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NUMERICAL MODELLING OF LIGHT TRANSMISSION IN A TEMPERATURE AND STRESS SENSITIVE OPTICAL ELEMENT

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A b s t r a c t

The subject of this research is numerical modelling of single pass light transmission through a general optical component subjected to thermal and structural loads. The light transmission is calculated numerically for an FE model considering both the thermal effect on the refractive index, and the structural distortions of the optical component. The thermal and the structural analyses of the optical component model are computed using the MSC/NASTRAN FE software.
The results of the MSC/NASTRAN FE analyses are used as input to the light transmission analysis 'IMU-POST' .
The output light beam is analysed with respect to wave-front deviation and distortion.

1. Introduction

There is a wide range of technological problems in optics and electro-optics which require investigation of the influence of thermal fields and structural distortions.
Highly sophisticated computer programs have been widely used for geometric ray tracing, lens design and images modelling. However, none of the existing packages takes into account the influence of non-uniform thermal fields and structural stresses on light propagation through an optical element which is sensitive to the above-mentioned factors.

The significance of ‘IMU-POST’ to this problem lies in the fact that it takes into account both the physical distortion of the optical element, and the thermal effect on refractive index. It is shown that the wave-front perturbations and the thermal focusing effect are significant although the temperature gradients are apparently small. Applying ‘IMU-POST’ improve the accuracy of the results by 30% to 80% depending on the material .

The software uses the flexibility and accuracy of the FE method to directly investigate the influence of the thermal and structural loads on the properties of an optical element.

The program uses a MSC/NASTRAN model so that the approach does not require an analytical description of the problem, but rather uses FE geometry as input.

The software can be used for GRIN optical elements and will take into account both types of refractive index variation: thermal variation, and the initial optical non-homogeneity of the material.

The proposed algorithm and software provide highly precise simulations. This was verified by numerous comparisons with solutions that were obtained both in numerical form using commercial optical design software results and with analytical results.

The results of the calculations represent a wide range of beam parameters including : Thermal focusing (or defocusing) and wave-front characteristics.

The algorithm and software can be adapted to take into account additional effects such as refractive index dependence on mechanical stress.

The software can easily be interfaced to existing software for computerised design and optimisation of optical devices.

2. Program description

The input for the IMU software program is the MSC/NASTRAN model, the thermal results and deformation results (Punch NASTRAN results), the analysis can be linear or non-linear for both thermal and structural analyses .

The transmitted light wave is calculated numerically for the FE model considering the temperature effect on the refractive index , wavelength and the structural distortions of the optical component.

Program output consists of: the optical path difference (OPD) the divergence of the light beam , the new focal position of the optical element , and wave-front description in terms of Zernike polynomial constants for use with standard optical design programs.

The software is available for IBM PC 486 or Pentium microcomputers. Its requirements for RAM and space on the hard disk are met by most standard computers of this type.

3. **Basic equation**

The spatial configuration of the single ray transmission through a medium with refraction index $n(x,y,z)$ can be described by equations [1]:

$$\frac{d}{ds}(n\frac{dx}{ds}) = \frac{fn}{fx}, \qquad \frac{d}{ds}(n\frac{dy}{ds}) = \frac{fn}{fy}, \qquad \frac{d}{ds}(n\frac{dz}{ds}) = \frac{fn}{fz},$$

where s is the natural coordinate measured along the ray.

The components of the ray direction vector are determined by first order derivatives: $v_x = \frac{dx}{ds}, \quad v_y = \frac{dy}{ds}, \quad v_z = \frac{dz}{ds};$

Although there is no general solution for these equations , it is known that the curvature of the trace at each point is proportional to the module of gradient of $\ln(n)$.

4. **Calculation algorithm**

The proposed numerical algorithm for creation of a spatial curve related to general equations (1) with prescribed starting point and initial direction consists of the following :

It is assumed that the ray trace is chosen as close as possible to the x_1 axis of the basic coordinate system, thus the variable x_1 is designated as the major coordinate.

