LINK MESH MODEL OF AN ELEMENTARY PANEL BAY FOR LINEAR MSC/NASTRAN/PARAMETRIC ANALYSIS OF STIFFENED SHELLS

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ABSTRACT

In light of the general theory of stiffened shells of least weight, the main chords and stiffeners are subjected to tensile and compressive stresses while the web-plates are subjected to shearing stress only.

The link mesh model incorporates ' Quad ' plate and ' Rod ' elements.

The ' Quad ' plate-element nodes, or, alternatively, the ' Beam ' element nodes of chords and stiffeners are in a 'half-wave-length-out-of-phase' configuration relative to the ' Quad ' plate-element nodes of the web. The 'out-of-phase' nodes of chords and stiffeners are connected to the web edge nodes with an array of visible ' Rod ' element links that are nearly parallel to chords and stiffeners. The fastener-like ' Rod ' elements are providing links between nodes of the web and the nodes of chords-stiffeners to diffuse web shear only. The cross-section area of each elastic 'Rod ' element link is determined by equating its axial stiffness to the combined shear stiffness of a real fastener, its bearing stiffness and also the bearing stiffness of the two fastened plates, all in series.

The main advantages of the link-mesh model of are:
[1] Inclusion of the elastic displacements of the fasteners themselves as a very significant component of the panel displacement integral, hence, improved correlation between modal analysis and measured natural frequencies.
[2] More accurate determination of the 'Load Transfer Factor' i.e. fatigue life, due to an improved read-out of the shear-flow loads applied to a fastener (even in the case of heterogene materials, different fastener diameters, or, non-uniform thickness of fastened plates).
[3] Web subjected to predominant shearing due to an exponential-like diffusion, while the axial load build-up within chords and stiffeners remains substantially linear.
[4] Simplicity required for parametric definition of complex structures.
[5] Elimination of the secondary elastic frame effect of an 'Overlaid Array of Merged-Node Quad Elements' mesh.

INTRODUCTION

The main reasons for development of the presented link mesh model were the following concerns:

[1] A relatively large number of all observed ultimate fractures and fatigue cracks emanates from locations of fasteners.
[2] Measured natural frequencies of real stiffened shells of least weight are always lower than the natural frequencies that are obtainable from the modal finite element method analysis.
[3] The historical methodology of stress substantiations of fasteners relies upon an elaborate engineering calculation of the stress level at the fastener location from the stress levels acting within its surroundings.

PROBLEM STATEMENT

According to the theory of stiffened shells the linear displacement of an elementary panel bay ABCD, in direction of the unit load vector application, is given by:

$$\Delta = \frac{q_1 * q_F * S}{G * t_W} + \sum_{k=1}^{k=m} \int_0^L \frac{F_1^k * F^k}{E * A_k} * dL$$

(References 1 and 2) . In this example m=4, i.e. there are four end-load members.
q1 and qF are shear flows due to unit and applied load respectively:
q1 = 1/h (lb/in) and qF = F/h (lb/in)
S: is the plane area of the plate (web) = p*h (in^2)
E: is the modulus of elasticity of chords and stiffeners (lb/sqin)
G: is the shear modulus of the web (lb/sqin)
F: is an externally applied load vector intensity, in this case a vertical load (lb)
F1: is an unit load that is collinear with 'F' (lb)
tw: is the web thickness (in)
Ak: are cross-sectional areas of the end-load members from k = 1 to k = 4

However, this relationship does not include very significant effects due to fastener flexibility. For example an almost zero value stiffness of fasteners would cause nearly infinite displacement.

ANALYSIS

The presented link mesh model incorporates a number of axially flexible 'Rod' elements connecting the web nodes to subsequent 'out-of-phase' nodes of chords and stiffeners, as shown on Page 8. The cross-section area of each elastic 'Rod' element link is determined by equating its axial stiffness (Hooke's Law) to the combined stifnesses of a real fastener.

Although Reference [4] takes into account an additional flexural component of the elastic 'S' shaped deformation of fasteners, in the case of relatively thin aero-space shell plates of the presented model, only predominant bearing and shearing stifness of fasteners and plates have been have been combined, in light of the 'Two Plates' concept of Reference [6].

Cross–Section Area of the Rod Element:

$$A_{rod} = \frac{L}{\left[\dfrac{2\,E_{ref}\left(t_{pch} + t_{pw}\right)}{3.14159\,G_b\,d^2} + \dfrac{E_{ref}\left(1 + \dfrac{E_{cb}}{E_{cch}}\right)}{E_{cb}\,t_{pch}} + \dfrac{E_{ref}\left(1 + \dfrac{E_{cb}}{E_{cw}}\right)}{E_{cb}\,t_{pw}}\right]} \qquad \left[in^2\right]$$

Reference 7

Where: *L : is the true length of the rod (link) element [in].*

Eref : is the reference modulus of the rod elememnt. For example if the largest number of fasteners is made out of Titanium, then Eref can be selected to be modulus of elasticity of Titanium [lb/in^2].

tpch : is the web attached chord flange plate thickness [in].

tpw : is the web plate thickness [in]

Gb : is the shear modulus of the fastener [lb/in^2].

Ecb : is the compression modulus of the bolt [lb/in^2].

Ecch : is the compression modulus of the chord [lb/in^2].

Ecw : is the compression modulus of the web [lb/in^2].

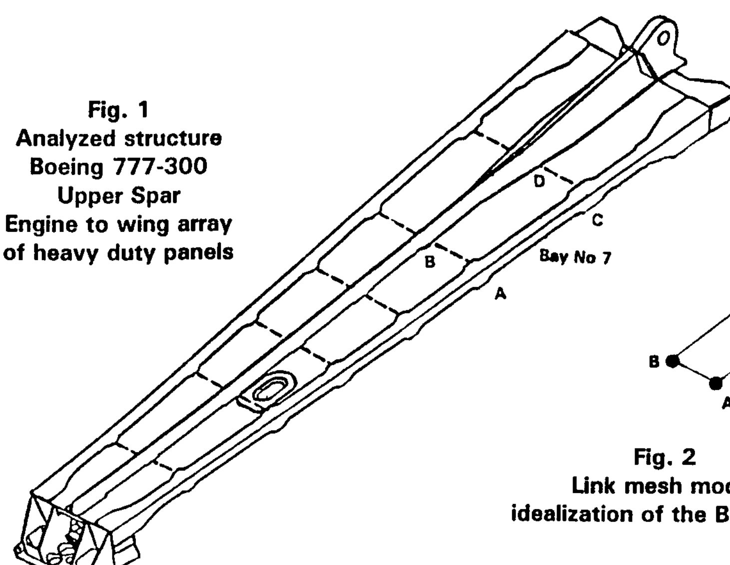


Fig. 1
Analyzed structure
Boeing 777-300
Upper Spar
Engine to wing array
of heavy duty panels

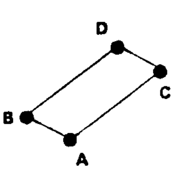


Fig. 2
Link mesh model
idealization of the Bay No 7

MSC/PATRAN MODEL DESCRIPTION

GEOMETRY

The presented elementary panel bay consists of two longitudinal 'T'shaped chords, two vertical 'L' shaped stiffeners and a web plate. The chords and stiffeners are represented with 'Quad' plate elements of uniform 2 in width. The chord 'Quad' element thickness is 0.375 in and the stiffener 'Quad' plate thickness is 0.1875 in. The web plate size is 19.00 in length along its 'pitch' and 9 in high. The web plate thickness is 0.125 in. Both chord and stiffener 'Quad' central lines are displaced by 1/2 of the web thickness (i.e. 0.0625 in) to improve visibility of the 'Rod' elements representing fasteners. The web is made out of AlAlloy 2024-T851 while chords and fasteners are made out of titanium. The fastener diameter is determined for simultaneous web-bearing and single shearing modes of failure. The calculated fastener diameter was rounded off to 0.25 in so the optimum fatigue strength spacing of fasteners =4*d=1.0 in. There are 38 fasteners (19 each row) attaching single chord to the web edge and 18 fasteners attaching stiffeners to the web, for simplicity.

MESHING

From locations of fasteners the web mapped mesh consists of 19*9 elements. The chord mesh element size is the same, only at all four corners there are 16 elements whose lengths are half of the web mesh element length, to provide the 'out-of-phase' locations of nodes, as shown on Page 8.

LOADS AND BOUNDARY CONDITIONS

The elementary bay was simply supported at 'A' (X and Y constrained) and at 'B' (X constrained). An external load acting vertically and amounting to 12,000 lb is applied at 'C'.

STRESS CONTOURS AND DISPLACEMENTS

The cross-section area of 'Rod' elements attaching chords (see equation and description of parametric variables on Page 3) was found to be: Arod-nominal=0.0128 in^2 and the cross-sectional area of 'Rod' links attaching stiffeners to the web was found to be almost the same. The panel displacement is shown on Page 9. On Page 10 the web shear stress distribution is shown that is remarkably close to the shear stress obtainable from the engineering theory of linear diffusion (St Venant, shear flow over web thickness): 12,000/9/0.125=10,666 psi. On Page 11 a local diffusion is shown together with an isometric view of fastener-like 'Rod' elements. On pages 12 and 14 a linear build-up of the chord and stiffener end loads are shown that are in rather good agreement with the general theory of shells. The calculated Arod-nominal was initially increased towards its maximum value that is equal to the entire 'welded' chord to web area. In this experiment the panel displacement was reduced towards its theoretical minimum (that can be calculated using the expression that is shown on Page 2: Delta =0.0899 in).

The fastener flexibility is then increased by reducing Arod-inadequate=0.00128 in^2, i.e. ten times smaller than its nominal value , consequently the panel displacement went up as shown on Page 18.

