Computation of Dynamic Loads on Aircraft Structure due to Continuous Gust Using MSC/NASTRAN

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Abstract

The computation of gust loads on the structure of an aircraft is part of the engineering work during the development and certification phases of a new project. The present work describes the methodology used at EMBRAER to compute dynamic loads caused by atmospheric continuous gusts. The mathematical formulation assumes that the gust phenomenon is described as a stationary random process and that the aircraft dynamics is linear. MSC/NASTRAN is used for obtaining the dynamic system modal data by means of SOL103 (normal modes solution), and the modal amplitudes necessary to generate the dynamic system frequency-response functions by means of SOL146 (aeroelastic response solution). An example is given in which the methodology is applied to a modern jet aircraft.

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1 Introduction

The computation of gust loads on the structure of an aircraft is part of the engineering work during the development and certification phases of a new project. The present work describes the methodology used at EMBRAER to compute dynamic loads caused by continuous gusts.

The paper begins with the basic mathematical formulae describing the methodology, followed by the two criteria defined in FAR Part 25 Appendix G (design envelope and mission analysis). Then, the mathematical formulation used to obtain the frequency response function will be described as well as the MSC/NASTRAN role in such formulation. Basically, the MSC/NASTRAN normal modes solution (SOL103) is used to obtain the eigenvalues and eigenvectors of the dynamic system, and the MSC/NASTRAN aeroelastic response (SOL146) is used to obtain the modal amplitudes necessary to compute the frequency response functions.

A practical case is then studied, where dynamic loads are calculated on a modern jet aircraft according to the design envelope and mission analysis criteria. Some considerations are then made regarding the potential influence of different reduced frequency discretizations on the computation of the aerodynamic matrices.

2 Mathematical Formulation

In gust loads theory the atmospheric turbulence can be represented as a stationary random process [1, 2]. In such representation, the shape of the Power Spectral Density (PSD) function for vertical and lateral gust velocities recommended in Appendix G of FAR Part 25 [4] is the von Kármán gust PSD given by

$$\Phi(\Omega) = \sigma_w^2 \frac{L}{\pi} \frac{1 + 8/3(1.339L\Omega)^2}{[1 + (1.339L\Omega)^2]^{11/6}}, \tag{1}$$

where $\Omega = 2\pi f/U$, f is the frequency in Hertz, U is the airplane speed, L is the scale of turbulence equal to 762 m (2500 ft), and σ_w is the root mean square of the gust velocity.

Assuming the aircraft dynamics is linear, Ref. [3] shows that the PSD function $\Phi_o(f)$ of a dynamic load induced on the structure by an atmospheric turbulence having a PSD function $\Phi_i(f)$ can be written as

$$\Phi_o(f) = |H(f)|^2 \, \Phi_i(f), \tag{2}$$

where $H(f)$ is the frequency response function of the corresponding dynamic load.

Given the PSD functions $\Phi_i(f)$ of the input, and $\Phi_o(f)$ of the output, the two quantities $\bar{A}$ and N_0 that will be used in defining the design envelope criterion and the mission analysis criterion are given by

$$\bar{A}^2 = \frac{\int_0^\infty \Phi_o(f)df}{\int_0^\infty \Phi_i(f)df}, \quad \text{and} \quad N_0^2 = \frac{\int_0^\infty f^2\Phi_o(f)df}{\int_0^\infty \Phi_o(f)df}. \tag{3}$$

In practice, the integrals defining $\bar{A}$ and N_0 are evaluated only up to a reasonable upper limit, beyond which the contribution to the integrals for computing $\bar{A}$ is negligible.

2.1 Design Envelope Criterion

The incremental load y_{design} according to the design envelope criterion is given by

$$y_{\text{design}} = \bar{A} U_\sigma, \qquad (4)$$

where the quantity $\bar{A}$ has been defined by Relation (3), and U_σ is the design gust velocity given in Figure 1, taken from Ref. [4]. In Figure 1, V_b is the design speed for maximum gust intensity, V_c is the design cruise speed, and V_d is the design dive speed.
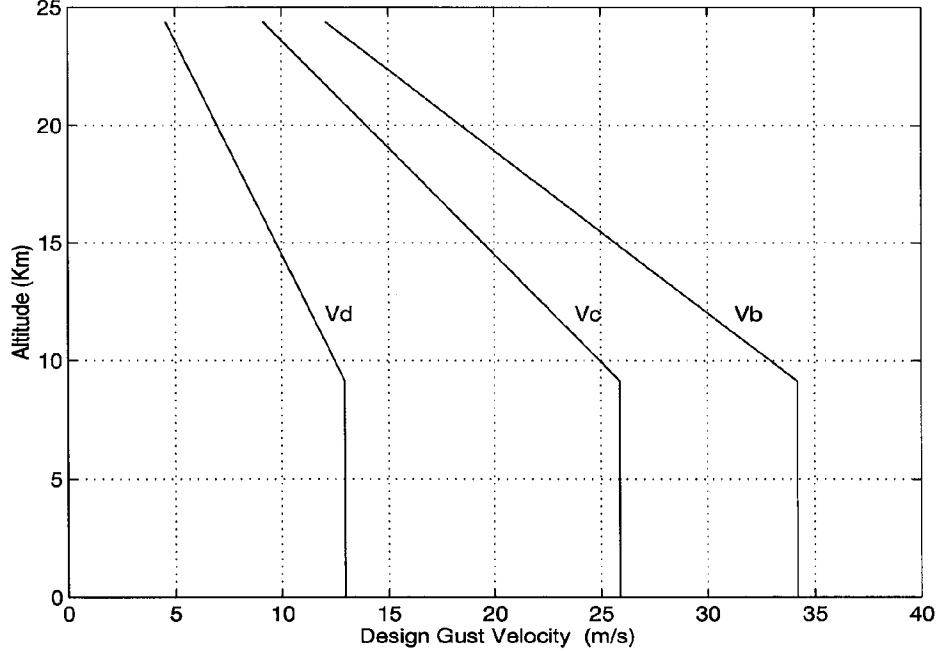

