

**THREE DIMENSIONAL MODEL OF AN ARRAY OF PANEL BAYS
INCORPORATING PRETENSIONED FASTENERS, PRECOMPRESSED
FASTENER SURROUNDINGS, SHELL-TO-SOLID AND BAY-TO-BAY
NASTRAN INTERFACE CONNECTORS**

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Abstract

This paper extends applicability of the finite element method to the analysis of interaction between the main components of a panel bay (chords, webs and stiffeners) and an array of elastic fasteners on elastic plate-foundations.

The pre-tensioned fasteners and the pre-compressed fastener-surroundings of this model are three-dimensional fine-mesh-solid islands that are integrated into the rest of the plate-element structure using the new shell-to-solid Nastran Interface Connectors.

The outer cylinder mesh nodes of fasteners are connected to the fastener-bore mesh nodes with an array of substantially non-linear radial gap elements, whose radial stiffness may have two magnitudes.

The first run of the model will be made with the least possible radial stiffness of the gap elements. The main objective of this run is determination of the maximum strain gauge element stresses. The strain gauge elements are located at the outermost and innermost plate-to-hole edges.

The second run of the model will be made with a relatively high magnitude of the gap element radial stiffness generating a 'Filled Hole' mesh. The main objective of the 'Filled Hole' mesh is the reference stress determination at the hole edge strain gauge elements.

By definition, the maximum plate-to-hole edges to the reference stress ratios are the stress concentration factors 'Kt max' and 'Kt min' at the outermost and innermost edges of the plates. This increase of the tensile and hoop stresses, relative to an inelastic fastener concept incorporating 'Kt mean' at the central lines of plates, is caused by an additional bearing stress component due to the fastener-to-fastened plate interference, in their elastically deformed configuration.

The Load Transfer Factor (LTF) will be calculated also in two stages. The first stage will be the same as that of the existing inelastic fastener methodology, i.e. the logarithmic magnitude of the LTF (that was substantiated with numerous tests) will be determined for a given ratio between bearing stress-flange thickness and the tensile stress-fastener diameter products.

Within the second stage a correction factor, to be applied to the LTF, will be established. This smaller than '1' correction factor will be found from increments of the the mean integral and the maximum 'Kt max' values of this model using the allowable stress for varying 'Kt', as obtainable from the Kinetic Theory of Solids and Reference [7].

[1] PROBLEM STATEMENT

Generally, all the engineering theories of the structural fatigue analysis, including the unique Boeing methodology of the design for fatigue, are based upon the fundamental Cumulative Damage Concept of Ref. 7. The calculated life utilization ratio, in light of Ref. 7, should be smaller than 0.7 to 2.2. Due to this uncertainty, the minimum fatigue safety factor should not be smaller than $2.2/0.7=3.14$. This minimum safety factor was increased from 3.14 to 5.0 by the best practices of the British Aircraft Corporation-Supersonic Transport Division, France's Aerospaciale and the United States Boeing Company.

The main objective of the full scale fatigue testing is to provide the final re-assurance that the actual fatigue safety factor remains at least equal to 5.0, as intended. However, if the full scale testing indicates certain amount of the safety factor reduction, or, alternatively, its convergence towards 1.0, there is a considerable amount of concern.

This convergence of the safety factor towards 1.0 was recorded during the recent full scale fatigue testing of the analyzed array of heavy duty panels 'Upper Spar', Bay No 5, fastener No 113.

During the analytical investigation of this event and the consequent re-analysis of the 'Upper Spar' the author and his colleagues applied the historical methodology and also the method of Ref. 6 but were unable to explain the causes for this rather substantial reduction of the fatigue safety factor.

In contrast, the presented exact solution and the fine mesh solid island interface element models are providing complete explanations and a new methodology for pre-analysis and elimination of the future concerns and may be liabilities.

[2] THE EXACT SOLUTION

With reference to the Attachment 'A-1', there is a local increase in bearing pressure due to interference between an elastically sheared fastener and the web-like fastened lug plate. This local increase in bearing pressure causes hoop stress that elevates the stress concentration factor and therefore the Load Transfer Factor 'LTF' will be reduced. Consequently the margin of safety becomes more critical than that obtainable from the historical methodology.

[3] A CONCISE DESCRIPTION OF THE THREE DIMENSIONAL ARRAY OF PANEL BAYS

Fig. 1 shows geometry of the analyzed Bay No 5. Fig 2 shows a side view of the Bay No 5. Fig. 3 shows details of the presented finite element model. Fig. 4 shows zoomed isometric view of the fine mesh solid island together with the interface element connectors to coarse meshes. Fig. 5 shows an exploded view of the Inter-Bay Interface Connectors. It may be of some interest to note that the web attached flange of the chord-to-chord interface connectors are off-set relative to the web-to-web interface connectors by a separation amounting one half the web attached flange thickness plus one half the web thickness.

[4] A CONCISE DESCRIPTION OF THE PRELIMINARY PROTOTYPE MODEL

The elastic bearing strains of the fastened plates and a fastener are parallel to the central lines of the plates therefore there is no interference between an elastically deformed fastener, due to bearing pressure, and its surroundings.

In contrast, the 'Unit Shear Strain' causes an angular displacement and consequently an interference with the fastener bore.

Since the main objective of this model is determination of the stresses that are induced by the interference, it was necessary to develop a method of the 'pre-tension-like' loading that would produce a condition of pure shear of a fastener. This method of loading is shown in Fig. 6.

With reference to the Fig. 7, the fine mesh of the fastener bore consists of 16 'Hex 8' elements per quadrant. This perimetric fine mesh is connected to the coarse 'Hex 8' mesh using the 'Wedge 6' elements. The fine mesh solid island consists of 16 'Hex 8' and 36 'Wedge 6' elements per quadrant. The coarse plate mesh consists of 5 'Hex 8' elements per quadrant. The fastener mesh consists of 80 'Hex 8' elements per quadrant.

The heights of the elements are:

Web plate: 0.040 in (6 layers)

Web-attached Chord: 0.040 in (4 layers)

The 'Hex 8' elements of the fastener bore and the outermost 'Hex 8' elements of the fastener itself are connected with 16 coincident Gap Elements per quadrant, as shown in Fig. 7.

