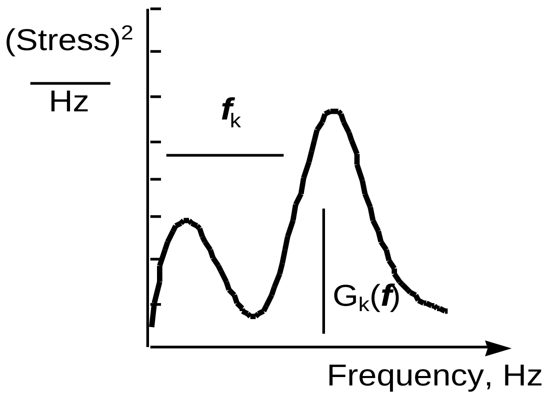

Figure 8. Moments from a PSD

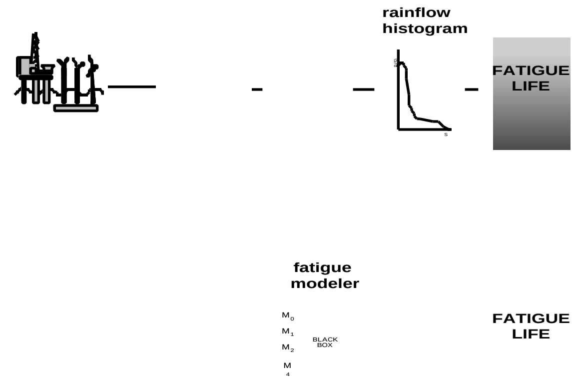

Figure 9. Fatigue analysis in the time and frequency domains

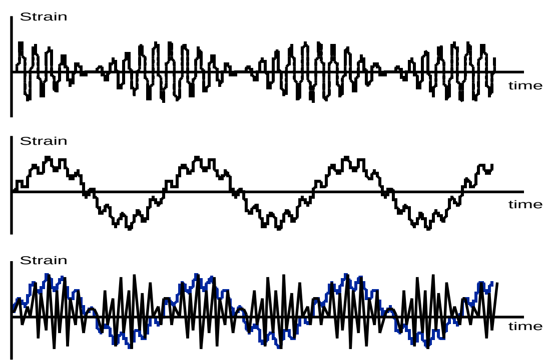

Figure 10. Consequences of the Narrow Band assumption

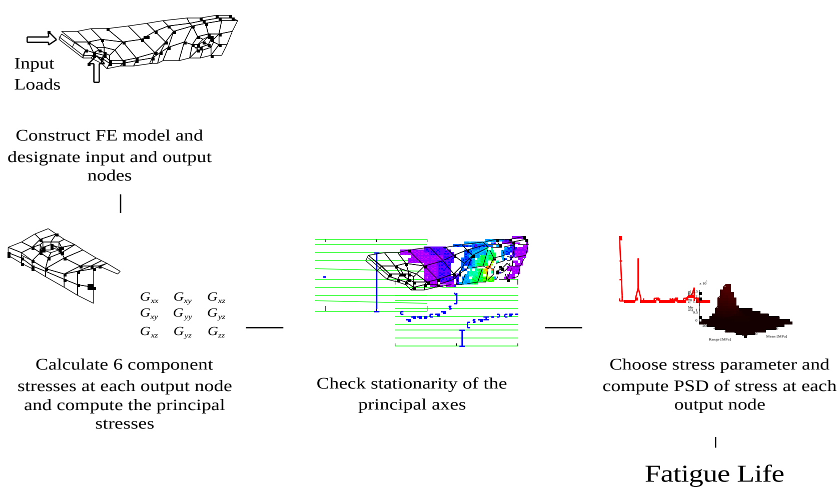

Figure 11. Overview of the vibration fatigue analysis procedure

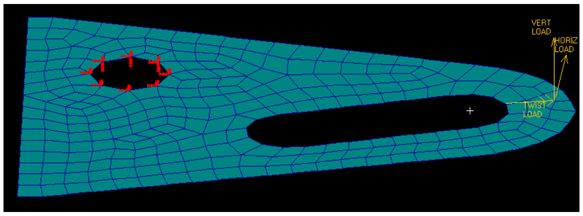

