Adjoint Sensitivity Analysis in MSC/NASTRAN

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Abstract

The Adjoint Method for sensitivity analysis can sometimes produce sensitivities at a fraction of the computer resources required by the Direct Method. This paper presents the motivation, theory, implementation and selected results from installing this technique in Version 70 of MSC/NASTRAN. The application of the method to large scale design tasks is seen to ‘‘enable’’ the practical solution to design tasks driven by NVH (noise, vibration and harshness) considerations. Concluding comments summarize the results and discuss possible further developments.

1.0 Introduction

Structural Design Optimization has been an integral feature of MSC / NASTRAN since Version 66 was released in 1989. Reference 1 provides a useful overview of the general area of design optimization and its implementation in MSC / NASTRAN. Design Optimization in MSC / NASTRAN is a gradient based procedure, with the gradients obtained in a "direct" method, as detailed below. It has long been recognized that the alternative "adjoint" is preferable in certain situations. Reference 2 is an early paper that contains a description of the two methods with what is referred to here as the Adjoint Method labeled the Virtual Load method and the Direct Method labeled the Design Space method.

This paper briefly provides the mathematical basis for the method and then motivates why it is sometimes useful. The bulk of the paper shows examples of the performance improvements and the concluding comments discuss possible further developments.

2.0 Basic Theory

A brief theoretical description of the adjoint method is given here. Since the primary application of the method is to frequency response tasks, a frequency response formulation is used. No distinction is made between direct and modal frequency analysis. Instead, a sketch of the method is presented and it should be clear that either frequency analysis method could be used for the underlying calculations. The description given here is adapted from Reference 3, where comparable equations from a static analysis are presented.

The standard equation for frequency response analysis is:

$$[-\omega^2 M + i\omega B + K]\{u\} = \{P\} \qquad \text{(Eq. 2-1)}$$

If the response for which sensitivity is desired is expressed as a function of the solution vector

$$r = f(u) \qquad \text{(Eq. 2-2)}$$

then the sensitivity of the response with respect to a design variable can be written as:

$$dr/dx = \{\delta f/\delta u\}^T\{\delta u/\delta x\} \qquad \text{(Eq. 2-3)}$$

As documented in Reference 1, MSC/NASTRAN makes the simplifying assumption for frequency sensitivity that the load vector {P} of (Eq. 2-1) is not a function of the design variable. In this case, the $\{\delta u/\delta x\}$ term can be obtained from

$$[-\omega^2 M + i\omega B + K]\{\delta u/\delta x\} = -[-\omega^2 \delta M/\delta x + i\omega\delta B/\delta x + \delta K/\delta x]\{u\} \quad \text{(Eq. 2-4)}$$

The Direct Method of sensitivity analysis requires the solution of (Eq. 2-4) for each design variable at each frequency of interest. The adjoint method avoids this by first solving for an adjoint expression of the form

$$[-\omega^2 M + i\omega B + K]\{\lambda\} = \{\delta f/\delta u\} \quad \text{(Eq. 2-5)}$$

If the $[-\omega^2 M + i\omega B + K]$ matrix is designated [FAC] and it is symmetric, then $[FAC]^{-1}[FAC] = [I]$ and (Eq. 2-3), (Eq. 2-4) and (Eq. 2-5) can be combined to give:

$$dr/dx = \{\lambda\}^T[-\omega^2 \delta M/\delta x + i\omega\delta B/\delta x + \delta K/\delta x]\{u\} \quad \text{(Eq. 2-6)}$$

Equation 2-6 represents the essence of the Adjoint Method. Its utility is discussed in the following section. This section is completed by touching on some of the assumptions that have been made. The first is that the applied loading is invariant with respect to the design variables. This is a limitation in dynamic response optimization in MSC/NASTRAN and must be kept in mind when using the procedure. Thermal and gravity loads are examples where this assumption breaks down, as is the case of loads that are a function of geometry in a shape optimization task.

Another assumption is that the FAC matrix is symmetric. A project that was performed in parallel with the Adjoint Sensitivity Method project was to "symmetrize" the acoustic analysis. The assumption on symmetry is therefore valid for the usual case, but users with special problems should again be aware of the limitation. Both of these limitations can be removed with additional development effort and do not invalidate the basic techniques developed here.

3.0 Motivation

The basic motivation for the new method is a desire for increased speed in the sensitivity analysis and a reduction in disk space. As mentioned, the Direct Method of (Eq. 2-4) requires the computation of pseudo load vectors equal in number to the number of load cases (nlc) multiplied by the number of design variables (ndv). In a statics analysis, the number of load cases is equal to the number of subcases; in a frequency response analysis, the number of load cases is equal to the number of subcases multiplied by the number of frequencies per subcase. Once the load vectors are obtained, they require solution for the perturbed displacements and then data recovery to extract the sensitivity information.