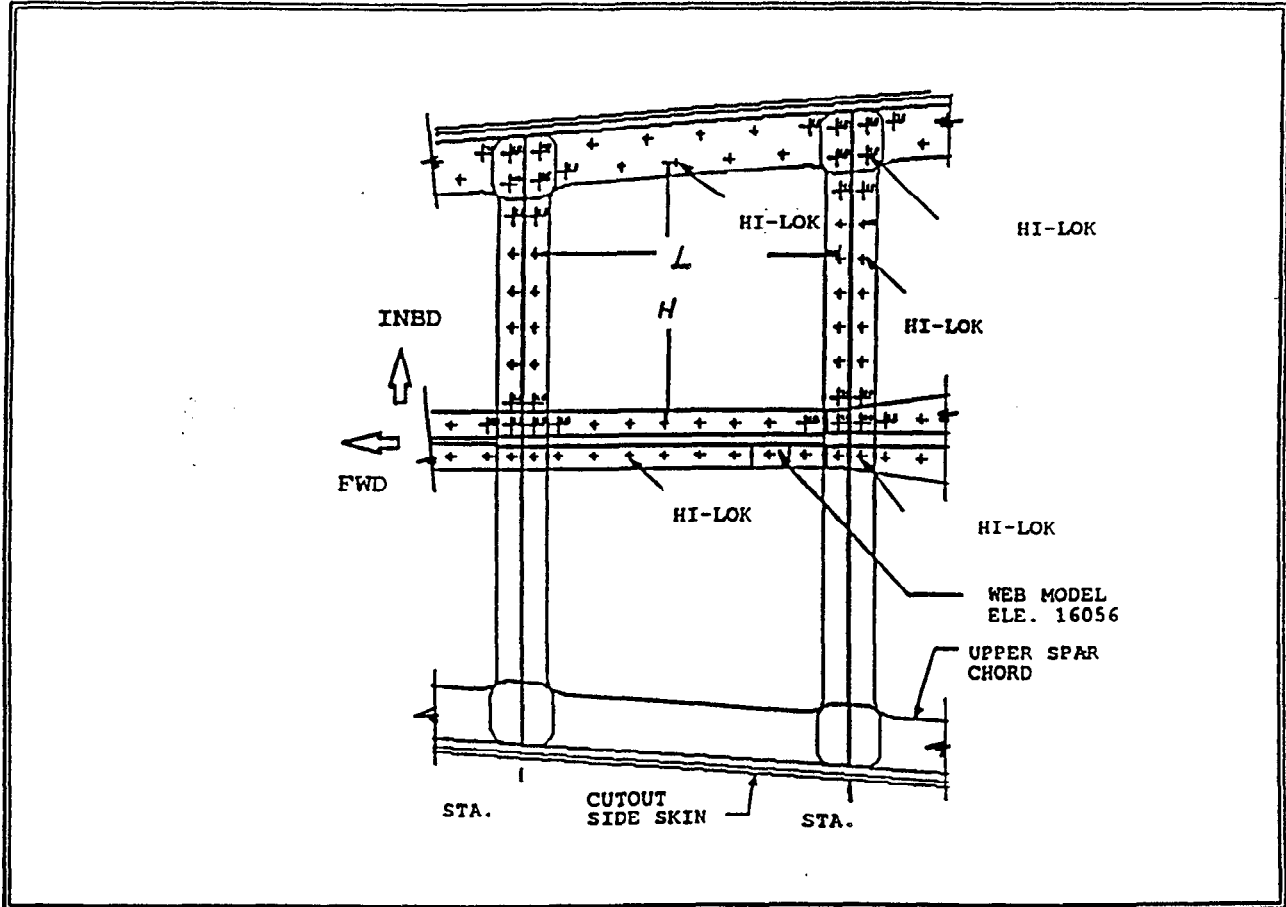
The numbers of elements per island are:

Web plate: $(16 + 36 + 5 + 16 + 80) * 4 * 6 = 3672$

Web attached Chord: $(16 + 36 + 5 + 16 + 80) * 4 * 4 = 2448$

Total number of elements per island: 6120

Fig. 1
Upper Spar
Geometry



The material name is (aluminum/low steel) _____ MatName =
 The web thickness is _____ tsk =
 The doubler thickness is _____ td =
 The doubler effectiveness is _____ Eff =
 The ratio of to moduli is _____ Re =

The actual plate thickness is _____ t = tsk + td
 The equivalent plate thickness is _____ teq = tsk + td*Re*Eff
 The fastener diameter is _____ d =
 The fastener pitch is _____ p =
 The number of fastener rows is _____ n =
 The row number from edge of part is _____ rn =

Fig. 2
Upper Spar
Bay No 5

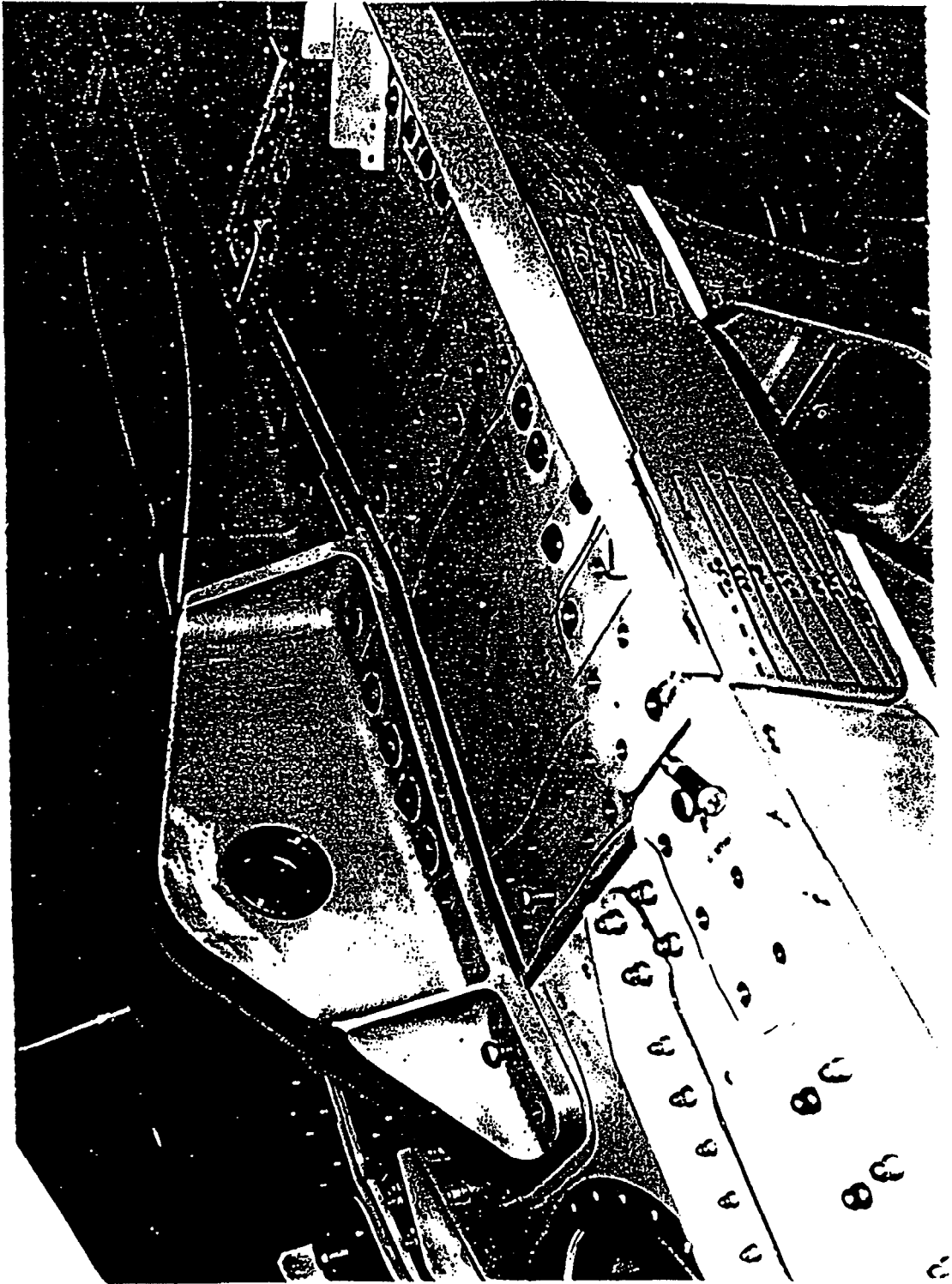


Fig. 3
Upper Spar
Bay No 5
Plane view of the presented finite element model

Symbols:

- ⊙ : are the Chord-to-Web Fasteners
- ⊛ : are the Stiffener-to-Web Fasteners

□ : are three-dimensional solid fine mesh Islands that are connected to a coarse mapped meshes of the Chords, Stiffeners and Webs using the Nastran Interface Elements (referred to an isometric view that is shown on the next page, Fig. 4).