At each incremental step along the ray, of length Δs, the basic coordinates obtain increments Δx_i, $i = 1,2,3$ in accordance with a parabolic dependence

$$\Delta x_i = a_i \quad (\Delta s)^2 + b_i ?\Delta s ,$$

where

$$a_i = \frac{1}{2n_0} ?(g_{0i} - f ?v_{0i})$$

$$b_i = v_{0i} ,$$

$$f = g_{01} ?v_{01} + g_{02} ?v_{02} + g_{03} ?v_{03} ,$$

$$(g_{01}, g_{02}, g_{03}) = grad(n(s_0)).$$

The values designated by index 0 are calculated relating to the first point of the current incremental step, i.e. for $s = s_0$. The first equation (1) yields

$$\Delta s = \frac{2\Delta x_1}{b_1 + \sqrt{b_1^{\,2} + 4 ?a_1 ?\Delta x_1}} ,$$

and this increment of natural coordinate determines the second and third components of the end point, relating to a new value of arc length $s = s_0 + \Delta s$:

$$x_1 = x_{01} + \Delta x_1 ,$$

$$x_2 = x_{02} + (v_{02} + a_2 ?\Delta s) ?\Delta s,$$

$$x_3 = x_{03} + (v_{03} + a_3 ?\Delta s) ?\Delta s$$

The new coordinates of the tangent vector are

$$v_i = v_{0i} + 2 ?a_i ?\Delta s$$

The optical path length on this interval is an increment

$$\Delta L = \Delta s ?¾ n_0 + \frac{1}{2} f ?\Delta s √.$$

5. **Refractive index calculation**

The element is considered as a thermally sensitive material.
The values of refractive index as a function of wave length and temperature are obtained based on the formula: [ref. 3]

$$n(\lambda, T) = n(\lambda, T_0) + \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot \left[D_0 \Delta T + D_1 \Delta T^2 + D_2 \Delta T^3 + \frac{E_0 \Delta T + E_1 \Delta T^2}{\lambda^2 - \lambda_{TK}^2} \right]$$

where

$$n^2(\lambda, T_0) = 1 + \left[\frac{B_1}{\lambda^2 - C_1} + \frac{B_2}{\lambda^2 - C_2} + \frac{B_3}{\lambda^2 - C_3} \right] \lambda^2,$$

temperatures are measured in $^{\circ}C$, λ is wavelength in microns,

6. <u>Finite Element Formulation</u>

A six-faced eight-nodal solid finite element of type CHEXA was used for presentation of the optical element in a format acceptable for MSC/NASTRAN software application.

The area Inside the single element is defined by basic Cartesian variables x, y, z and by element coordinates ξ, η, ζ , which are mapped onto a standard interval [-1 ,1]. The relationship of basic coordinates to element coordinates is given by

$$x = \sum_i x_i \cdot N_i(\xi, \eta, \zeta, \xi_i, \eta_i, \zeta_i),$$

$$y = \sum_i y_i \cdot N_i(\xi, \eta, \zeta, \xi_i, \eta_i, \zeta_i), \qquad \text{(eq.1)}$$

$$z = \sum_i z_i \cdot N_i(\xi, \eta, \zeta, \xi_i, \eta_i, \zeta_i),$$

where ξ_i, η_i, ζ_i and x_i, y_i, z_i $(i = 1...8)$ are coordinates of i-th node in the element and basic systems respectively,

$$N_i = \frac{1}{8} \cdot (1 + \xi\xi_i)(1 + \eta\eta_i)(1 + \zeta\zeta_i)$$

Similar to (eq. 1) the dependencies of refraction index on temperatures are assumed. The properties for calculation of the values $n(P)$ and $T(P)$ are known in the current point $P(x, y, z)$ and are considered in the finite element having element coordinates ξ, η, ζ :

$$n(P) = \sum_i n_i \cdot N_i(\xi, \eta, \zeta, \xi_i, \eta_i, \zeta_i),$$

$$T(P) = \sum_i T_i \cdot N_i(\xi, \eta, \zeta, \xi_i, \eta_i, \zeta_i),$$

where n_i and T_i are nodal values of these functions..