Figure 1: Velocity Profiles for the Design Envelope Criterion

2.2 Mission Analysis Criterion

The mission analysis criterion is based on the frequency of exceedance equation given by

$$N(y) = \sum_{j=1}^{n_p} t(j) N_0(j) \left[P_1(j) \exp\left(-\frac{|y - y_{1-g}(j)|}{\bar{A}(j)b_1(j)}\right) + P_2(j) \exp\left(-\frac{|y - y_{1-g}(j)|}{\bar{A}(j)b_2(j)}\right) \right], \qquad (5)$$

where $N(y)$ is the number of peaks (or cycles) per unit time in excess of load y; n_p is the number of segments in the mission profile being analyzed; $t(j)$ is the fraction of time in segment j relative to the sum of all other segments; $P_1(j)$, $b_1(j)$, $P_2(j)$, and $b_2(j)$ are functions of the flight altitude of segment j, as shown in Figures 2 and 3; $y_{1-g}(j)$ is the corresponding one-g load for segment j; and $\bar{A}(j)$ and $N_0(j)$ have been defined by Relation (3). The design load y_{mission} according to the mission analysis criterion is the root of Equation (5) with the left-hand side equal to 2×10^{-5} cycles/hour, that is,

$$N(y_{\text{mission}}) = 2 \times 10^{-5} \text{ cycles / hour.} \qquad (6)$$
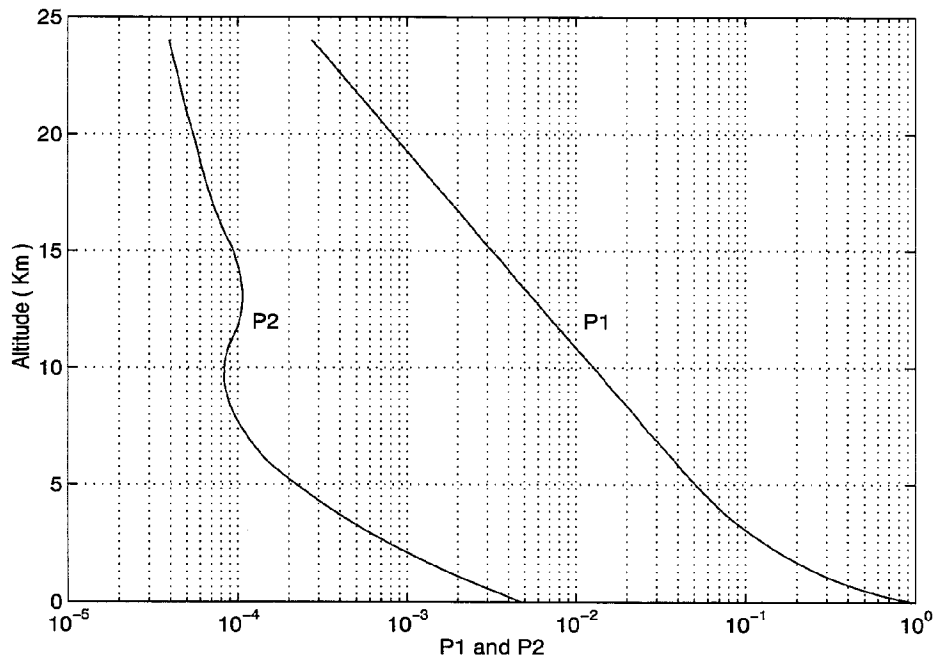
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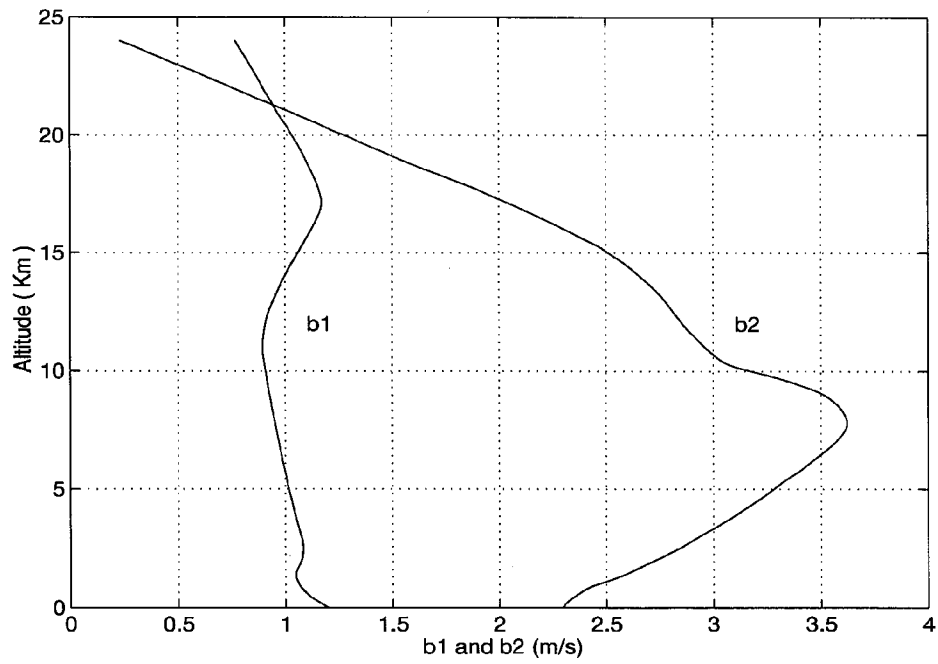Figure 2: P_1 and P_2 Functions

Figure 3: b_1 and b_2 Functions

2.3 Frequency-Response Computation on Airplanes

The system of differential equations governing the elastic deformation of an airplane can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) \;=\; \mathcal{F}(t), \tag{7}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathcal{F}(t)$ is the vector of external forces on the structure generated by the gust field, and $\mathbf{x}(t)$ is the vector of elastic displacements on the structure.

To obtain the frequency response function $H(f)$, the system will be subjected to a sinusoidal gust field that induces a vertical disturbance velocity $w(t)$ of the form

$$w(t) \;=\; a\,e^{I2\pi ft}, \tag{8}$$

where a is a real constant and I is the imaginary number equal to $\sqrt{-1}$. Such disturbance field will generate a load vector $\mathcal{F}(t)$ on the structure given by

$$\mathcal{F}(t) \;=\; \bar{\mathbf{F}}(f)\,e^{I2\pi ft}. \tag{9}$$

Substituting Relation (9) into Equation (7) one gets the following equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) \;=\; \bar{\mathbf{F}}(f)\,e^{I2\pi ft}, \tag{10}$$

and its corresponding steady-state solution can be written as

$$\mathbf{x}(t) = \bar{\mathbf{x}}(f)\,e^{I2\pi ft}. \tag{11}$$

Substituting Solution (11) and its time derivatives into Equation (10) one gets

$$\left(-4\pi^2 f^2\,\mathbf{M} + I\,2\pi f\,\mathbf{C} + \mathbf{K}\right)\bar{\mathbf{x}}(f)\,e^{I2\pi ft} \;=\; \bar{\mathbf{F}}(f)\,e^{I2\pi ft}. \tag{12}$$

The objective here is the computation of the elastic forces $\mathcal{F}_e(f)$ acting on the structure given by

$$\mathcal{F}_e(f) \;=\; \mathbf{K}\,\bar{\mathbf{x}}(f)\,e^{I2\pi ft}. \tag{13}$$

Such forces are the dynamic response to the gust field given by Relation (8). Therefore, the frequency-response of the elastic forces associated with grid points can be written as the ratio of Relations (13) and (8), that is,

$$\mathbf{H}(f) \;=\; \frac{\mathbf{K}\,\bar{\mathbf{x}}(f)}{a}, \tag{14}$$

where a is the gust intensity.