Figure 12. The FEA model used for the analysis

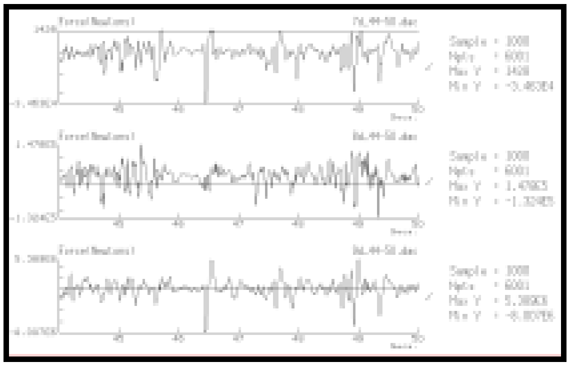

Figure 13(a). The 3 loading time signals

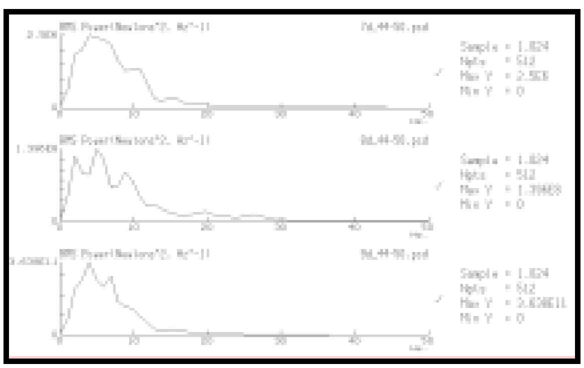

Figure 13(b). The 3 loading PSD's

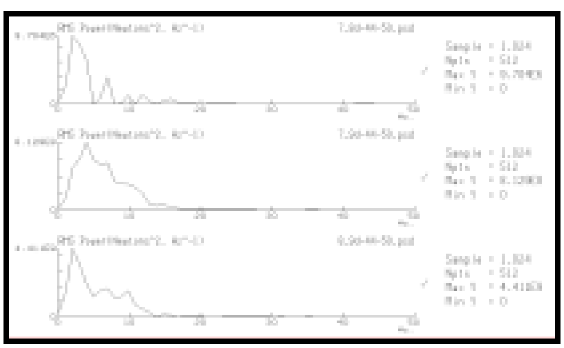

Figure 13(c). The 3 loading cross PSD's

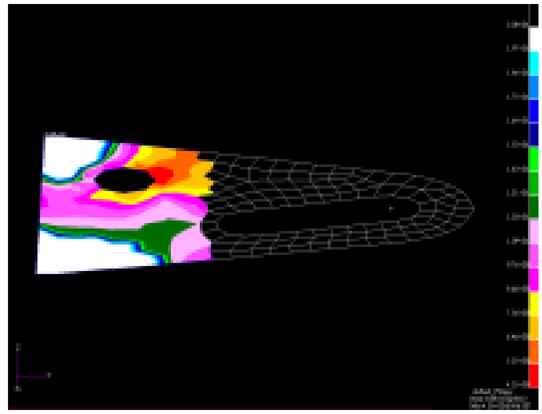

Figure 14. Fatigue life for combined inputs from pseudo static analysis – static model