The partial derivatives of the scalar function $n(P)$ with respect to basic coordinates were calculated as

$$\frac{\partial n}{\partial x} = \sum n_i \cdot \left(\frac{\partial N_i}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial N_i}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} + \frac{\partial N_i}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial x} \right)$$

The components $\frac{\partial \xi}{\partial x}, \frac{\partial \eta}{\partial x}, \frac{\partial \zeta}{\partial x}$ are obtained as solutions to the following equations resulting from (1) by differentiation with respect to x :

$$(a_x \eta\zeta + b_x \eta + c_x \zeta + e_x) \cdot \frac{\partial \xi}{\partial x} + (a_x \xi\zeta + b_x \xi + d_x \zeta + f_x) \cdot \frac{\partial \eta}{\partial x} +$$

$$+ (a_x \xi\eta + c_x \zeta + d_x \eta + g_x) \cdot \frac{\partial \zeta}{\partial x} = 8,$$

$$(a_y \eta\zeta + b_y \eta + c_y \zeta + e_y) \cdot \frac{\partial \xi}{\partial x} + (a_y \xi\zeta + b_y \xi + d_y \zeta + f_y) \cdot \frac{\partial \eta}{\partial x} +$$

$$+ (a_y \xi\eta + c_y \zeta + d_y \eta + g_y) \cdot \frac{\partial \zeta}{\partial x} = 0,$$

$$(a_z \eta\zeta + b_z \eta + c_z \zeta + e_z) \cdot \frac{\partial \xi}{\partial x} + (a_z \xi\zeta + b_z \xi + d_z \zeta + f_z) \cdot \frac{\partial \eta}{\partial x} +$$

$$+ (a_z \xi\eta + c_z \zeta + d_z \eta + g_z) \cdot \frac{\partial \zeta}{\partial x} = 0,$$

where

$$a_x = \sum \xi_i \eta_i \zeta_i x_i, \quad a_y = \sum \xi_i \eta_i \zeta_i y_i, \quad a_z = \sum \xi_i \eta_i \zeta_i z_i,$$

$$b_x = \sum \xi_i \eta_i x_i, \quad b_y = \sum \xi_i \eta_i y_i, \quad b_z = \sum \xi_i \eta_i z_i,$$

$$c_x = \sum \xi_i \zeta_i x_i, \quad c_y = \sum \xi_i \zeta_i y_i, \quad c_z = \sum \xi_i \zeta_i z_i,$$

$$d_x = \sum \eta_i \zeta_i x_i, \quad d_y = \sum \eta_i \zeta_i y_i, \quad d_z = \sum \eta_i \zeta_i z_i,$$

$$e_x = \sum \xi_i x_i, \quad e_y = \sum \xi_i y_i, \quad e_z = \sum \xi_i z_i,$$

$$f_x = \eta_i x_i, \qquad f_y = \eta_i y_i, \qquad f_z = \eta_i z_i,$$

$$g_x = \zeta_i x_i, \qquad g_y = \zeta_i y_i, \qquad g_z = \zeta_i z_i.$$

The derivatives $\frac{\partial\xi}{\partial y}, \frac{\partial\eta}{\partial y}, \frac{\partial\zeta}{\partial y}$, and $\frac{\partial\xi}{\partial z}, \frac{\partial\eta}{\partial z}, \frac{\partial\zeta}{\partial z}$ .are obtained in a similar manner.

All these partial derivatives calculated at the current point are components of the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial\xi}{\partial x} & \frac{\partial\eta}{\partial x} & \frac{\partial\zeta}{\partial x} \\ \frac{\partial\xi}{\partial y} & \frac{\partial\eta}{\partial y} & \frac{\partial\zeta}{\partial y} \\ \frac{\partial\xi}{\partial z} & \frac{\partial\eta}{\partial z} & \frac{\partial\zeta}{\partial z} \end{pmatrix},$$

which is an important component in the calculation of the vector $grad\ n$. It is the major factor in the numerical ray tracing.

7. **Sample problem**

A cube shaped optical element made of BK7 glass is subjected to regulated heating on one cube face. On the other faces there are both radiative and convective heat transfer .

In the first stage, the temperature distribution within the optical element is calculated as a function of time using NASTRAN (see Fig 1) .