To cast Equation (14) in a more appropriate format, consider the free vibration of a conservative mass-spring system given by

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) \;=\; \{0\}. \tag{15}$$

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Assuming an exponential solution given by

$$\mathbf{x}(t) = \boldsymbol{\eta}(f)\,e^{I2\pi ft}, \tag{16}$$

one can compute n vectors $\boldsymbol{\eta}_j(f)$ $(j = 1, 2, \cdots, n)$ such that Equation (16) is satisfied, where n is the order of the system given by Eq. (15).

Substituting Relation (16) into Equation (15) one gets

$$(-4\pi^2 f_j^2\, \mathbf{M}\, \boldsymbol{\eta}_j(f) + \mathbf{K}\, \boldsymbol{\eta}_j(f))e^{I2\pi ft} = \{0\}, \quad \text{for} \quad j = 1, 2, \cdots, n \tag{17}$$

that is,

$$-4\pi^2 f_j^2\, \mathbf{M}\, \boldsymbol{\eta}_j(f) + \mathbf{K}\, \boldsymbol{\eta}_j(f) = \{0\}, \quad \text{for} \quad j = 1, 2, \cdots, n \tag{18}$$

where $4\pi^2 f_j^2$ are the eigenvalues, and $\boldsymbol{\eta}_j(f)$ are the eigenvectors associated with the system.

Equation (18) can be rewritten as

$$\mathbf{K}\,\boldsymbol{\Phi} = \mathbf{M}\,\boldsymbol{\Phi}\,\boldsymbol{\lambda}, \tag{19}$$

where $\boldsymbol{\Phi}$ is the eigenvector matrix, defined as

$$\boldsymbol{\Phi} = \left[\; \boldsymbol{\eta}_1(f)\;\middle|\;\boldsymbol{\eta}_2(f)\;\middle|\;\cdots\;\middle|\;\boldsymbol{\eta}_n(f)\;\right], \tag{20}$$

and $\boldsymbol{\lambda}$ is the diagonal matrix of the eigenvalues, that is,

$$\boldsymbol{\lambda} = 4\pi^2 \begin{bmatrix} f_1^2 & 0 & \cdots & 0 \\ 0 & f_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & f_n^2 \end{bmatrix}. \tag{21}$$

Expression (19) relates the mass and stiffness matrices using the eigenvector and eigenvalue matrices. Using a modal transformation of coordinates, the physical elastic displacement vector $\bar{\mathbf{x}}(f)$ of Equation (14) can be written as

$$\bar{\mathbf{x}}(f) = \boldsymbol{\Phi}\,\boldsymbol{\xi}(f), \tag{22}$$

where $\boldsymbol{\xi}(f)$ is the so-called vector of modal amplitudes. Substituting Equations (22) and (19) into Equation (14), one finds that the frequency-response expression associated with grid points is given by

$$\mathbf{H}(f) = \frac{\mathbf{M}\,\boldsymbol{\Phi}\,\boldsymbol{\lambda}\,\boldsymbol{\xi}(f)}{a}. \tag{23}$$

The above frequency-response gives the grid loads, where for each grid point j there are six load components:

- Three Forces: F_x, F_y, F_z and
- Three Moments: M_x, M_y, M_z.

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The frequency response function associated with grid point j is given by

$$\mathbf{H}_j(f) \;=\; \left\{ \begin{array}{c} \mathbf{F}_j(f) \\ \mathbf{M}_j(f) \end{array} \right\}, \tag{24}$$

where $\mathbf{F}_j(f) = [\, F_x \; F_y \; F_z \,]^T$ and $\mathbf{M}_j(f) = [\, M_x \; M_y \; M_z \,]^T$.

Therefore, the frequency response expression in Equation (23) can be rewritten as

$$\mathbf{H}(f) = \left[\begin{array}{c} \mathbf{H}_1(f) \\ \mathbf{H}_2(f) \\ \vdots \\ \mathbf{H}_m(f) \end{array} \right], \tag{25}$$

where m is the number of grid points having mass associated with them. Note that, in general, one shall have $n = 6m$, where n is the order of the dynamic system given by Equation (7).

To get the frequency-response $\mathcal{H}(f)$ associated with a structural section (a group of zero or more grid points with mass), one must take the necessary grid points with mass involved, make the load transport using the relationships from statics, and add all the components. Consider, for example, a structural section k on the aircraft right wing located at coordinate y_k, where $y = 0$ at the wing root and y increases towards the wing tip. The frequency-response $\mathcal{H}_k(f)$ will be given by

$$\mathcal{H}_k(f) \;=\; \left\{ \begin{array}{c} \displaystyle\sum_{j=1}^{p} \mathbf{F}_j(f) \\ \displaystyle\sum_{j=1}^{p} \mathbf{M}_j(f) + \mathbf{r}_{kj} \times \mathbf{F}_j(f) \end{array} \right\} \quad \text{for} \quad y_j > y_k, \tag{26}$$

where the index j represents the grid points on the right wing containing concentrated mass for which $y_j > y_k$, $\mathbf{r}_{kj}$ is the position vector of grid j relative to the point of interest in the edge of section k, and p is total number of grids satisfying this condition.

Note that the summation in Equation (26) is possible since the modal formulation guarantees that all components are in the same phase.

The frequency response function $\mathcal{H}_k(f)$ associated with a section k is complex, that is,

$$\mathcal{H}_k(f) \;=\; \mathcal{H}_{k_{\text{re}}}(f) \;+\; I\,\mathcal{H}_{k_{\text{im}}}(f). \tag{27}$$

Once the frequency response function $\mathcal{H}_k(f)$ of interest has been computed, Equation (2) for a particular section k becomes

$$\{\Phi_o(f)_k\} \;=\; (\,\mathcal{H}_{k_{\text{re}}}^2(f) + \mathcal{H}_{k_{\text{im}}}^2(f)\,)\,\Phi_i(f), \tag{28}$$

where $\{\Phi_o(f)\}_k$ is a 6th-dimensional vector containing the PSD functions of the elastic loads at section k.

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3 Problem Definition

The aeroelasticity group at EMBRAER represents the EMB-145 airplane by a stick model, using MSC/NASTRAN as the modeling platform. The dynamic model has nearly 400 lumped mass points, and the aerodynamic data for computations are obtained using the Doublet-Lattice theory. Figure 4 shows the panels of the aerodynamic model used for vertical gust load calculations.