DEFINITION OF THE LINK MESH 'HALF-WAVE-OUT-OF-PHASE' CONFIGURATION

Fig. 3

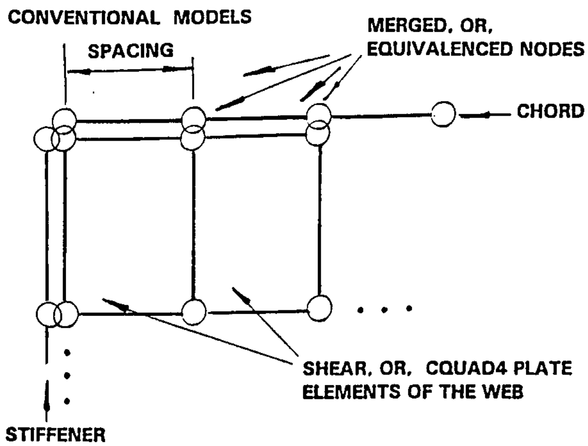
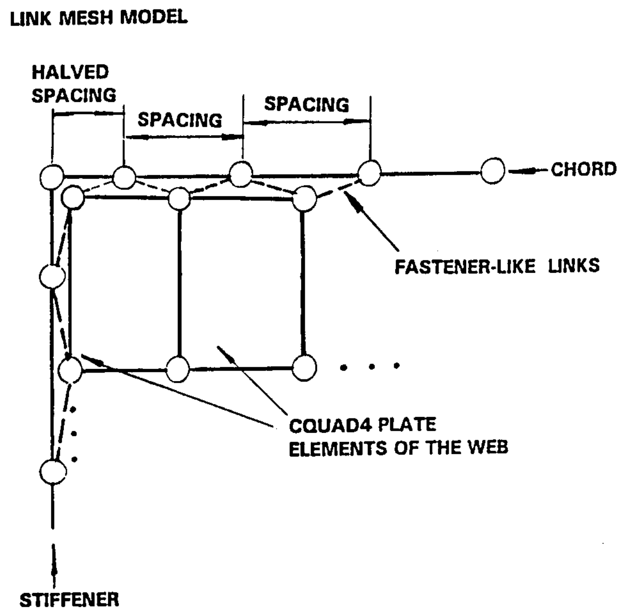


Fig. 4



The spacing of web nodes along chords, in the case of the link mesh model, is equal twice the calculated optimum distance between central lines of two subsequent fasteners when all the fasteners are lined-up within one single row, and it is equal to the optimum distance in the case of two rows of fasteners. This is due to twin links per node. Generally, the spacing of web nodes along stiffeners is somewhat greater than that along chords (Reference 1).

SUMMARY OF CONCLUSIONS

From equality between integrated strain energies of all link-rod elements; chords; stiffeners; web and the potential energy, or, work done due to an externally applied concentrated load, it was possible to conclude that the ratio between an elastic deformation of the entire panel due to elastic deformation of all fasteners, whose link-rod element cross-sectional area is reduced to its 'inadequate' magnitude (in this example 0.00128 in^2), to the elastic displacement of the panel due to elastic deformation of fasteners whose link-rod element cross-sectional area remained nominal, is proportional to reciprocals of respective cross-sectional areas.

The reference displacement 'Delta' (See page 2), consists of two component. Its first component, due elastic deformation of the web, was found to be 0.05066 in and the second, due to elastic deformation of end-load members, was found to be 0.0388 in. However due to locally increased stress levels this theoretical value, amounting to 0.08946 in was found from FEM results to be 0.1137 in (See page 9). This method was used at the Boeing Corporation during analysis of causes for fatigue failures of panels that were test-recorded at two and half design life-times.

A CONCISE DISCUSSION OF FIVE ADVANTAGES

[1] The presented method extends applicability of the engineering theory of stiffened shells to include a very significant component of the panel displacement integral that is due to an elastic deformation of fasteners themselves as well as their pin loaded holes by bearing pressures. In this example a reduced bolt diameter and decreased fastener reference modulus almost re-doubled the elastic deformation of the entire panel, consequently, its natural frequency would be reduced by 41 percent.

[2] The load transfer factor can be more accurately determined from the individual bearing stress of fasteners to locally increased chord tensile stress ratio.

[3] The exponential-like diffusion indicates regions of an elevated web shear stress.

[4] Adjacent panel bays can be joined at corner nodes A,B,C and D.

[5] This method eliminates the secondary 'Elastic Frame' effect. According to Reference 1, this secondary effect should be excluded from the static strength substantiation.

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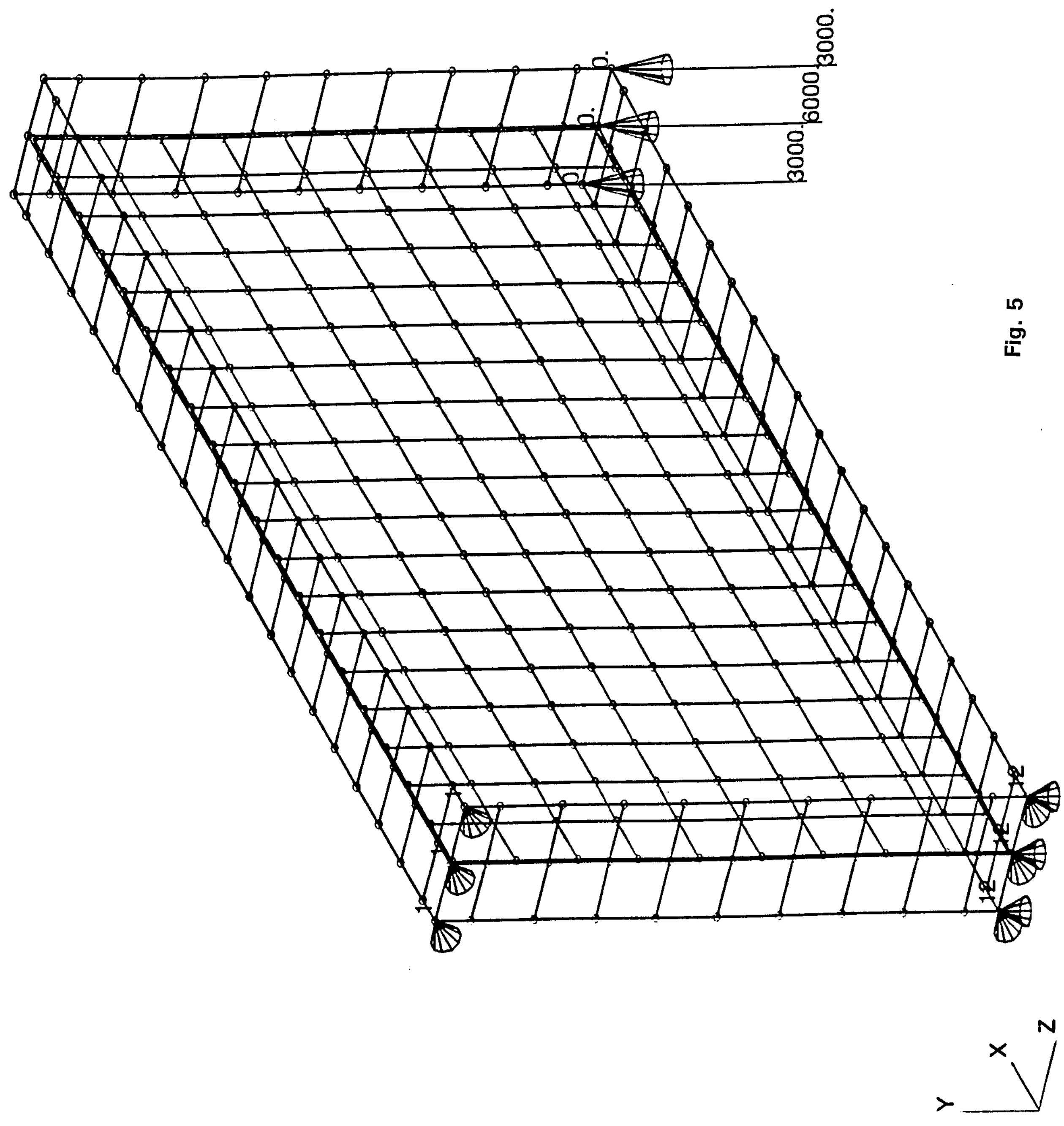