Temperature results (MSC/NASTRAN) [°K]

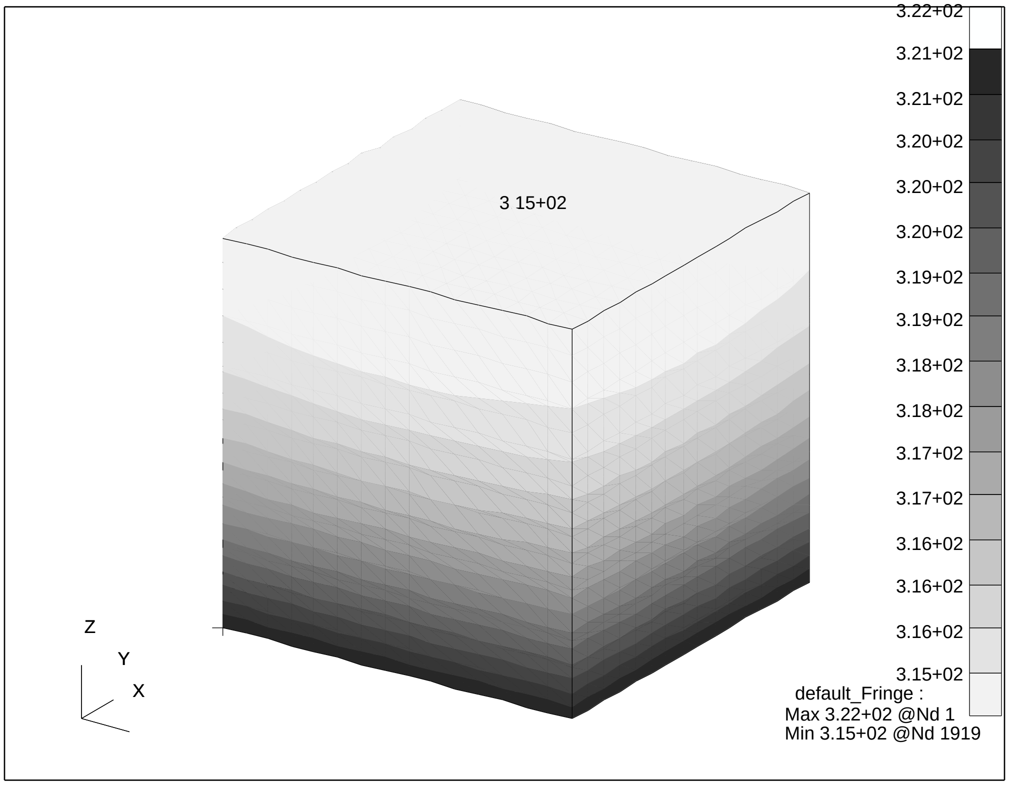

Fig 1 :MSC/PATRAN plot for time step 20

The second stage consist of the simulation of the deformation on the same model using the temperatures calculated in the first stage, with appropriate boundary condition and loads (see Fig 2) .

Deformation results (MSC/NASTRAN) [mm]