A computational program named CGUST (Continuous Gust) has been written at EMBRAER to compute the incremental dynamic loads due to continuous gusts according to the design envelope and mission analysis criteria described previously.

The airplane data necessary to use the mathematical formulation prescribed previously in order to calculate dynamic loads at section k is the frequency response function $\mathcal{H}_k(f)$ given by Equation (26), and that is obtained from Equation (23) by knowing the mass matrix $\mathbf{M}$, the modal matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\lambda}$, and the matrix of modal amplitudes $\boldsymbol{\xi}$.

In CGUST the mass matrix $\mathbf{M}$ is assembled by reading the mass cards directly from the MSC/NASTRAN bulk data file, the modal matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\lambda}$ are obtained by means of the MSC/NASTRAN normal modes solution (SOL103), and the matrix of modal amplitudes $\boldsymbol{\xi}$ is obtained by means of the MSC/NASTRAN aeroelastic response solution (SOL146).

To cover all selected cases necessary to generate the incremental loads according to the design envelope or mission analysis criteria, several aeroelastic solutions SOL146 must be obtained. A parameter in the bulk data having a strong influence on the execution time is the number n_s of reduced frequencies k_s ($s = 1, 2, \cdots, n_s$) that will be used to compute the aerodynamic matrices. Therefore, it is desirable to keep the number of selected reduced frequencies as low as possible without compromising the results.
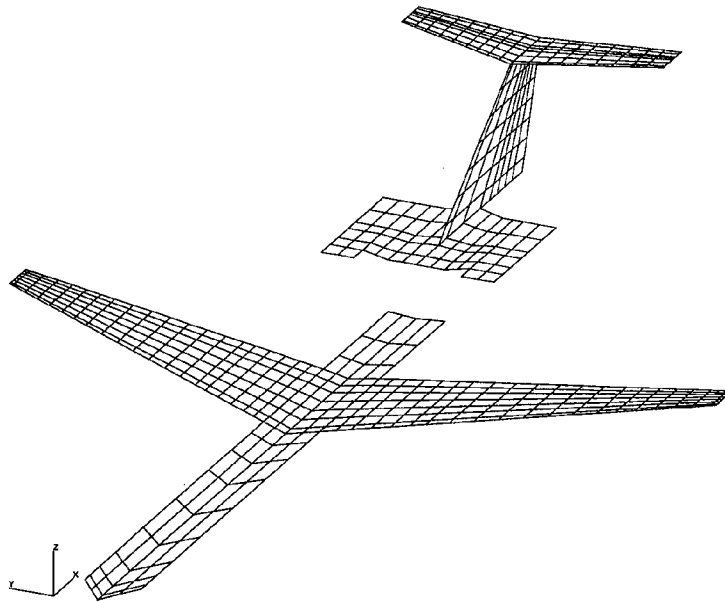

Figure 4: Doublet-Lattice Panel Model for Vertical Gust Loads

FREQ1 cards are used for selecting the frequencies in which the modal amplitudes are desired. Since the number n_r of selected frequencies k_r can be much higher than the number of points in which the aerodynamic matrices are computed ($n_r \gg n_s$), the special linear interpolation method described in Reference [5] is used to interpolate the aerodynamic matrices at reduced frequencies k_r based on the knowledge of the aerodynamic matrices at reduced frequencies k_s. Some cards of a typical SOL146 are listed below, with thirteen reduced frequencies selected by the MKAERO1 cards covering a corresponding range of frequencies up to 50 Hz.

```
$$-----------------------------------------------------------------$$
$$--- CARDS FROM THE MSC/NASTRAN BULK DATA SECTION (CASE YCH3F50) ---$$
$$-----------------------------------------------------------------$$
EIGR        200    MGIV    0.0    50.0                              +EIGR50
+EIGR50     MAX
GUST        300     300 .005285  .00000 189.200
RLOAD1      300     146                     147
DAREA       146   10000       3  1.000
TABLED1     147                                                    +TD1001
+TD1001   0.000   0.000   0.010  1.000  50.000  1.000    ENDT
TABDMP1     400                                                     +DAMP1
+DAMP1      0.0   0.010    100.  0.010    ENDT
FREQ1       500   0.000   0.005    200
FREQ1       500   1.075   0.075    119
FREQ1       500   10.25   0.250    159
$$-----------------------------------------------------------------$$
$$---------- BEGIN MKAERO1 CARDS ( 13 REDUCED FREQUENCIES ) --------$$
$$-----------------------------------------------------------------$$
$& FREQ=   .1411    .5000  1.0000  5.0000 10.0000 15.0000 20.0000 25.0000
MKAERO1    .57615                                                  +MKAER1
+MKAER1    .00671   .02379  .04757  .23786  .47572  .71358  .95144 1.18931
$& FREQ= 30.0000 35.0000 40.0000 45.0000 50.0000
MKAERO1    .57615                                                  +MKAER2
+MKAER2   1.42717 1.66503 1.90289 2.14075 2.37861
$$-----------------------------------------------------------------$$
$$-------- END OF MKAERO1 CARDS ( 13 REDUCED FREQUENCIES ) --------$$
$$-----------------------------------------------------------------$$
```

The strategy for selecting the reduced frequencies for each condition is the following: Given the flight speed U, the upper limit of eigenvalue extraction f_u, and the reference length c, the selected reduced frequencies k_s are computed by

$$k_s = \frac{\pi f_s c}{U} \tag{29}$$

based on a set of predefined frequencies f_s ($s = 1, 2, \cdots, n_s$), with $f_{n_s} = f_u$. The objective of such strategy is to select reduced frequencies only within the range of interest, where the upper limit is the maximum frequency of eigenvalue extraction.

Two examples are now given in which the methodology being described in the paper was applied to the preliminary phase of loads calculation of the EMB-145 jet aircraft, according to the design envelope and mission analysis criteria.

3.1 Design Envelope

To compute incremental dynamic loads according to the design envelope criterion, the four points of the flight envelope showed in Table 1 were selected. The mass configuration selected has 17100 Kg and corresponds to the MZFW (Maximum Zero Fuel Weight). The upper limit of the integrals used to compute $\bar{A}$ was set to 50 Hz (Condition F50).

Table 1: Flight Condition Nomenclature

Flight Condition	Altitude (m)	True Airspeed (m/s)	Dynamic Pressure (N/m²)	Mach Number
CH3	3048.	189.2	16191.5	0.576
CH4	6096.	218.8	15623.3	0.692
DH3	3048.	220.9	22071.7	0.673
DH4	6096.	254.0	21054.6	0.804

For each selected point in the flight envelope, two sets of MKAERO1 cards were used: the first set with 13 reduced frequencies (Condition Y) and a second set with 26 reduced frequencies (Condition V).