Fig. 5

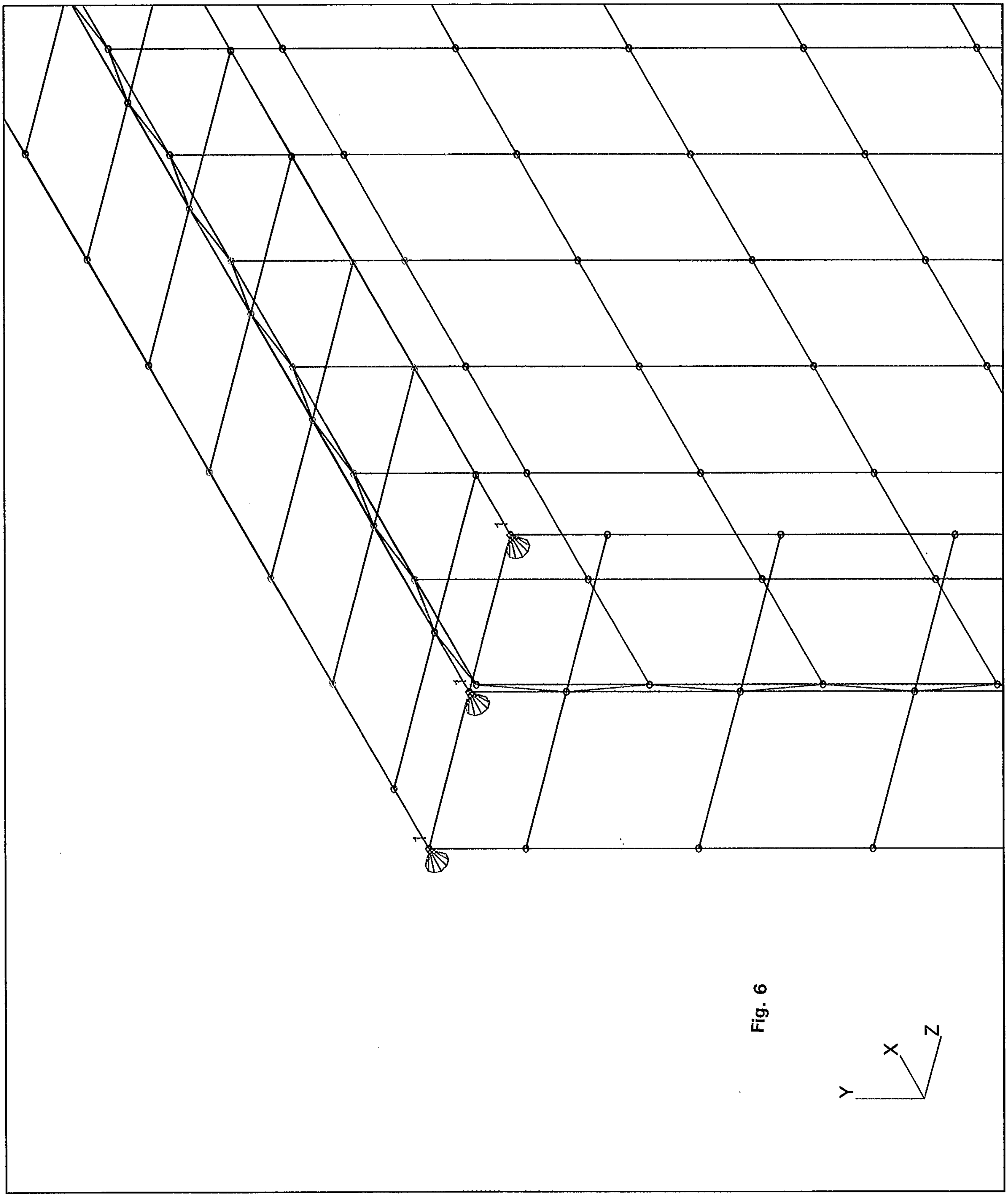

Fig. 6

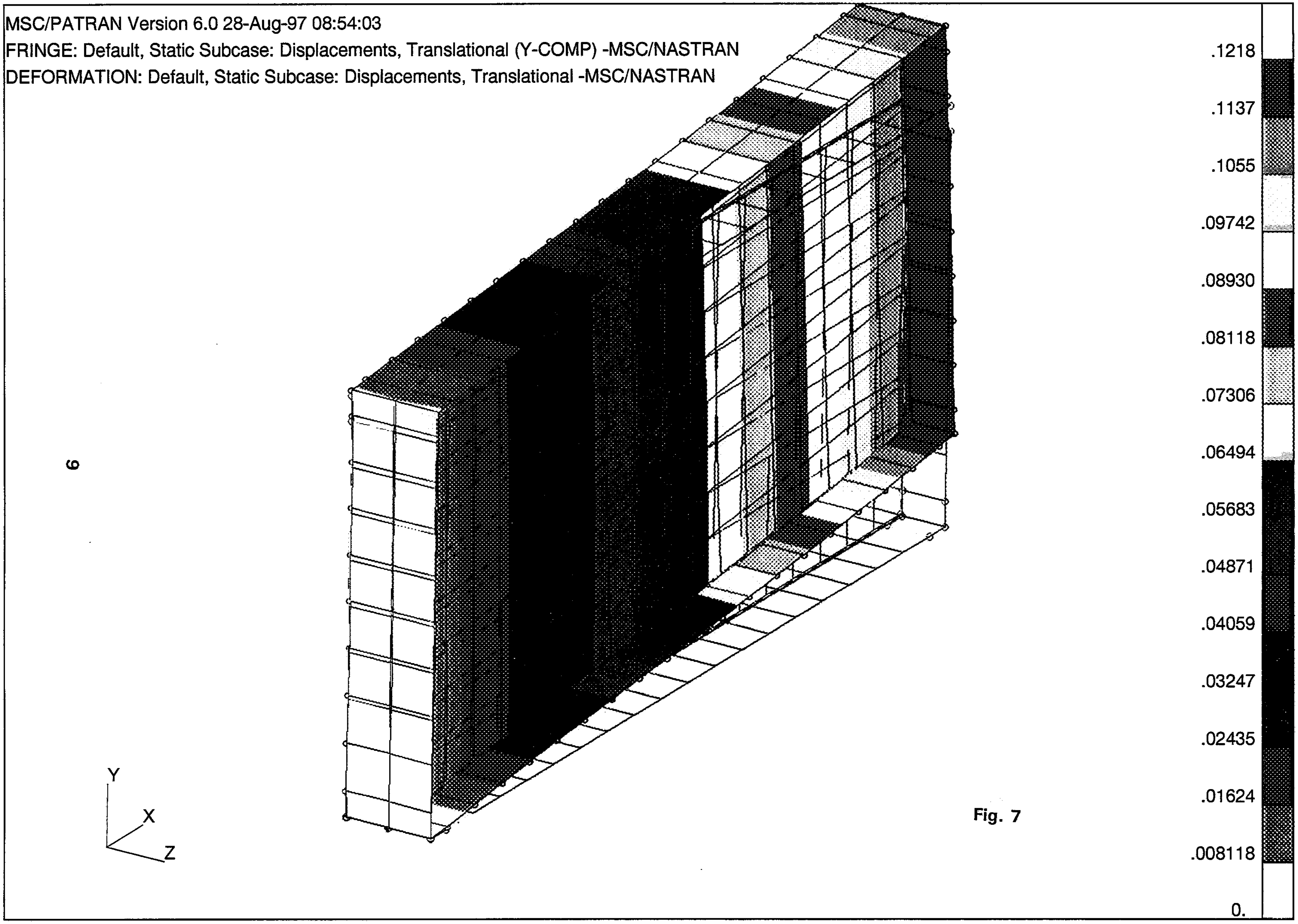
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Fig. 7

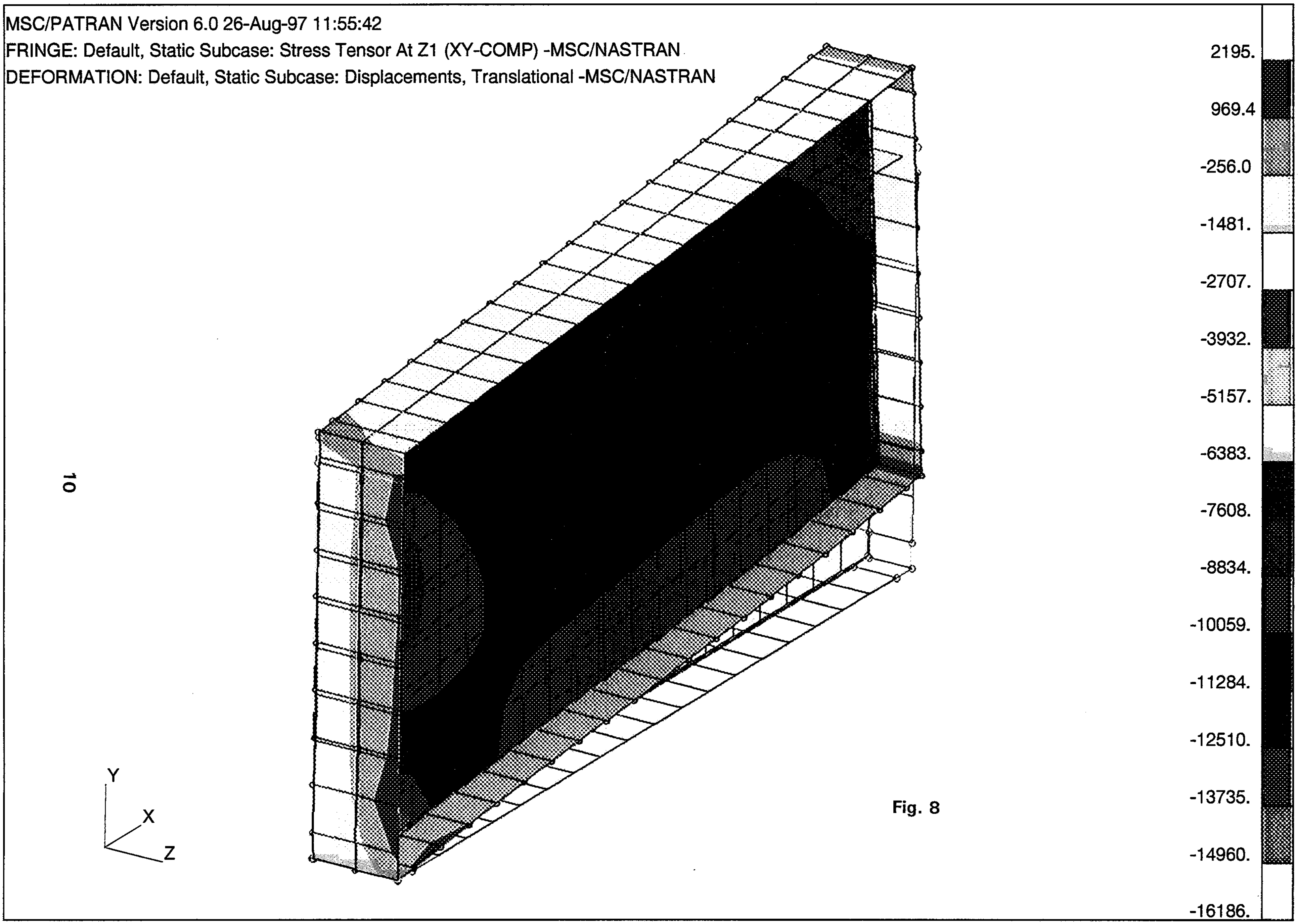
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Fig. 8

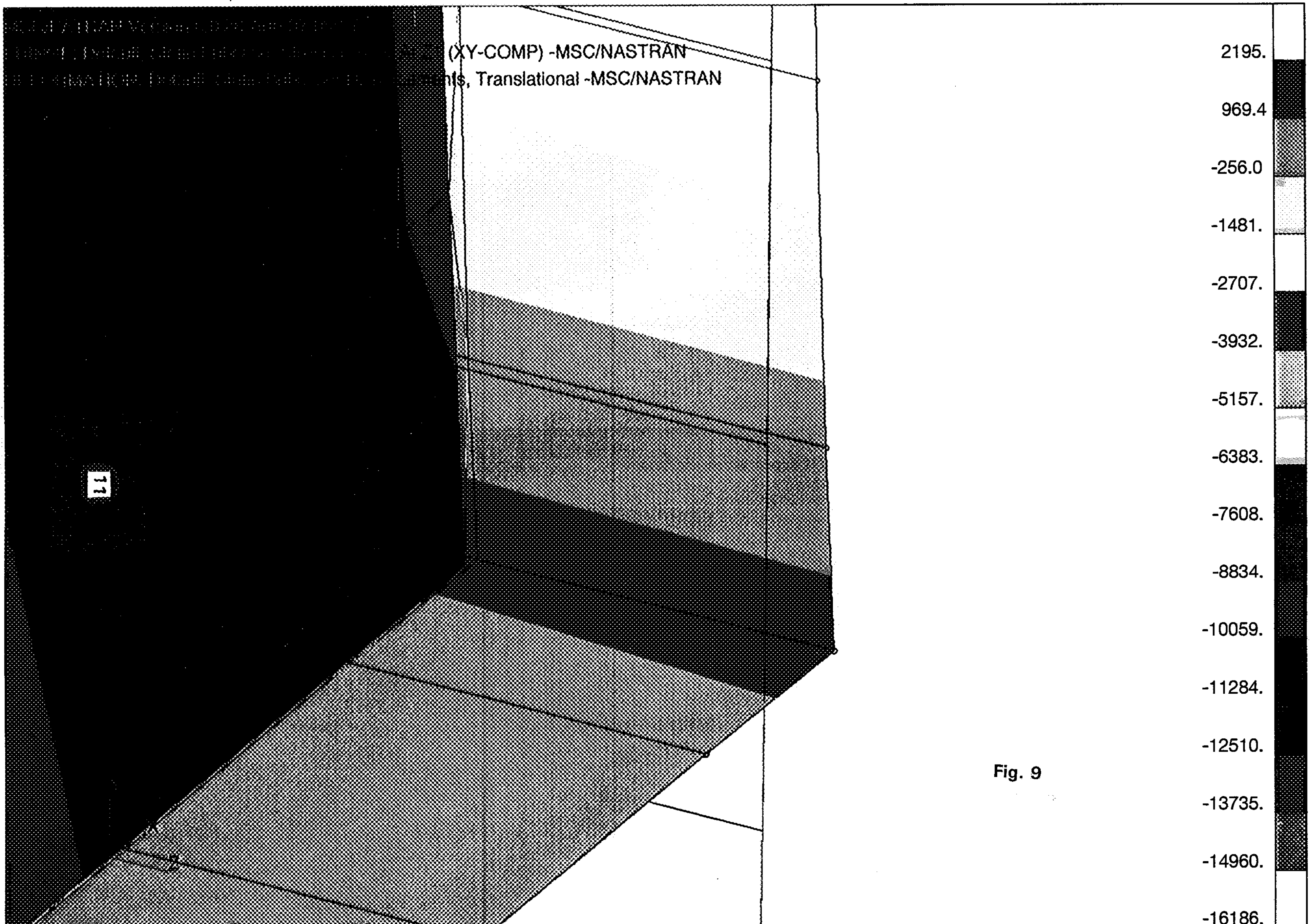
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Fig. 9