In contrast, the Adjoint Method of (Eq. 2-6) requires the computation of adjoint vectors equal in number to the number of retained responses (NRESP) in the optimization task. In its simplest form, the adjoint method requires fewer operations whenever

$$nresp < ndv \times nlc \qquad \text{(Eq. 3-1)}$$

The adjoint method and the above inequality have long been recognized. The original choice of the direct method for implementation in MSC / NASTRAN was correctly made on the assumption that there are typically many active responses per load case. This is not always true, however, and there are cases where *nresp* is much less than $ndv \times nlc$. In particular, NVH (noise, vibration and harshness) design studies are typically performed over a broad frequency range and often involve large number of design variables. This makes $ndv \times nlc$ (where each frequency is a load case) much larger than the handful of pressure responses that are monitored to control the sound level in the NVH design task.

Another factor that inhibits the use of the Adjoint Method is that the $\{\delta f / \delta u\}$ term required by (Eq. 2-5) is often difficult to formulate. For element responses, an exact expression is unavailable for some elements and responses. For this reason, the implementation has been limited to grid responses only.

4.0 User Interface

There are no user inputs required to use the Adjoint Method. Instead, a decision is made at the end of the constraint screening process as to whether the adjoint or the direct method is the most appropriate. A separate decision is made for statics and frequency analyses. In some cases, a dual approach is selected whereby some load cases use the Direct Method for sensitivity analysis while other subcases use the Adjoint Method.

5.0 Limitations

For the Adjoint Method to be selected it must satisfy a number of criteria before the $nresp < ndv \times nlc$ criterion is tested. The prerequisite criteria include:

- Only grid responses can utilize the adjoint method (i.e., of the TYPES's available on the DRESP1 Bulk Data entry, only DISP, FRDISP, FRVELO and FRACCL are candidates for the adjoint method). If a load case is found to have active element responses or an FRSPCF response, it is disqualified from using the adjoint method. A practical consideration in this regard is that it is unlikely that a design task that included element responses would pass the $nresp < ndv \times nlc$.

- The method is not applied in transient analysis, static aeroelastic analysis, and static analysis with thermal loads or in static analysis with inertia relief.
- The method is not available with p-element shape optimization.
- The method can be applied to superelement models only if the design model resides in the residual superelement.

6.0 Examples

Since there are no user actions required to use the method, it is not necessary to show actual test cases. Instead, this section describes one example by stating the problem addressed and comparing statistics on the performance results when using the Adjoint and Direct Methods to sensitivity analysis. A second example demonstrates the application for a very large model that could not realistically be addressed without the adjoint capability.

6.1 Visual Sensor

The finite element model shown in Figure 1 is a dual gimballed visual sensor. The sensor is gimballed about the pitch axis and the roll axis to provide for optimal tracking motion. The sensor consists of the following components: the base including gussets to increase stiffness and inner race of roll bearing; the roll housing including the outer race of roll bearing, end caps, and outer race of pitch bearing; and the pitch shaft including the inner race of pitch bearing, telescope and counterweights. In addition, a “mock” optical path of the sensor is modeled including the focusing optics, prism, and focal plane array. The optical components are assumed to be rigid.