The loads computed according to the design envelope criterion are presented in Table 2 which gives the incremental vertical force and bending moment for a section on the fuselage and another on the wing.

Table 2: Incremental Loads due to Continuous Gust (Design Envelope)

Flight Condition	Fuselage Section		Wing Section	
	FZ (N)	MY(N*m)	FZ (N)	MX(N*m)
VCH3F50	7.3049E+4	3.1470E+5	7.4352E+4	4.2360E+5
YCH3F50	7.3052E+4	3.1455E+5	7.4370E+4	4.2370E+5
VCH4F50	7.0415E+4	3.0418E+5	7.2850E+4	4.1724E+5
YCH4F50	7.0459E+4	3.0430E+5	7.2880E+4	4.1742E+5
VDH3F50	4.2420E+4	1.8245E+5	4.3464E+4	2.4774E+5
YDH3F50	4.2429E+4	1.8251E+5	4.3475E+4	2.4782E+5
VDH4F50	4.1411E+4	1.7935E+5	4.2782E+4	2.4503E+5
YDH4F50	4.1727E+4	1.8033E+5	4.3531E+4	2.4969E+5

Figure 5 gives the PSD function of the vertical force on the fuselage, and Figure 6 shows the convergence of the corresponding $\bar{A}$ parameter as function of the integrals in Relation (3). Figures 7 and 8 show the PSD function of vertical force and the convergence of $\bar{A}$, respectively, for the station on the wing.

Figure 9 and 10 give the components of the modal amplitude vector $\xi(f)$ for conditions YCH3F50 and VCH3F50 corresponding, respectively, to a fuselage bending mode of 5.247 Hz and to a wing bending mode of 6.183 Hz.
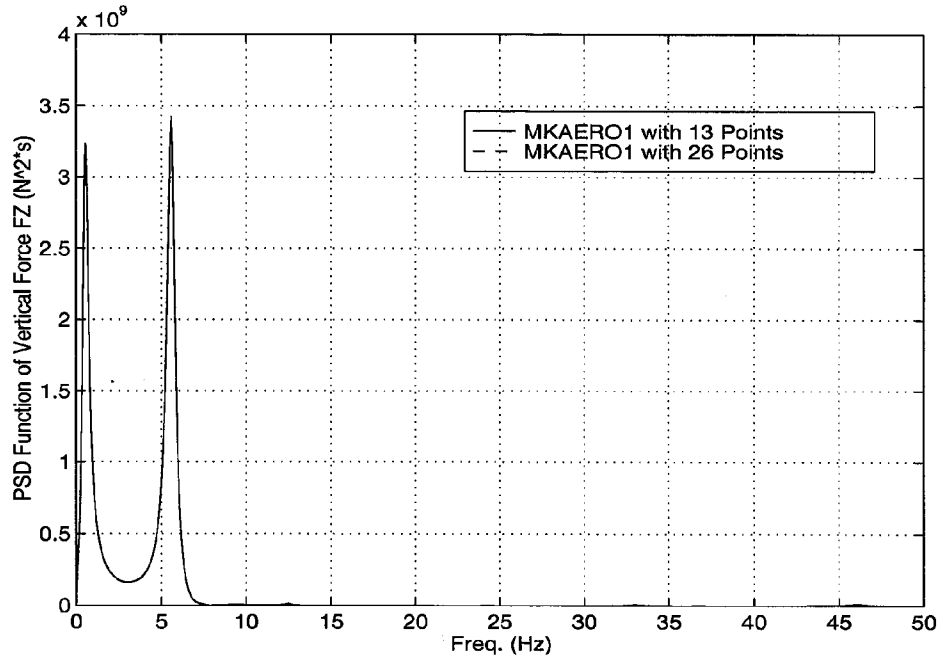
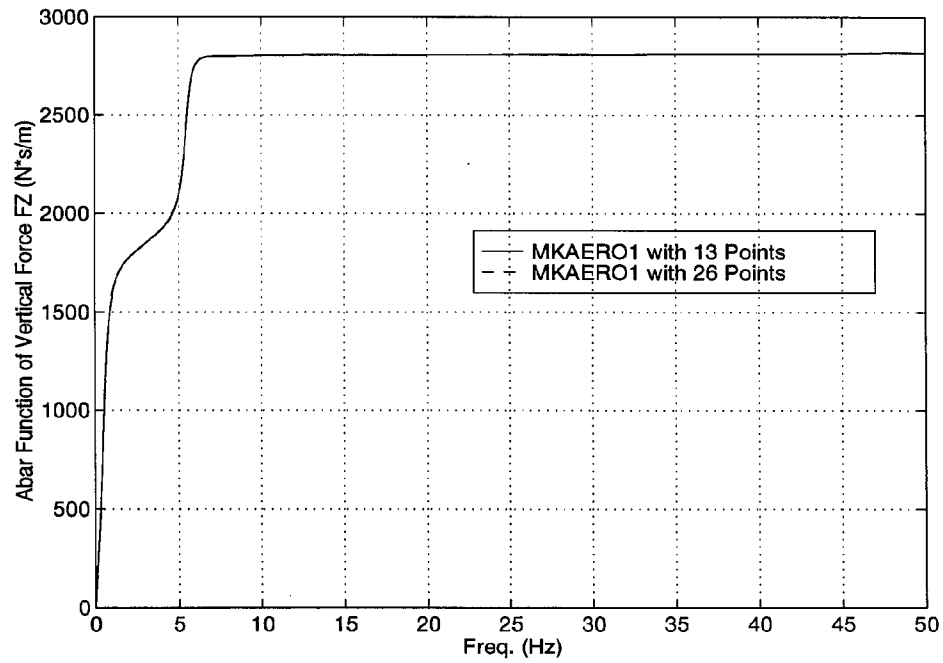Figure 5: PSD at Fuselage Station - Condition VCH3F50 / FZ