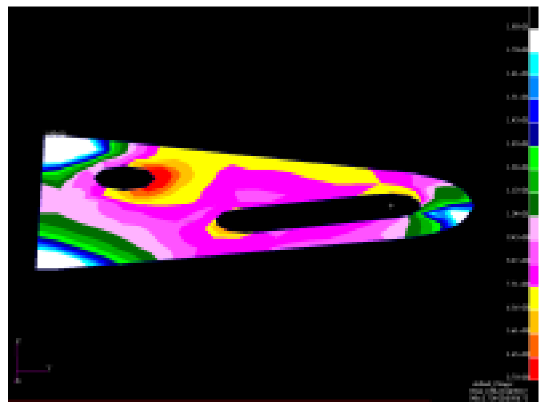

Figure 15. Fatigue life for combined inputs from PSD analysis – static model

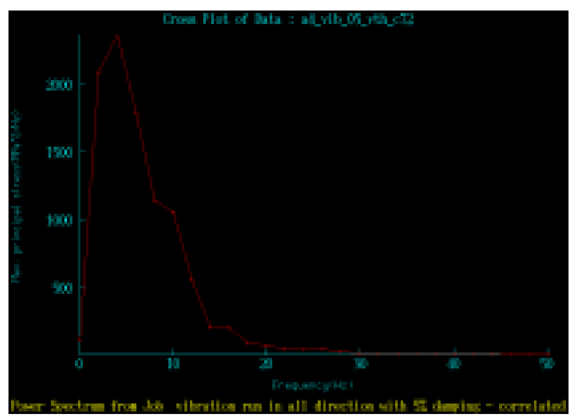

Figure 16. Output from critical node – PSD analysis, static model

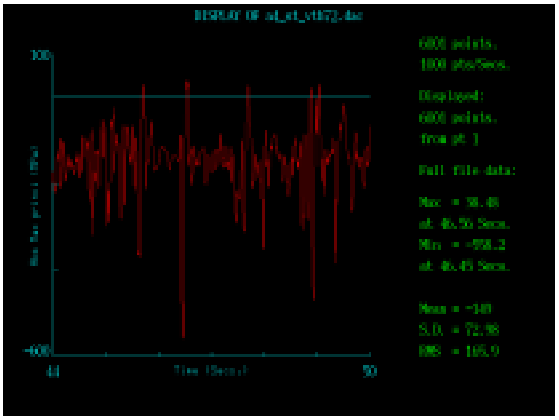

Figure 17. Output from critical node – pseudo static analysis, static model

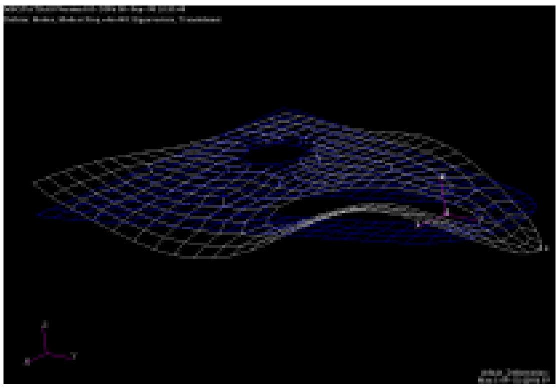

Figure 18. Mode 6 at 46Hz

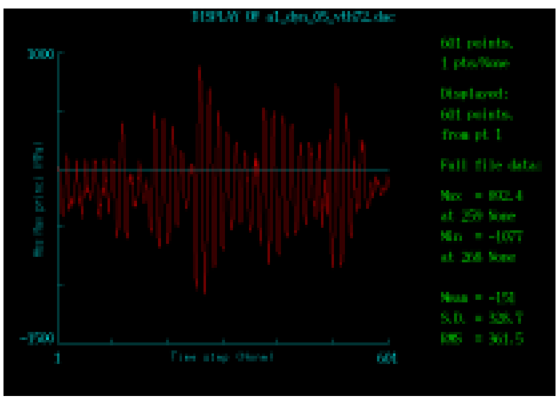

Figure 19. Output from critical node – transient analysis, dynamic model

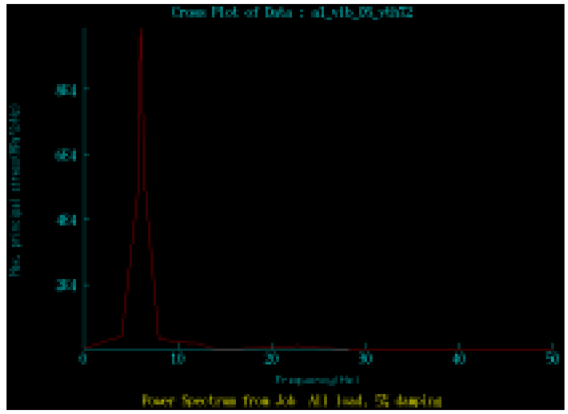

Figure 20. Output from critical node – PSD analysis, dynamic model

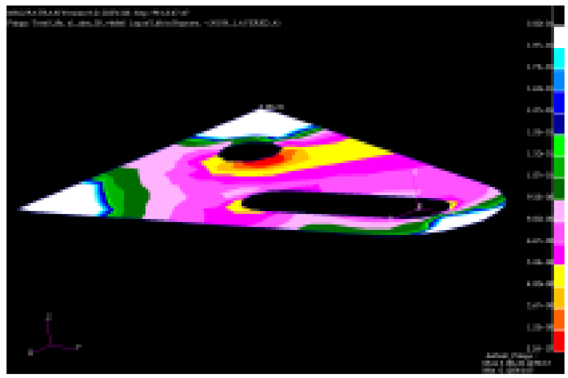

Figure 21. Fatigue life for combined inputs from transient analysis – dynamic model

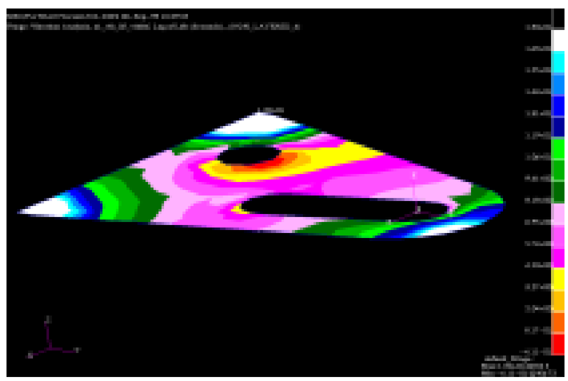

Figure 22. Fatigue life for combined inputs from PSD analysis – dynamic model