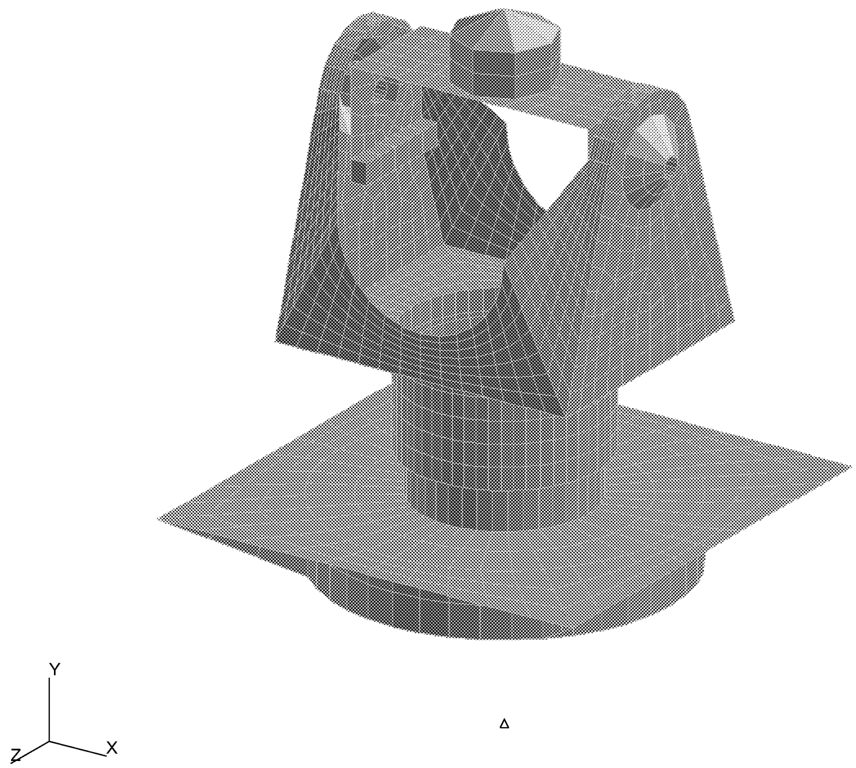

Figure 1. Visual Sensor Model.

The critical performance criteria of a sensor is its ability to track targets while excited by external loads, usually random vibration loads. The critical parameter is sensor jitter, or the motion of energy as it is passes through the sensor's optical path. Rotation and translation of the components within the optical path, i.e., the telescope or prism, affect the energy focus when it arrives at the focal plane array. If the image on the focal plane array is fuzzy, the sensor jitter is high and hence tracking capabilities are impaired. The overall motion of the optical path was accounted for by writing a multi point constraint equation (MPC) that sums up the total motion of the optical path including optical power factors.

MSC / NASTRAN Design Sensitivity and Optimization was used to minimize the transfer function between a 1 G sinusoidal acceleration applied at the base of the sensor and the over all sensor jitter. Because the response of a linear system to a gaussian white random vibration input is the magnitude of the transfer function squared multiplied by the input, this approach worked well to minimize the sensor jitter RMS response to a base input random vibration loading that is constant across the range of excitation frequencies.

The model has the following statistics:

Table 1: Visual Sensor Statistics

Number of grids:	2335
Number of elements:	2129
Number of subcases:	1
Number of excitation frequencies:	101
Number of design variables:	13
Number of responses:	91

The inequality expressed in (Eq. 3-1) clearly favors the adjoint method in this case: $91 < (13 \cdot 91)$, where only 91 of the 101 excitation frequencies are included in the rms calculation. Constraints were also imposed that the weight could not exceed 8.5 units and that the first two eigenvalues must be greater than 400,000 $(\mathrm{rad/sec})^2$ (100.66 Hz.)

Figure 2 shows the objective function history for this example and Figure 3 shows the maximum constraint history. These plots were produced using the optimization post processing capability available in MSC / PATRAN Version 7.0 (Reference 4). It is seen that the initial design is infeasible (the first two natural frequencies are 73.6 and 78.9 Hz.). MSC / NASTRAN is able to simultaneously overcome the constraint violation and reduce the objective rms response from 2.224 to 1.386 in 9 design iterations. Figure 4 displays this improvement graphically by depicting the frequency response of the initial and final design cycles. The areas under these curves can be thought of as the objective functions.

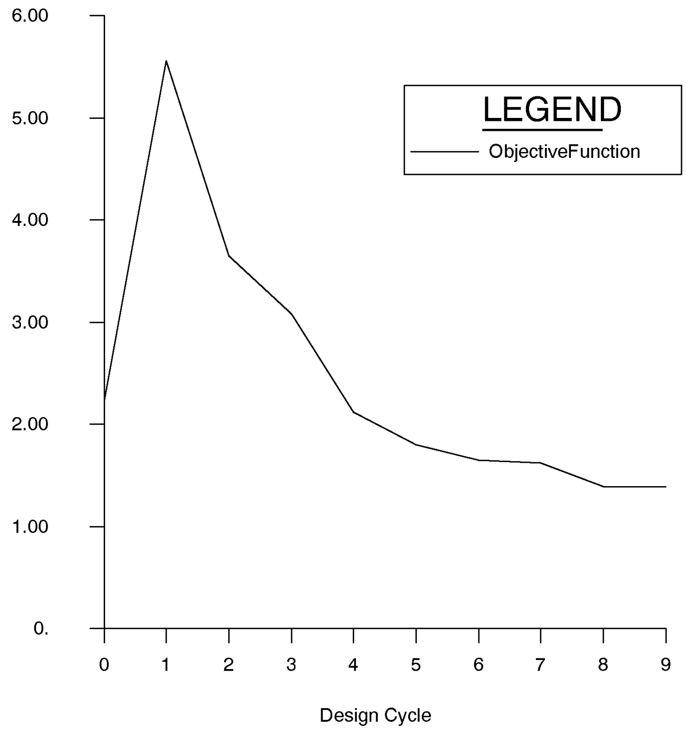

Figure 2. Objective Functions vs. Design Cycle.