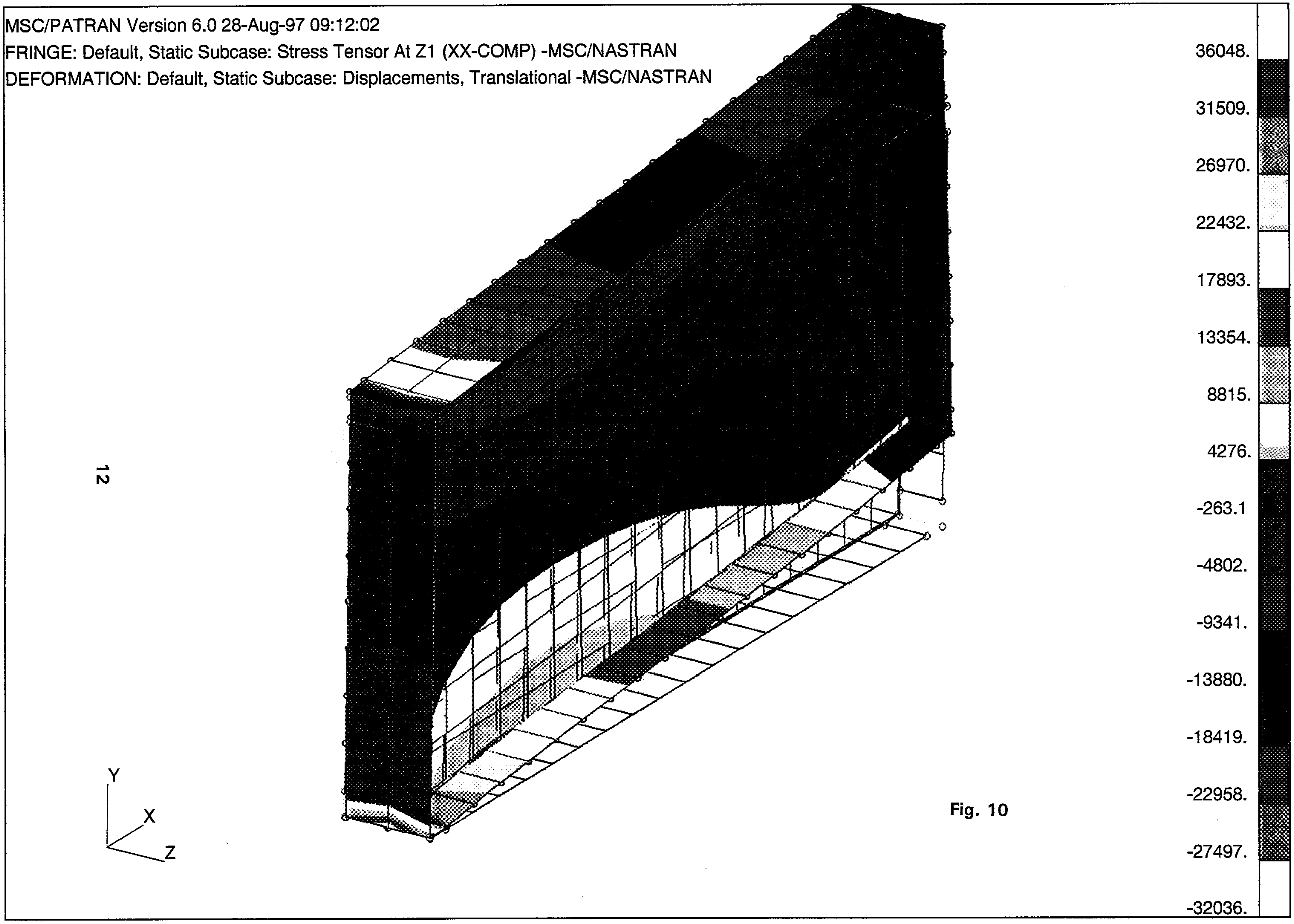
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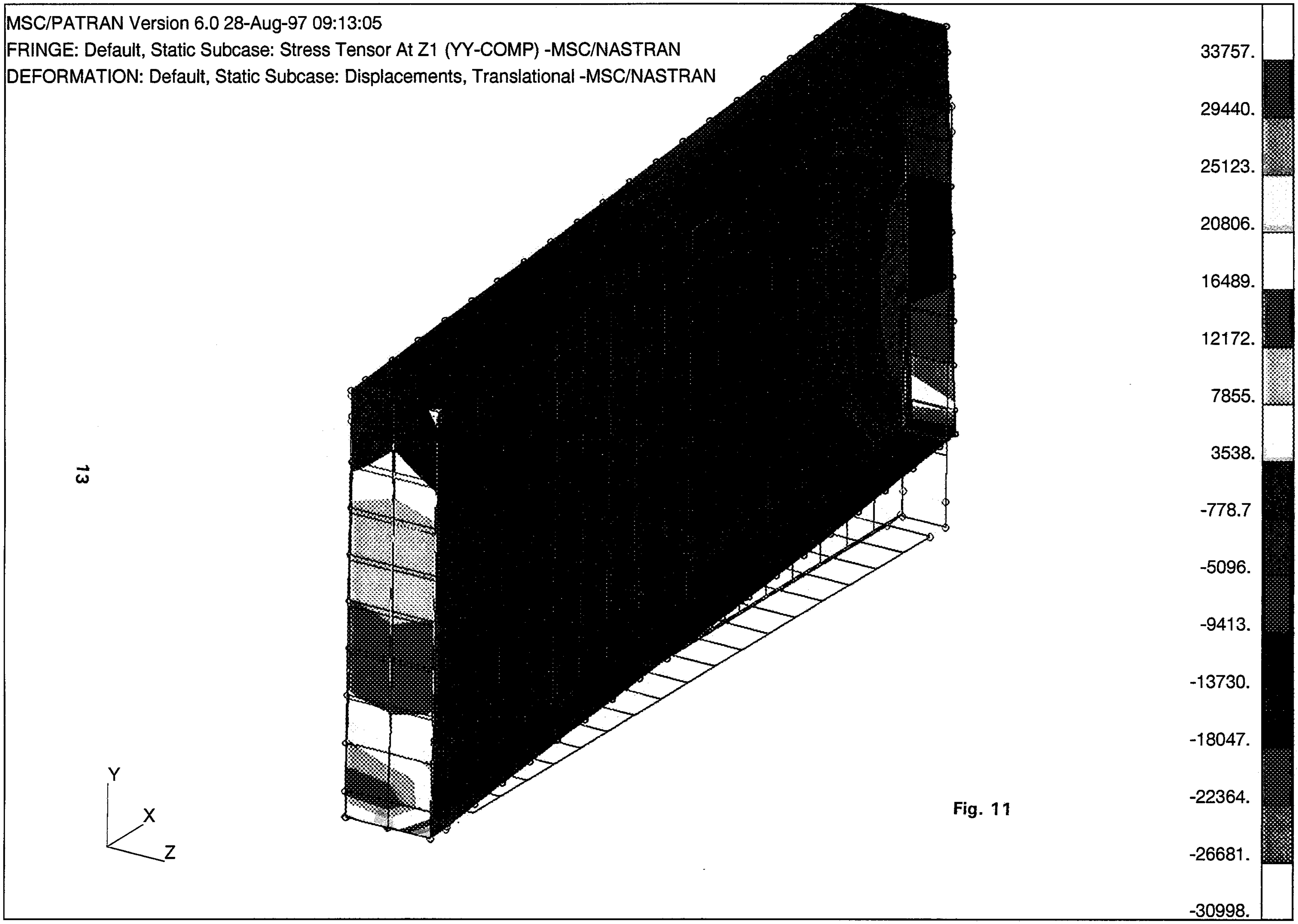
Fig. 10