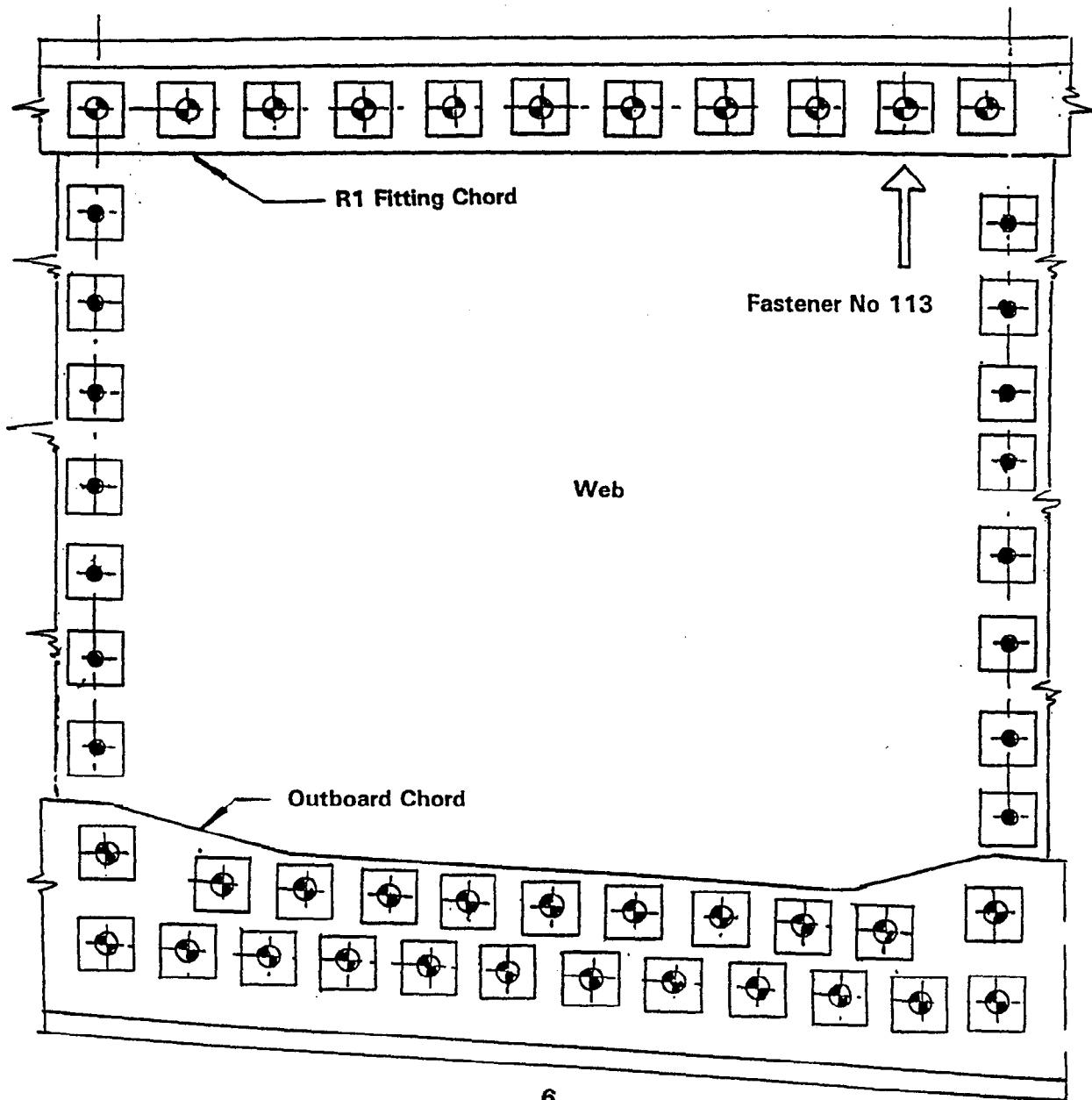


Fig. 4 CHORD AND WEB SOLID ISLAND CONNECTIONS
TO THE COARSE PLATE MESH USING NASTRAN
INTERFACE ELEMENTS

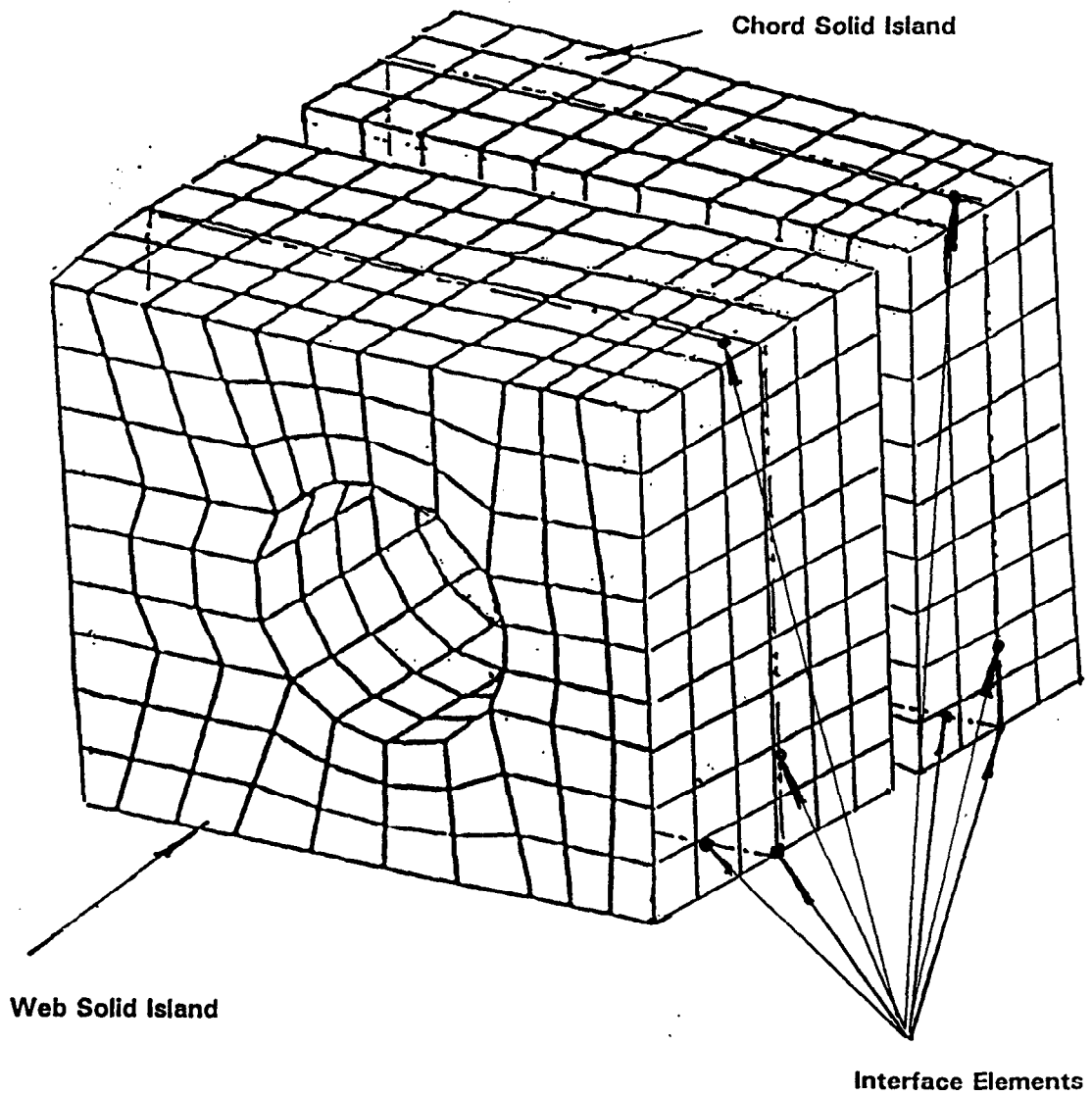
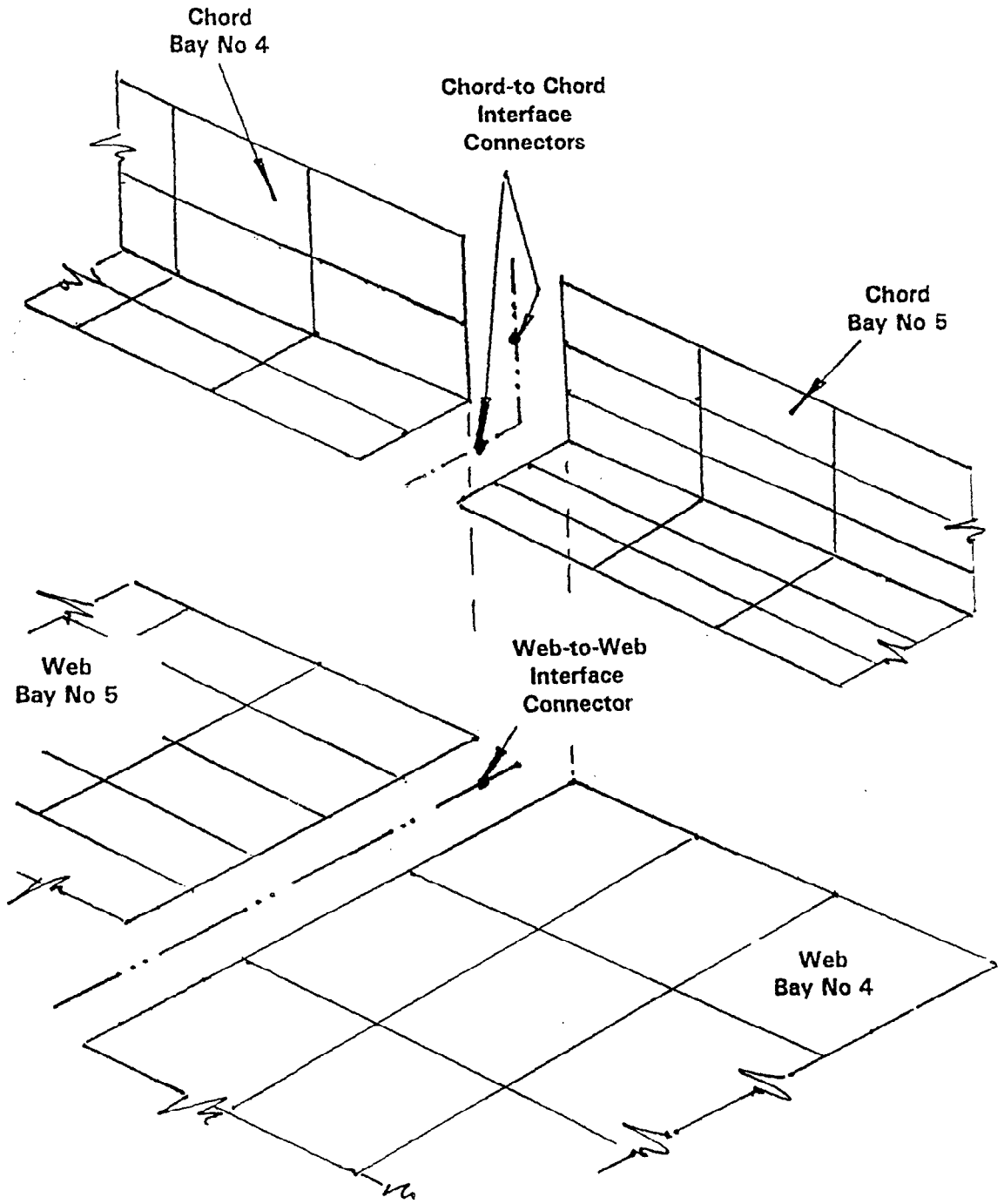