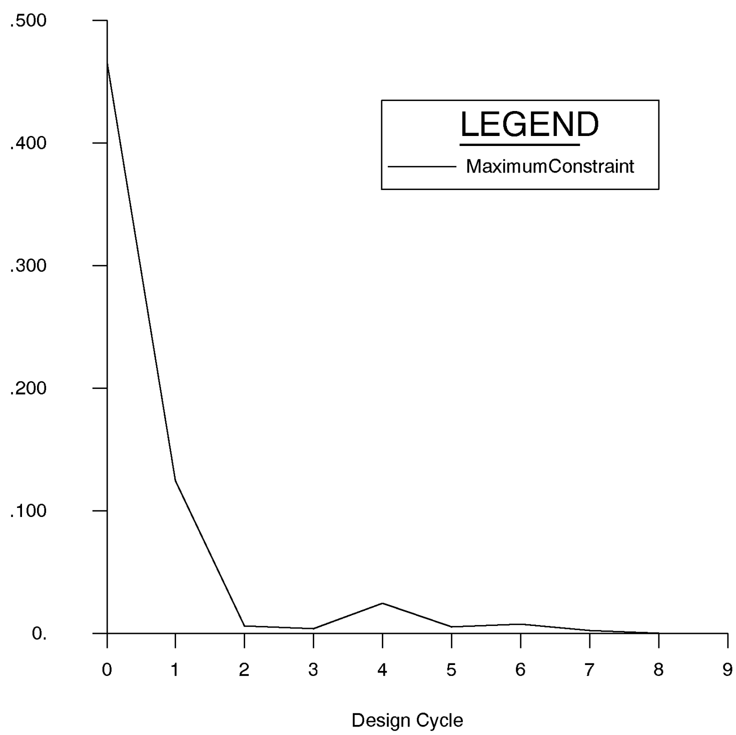

Figure 3. Maximum Constraint vs. Design Cycle.

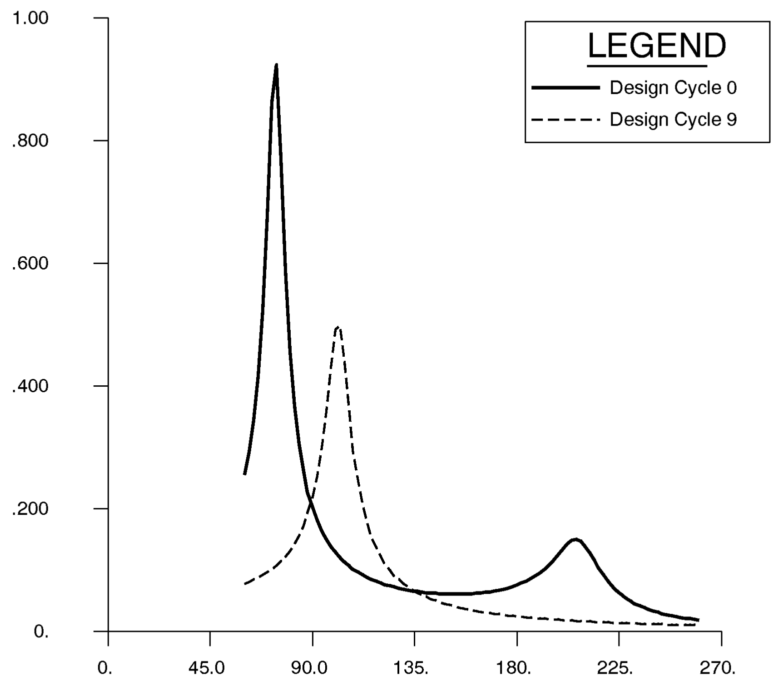

Figure 4. Frequency Response Curve for the Initial and Final Design Cycle.

The results from this example that are of the most interest here are the resources required to solve this problem in V69.1 and V70. Table 2 compares these results. The adjoint method provides a dramatic reduction in both the CPU time required and the amount of disk space used. The decrease in the final objective and the increase in the number of design cycles are not considered significant.

Table 2: Visual Sensor Results.