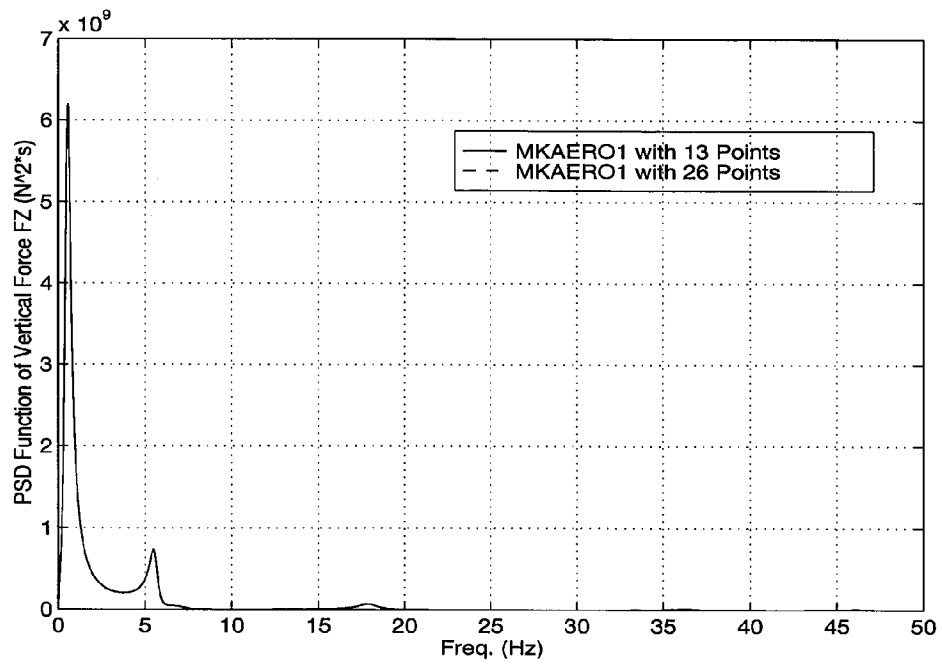Figure 6: $\bar{A}$ at Fuselage Station - Condition VCH3F50 / FZ

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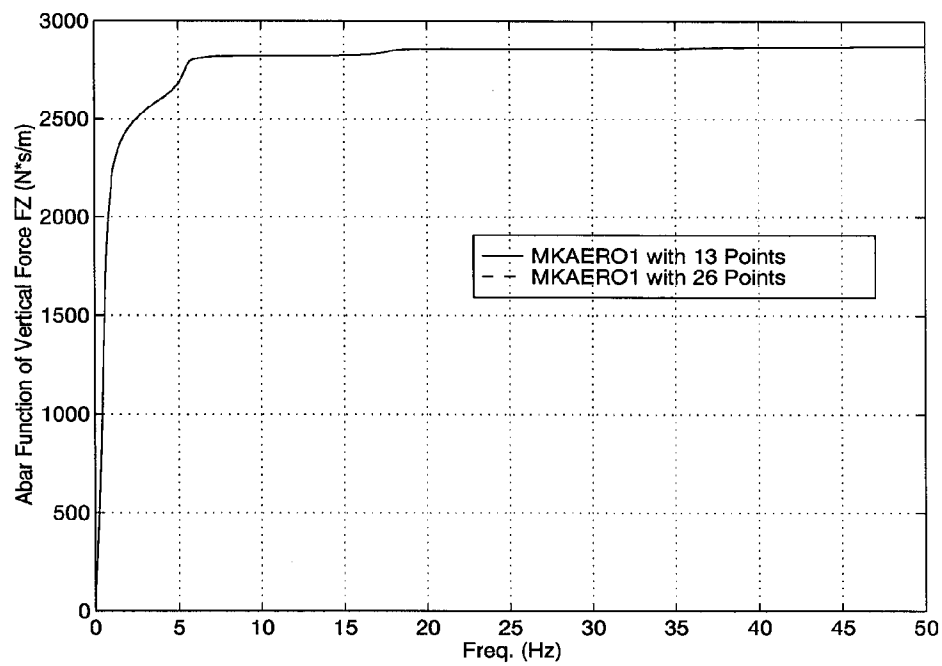Figure 7: PSD at Wing Station - Condition VCH3F50 / FZ

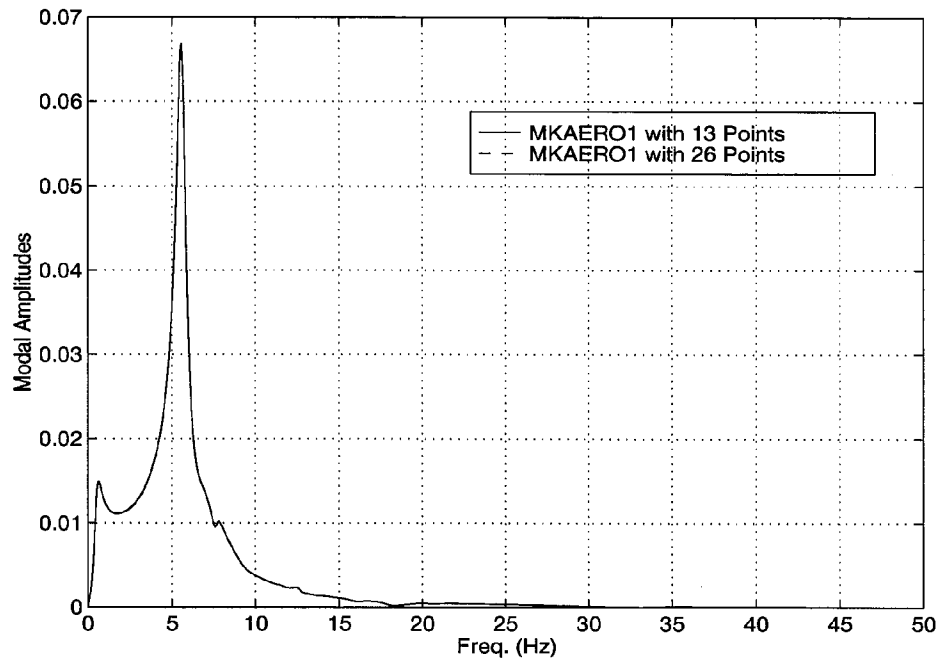Figure 8: $\bar{A}$ at Wing Station - Condition VCH3F50 / FZ

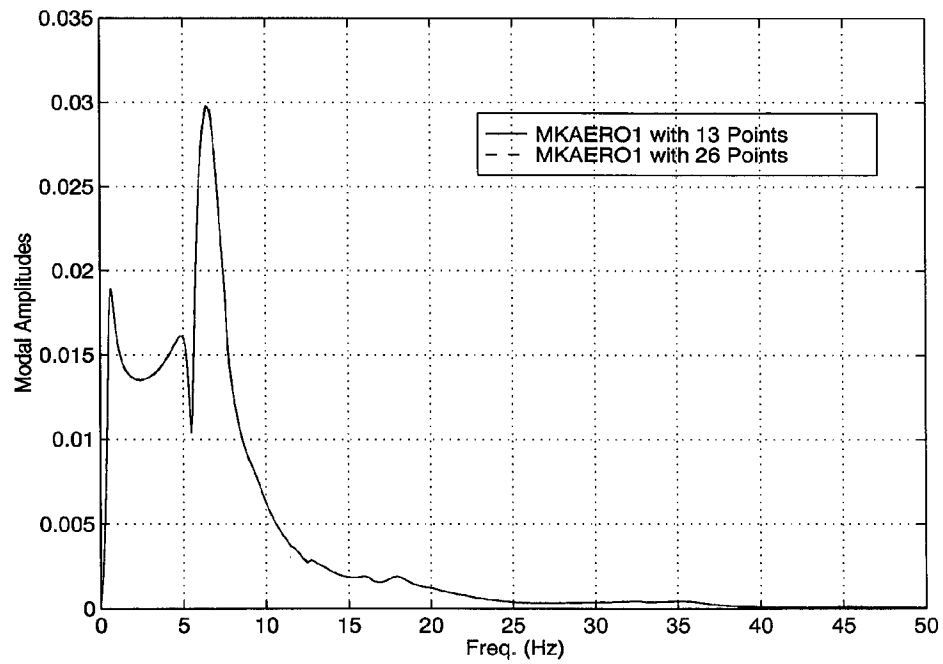Figure 9: Modal Amplitudes for Fuselage Bending Mode of 5.247 Hz