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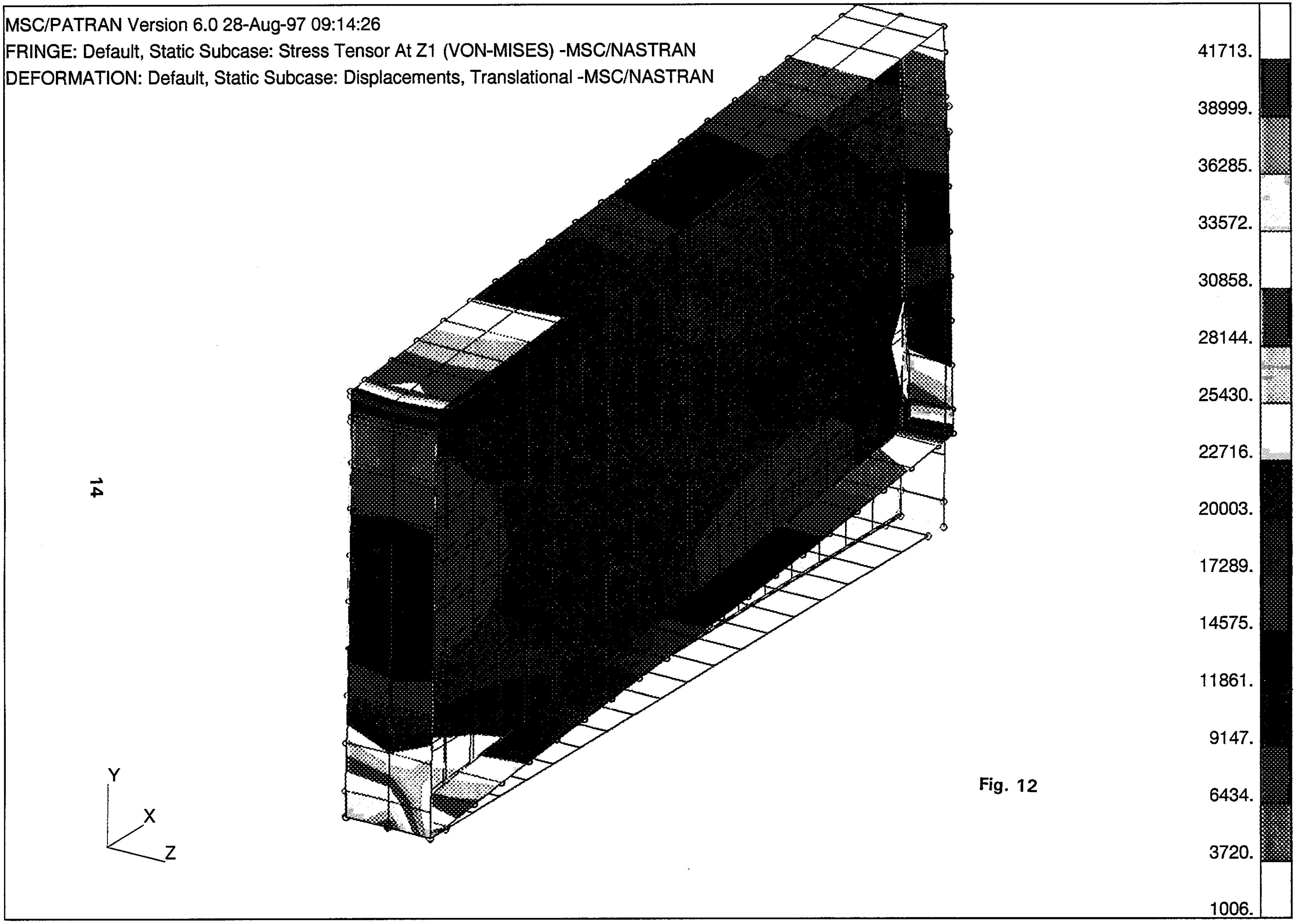
Z

Fig. 11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Y
X
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Fig. 12

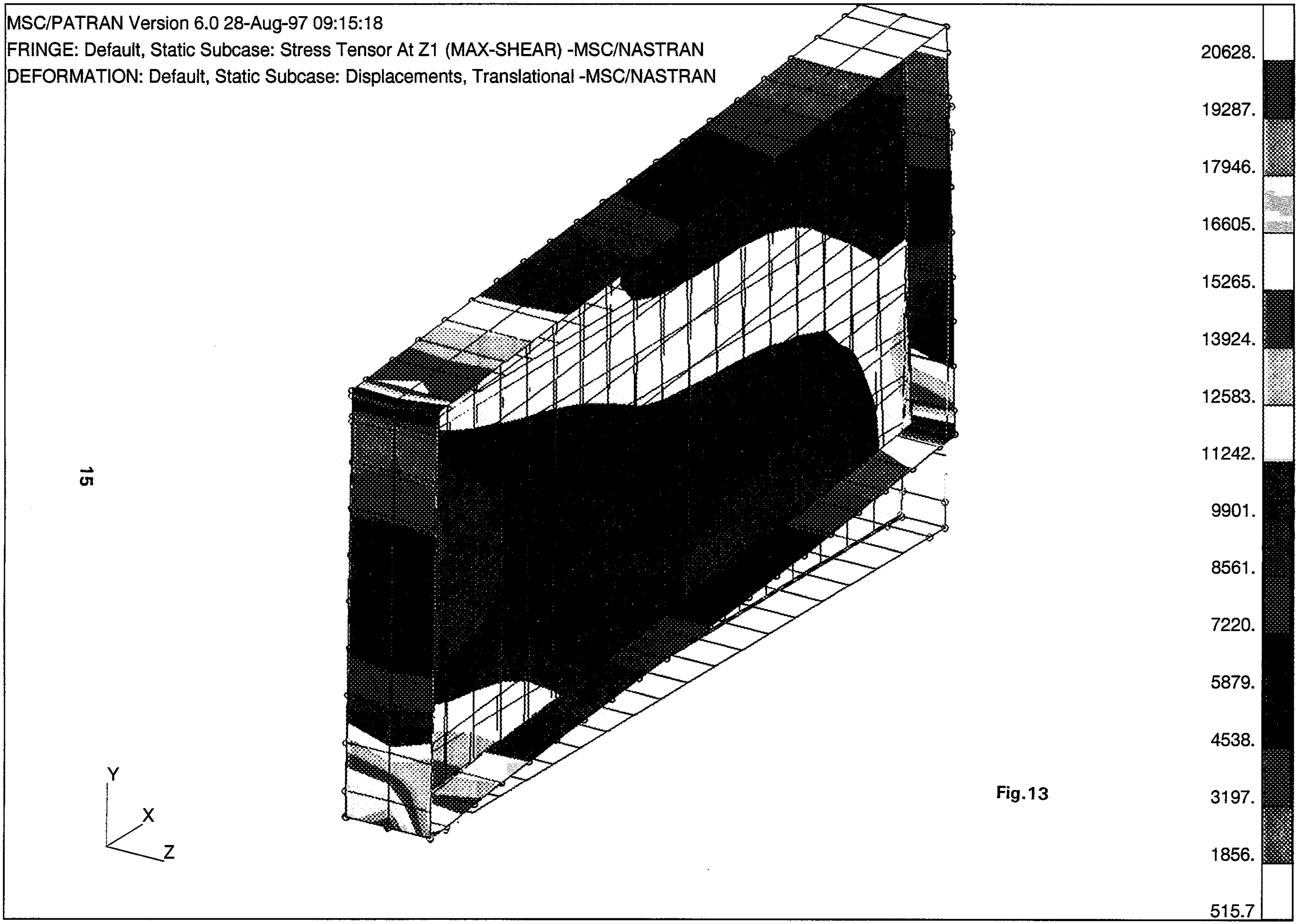
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Fig.13

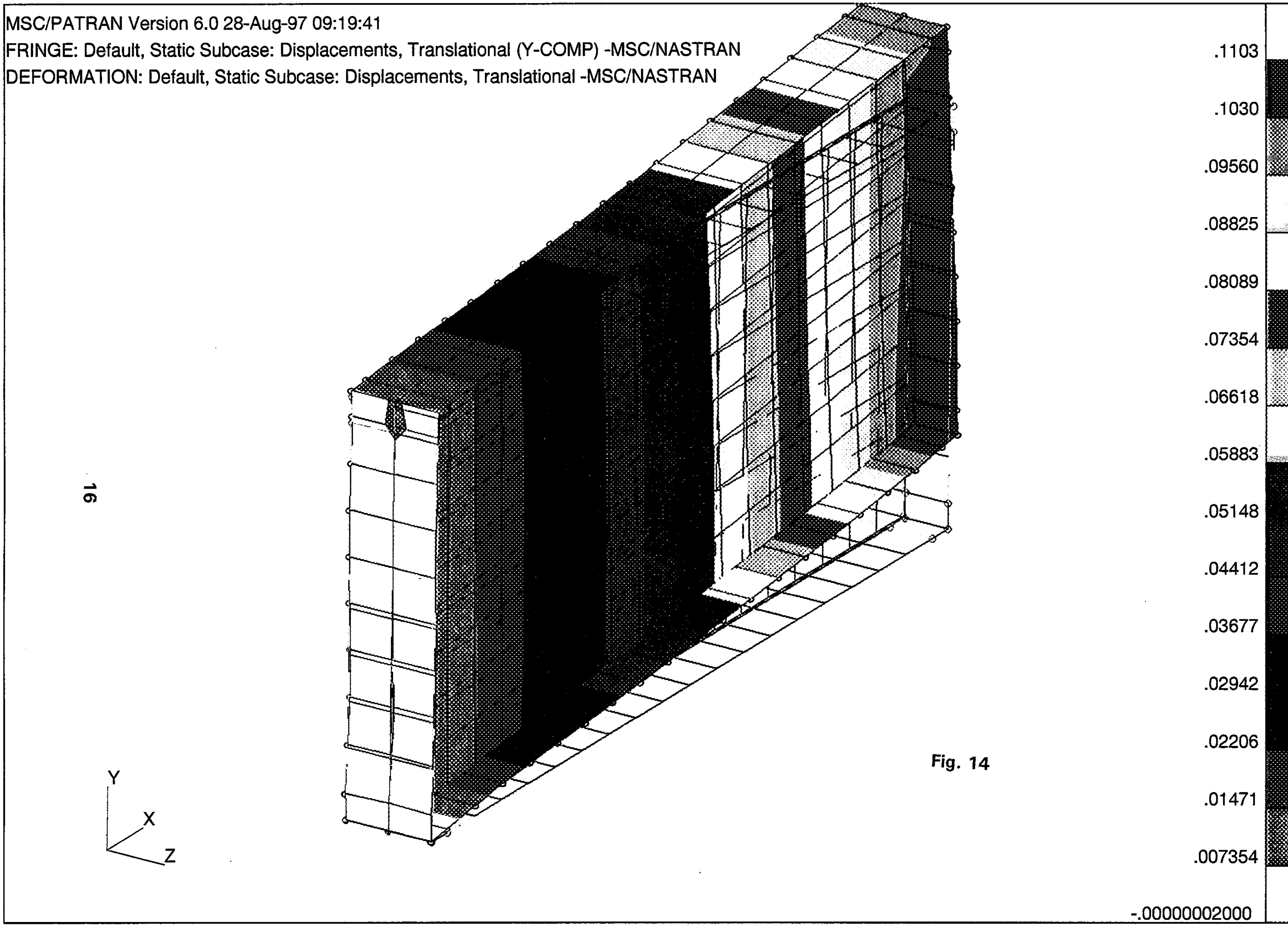
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Fig. 14