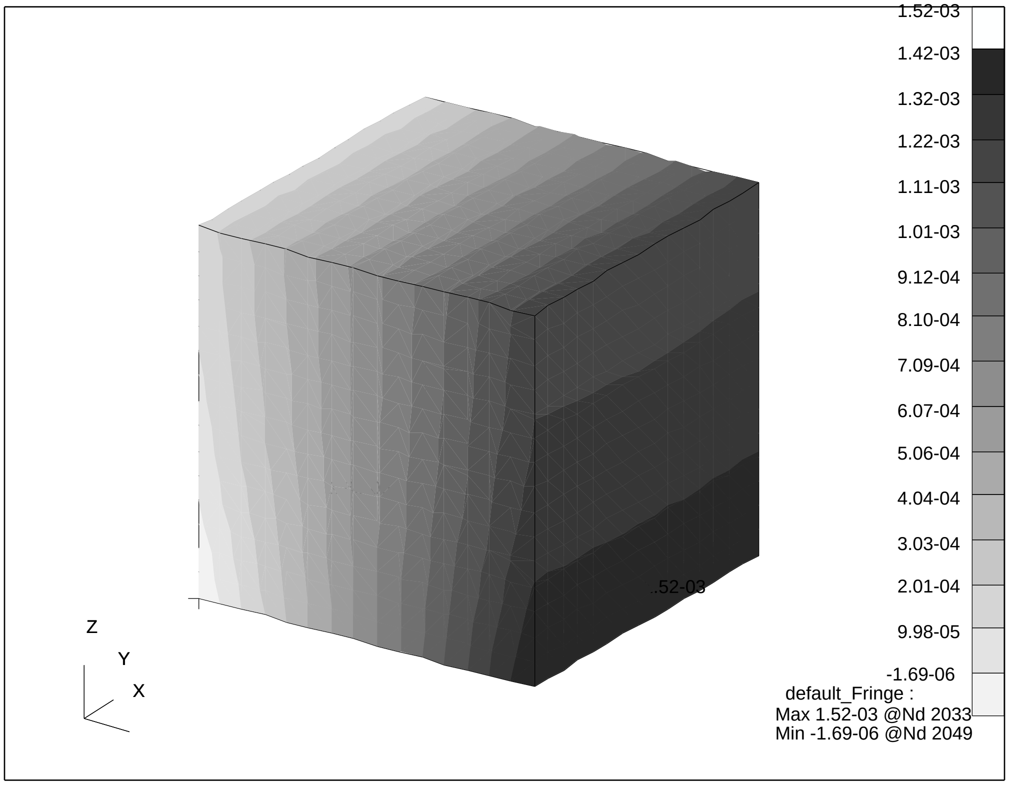

Fig 2 : MSC/PATRAN plot for time step 20

The “IMU-POST” software uses a NASTRAN model , the temperature simulation punch file results, the deformation punch file results and the optical properties of the glass to calculate: the OPD (see Fig 3), the average direction angle (grad), OPD standard deviation, focal point coordinates, and the Zernike coefficients.

Data for BK7 optical glass: [3]

temperatures are measured in ^{Y}C , λ is wavelength in microns,

$T_0 = 20^{Y}C, \quad \Delta T = T - T_0, \quad \lambda_{TK} = 0.17\,\mu m,$

$B_1 = 1.03961212, \quad B_2 = 2.31792344 ? 10^{-1}, \quad B_3 = 1.01046945,$

$C_1 = 6.0069867 ? 10^{-3}, \quad C_2 = 2.00179144 ? 10^{-2}, \quad C_3 = 1.03560653 ? 10^{2},$

$D_0 = 1.86 ? 10^{-6}, \quad D_1 = 1.31 ? 10^{-8}, \quad D_2 = -1.37 ? 10^{-11},$

$E_0 = 4.34 ? 10^{-7}, \quad E_1 = 6.27 ? 10^{-10}.$

Program output:

(IMU POST) United Table of Ray Tracing Results   20-th step

Range of OPL (mm): [15.15327522 15.15349657]

Full difference (mm):   .00022135

Additive component of OPL (mm) 15.15275400 (=   23938 wl)

Full OPD interval (wl): [.82341   1.17310]

Optical Paths Differences (wl):

