Improved Identification of Contributing Modes in Superelement Modal Frequency Response Analysis, V70

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Abstract

In parallel with experimental methods, modern-day cars are designed by advanced dynamic analyses using the Finite Element Method. These analyses help identify components sensitive to vibrations that could possibly lead to deterioration of ride comfort and safety. For correct simulation of the real dynamic behavior of a car all parts relevant to vibrations have to be taken into consideration when assembling a full-vehicle simulation model. Since this model contains many viscous dampers, such as shock absorbers, the steering damper, and several hydro-mounts for the engine/gear unit and the rear-axle support - when the modal transformation is applied for reduction purposes - the generalized damping matrix is not diagonal; that is, the modal equations remain coupled by the damping coefficients. Consequently, the dynamic response is composed of various contributions resulting from eigenvectors. Knowing the contributions from each mode shape is a very important requirement for taking suitable measures in order to decrease obnoxious vibration amplitudes, i.e., the modal contributors are a means of finding suitable locations for modifications aimed at improving the structure's dynamics. It is worthwhile to notice, that by the use of the presented procedure innovative redesigns may be found by the analyst which the automatic stuctural optimizers are never able to find. Guidelines on how to influence contributing modes in order to decrease high resonance peaks are given along with a V70 SOL 111 DMAP alter for the identification of contributing modes.

1. Introduction and Problem Definition

In modal frequency response analysis of a full-vehicle simulation model the following analysis steps

- calculating frequency response predictions,
- determining contributing modes at critical frequencies,
- identifying relevant residual structure mode shapes,
- animating relevant mode shapes on the screen,
- applying the vehicle dynamicist's creativity leading to an innovative redesign

form a complete cycle in the development process. The sequence of pictures in Fig. 1 is the schematic representation of "design optimization procedure" under consideration.

The dynamicist's major goal is to provide the driver and the passengers with a smooth comfortable ride. Displacement or acceleration vs. frequency plots obtained in the first step indicate the critical resonance peaks of the comfort evaluation points.

Since a full-vehicle simulation model contains many viscous dampers, such as shock absorbers, the steering damper, and several hydro-mounts for the engine/gear unit and rear axle support - when the modal transformation is applied - the generalized damping matrix is not diagonal; that is, the modal equations remain coupled by the damping coefficients. Consequently, the response must be obtained by integrating the equations simultaniously rather than individually (coupled solution algorithm). Certainly, it is more efficient to perform the integration for a small number of coupled modal-coordinates equations than for the original coupled equation system. Moreover, the dynamic physical response is composed of various physical contributions resulting from the eigenvectors. For a better understanding of the resonance peaks, it is necessary to know the contributions from each of the mode shapes. Therefore, in the second step, in an analysis using loading with resonance frequency and a DMAP alter the modal contributors are calculated for any desired physical coordinates.

Next, the mode shapes the modal contributors refer to have to be identified quickly and animated on the screen. Knowing the modal characteristics of the mechanical, conservative system (eigenmodes and eigenfrequencies) is a requisite for taking measures to decrease vibration amplitudes, i.e., the modal contributors are a means of finding suitable sites for modifications aimed at improving the structure's dynamics.

Finally, having this information the vehicle dynamicist's creativity is required for working on a local redesign.

In some cases, the modal contributors make it possible to expose unwelcome resonance amplitudes to be due to modeling anomalies.

Many efforts into the identification of contributing modes have been made by several investigators in the past, Ref. (1), (2). However, the identification methods have relied upon V64 rigid formats. The knowledge of contributing modes is a very important requirement and is considered a necessary tool for almost every dynamicist. Thus, considerable joint efforts of Daimler-Benz AG and MacNeal-Schwendler Corporation have led to an elegant DMAP alter for calculating the contributing modes in Superelement Modal Frequency Response, SOL 111, V70.

In automotive applications, responses of high quality are only obtained if both the dynamic mode shapes and the relevant quasi-static mode shapes are present in the analysis of the residual structure, Ref. (7), (11), (12). Both types of mode shapes are obtained via the Component Mode Reduction of the Residual Structure (CMR of RS). Consequently, the CMR of RS has to be applied in all subsequent analyses.

Additionally, guidelines on how to influence the contributing modes in order to decrease obnoxious vibration amplitudes are given to the reader. It is worthwhile to notice that by the use of this procedure innovative redesigns may be found by the analyst the automatic structural optimizers are never able to find.

2. Review of Theory

2.1 Modal Response Analysis

A modal analysis can be separated into four phases:

- Real Eigenanalysis,
- Modal Formulation,
- Solution of the Equations of Motion in Modal Subspace,
- Transformation from Modal to Physical Coordinates.