Figure 10: Modal Amplitudes for Wing Bending Mode of 6.183 Hz

3.2 Mission Analysis

The mission analysis criterion requires the definition of a mission profile divided into segments that are representative of the aircraft usage. The mission profile selected for the EMB-145 jet during the preliminary loads calculation is shown in Table 3.

The program CGUST was used to compute the quantities $\bar{A}(j)$ and $N_0(j)$ for each segment j, $(j = 1, 2, \cdots, 14)$. By knowing the time fraction $t(j)$ and altitude $h(j)$ of each segment j given on the third and fourth columns of Table 3, respectively, and setting the one-g loads $y_{1-g}(j) = 0$, Equation (5) is used to compute the frequency of exceedance curves $N(y)$ corresponding to the incremental vertical forces for stations on the fuselage, given in Figure 11, and on the wing, given in Figure 12.

The upper limits of the integrals in Relation (3) were determined using Houbolt's criterion [6] which states that the upper limit to be chosen, called the cutoff frequency, is taken as the frequency in which the corresponding $\bar{A}$ has reached 98% of its converged value.

Table 3: Summary of Lumped Flight Segments for Mission Analysis

NO.	SEGMENT DESCRIPTION	TIME (%)	ALTITUDE (m)	TAS (m/s)	WING FUEL (Kg)	AIRCRAFT MASS (Kg)
1	CLIMB-1	8.20	3048.	119.2	1745.	17260.
2	CLIMB-2	9.56	6096.	135.8	1745.	17260.
3	CLIMB-3	5.36	9144.	152.2	2645.	18160.
4	CRUISE-1	13.74	7925.	218.7	975.	16490.
5	CRUISE-2	29.22	10668.	221.1	1745.	17260.
6	CRUISE-3	6.15	11278.	210.2	2645.	18160.
7	CRUISE-4	9.15	11278.	210.2	1745.	17260.
8	CRUISE-5	0.32	6096.	210.8	965.	12910.
9	CRUISE-6	0.29	1524.	138.5	965.	12910.
10	DESCENT-1	1.84	9144.	236.4	975.	16490.
11	DESCENT-2	6.36	6096.	201.9	975.	16490.
12	DESCENT-3	9.56	3048.	175.8	975.	16490.
13	DESCENT-4	0.18	3048.	183.6	965.	12910.
14	DESCENT-5	0.07	762.	133.4	965.	12910.

From Figure 11 one can get the incremental vertical force y_{fuselage} on the fuselage station corresponding to $N(y_{\text{fuselage}}) = 2 \times 10^{-5}$ cycles/hour as

$$y_{\text{fuselage}} = 66373 \text{ N},$$

and from Figure 12 one can get the incremental vertical force y_{wing} on the wing station corresponding to $N(y_{\text{wing}}) = 2 \times 10^{-5}$ cycles/hour as
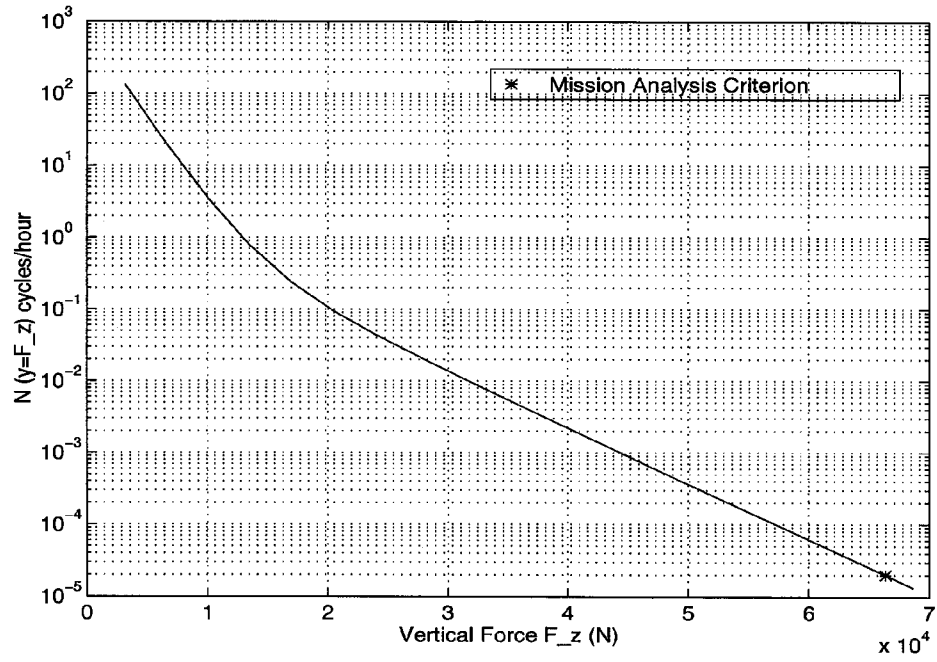
$$y_{\text{wing}} = 89221 \text{ N}.$$