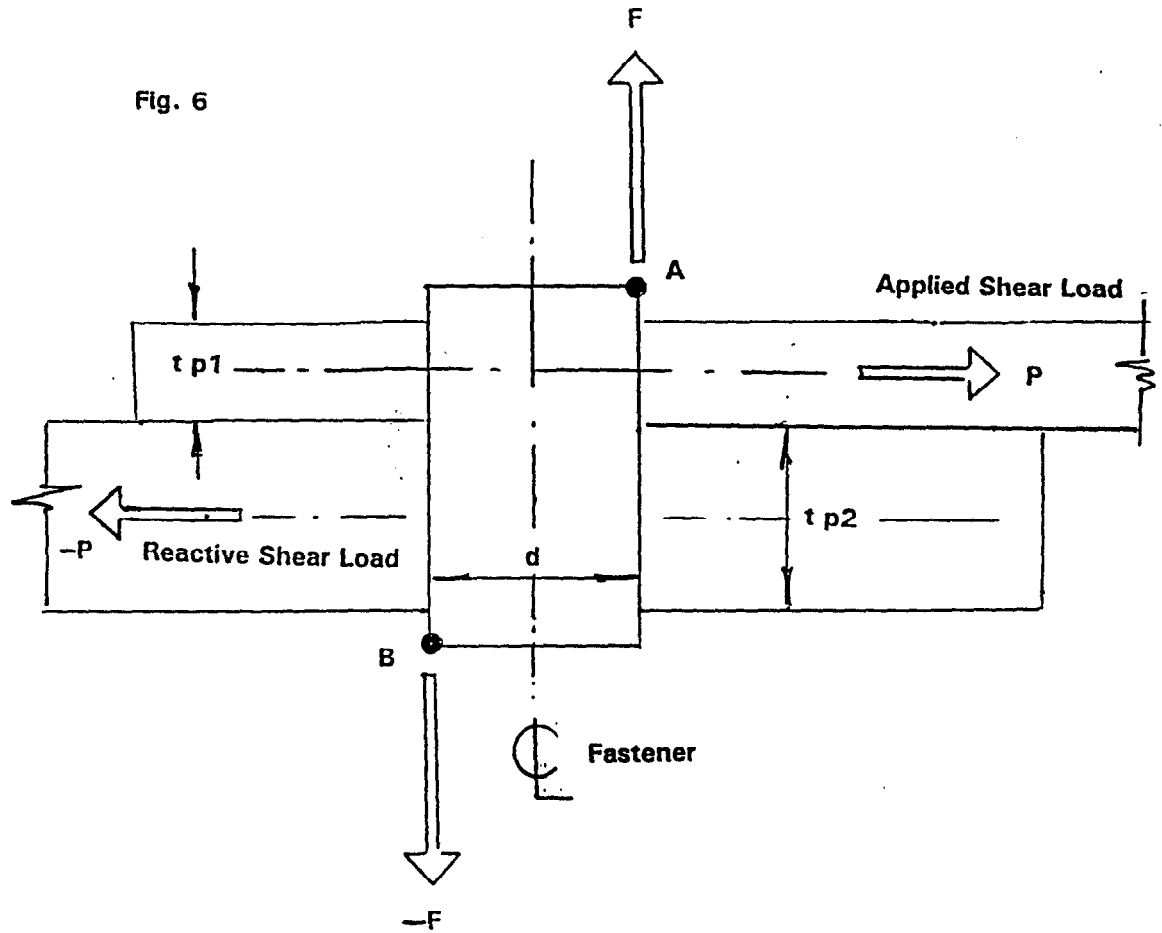


Fig. 5
Upper Spar
Bays No 4 and No 5
Inter-Bay Interface Connectors



THE UNIT SHEAR STRAIN OF FASTENERS

A condition of pure shear of fasteners may be produced by the following method of the pretension-like loading:



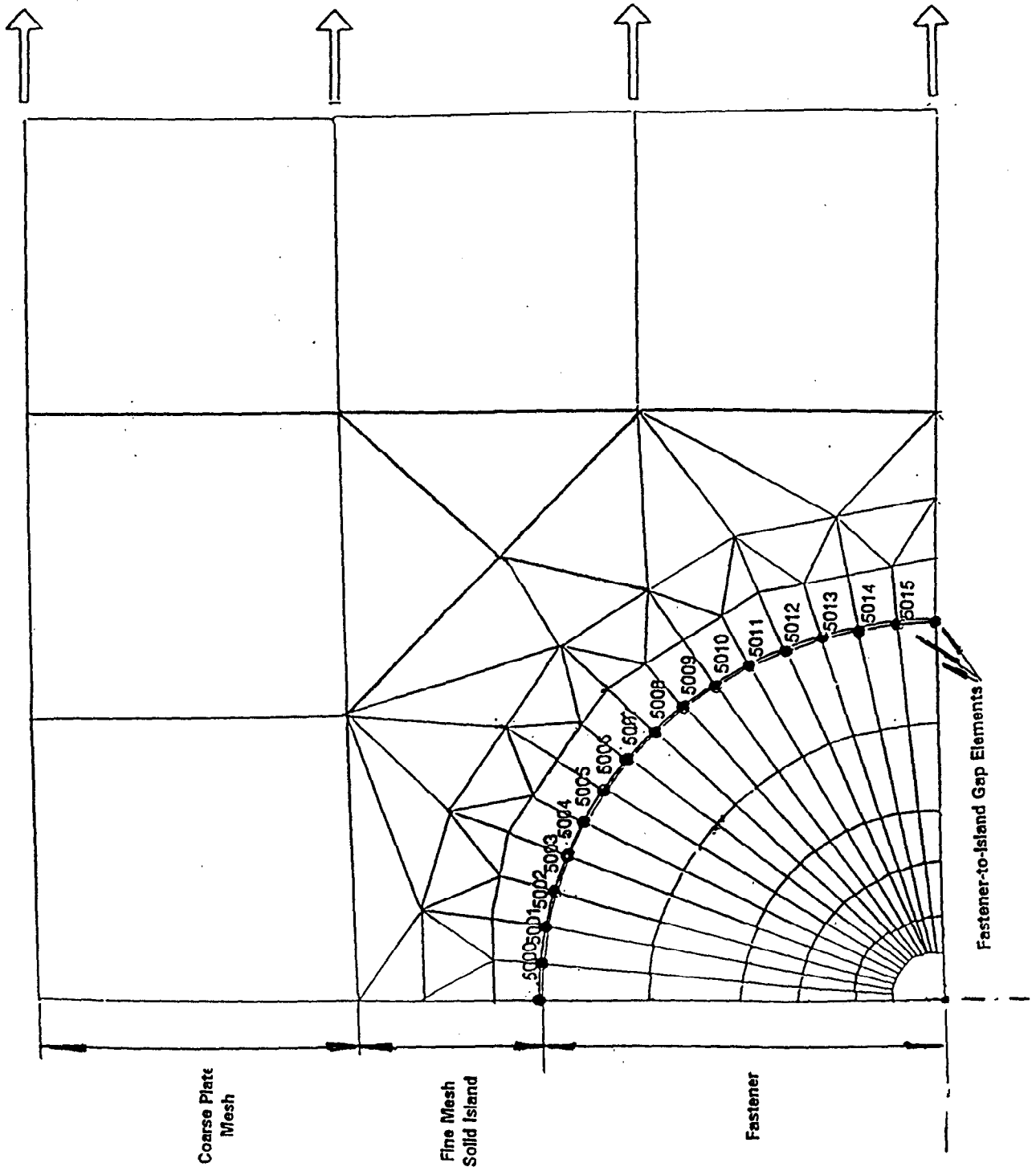
From $\sum M_B = 0$

$$P \frac{(t p1 + t p2)}{2} = F d$$

$$F = P \frac{(t p1 + t p2)}{2 d}$$

Fig. 7
Upper Spar

Preliminary Prototype of the Fine Mesh Solid Island



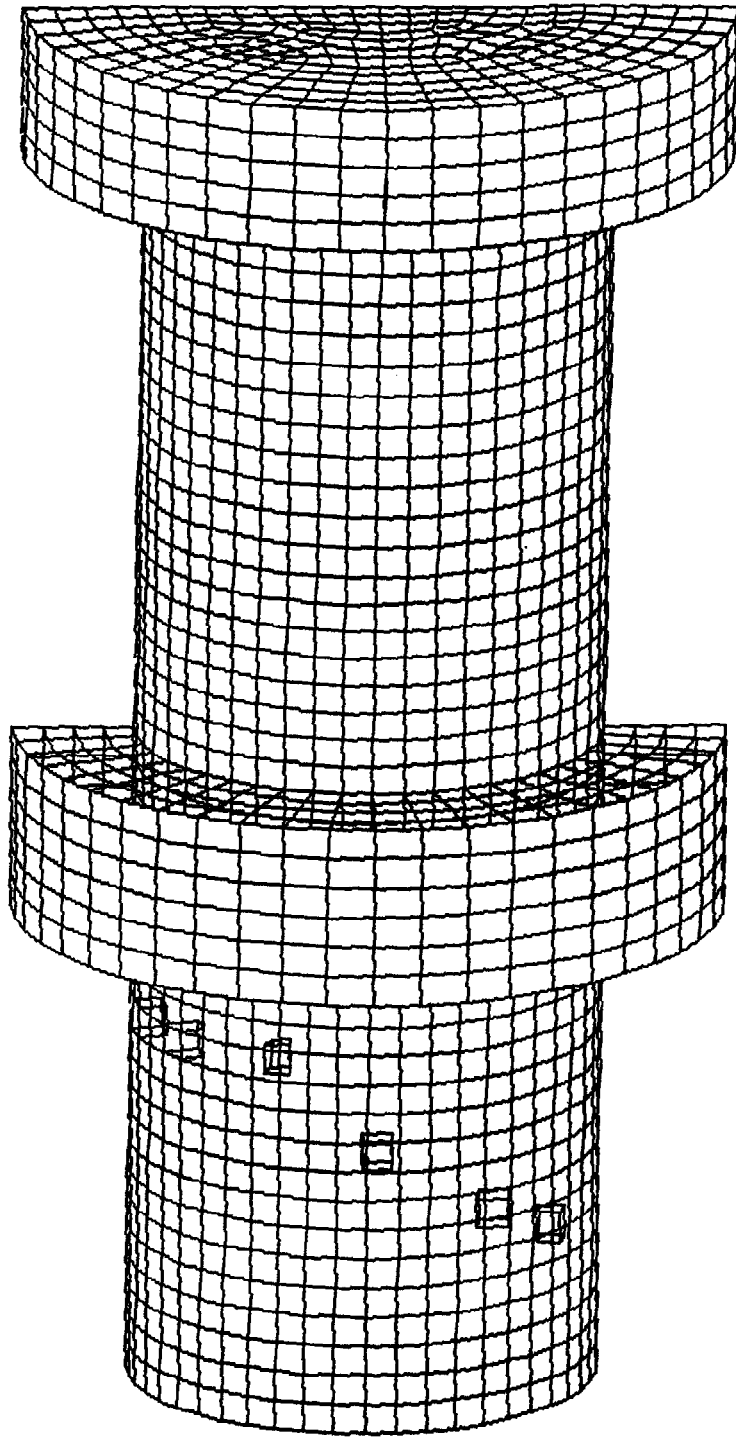
Coarse Plate
Mesh

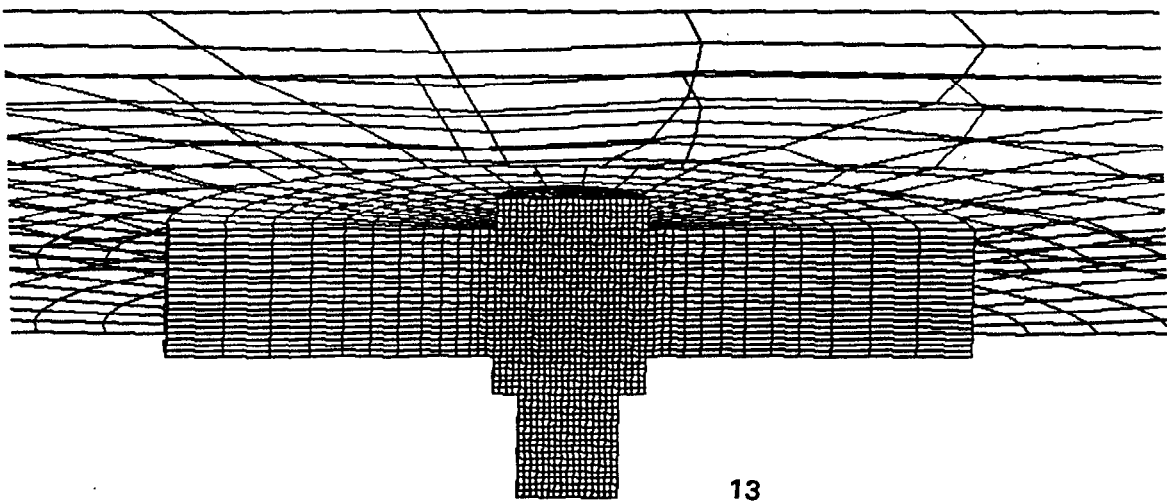
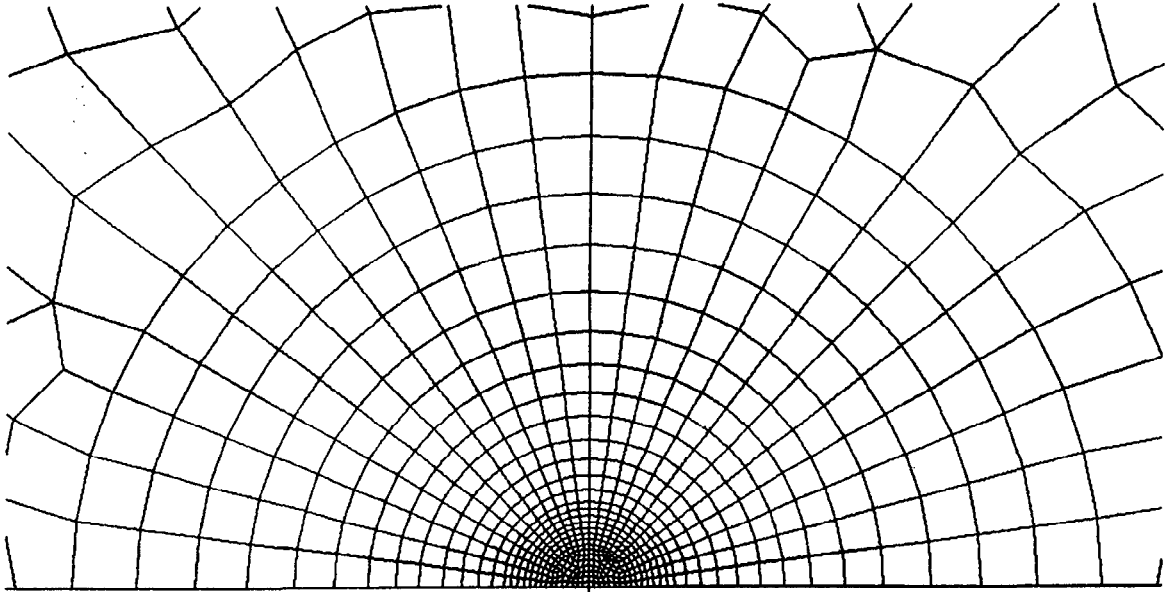
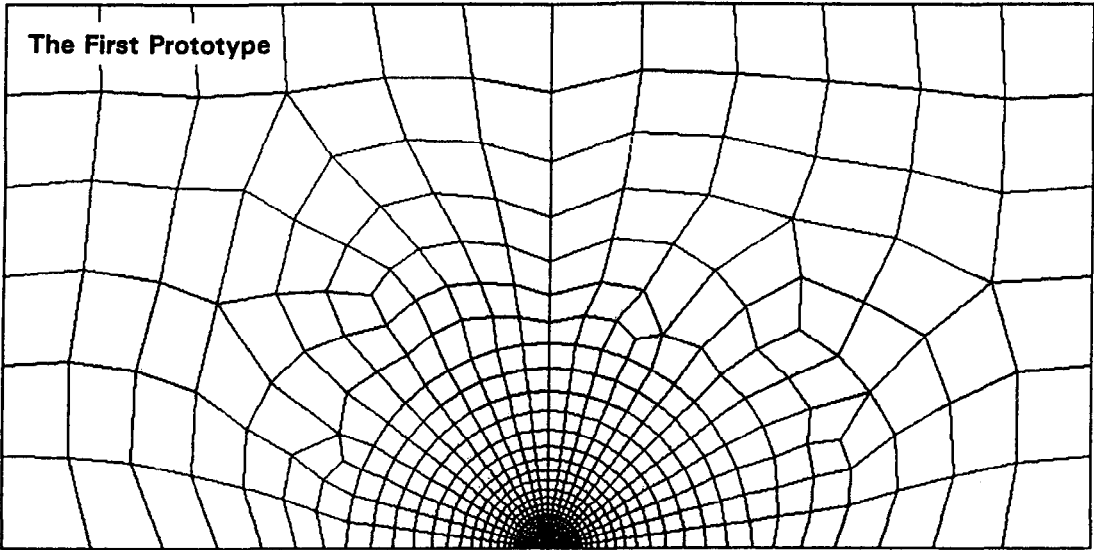
Fine Mesh
Solid Island

Fastener

Fastener-to-Island Gap Elements

The First Prototype
fastener





[6] ATTACHMENT A-1

The main objectives of this attachment are :

- 1) Determination of an exact expression for the complementary Bearing stress due to interference between an elastically sheared fastener and two semi-circular lugs.
- 2) Calculation of the Load Transfer Factor correction coefficient.
- 3) Reanalysis of the critical Margin of Safety for the fastener No:113 of a heavy duty panel bay.

Nomenclature

- tp1: Thickness of a web-like semi-circular lug plate [in.]
tp2: Thickness of a chord-like semi-circular lug plate [in.]
 γ : The unit Shear strain [Radians]
d: Fastener Diameter [in.]
 G_F : Shear Modulus of the Fastener [Psi]
P: Shear Load [lb]
 f_{br} : Bearing Stress [Psi]
 f_{th} : Hoop Stress [Psi]
 E_{CP1} : Compression Modulus of the Web-like Lug Plate [Psi]
 E_{CP2} : Compression Modulus of the Chord-like Lug Plate [Psi]
 E_{P1} : Modulus of Elasticity of the Web-like Lug Plate [Psi]
 E_{P2} : Modulus of Elasticity of the Chord-like Lug Plate [Psi]