2.2 Real Eigenanalysis

The real eigenanalysis provides information on the structure. For instance, the mode shapes of the structure contain characteristics for the identification of the mechanical behavior of the structure without referring to detailed masses and stiffnesses. The eigenvalue problem can be written as

$$([K_{aa}] - \omega_i^2 [M_{aa}])\{\Phi_{ai}\} = 0 \quad i = 1, 2, 3, ... \qquad (1)$$

where

$[K_{aa}]$ = stiffness matrix
$[M_{aa}]$ = mass matrix
ω_i^2 = eigenvalue
$\{\Phi_{ai}\}$ = eigenvector

a = number of analysis degrees of freedom

The i-th natural frequency f_i and the i-th eigenvector $\{\Phi_{ai}\}$ define a free vibration mode of the structure. The i-th eigenvalue ω_i^2 is related to the i-th natural frequency as follows:

$$f_i = \omega_i / 2\pi \tag{2}$$

The eigenvectors are also known as normal modes, characteristic vectors, proper vectors, or latent vectors. Clearly, all eigenvectors used in the analysis can be arranged in a matrix, the so-called eigenvector matrix or mode-shape matrix or modal matrix:

$$[\Phi_{ai}] = [\{\Phi_{a1}\}\{\Phi_{a2}\}\{\Phi_{a3}\}\ldots\{\Phi_{ai}\}] \tag{3}$$

where

a = number of analysis degrees of freedom

i = number of eigenvectors used in the analysis.

The eigenvectors are orthogonal with respect to mass and stiffness matrix.

Moreover, the modal matrix can also be written in a somewhat different form, viz., as an accummulation of row vectors:

$$[\Phi_{ai}] = \begin{matrix} [\Phi_{1i}] \\ [\Phi_{2i}] \\ . \\ [\Phi_{li}] \\ . \\ [\Phi_{ai}] \end{matrix} \tag{4}$$

The l-th row of $[\Phi_{ai}]$, $[\Phi_{li}]$, represents the displacements of the arbitrary physical degree of freedom l at i eigenvectors.

2.3 Modal Formulation

The eigenvalues and eigenvectors produced by the real eigenvalue analysis modules may also be used to generate modal coordinates, $\{\xi_i\}$, for additional dynamic analyses by the modal method in order to reduce the number of equations of motion.

The modal transformation is given by

$$\{u_a\} = [\Phi_{ai}]\{\xi_i\} \tag{5}$$

If extra points, $\{u_e\}$, are included (for instance, to formulate differential equations) then the modal coordinates, $\{\xi_i\}$, are augmented by the non-structural variables, $\{u_e\}$, and thus we obtain the $\{u_h\}$ vector:

$$\{u_h\} = \begin{Bmatrix} \xi_i \\ u_e \end{Bmatrix} \tag{6}$$

Similarly, the physical degrees of freedom are augmented by the $\{u_e\}$ vector and form the $\{u_d\}$ vector:

$$\{u_d\} = \begin{Bmatrix} u_a \\ u_e \end{Bmatrix} \tag{7}$$

Thus, the transformation equation becomes

$$\{u_d\} = [\Phi_{dh}]\{u_h\} = \begin{bmatrix} \Phi_{ai} & 0_{ae} \\ 0_{ei} & I_{ee} \end{bmatrix} \begin{Bmatrix} \xi_i \\ u_e \end{Bmatrix} \tag{8}$$

In the modal equations of motion, the appropriate mass, damping and stiffness matrices are of size h x h.

2.4 Solution of the Equations of Motion in Modal Subspace

The equations to be solved in Modal Frequency Response Analysis can be written as

$$(-\omega^2[M_{hh}] + i\omega[B_{hh}] + [K_{hh}])\{u_h\} = \{P_h\} = [\Phi_{dh}]^T \begin{Bmatrix} P_a \\ P_e \end{Bmatrix} \tag{9}$$

where

$[M_{hh}]$ = dynamic mass matrix expressed in modal coordinates
$[B_{hh}]$ = dynamic damping matrix expressed in modal coordinates
$[K_{hh}]$ = dynamic stiffness matrix expressed in modal coordinates
$\{P_h\}$ = dynamic load vector expressed in modal coordinates

2.5 Transformation from Modal to Physical Coordinates

The modal quantities are transformed back to physical coordinates according to the modal formulation, equation (8),

$$\{u_d\} = [\Phi_{dh}]\{u_h\} = \begin{bmatrix} \Phi_{ai} & 0_{ae} \\ 0_{ei} & I_{ee} \end{bmatrix} \begin{Bmatrix} \xi_i \\ u_e \end{Bmatrix} \tag{10}$$

2.6 Physical Contributions Resulting from Eigenvectors

For simplicity, in the following the extra points are not considered.