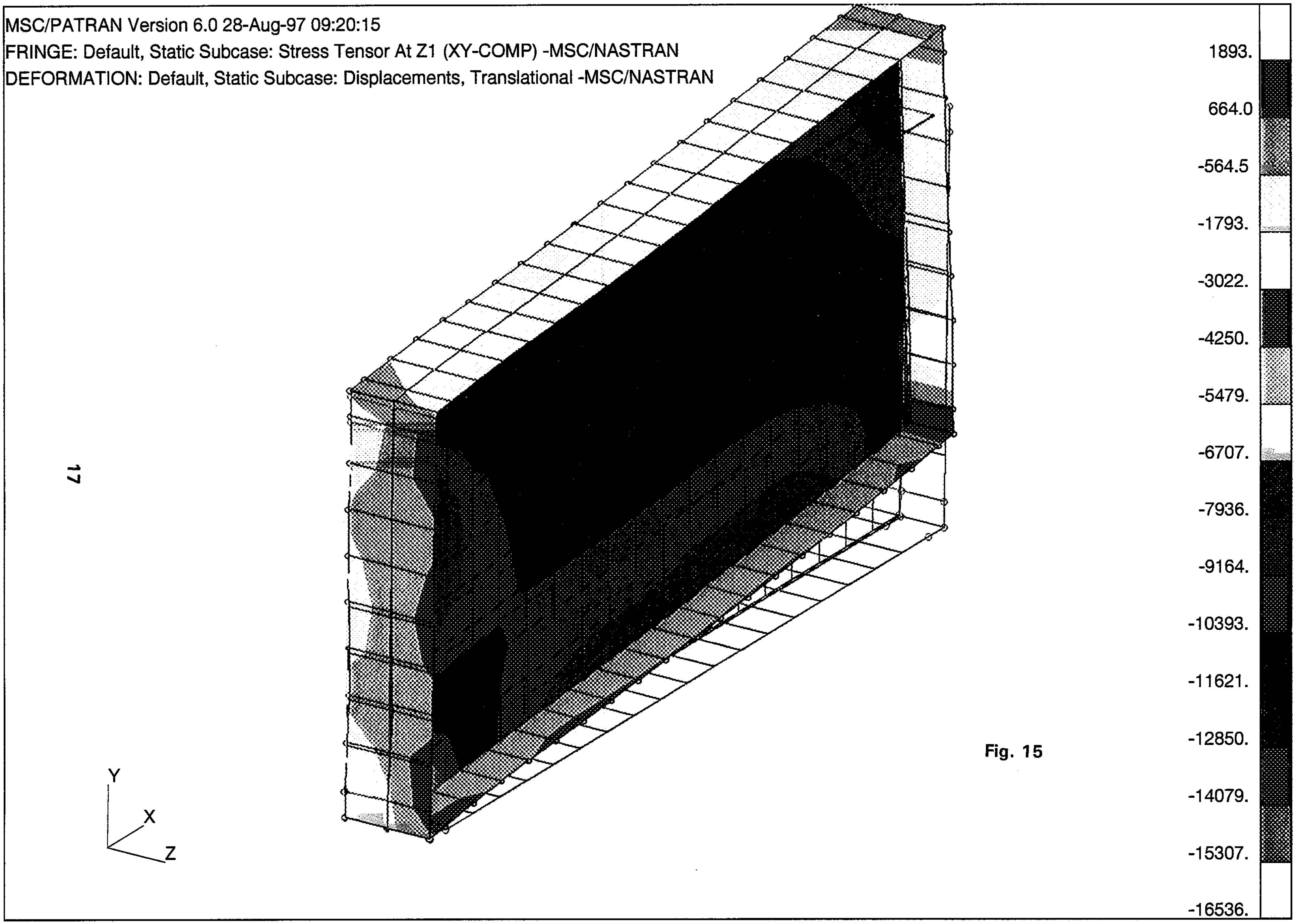
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Fig. 15

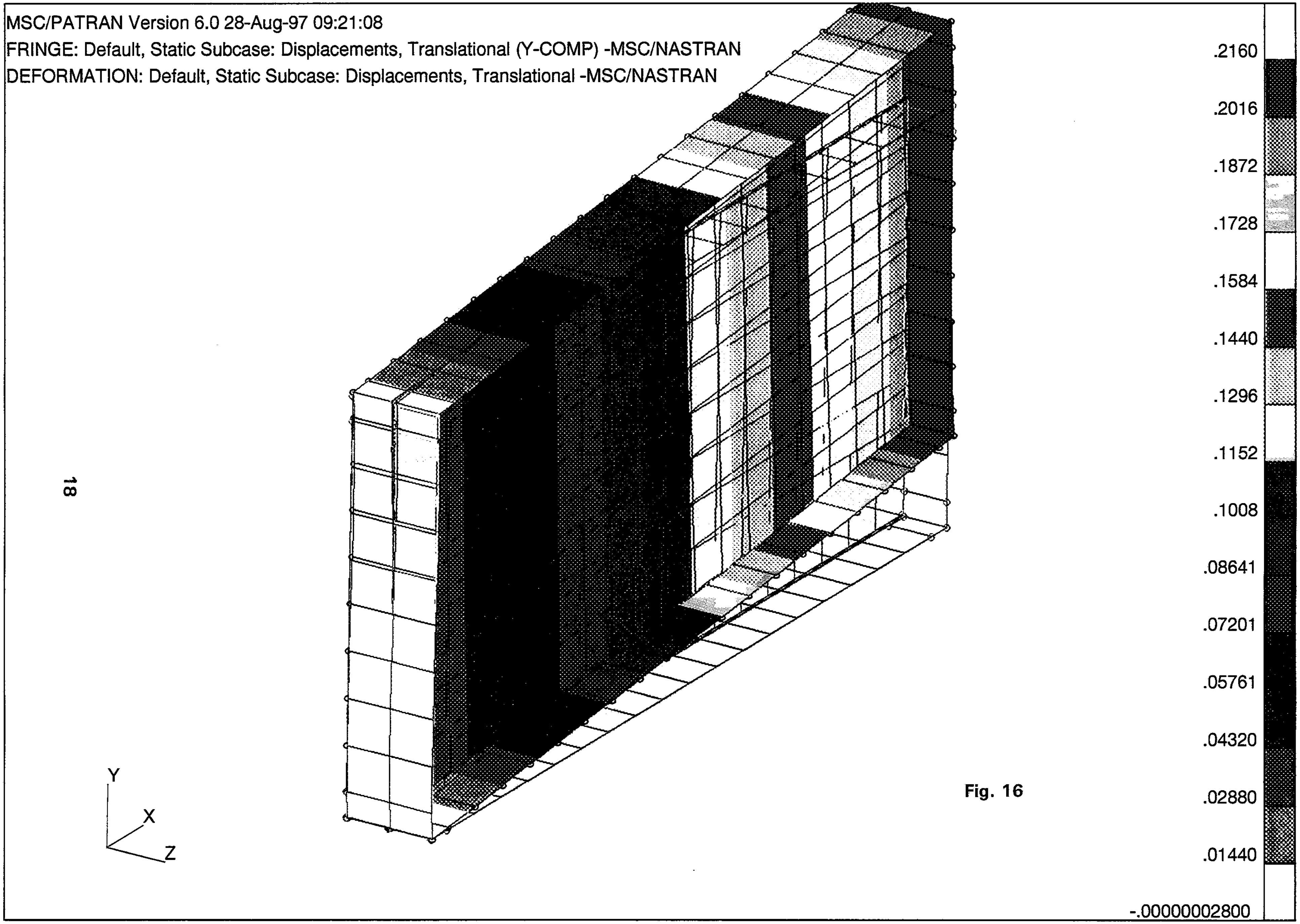
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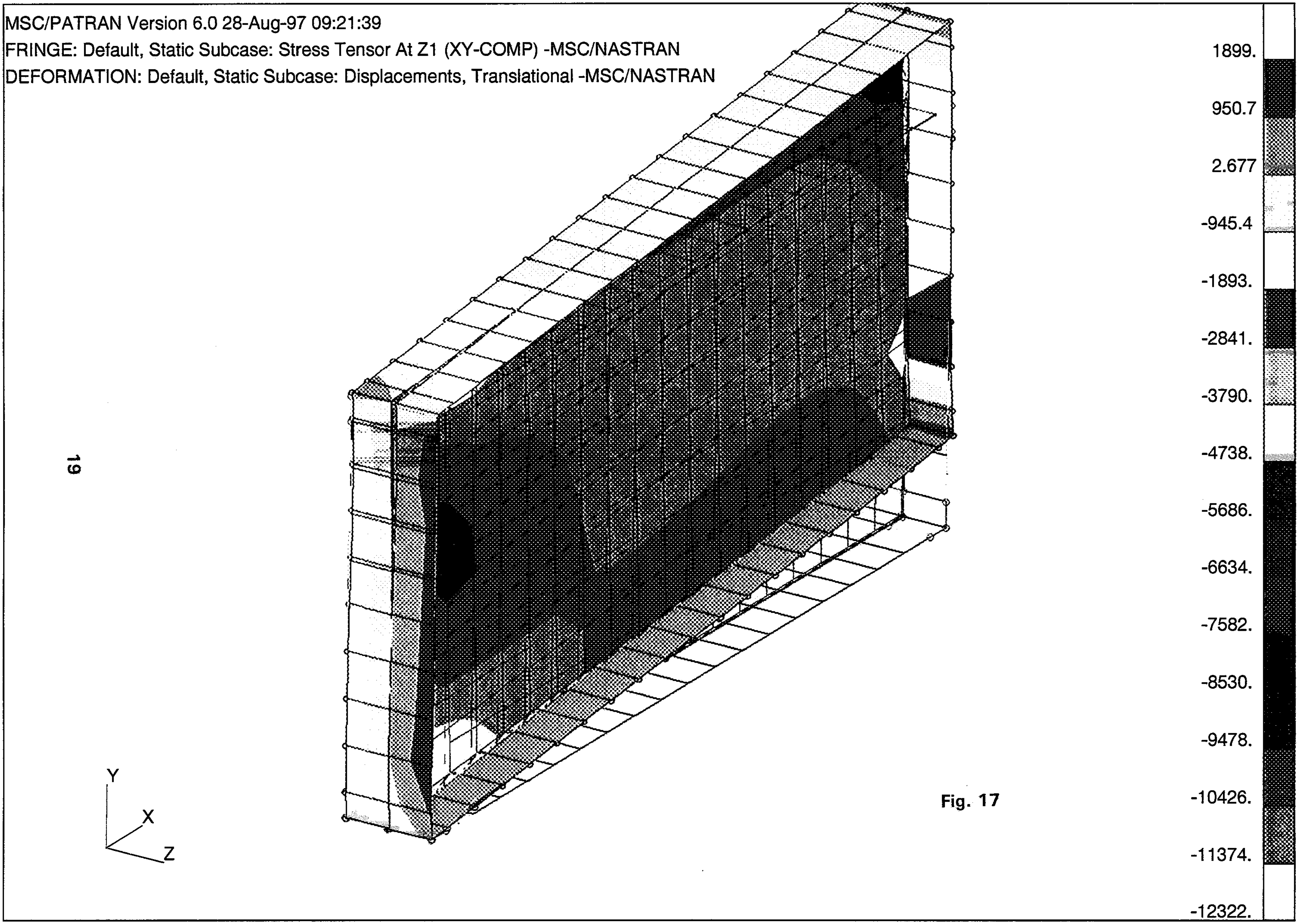
Fig. 16



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-9478.
-10426.
-11374.
-12322.
Y
X
Z

Fig. 17

OPTIMIZATION NOTES AND METHODOLOGY OF APPLICATIONS

Abstract

The presented link mesh model defines the following design variables:
[1] The strain energy density of fasteners.
[2] An optimum ratio between strain energy density of fasteners to the strain energy density of the panel bay.
[3] An optimum fastener diameter and spacing.
[4] A preferred modulus of elasticity of fasteners, web and chords
[5] The elastic displacement of the panel bay components without necessity to compute the displacement integrals that are shown on page No 2.
[6] An optimum web-attached chord flange thickness.
[7] An optimum amount of the web chemical milling, i.e. an optimum chord flange-attached web thickness.