The Unit Shear Strain ' γ ' and the shear stress are related:

$$\gamma G_F = P / (d^2 \pi / 4) \quad \text{Ref:}$$

Hence the Unit Shear Strain is

$$\gamma = 4P / (\pi G_F d^2) \quad \text{[Radians]}$$

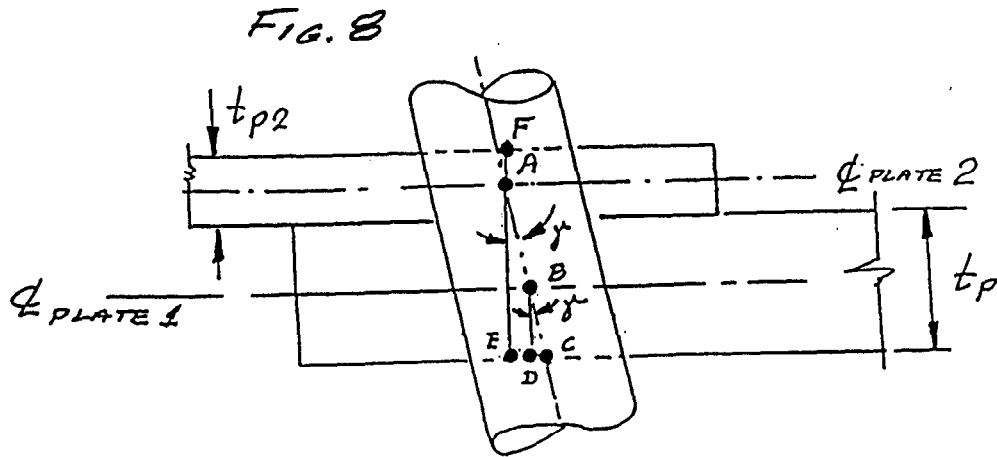
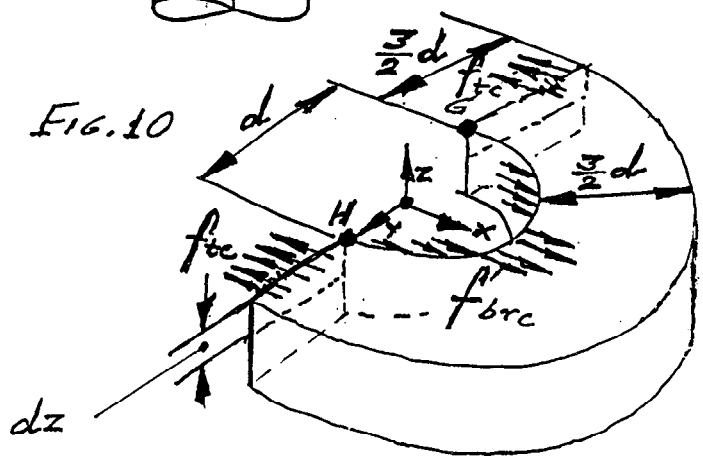
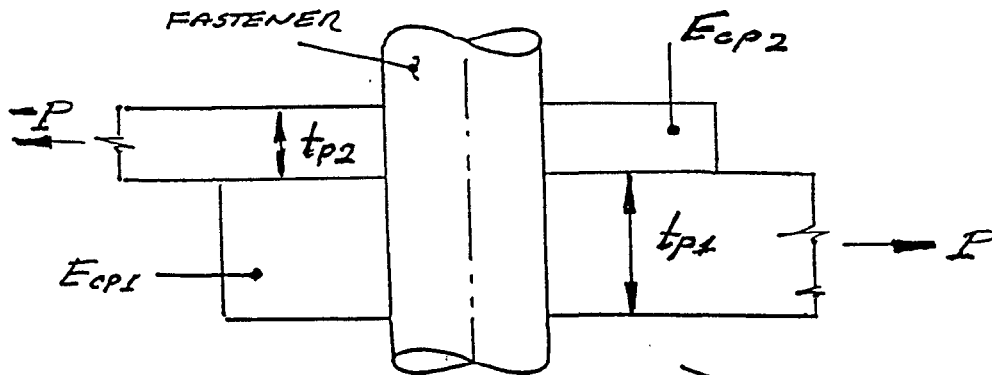
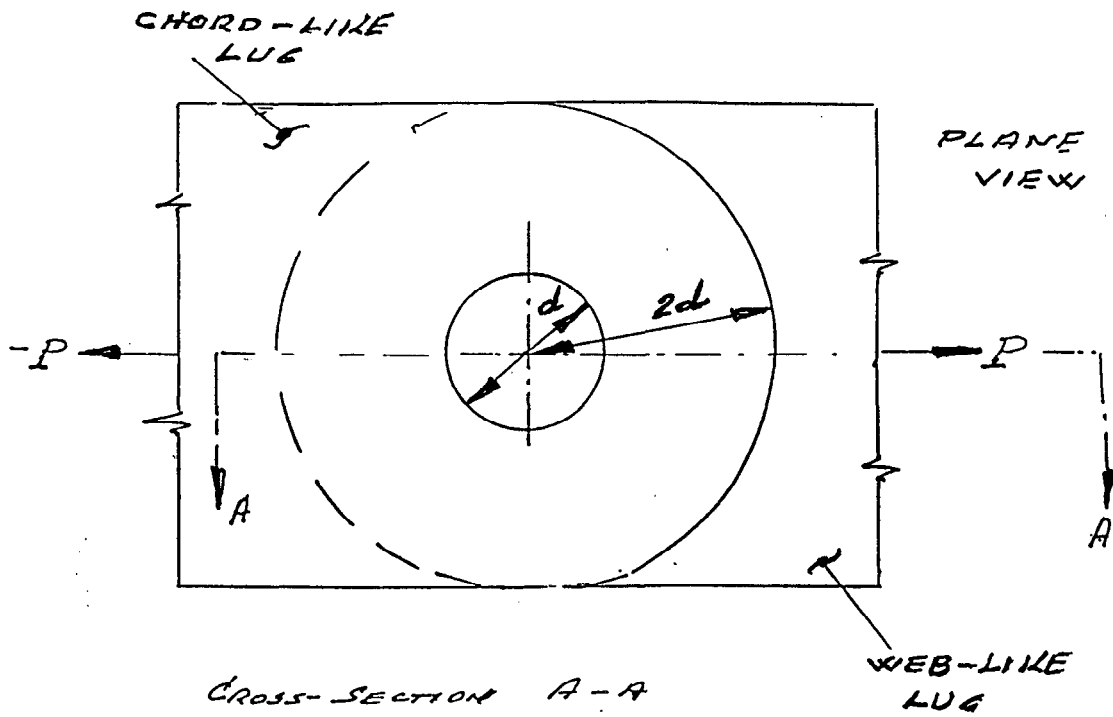


Fig. 9 Geometry and Notations



The historical methodology and Ref: 6 focuses onto an elastic fastener connecting the control line of the plate 2, point 'A'. The central line of the plate 1, point 'B' i.e. the historical methodology and the method of Ref:6, eliminates two outermost regions of both plates 'BD' and 'AF'. According to the Eddy Current Monitor Data, almost all fatigue fractures tend to occur at the outermost regions of the plates 'BD' and 'AF'.

In contrast, this methodology includes effects of the complementary bearing stress due to the greatest possible interference 'DC' of the web plate, assuming that the compression modulus of the fastener is higher than that of the web-like lug.

Plate 1:

Since:

$$\frac{DC}{BD} \approx \gamma$$

and where

$$BD = t_{p1}/2$$

and hence

$$DC = (t_{p1}/2) * \gamma$$

Substituting the Unit Shear Strain γ as obtained from the first principle:

$$DC = (t_{p1}/2) * (4P/\pi G_F d^2) = (2Pt_{p1}) / (\pi G_F d^2)$$

The physical meaning of the interference 'DC' is the bore diameter increase Δd . Therefore the complimentary bearing stress is

$$(\Delta d/d) * E_{CP1} = f_{brc} \quad (\text{Plate 1})$$

Substituting $\Delta d = DC$, then

$$f_{brc} = (E_{CP1}/G_F) * (2Pt_{p1}/\pi d^3)$$

This complementary bearing stress generates the hoop tensile stress (Ref:9) at two points of the maximum stress concentration factor.