The displacement of any arbitrary geometric coordinate l is

$$u_l = [\Phi_{li}]\{\xi_i\} \tag{11}$$

It is apparent that the row from the mode-shape matrix, $[\Phi_{li}]$, serves to transform from the modal coordinates, $\{\xi_i\}$, to the geometric coordinate u_l, or, in other words, the modal solution vector (column vector), $\{\xi_i\}$, is weighed by the appropriate row of the modal matrix, $[\Phi_{li}]$.

The total displacement of the arbitrary degree of freedom l, u_l, is then obtained as the sum of the modal components

$$u_l = \varphi_{l1}\xi_1 + \varphi_{l2}\xi_2 + \ldots + \varphi_{li}\xi_i = \sum_{j=1}^{l} \varphi_{lj}\xi_{lj} = \sum_{j=1}^{i} u_{lj} \tag{12}$$

The components $\varphi_{li}\xi_i$ are herein defined to be

The Physical Contributions Resulting from Eigenvectors, or
The Modal Contributions, or
The Modal Contributors.

They are calculated using the special Element-by-Element-Multiplication, $\overset{E}{\underset{E}{*}}$. Thus,

$$\{u_{li}\} = \{\Phi_{li}\} \overset{E}{\underset{E}{*}} \{\xi_i\} \tag{13}$$

and, summing up the modal contributions

$$u_l = [111..1]\{u_{li}\} \tag{14}$$

2.7 Physical Contributions Resulting from Eigenvectors in Modal Frequency Response Analysis

In frequency response calculations, the structure is loaded harmonically, however, different phase angles are allowed between several grid points. The steady-state response is harmonic, too.

The modal contribution to the displacement of the arbitrary physical degree of freedom l at a distinct excitation frequency f becomes

$$\left\{u_{li}(f)\right\} = \begin{bmatrix} \varphi_{l1} \\ \varphi_{l2} \\ . \\ . \\ \varphi_{li} \end{bmatrix} * \begin{bmatrix} \xi_1(f) \\ \xi_2(f) \\ . \\ . \\ \xi_i(f) \end{bmatrix} = \left\{\Phi_{li}\right\} * \left\{\xi_i(f)\right\} \tag{15}$$

If modal contibutions to displacements of several degrees of freedom, $l, m, n, ...,$ are to be calculated then the matrix equation becomes

$$\left[\left\{u_l(f)\right\}\left\{u_m(f)\right\}\left\{u_n(f)\right\}...\right] = \left[\left\{\Phi_{li}\right\}\left\{\Phi_{mi}\right\}\left\{\Phi_{ni}\right\}...\right] * \left[\left\{\xi_i(f)\right\}\left\{\xi_i(f)\right\}\left\{\xi_i(f)\right\}...\right] \tag{16}$$

The contributions are summed up resulting in the physical, complex displacements:

$$\left[u_l(f)\, u_m(f)\, u_n(f) ...\right] = \left[111...1\right]\left[\left\{u_l(f)\right\}\left\{u_m(f)\right\}\left\{u_n(f)\right\}...\right] \tag{17}$$

It is reasonable to replace the last row of the modal contributions by the resultants, i.e. by the row vector from left-hand side of equation (17):

$$\left[MC\right] = \begin{bmatrix} mc_{l,1} & mc_{m,1} & mc_{n,1} & \\ mc_{l,2} & mc_{m,2} & mc_{n,2} & \\ mc_{l,3} & mc_{m,3} & mc_{n,3} & \\ . & . & . & ... \\ . & . & . & \\ mc_{l,i-1} & mc_{m,i-1} & mc_{n,i-1} & \\ u_l & u_m & u_n & \end{bmatrix} \tag{18}$$

Consequently, each column can be normalized with respect to the relating resultant $u_l,\ u_m,\ u_n, ...$

Finally, the normalized contributions become:

$$\left[MC\right]^{norm} = \begin{bmatrix} mc^{norm}_{l,1} & mc^{norm}_{m,1} & mc^{norm}_{n,1} & \\ mc^{norm}_{l,2} & mc^{norm}_{m,2} & mc^{norm}_{n,2} & \\ mc^{norm}_{l,3} & mc^{norm}_{m,3} & mc^{norm}_{n,3} & \\ . & . & . & ... \\ . & . & . & \\ mc^{norm}_{l,i-1} & mc^{norm}_{m,i-1} & mc^{norm}_{n,i-1} & \\ 1.0,0.0 & 1.0,0.0 & 1.0,0.0 & \end{bmatrix} \tag{19}$$

Since the matrices $[MC]$ and $[MC]^{norm}$ consist of h rows, the VDR module may be used to output the modal contributions in successive passes. In the related Case Control command

SDISP (SORT1,PRINT,PHASE) = ALL

the polar form, i.e. the magnitude/phase representation, of the complex modal contributors may be selected advantageously in order to quantify the modal contributors.