[1] THE STRAIN ENERGY DENSITY OF FASTENERS

On the next page the total displacement is shown, that consists of the elastic displacement of the panel $\underline{\Delta}_o$ FEM and the elastic displacement of the panel due to elastic deformation of fasteners (from $\underline{\Delta}_o$ FEM horizontal line to the hyperbola) for varying cross-sectional areas if the link tie rod elements. For example, the initial solution may be obtained with the nominal cross-sectional area of the rod element link as calculable from its definition (see page No 3). The initial solution in this case was denoted with the number '1'. By increasing and decreasing the initial cross-sectional area, five more points '2','3','4','5' and '6' had been obtained to define a hyperbolic total displacement variance. This variance approaches $\underline{\Delta}_o$ FEM asymptotically as A rod increases and it approaches infinity as A rod approaches zero value.

The elastic displacement of the panel due to elastic deformation of fasteners

$$\overline{\underline{\Delta}_o \text{ FEM '1'}}$$

multiplied by intensity of an externally applied load 'P' and divided by the total volume of all fasteners 'V fast ' defines an equivalent work done, or, the strain energy density of fasteners:

$$\frac{P \times \overline{\underline{\Delta}_o \text{ FEM '1'}}}{2 \text{ V fast}}$$

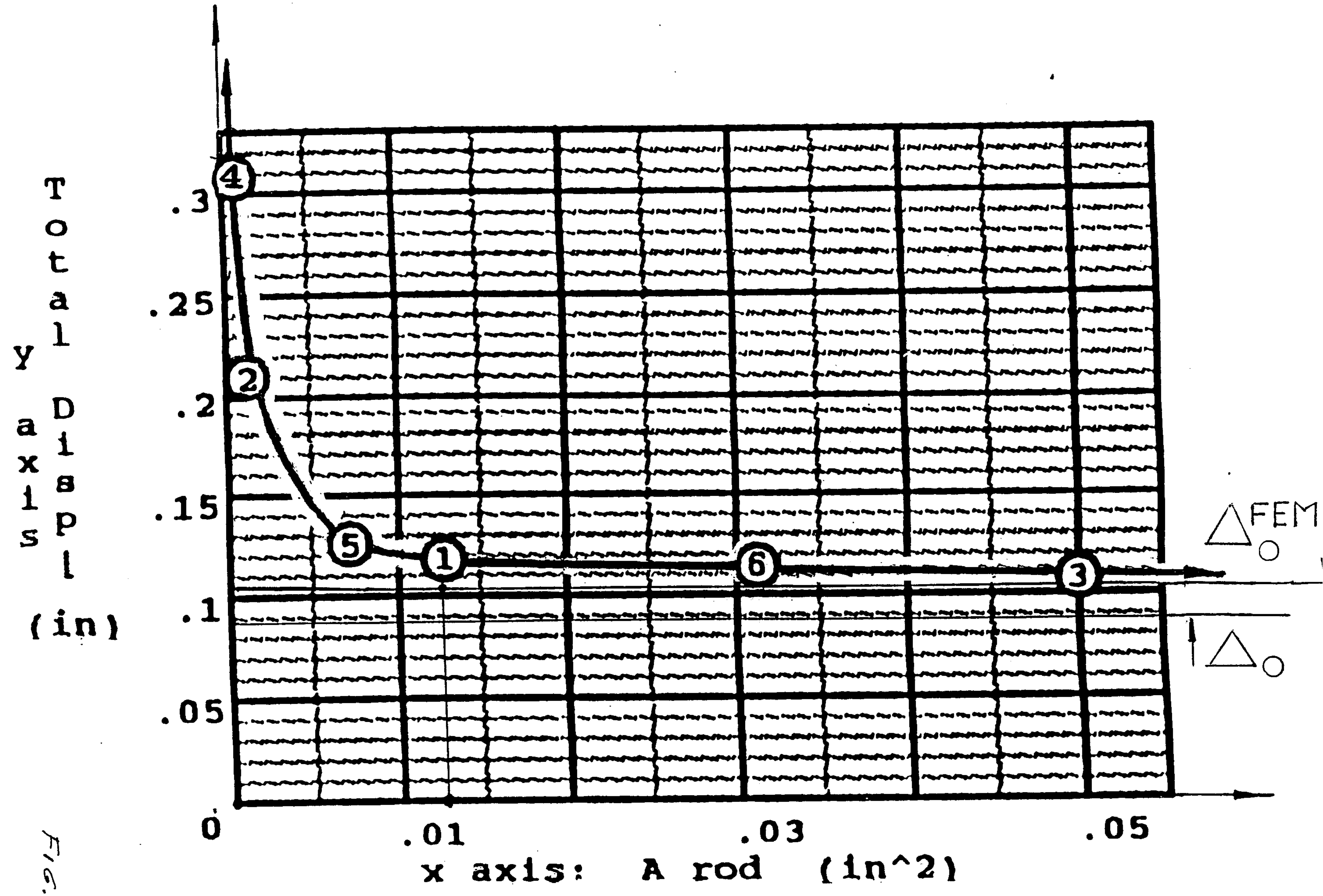
Total Displ
y axis
(in)
.3
.25
.2
.15
.1
.05
0
.01
.03
.05
x axis: A rod (in^2)
4
2
5
1
6
3
Δ_o^{FEM}
Δ_o

FIG. 18

[2] AN OPTIMUM RATIO BETWEEN STRAIN ENERGY DENSITY OF FASTENERS TO THE STRAIN ENERGY DENSITY OF THE PANEL BAY

If, for example, the panel bay has a very large stiffness, then the entire elastic deformation will be caused by the elastic deformation of fasteners, hence the work done of an externally applied load 'P' will be:

$$\frac{\text{P} \times \overline{\Delta_{o}^{\text{FEM '1'}}}}{2} = \text{Ufast}$$

and the strain energy density of fasteners:

$$\frac{\text{Ufast}}{\text{Vfast}} = \frac{\text{P} \times \overline{\Delta_{o}^{\text{FEM '1'}}}}{2\ \text{Vfast}}$$

Likewise, if the shear-bearing stiffness of fasteners would be much higher that that of the panel bay then the work done of an externally applied load 'P' will be:

$$\frac{\text{P} \times \Delta_{o}^{\text{FEM}}}{2} = \text{Upanel}$$

and the strain energy density of the panel bay

$$\frac{\text{Upanel}}{\text{Vpanel}} = \frac{\text{P} \times \Delta_{o}^{\text{FEM}}}{2\ \text{Vpanel}}$$

Consequently the ratio of the strain energy densities will be

$$\frac{\dfrac{\text{Ufast}}{\text{Vfast}}}{\dfrac{\text{Upanel}}{\text{Vpanel}}} = \frac{\dfrac{\overline{\Delta_{o}^{\text{FEM '1'}}}}{\text{V fast}}}{\dfrac{\Delta_{o}^{\text{FEM}}}{\text{Vpanel}}} = \text{X} \qquad (x < 1)$$

Analysis of the denominator

The elastic displacement of the panel Δ_{o}^{FEM} is calculable from any two subsequent executions of the link mesh model (see the next page). The volume of the panel is known from the first design stage dimensions. Since the chords and stiffeners are subjected to axial stresses and the web is subjected to shearing it is possible to determine a maximum strain energy density level that is based upon the uniaxial yield stress.

[3] AN OPTIMUM FASTENER DIAMETER

However all the fasteners are subjected to shearing in presence of bearing pressure generated compression. This is true also in the case of the surroundings of fastener holes, therefore, it would be reasonable to assume, in light of Rankine's Theory of Failure, that the principal stress should not exceed the yield stress (see Appendix A___) hence X=0.38.

Since $\overline{\Delta_o \text{FEM '1'}}$ is obtainable from initial execution of the link mesh model, the required volume of fasteners Vfast

$$\text{Vfast} = \frac{\overline{\Delta_o \text{FEM '1'}} \text{ x Vpanel}}{\Delta_o \text{FEM x } 0.38} \text{ in}\hat{\ }3$$

can now be compared with assumed volume of fasteners corresponding to the execution No 1. If the required Vfast is grater than the assumed, the fastener diameter has to be increased. In the case of the presented example Δ_oFEM=0.1089 in and the total displacement at the point No 1 =0.1175 hence $\overline{\Delta_o \text{FEM '1'}}$=0.1175--0.1089=0.0086 in
and Vpanel =58.41 in^3, consequently the required volume of fasteners will be:

$$\text{Vfast} = \frac{0.0086 \text{ x } 58.41}{0.1089 \text{ x } 0.38} = 12.14 \text{ in}\hat{\ }3$$

Since actual volume of the selected fasteners(1/4 in dia) is only 8.09 in^3, their optimum diameter will be 5/16 in dia.

[4] A PREFERRED MODULUS OF ELASTICITY OF FASTENERS, WEB AND CHORDS

The initial cross sectional area A rod (see page No 3) can be also increased by suitable selection of modulii of elasticity of standard aerospace materials.