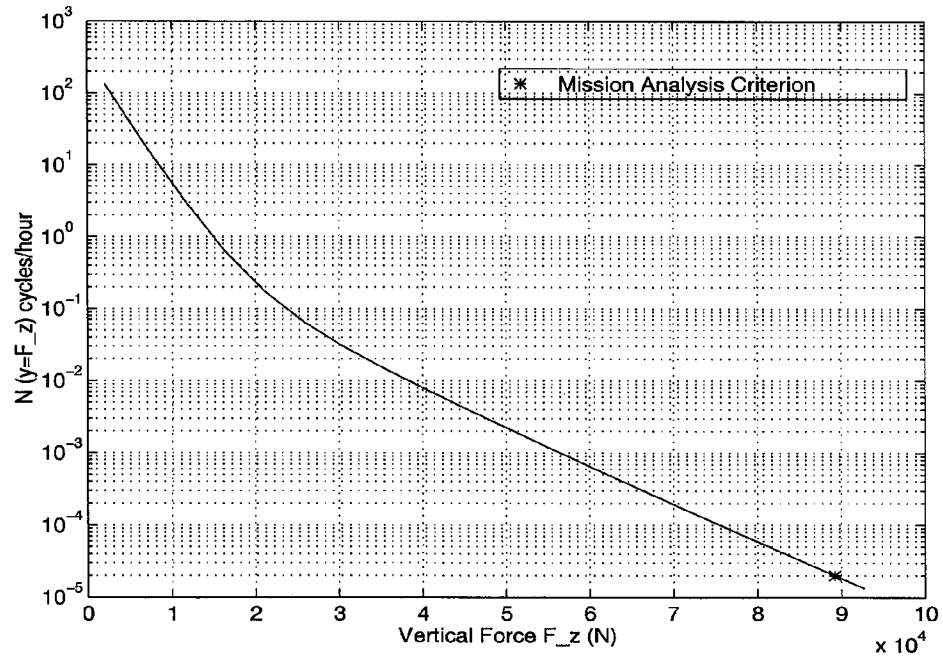Figure 11: Frequency of Exceedance for the Fuselage Station

Figure 12: Frequency of Exceedance for the Wing Station

4 Analysis

Table 2 shows that for conditions CH3, CH4, DH3, and DH4 listed in Table 1 the use of MKAERO1 cards with 13 (condition Y) or 26 (condition V) reduced frequencies leads to small differences between the loads computed according to the design envelope criterion. Such evidence can be confirmed for condition CH3F50 in Figures 5 to 10 where the curves corresponding to condition Y are right on the top of the curves corresponding to condition V.

Nevertheless, there were few cases in which the use of different discretizations of reduced frequencies in the MKAERO1 cards would lead to different loads. When such behavior was detected the technical support team of MSC/NASTRAN was contacted by EMBRAER, and the explanation is given next.

As described in the MSC/NASTRAN Aeroelastic Analysis User's Guide [5], the aerodynamic matrix Q_{hh} at reduced frequency k_{est} using the specialized linear interpolation method is given by

$$Q_{hh}(k_{est}) = \sum_{j=1}^{n_s} C(j) \left[Q_{hh}^{re}(k_j) + \frac{I}{k_j} Q_{hh}^{im}(k_j) \right], \tag{30}$$

where k_j $(j = 1, 2, \cdots, n_s)$ are the reduced frequencies selected in the MKAERO1 cards and $C(j)$ is the jth component of the weighting vector $\mathbf{C}$ given by

$$\mathbf{C} = \mathbf{A}^{-1} \mathbf{B}, \tag{31}$$

with the matrix $\mathbf{A}$ and the vector $\mathbf{B}$ defined by

$$A(i,j) = \left\{ \begin{array}{ll} |k_i - k_j|^3 + |k_i + k_j|^3 & \text{for} \quad i \text{ and } j \leq n_s \\ 0 & \text{for} \quad i = j = n_s + 1 \\ 1 & \text{for} \quad i = n_s + 1 \text{ or} \\ & \quad j = n_s + 1, \text{ with } i \neq j \end{array} \right. \quad \text{and}$$

$$B(j) = \left\{ \begin{array}{ll} |k_{est} - k_j|^3 + |k_{est} + k_j|^3 & \text{for} \quad j \leq n_s \\ 1 & \text{for} \quad j = n_s + 1. \end{array} \right.$$

It can be observed that if the selected frequencies are given in ascending order, the structure of matrix $\mathbf{A}$ is such that it will have small values on the upper left corner and large values on the lower right corner. For example, the set of reduced frequencies $[\, k_1 = 0.001 \quad k_2 = 0.010 \quad k_3 = 0.100 \quad k_4 = 1.000 \quad k_5 = 10.00 \,]$ generates an ill-conditioned $\mathbf{A}$ matrix given by

$$\mathbf{A} = \begin{bmatrix} 8.0000e{-}9 & 2.0600e{-}6 & 2.0006e{-}3 & 2.0000e{+}0 & 2.0000e{+}3 & 1.0000e{+}0 \\ 2.0600e{-}6 & 8.0000e{-}6 & 2.0600e{-}3 & 2.0006e{+}0 & 2.0000e{+}3 & 1.0000e{+}0 \\ 2.0006e{-}3 & 2.0600e{-}3 & 8.0000e{-}3 & 2.0600e{+}0 & 2.0006e{+}3 & 1.0000e{+}0 \\ 2.0000e{+}0 & 2.0006e{+}0 & 2.0600e{+}0 & 8.0000e{+}0 & 2.0600e{+}3 & 1.0000e{+}0 \\ 2.0000e{+}3 & 2.0000e{+}3 & 2.0006e{+}3 & 2.0600e{+}3 & 8.0000e{+}3 & 1.0000e{+}0 \\ 1.0000e{+}0 & 1.0000e{+}0 & 1.0000e{+}0 & 1.0000e{+}0 & 1.0000e{+}0 & 0 \end{bmatrix} .$$

In such case, the inversion of matrix $\mathbf{A}$ could generate a poor inverse depending on the machine precision where MSC/NASTRAN is implemented, and the computation of the interpolation coefficients in vector $\mathbf{C}$ could be compromised.

In CGUST a subroutine was added to compute the matrix $\mathbf{A}$ mentioned above for a given set of reduced frequencies. The inverse $\mathbf{A}^{-1}$ is computed in single precision, and the product $\mathbf{A}\mathbf{A}^{-1}$ is compared to the identity matrix $\mathbf{I}$. If the norm of the matrix $(\mathbf{A}\mathbf{A}^{-1}-\mathbf{I})$ is above a selected value, then a warning message is issued. The use of such subroutine is useful for avoiding bad selections of reduced frequencies on the MKAERO1 cards. Once a good selection of reduced frequencies has been made, the envelope of loads obtained by using the design envelope and mission analysis criteria shows compatible results with other existing criteria (e.g., the static gust criterion, and the tuned discrete gust criterion).

5 Conclusions

A procedure for computing dynamic loads on aircraft structures caused by atmospheric continuous gusts in the atmosphere using MSC/NASTRAN has been presented. The procedure uses the normal modes solution (SOL103) to obtain the eigenvalues and eigenvectors of the dynamic system and the aeroelastic solution (SOL146) to obtain the modal amplitudes for generating the dynamic system frequency response function.

Two examples have been presented in the computation of dynamic loads on a modern jet aircraft according to the design envelope and the mission analysis criteria.

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