3. Guidelines for Decreasing High Resonance Peaks

Using this tool, the relevant mode shapes of the contributors are identified. It can be seen from the graphs which modal contributors do contribute to the resultant and which modal contributors cancel each other out, or have phase lags of approximately 90 or 270 degrees. Thus, the method shows which is the direction to change the structure. It is straightforward that an undesired high resonance peak may be decreased by altering the magnitudes and/or phases of the dominant modal contributors and, additionally, by splitting clusters of frequencies.

Selected coefficients in the mass and stiffness matrices are to be changed only. The modifications at appropriate parts of the structure may be achieved by means of introducing additional elements locally to the model in that manner that most of the mode shapes and eigenfrequencies are kept unchanged with respect to the original model but the dominant contributing mode shapes are changed.

- For instance, additional shell elements may partly close (shut) a U-profile (such as a cardan tunnel). Or, bar elements may be used for a strutting.
- Furthermore, additional elements, such as stiffeners, may have a greater increase in stiffness than in mass.
- Also, dynamic vibration absorbers may be added to the model.
- Finally, viscous damping elements couple the equations of motion and may influence the phase angle of the dominant contributors in that manner that they are directed to the opposite side of the resultant and thus decrease the resonance peak.

All the elements mentioned above may be introduced by means of the dynamicist's expertise. It is seen that automatic structural optimizers are never able to introduce additional elements.

4. Modal Contributors for Detecting Modeling Anomalies

In some cases, by means of modal contributors unwelcome resonance amplitudes may be exposed to be due to modeling anomalies. For instance, if nonstructural masses, are, by mistake, put on sheet metals, or, if nonstructural masses are correctly put on a set of sheet metals, which are not, by mistake, tied together, then one-mass vibrators occur which contribute to resonance amplitudes. It are the modal contributors that refer to such one-mass vibrators hidden in a shell structure.

5. DMAP Output

The V70 SOL 111 DMAP alter given in the V70.5 sssalter directory provides the dynamicist with relative modal contributors in form of matrix prints and absolute modal contributors in form of formatted outputs in the polar format (magnitude/phase representation). The sequence of prints relates to the internal sort of the chosen degrees of freedom. It is given in an additional matrix print.
Rotational physical coordinates are handled as well as translational coordinates. The technique has practically no limitation with respect to number of physical coordinates.

6. Solution of a Real-Life Problem

To prove the validity of the novel procedure and to motivite by characteristic applications than by just giving practical advice, a superelement modal frequency response analysis, Solution 111, V70, was performed on a full-vehicle simulation model using Component Mode Reduction of Residual Structure.

The objective of this application was to reduce the vibration at the driver's seat point due to an imbalace at the rear wheels. In the automotive industry, comfort and safety of the driver are of prime importance. Our goal is to provide the driver and passengers with the best possible ride comfort. The full-vehicle model was subjected to first order excitation forces (imbalance) versus frequency which were applied to the rear wheels. The fequency range of interest for this problem was 10 to 30 Hz which corresponds to approximately 67 to 201 km/h. The acceleration responses in vertical and longitudinal directions versus frequency at the driver's seat, at position front left, were calculated at the residual structure level. The respective responses were shown in Figure 2, run (a). Then, when analysing the acceleration response curves it was found that excessive resonance amplitudes had been generated at 27.5 Hz.

Next, a restart run was performed including

- the developed alter named 'Contributions Resulting from Eigenvectors',
- loading with resonance frequency 27.5 Hz via a FREQ entry,
- the Grid-Components of the driver's seat point, at position front left, via the USET U1 entry.

The graphical representation of the displacement response in vertical direction of the driver's seat point and its contributions from mode shapes were given in Figure 3.

The great demand for a graphical interpretation of modal contributors of even very large models resulted in the development of an invaluable post-processor. It only requires the supply of the f06 file and provides a condensed printed output and an automated display as illustrated in Figure 4. The vector display in the complex plane allows to determine the major contributions from mode shapes at a glance.

In the present case, it is seen from the graphs in Figures 3 and 4, respectively, that the response dominantly consists of the contributions 19 and 34. Also, the contributor 31 is approximately orthogonal with respect to the resultant and therefore has less effect.

Furthermore, the labels 19 and 34 indicate that there is a very broad coupling of equations of motion, viz., coupling between modal equations 19 and 34, due to the viscous damping coefficients. The viscous dampers were used in the subframe support and in the shock absorber models in the rear.
Afterwards, the mode shapes of the modal contributors were identified by means of plots of "Modal Translational and Rotational Kinetic Energies" and the printed output of the filtered "Modal Kinetic Energy Matrix".