This complementary hoop tensile stresses due to f_{brc} is

$$f_{brc} * d * dz = 2 * dz * (3/2)d * f_{tc}$$

Consequently the complementary tensile stress at points 'G' and 'H':

$$f_{tc} = f_{brc}/3 \quad (\text{Psi})$$

Substituting for f_{brc} yields

$$f_{tc} = (2/3) * (E_{CP1}/G_F) * (Pt_{p1}/\pi d^3) \quad [\text{Psi}]$$

Example 1: The critical fastener No:113 Data

Ultimate Load:	P = 3850 Lb.
Fastener Diameter:	d = 0.3125 in.
Web-like Lug Plate Thickness	$t_{p1} = 0.25$ in.
Fastener Material	Titanium
Web-like Plate Material	Al Alloy

$$(E_{CP1}/G_F) \cong (11.0 \times 10^6)/(6 \times 10) = 1.83$$

$$\begin{aligned} \text{Complimentary tensile stress} = f_{tc} &= (2/3) * (1.83) * (3850 * 0.25) / (\pi * 0.3125^3) \\ &= 12248 \text{ Psi} \end{aligned}$$

The web-like lug plate far field stress will be

$$P/(4dtp1) = 3850/(4 * 0.3125 * 0.25) = 12320 \text{ Psi}$$

It will be assumed that the stress concentration at the pin hole cross-section points 'G' and 'H' will be $K_t = 3.1$ Therefore stresses at the points 'G' and 'H' without complementary stress will be

$$f_t = 3.1 * 12320 = 38192 \text{ Psi}$$

And by taking into account the complementary stress:

$$f_t = (3.1 * 12320 + 12248) \text{ Psi}$$

By taking the far field stress in front of the parentheses,

$$f_t = 12320 * (K_t + (12248/12320))$$

or generally,

$$f_t = f_{t \text{ farfield}} * (K_t + (f_{tc} / f_{t \text{ farfield}}))$$

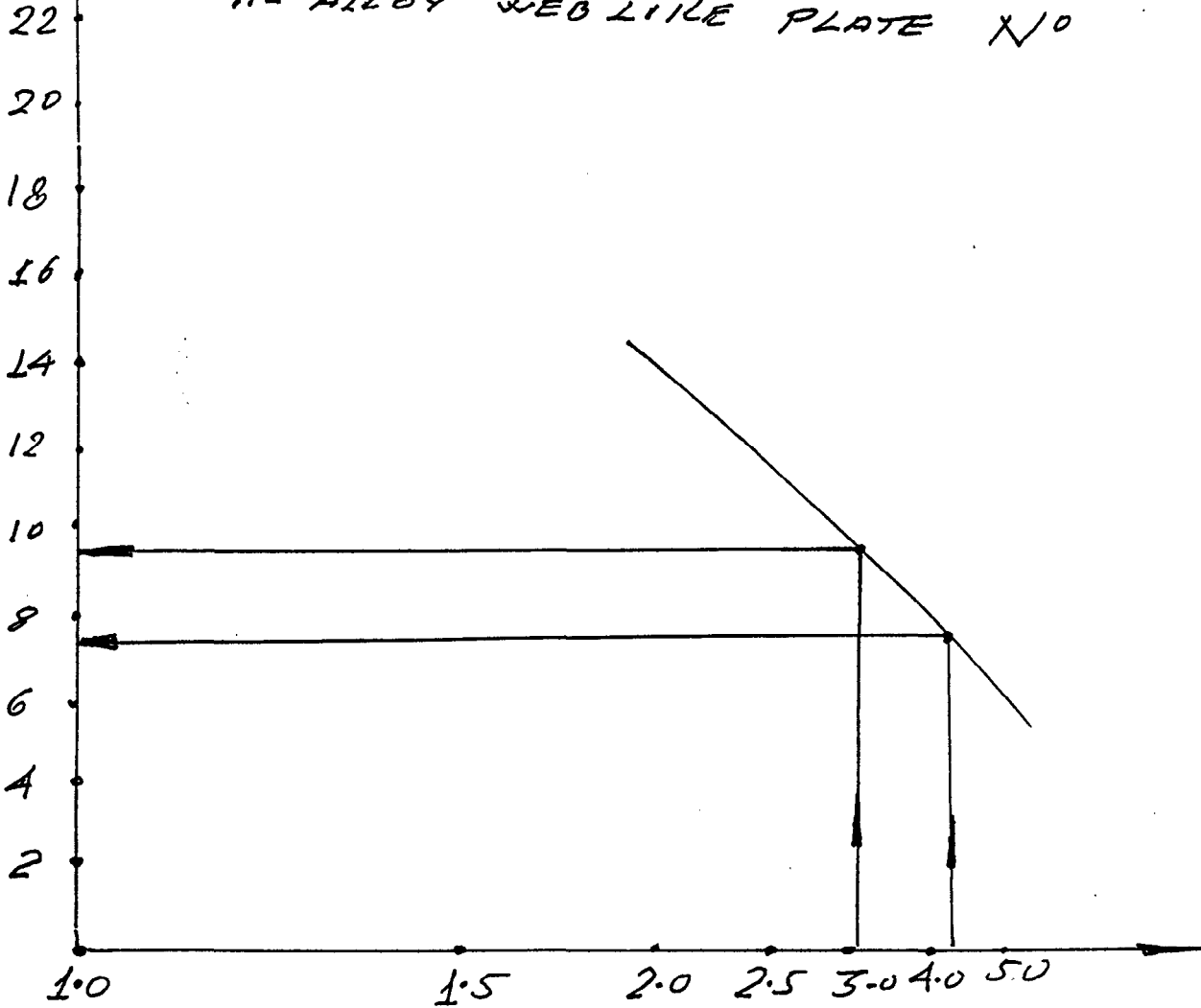
The second term within parentheses will be defined as an increment of the stress concentration factor K_t due to elastically sheared fastener interference:

$$(f_{tc} / f_{t \text{ farfield}}) = \Delta K_t$$

FIG. 11

σ ALL
ALT.

ALLOWABLE STRESS IN LIGHT
OF THE KINETIC THEORY OF SOLIDS AND
THE REFERENCE Δ [KSI]
AL ALLOY WEB LIKE PLATE NO



STRESS CONCENTRATION
FACTOR K_t

ΔK_t may be found from the stress contours of the proposed finite element models, or alternatively from the following attachment:

The detail fatigue rating (base) can be found in the following two stages:

The First Stage:

Find the $\sigma_{ALL\ CL}$ at the central lines of plates points 'A' and 'B' corresponding to $K_t = 3.1$ (Fig. 11)

$$\sigma_{ALL\ CL} = 9000 \text{ Psi}$$

The Second Stage:

Find the $\sigma_{ALL\ OS}$ at the outer surface of the web-like lug plate corresponding to $(K_t + \Delta K_t) = (3.1 + 1.0) = 4.1$ (Fig.).

$$\sigma_{ALL\ OS} = 7200 \text{ Psi}$$

Since the load transfer factor 'LTF' is the one among co-factors of the allowable base, the final expression for the load transfer factor becomes:

$$(LTF)_{\text{Point 'C'}} = (LTF)_{CL} * \sigma_{ALL\ OS} / \sigma_{ALL\ CL}$$

In this case (Fastener No:1), the historical methodology and Ref. 6 at the Central Line of the plates defines the load transfer factor:

$$(LTF)_{CL} = 0.78$$

$$\text{Margin of Safety} = 0.26$$

In contrast the presented new method defines a local load transfer factor at the point 'C':

$$(LTF)_{\text{Point 'C'}} = 0.78 * (7100/9000) = 0.61$$

$$\text{Margin of Safety} = 0.00$$

The events recorded during the recent fatigue testing of an array of heavy duty panels, incorporating Fastener No:1, are in complete agreement with the Margin of Safety=0.00.

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This development might increase accuracy of the Fatigue and the Static Strength Analysis of Aerospace Structures to the highest possible level.

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