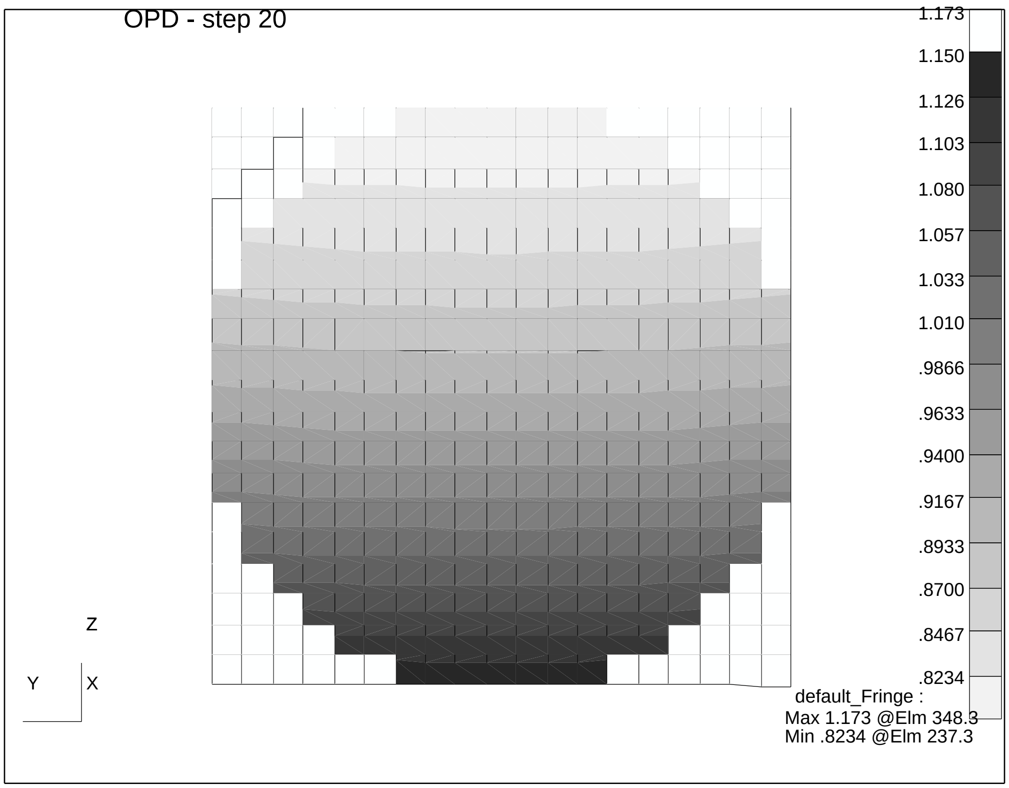

Fig 3 : MSC/PATRAN plot   for time step 20

Average value of relative OPD: .95989993

Standard deviation: 5.2356E-03

Parameters of thermal perturbation of rays:

Range of   y-components (mm): [-2.0261E-06 2.0261E-06]

z-components (mm): [-5.8103E-05 -1.3135E-05]

Average direction: vy   .000000   vz -.000035   Angle (grad):   .0020

Standard deviations: vy   6.3793E-08 vz   6.6655E-07

"Focal distance" from exit plane (mm) -1.5192E+05

Coordinates of "Focal point" (mm): y= 3.4292E-07 z= 1.0287E+01

Zernike coefficients:

1: 9.606795E-01	2: -1.922650E-01	3: 6.340066E-09	4: 1.895258E-02
5: 3.131207E-02	6: -7.669243E-10	7: -2.093297E-04	8: -3.842073E-10
9: -4.900679E-04	10: -7.738388E-04	11: 5.225382E-09	12: -5.879597E-04
13: -1.362501E-10	14: -1.938652E-04	15: -2.210750E-09	16: 3.701356E-05
17: -2.711043E-04	18: 3.211257E-09	19: -4.639226E-05	20: 4.026580E-09
21: 1.221731E-04	22: 6.442737E-09	23: 8.665901E-05	24: 7.096137E-09
25: -2.786541E-05	26: -7.211061E-05	27: -2.011191E-09	28: -2.420623E-04
29: 4.513378E-10	30: -1.825242E-04	31: 3.249907E-09	32: 1.461972E-04
33: 1.148000E-11	34: -4.504286E-05	35: 8.772098E-09	36: 3.337472E-05

Residuals: max deviation 2.64405381E-04

standard deviation 9.69335267E-05

Full range of Zernike approximation (wl): [8.159624E-01 1.203365E+00]

8. ACKNOWLEDGEMENTS

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9. REFERENCES

1. Born M. and Wolf E. "Principles of Optics". Pergamon Press, Oxford, 808 p. (1980).

2. Cornbleet S. "Microwave and Optical Ray Geometry". Wiley, Chichester, 152 p,. (1984).

3. SCHOTT Optical Glass catalog

4. Sharma A. "Computing Optical Path Length in Gradient Index Media: a Fast and Accurate Method", Applied Optics, 24, 4367 (1985).

5. Sharma A., Kumar D.V. and Ghatak A.K. "Tracing Rays through Gradient Index Media: a new Method". Applied Optics, 21, 984 (1982).

6. Montagnino L. "Ray Tracing in Inhomogeneous Media", Journal of Optical Society of America, 58, 1667 (1968).

7. Richerzhagen B. “Finite Element Ray Tracing: a New Method for Ray Tracing in Gradient-Index Media”. Applied Optics, 35, 6186, (1996).