In deed, as it could be seen from the the modal kinetic enery plot, mode shape 19 was identified with the rear axle mode shape. As to mode shape 34, the dominant kinetic energy contribution referred to SPOINT 95006 which represented the 6th mode shape of the car body superelement.

Now, the animation of the car body's (SE) 6th mode shape on the screen revealed that, in the primary design, an additional structure (annex) had been, against our better judgement, attached too weakly to the car body ground. Clearly, that annex had obeyed the rules of a badly tuned mass damper (dynamic absorber, vibration absorber) that had been attached to the primary mass and that had caused the creation of an additional eigenfrequency and eigenvector.

After the identification of mode shapes the design change had to be considered. In this case, a proper correction meassure was found. The only logical thing to do in this situation was to rigidly attach the annex to the car body ground.

The new responses obtained in the restart run with the redesigned car body superelement showed that the obnoxious resonance at 27.5 Hz had disappeared (Figure 2, run (b)).

7. Conclusion

Modal frequency response procedures for evaluating modal contributors to physical displacements, velocities, and accelerations have been described and demonstrated in detail. It can be seen that they allow an important, deep insight into the dynamic behavior of very complicated systems such as a full-vehicle simulation model having many viscous dampers. It is they that couple the equations of motion, and, consequently, the translational, rotational, and coupled translational and rotational vibrations. Most important, the procedures help find innovative redesigns by the analyst which automatic structural optimizers are never able to find.

For visualising "Modal Contributors and their Resultant" it is recommended to implement the "Modal Contributors Display in the Complex Plane" into the MSC/NASTRAN general purpose program. An classic example was given in this paper.

Acknowledgement

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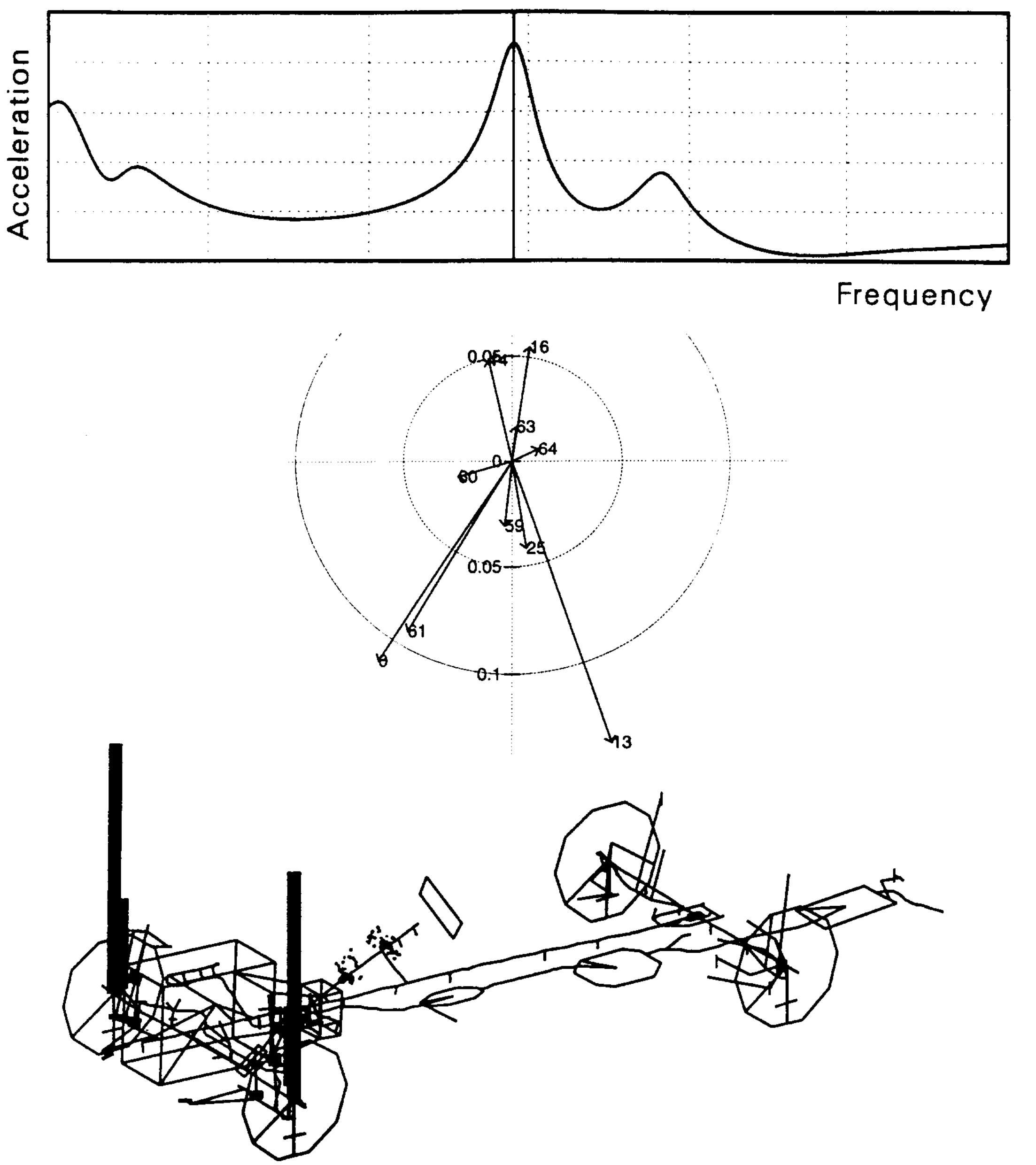
a)