Parameter	V70	V69.1
CPU Time	1913.4 secs	4709.9
Scratch Space	90.5 MB	608.4
SCR 300 Space	70.4 MB	356.6
No. of Design Cycles	9	8
Final Objective	1.3862	1.559

6.2 Van Body with Frequency Response

This second example is a very large finite element model that is considered representative of a state of the art design task. The model shown in Figure 5 was provided by a client and has the following statistics.

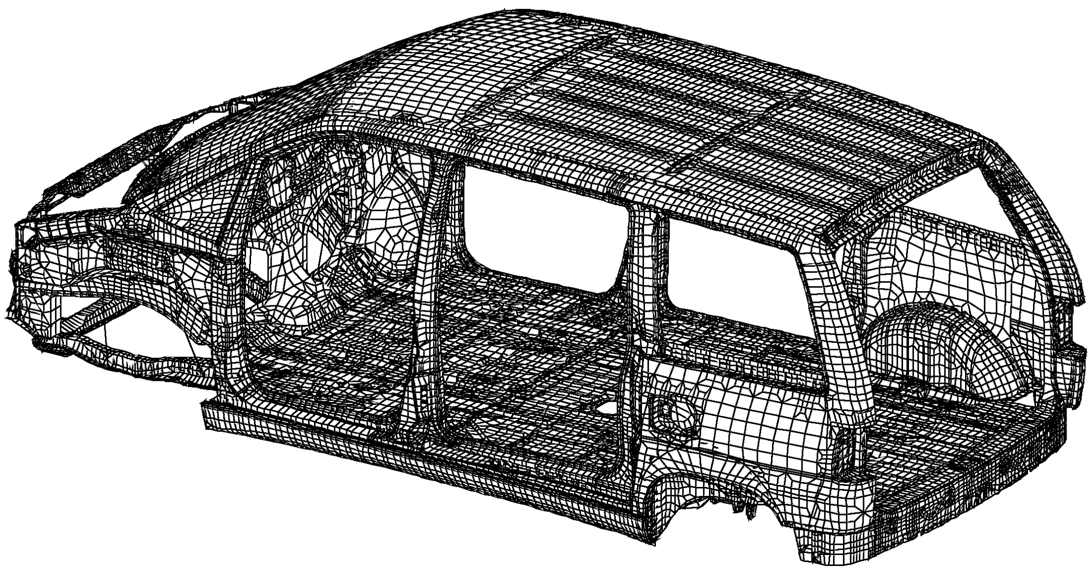

Figure 5. Van Body Model (courtesy of PSA Peugot, Citroën).

Table 3: Van Body Statistics

Number of grids:	102891
Number of elements:	91378
Number of subcases:	1
Number of frequencies:	705
Number of modes:	269
Number of design variables:	111
Number of responses:	618

The point of exercising this model was simply to demonstrate that a problem of this size could be run. It is seen that with over 617346 degrees of freedom, 705 frequencies and 111 design variables that the direct method would require 617346*705*111*16 bytes / term = 773 gigabytes for a single set of sensitivity vectors. Statistics from exercising this job on an HP Exemplar computer available at MSC are given in Table 4. It is seen that the hiwater disk usage is less than 4% of the space required by the sensitivity portion of the direct solution.

Table 4: Modal Frequency Car Body Results

Parameter	V70
CPU Time	13.82 hours
Scratch Space	23.3 GB
SCR 300 Space	5.0 GB
Hiwater Disk Usage	27.6 GB
No. of Design Cycles	1

Clearly, this problem is intractable without the availability of the adjoint method and is still only accessible to powerful computers with extensive available disk space.

7.0 Concluding Remarks

The implementation of the adjoint method is felt to be a significant enhancement to MSC's ability to perform design sensitivity and optimization. Dramatic reductions in the required CPU times and disk space are considered enabling for practical NVH optimization tasks. To date, there has been marked enthusiasm for the capability but little concrete feedback from client application of the method. It is anticipated that this feedback will be forthcoming as clients begin to utilize Version 70.

This feedback can be expected to direct what further developments are warranted. Section 5.0 lists a number of limitations to the method that could be removed if the demand was sufficient. However, with the exception of the restriction to the residual superelement, none of the limitations are deemed critical at this time.

8.0 Acknowledgments

Much of the impetus for performing this work came from a similar implementation of the Adjoint Method in Version 68X of CDH / NASTRAN. Mladen Chargin of CDH was quite helpful and forthcoming in describing his pioneering efforts. Dao Mong of MSC assisted in the coding tasks associated with the development while Mike Reymond performed a similar function for the DMAP aspects of the project. The Visual Sensor model was developed by Ron Hopkins, formerly of MSC, while the Van Body was provided by Peugeot, SA, via Alain Jacq of MSC's Paris office.

9.0 References

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