b)

c)



a) Calculating Modal Frequency Response Predictions

b) Determining Contributing Modes at Resonance Frequency

c) Identifying Relevant Mode Shapes

d) Animating Relevant Mode Shapes on the Screen

e) Applying the Dynamicist's Creativity Leading to a Redesign

Figure 1: Development process loop

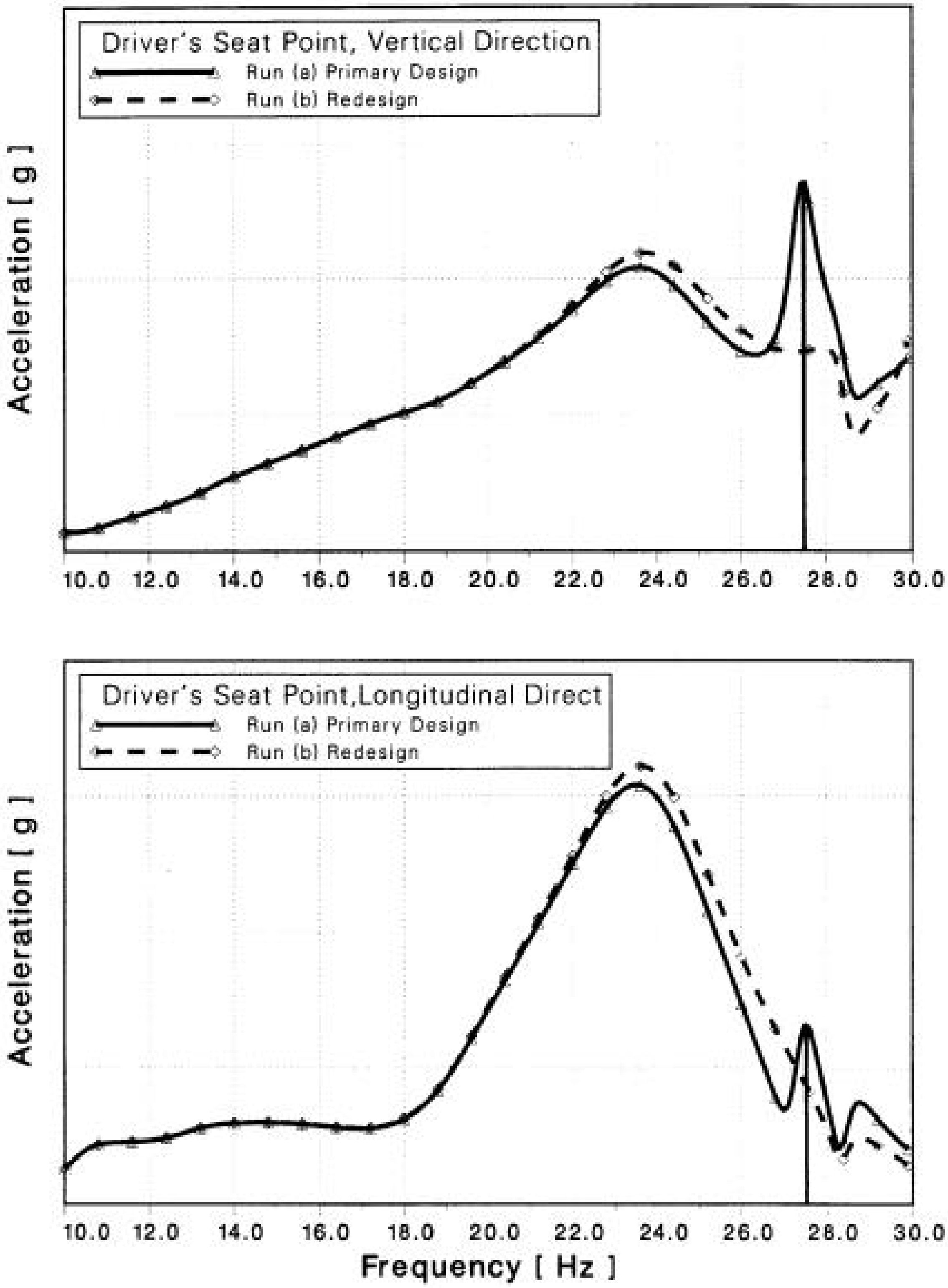


Figure 2: Acceleration vs. Frequency plots

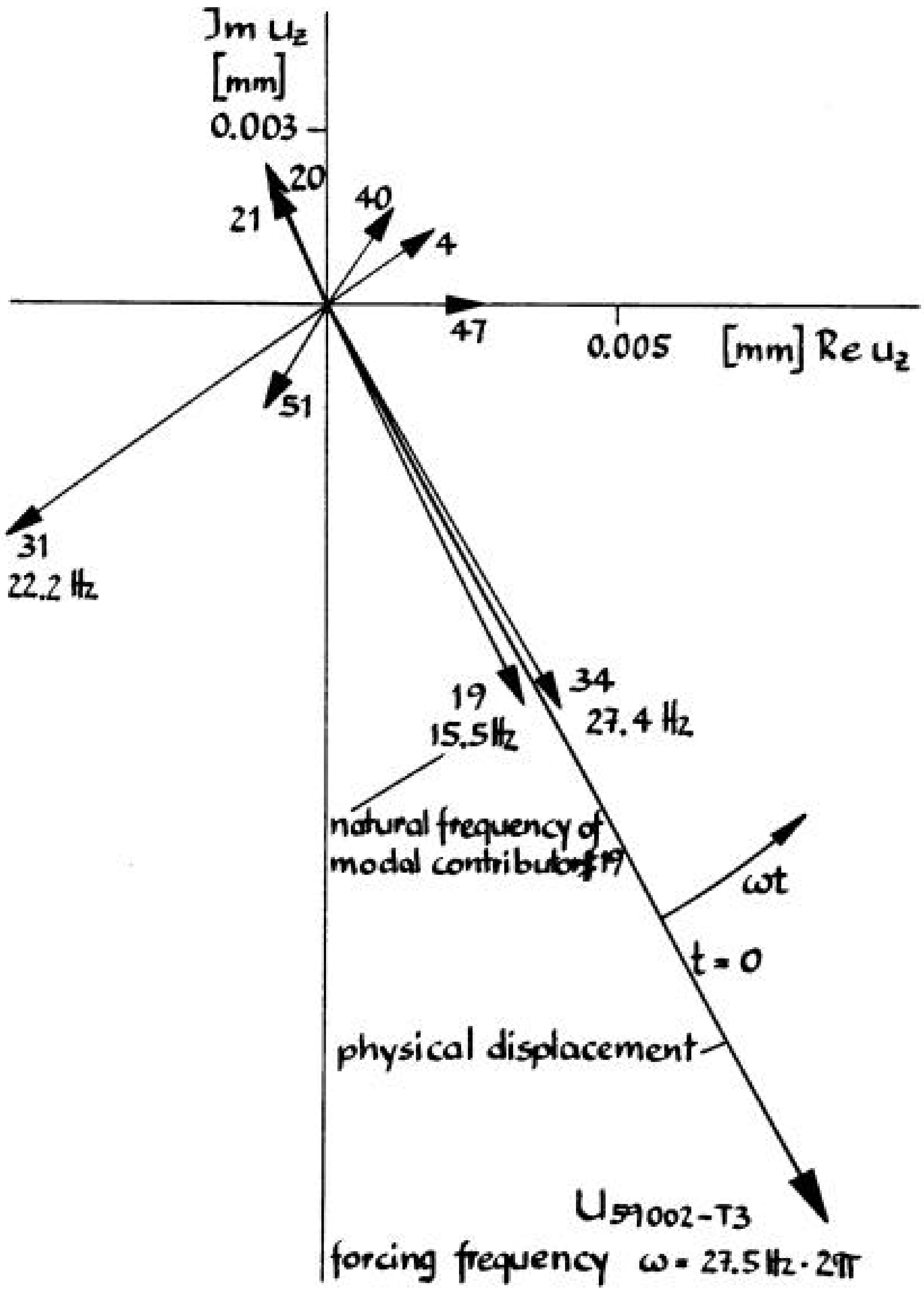


Figure 3: Displacement response and its physical contributions resulting from eigen-vectors; display in the complex plane; excitation frequency f = 27.5 Hz

```
# Total Solution of Grid    59002 Component  T3
#
#    REL. MC     ID  EXC FREQ   MAGNITUDE     PHASE
  1.0000E+00      0    27.500  1.7869e-02   298.720
#
# Modal Contributors, Magnitude >  0.1 * Magnitude(Total Solution)
#
#                                      ABS. MC
#    REL. MC     ID  NAT FREQ   MAGNITUDE     PHASE
  0.44549000     34    27.445  7.9620e-03   299.778
  0.42596000     19    15.461  7.6165e-03   296.661
  0.07032200     47    34.678  2.6979e-03     0.960
  0.06163000     51    37.742  2.0606e-03   241.026
  0.05031200     31    22.219  6.6932e-03   216.440
 -0.01527200      4     2.459  2.2988e-03    35.538
 -0.05042800     40    33.437  2.0308e-03    55.062
 -0.13039000     21    15.701  2.3390e-03   113.685
 -0.14827000     20    15.601  2.6628e-03   113.011
```

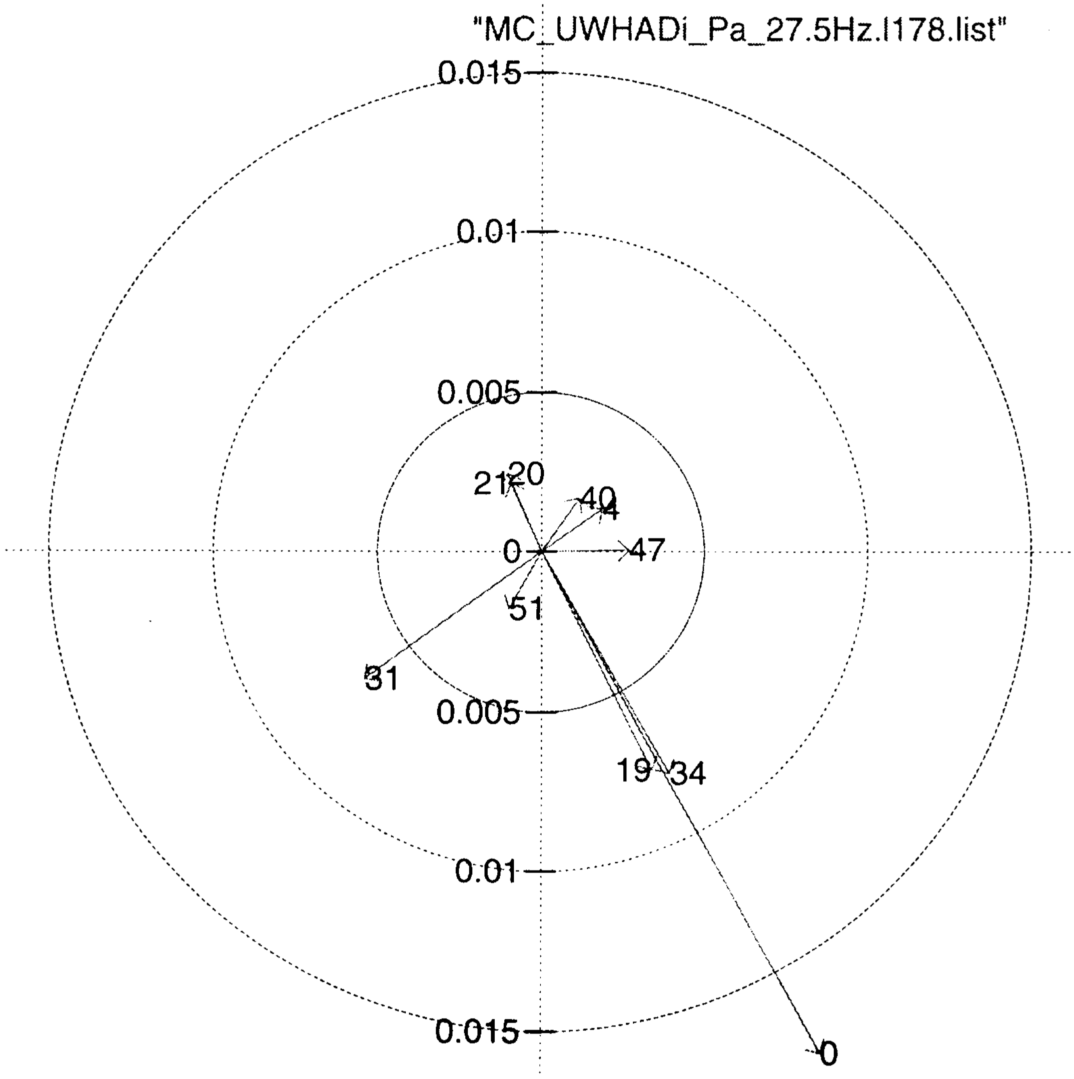


Figure 4: Displacement response and its physical contributions resulting from eigenvectors; display in the complex plane; automated representation of results; excitation frequency f = 27.5 Hz