[5] THE ELASTIC DISPLACEMENT OF THE PANEL BAY COMPONENTS WITHOUT NECESSITY COMPUTE THE DISPLACEMENT INTEGRALS THAT ARE SHOWN ON PAGE No 2

With reference to the 'Summary of Conclusions' (see page No 5), denoting the results of first execution of a link mesh model, whose rod element cross-sectional area is 'Arod1', with subscript '1' and denoting the results of the second execution with subscript '2', then the displacements and stresses applied to the link ties are given by, according to the strain energy interpretation of the linear model:

(continued)

$$\frac{(\Delta_{\Sigma} FEM\ 1 - \Delta FEM)}{(\Delta_{\Sigma} FEM\ 2 - \Delta FEM)} = \frac{\frac{1}{A\ rod\ 1}}{\frac{1}{A\ rod\ 2}}$$

Solving this equation in respect to ' $\Delta_{o} FEM$ '

$$\Delta_{o} FEM = \frac{\left(\frac{A\ rod\ 2}{A\ rod\ 1} \times \Delta_{\Sigma} FEM\ 2 - \Delta_{\Sigma} FEM\ 1\right)}{\left(\frac{A\ rod\ 2}{A\ rod\ 1} - 1\right)} \quad in$$

Example:

Point No 1: A rod 1 = 0.0128 in^2

Δ_{Σ} FEM 1 = 0.11775 in (computer output, total displacement)

Point No 5: A rod 5 = 0.00704 in^2

Δ_{Σ} FEM 5 = 0.12644 in (computer output, total displacement)

Substituting data:

$$\Delta_{o} FEM = \frac{\left(\frac{0.00704}{0.0128} \times 0.12644 - 0.11775\right)}{\left(\frac{0.00704}{0.0128} - 1\right)} = 0.107128 \quad in$$

This was verified for five more pairs of points and averaged to obtain :

Δ_{o} FEM = 0.10809 in (horizontal asymptote)

[6] AN OPTIMUM WEB ATTACHED CHORD THICKNESS

With reference to the appendix A______, from a general condition for existence of the first critical point of a function of two variables, an optimum web attached flange thickness was found:

$$t_{pch} = d\sqrt{\frac{\left(1 + \frac{E_{cb}}{E_{cch}}\right)}{\frac{4(1+\nu)}{\pi}}} \qquad (in)$$

[7] AN OPTIMUM AMOUNT OF THE WEB CHEMICAL MILLING, I.E. AN OPTIMUM CHORD-FLANGE ATTACHED WEB THICKNESS

With reference to the appendix A_______, from a general condition for existence of the second critical point of a function of two variables, on optimum chord flange attached web thickness was found:

$$t_{pw} = d\sqrt{\frac{\left(1 + \frac{E_{cb}}{E_{cw}}\right)}{\frac{4(1+\nu)}{\pi}}} \qquad (in)$$

$$\therefore$$

$$\left(\frac{t_{pch}}{t_{pw}}\right)_{OPT} = \sqrt{\frac{\left(1 + \frac{E_{cb}}{E_{cCH}}\right)}{\left(1 + \frac{E_{cb}}{E_{cw}}\right)}} \doteq 0.88\dot{8} \quad \text{(EXAMPLE MODEL)}$$

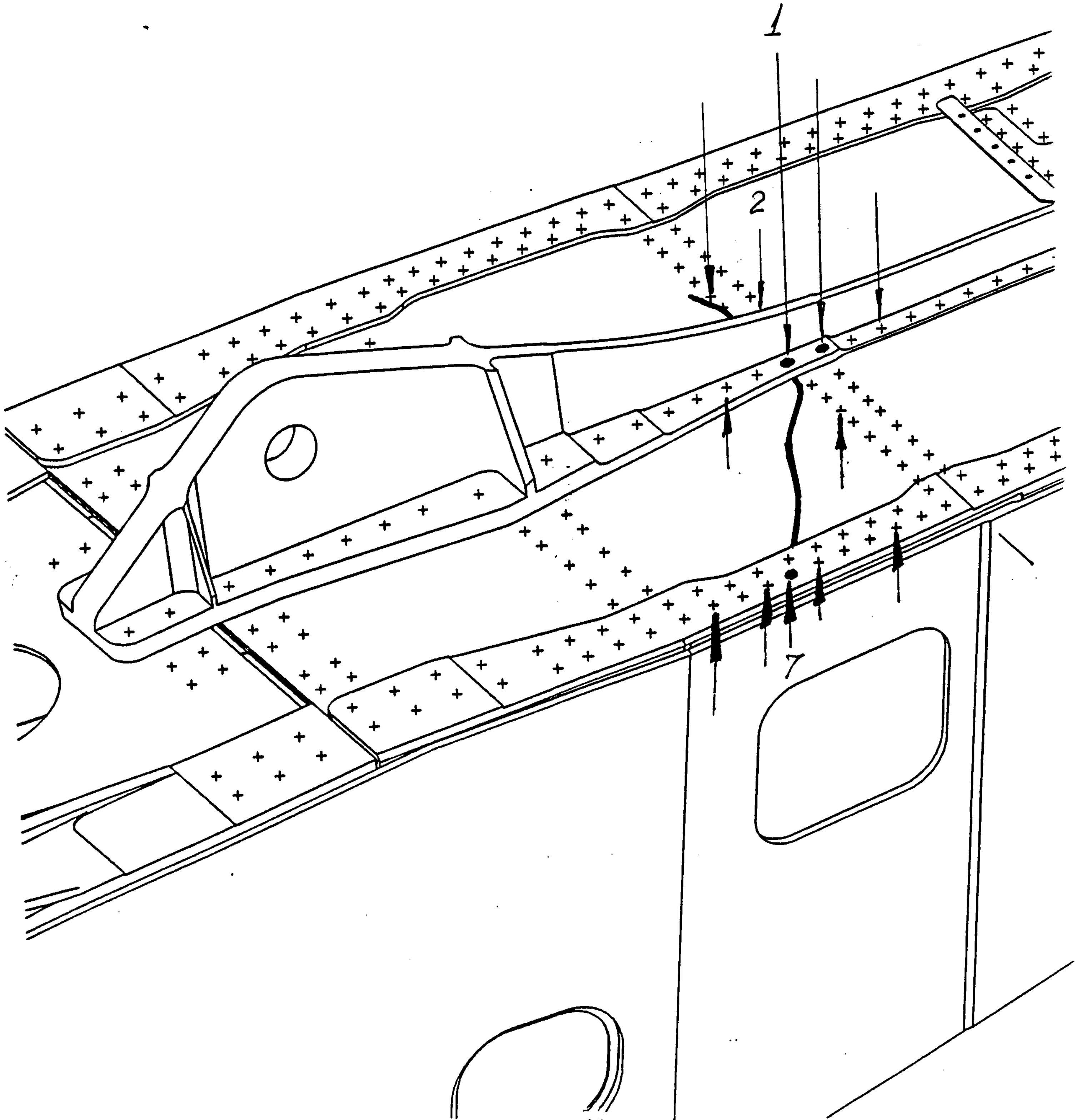

Fig 19

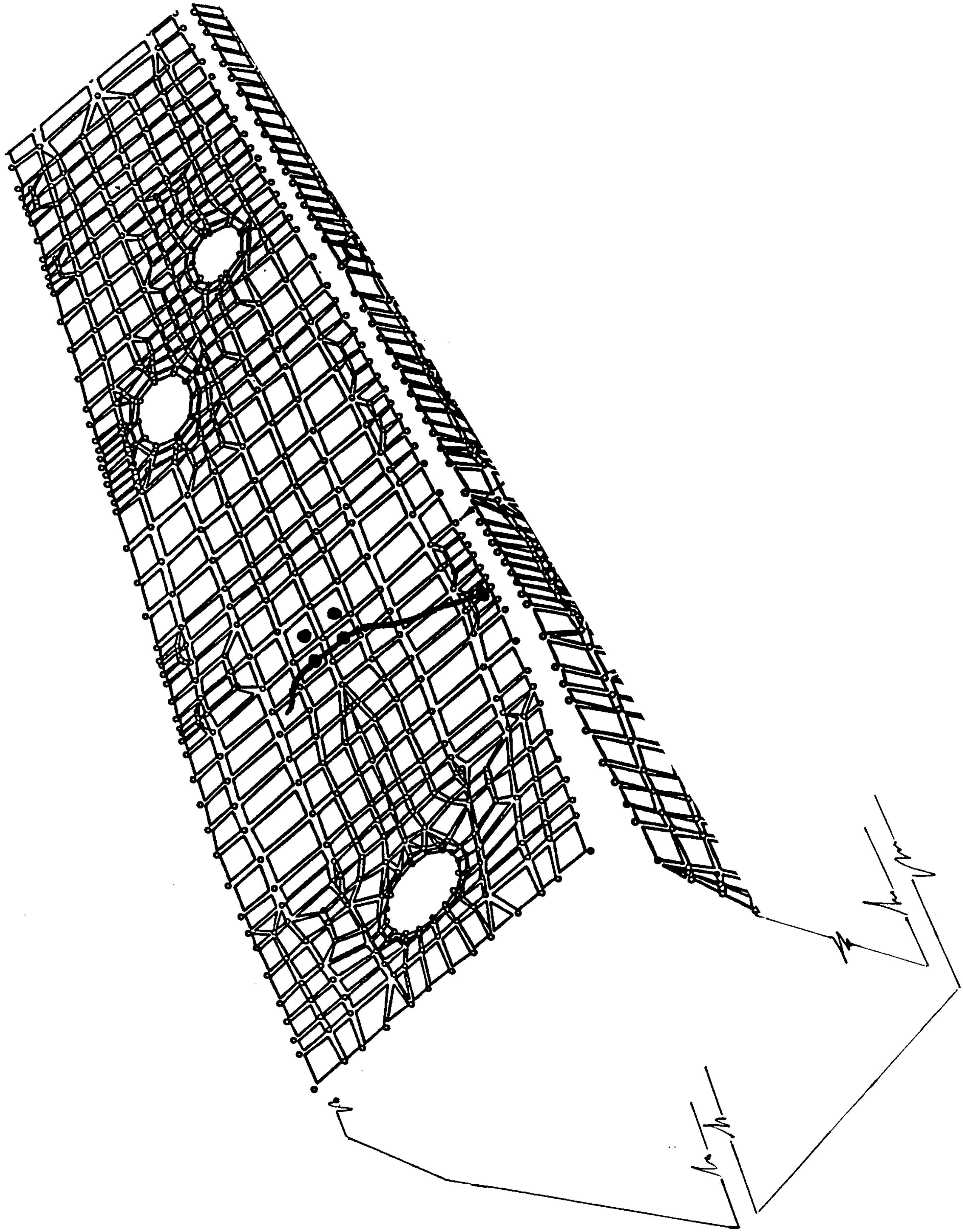

Fig 20

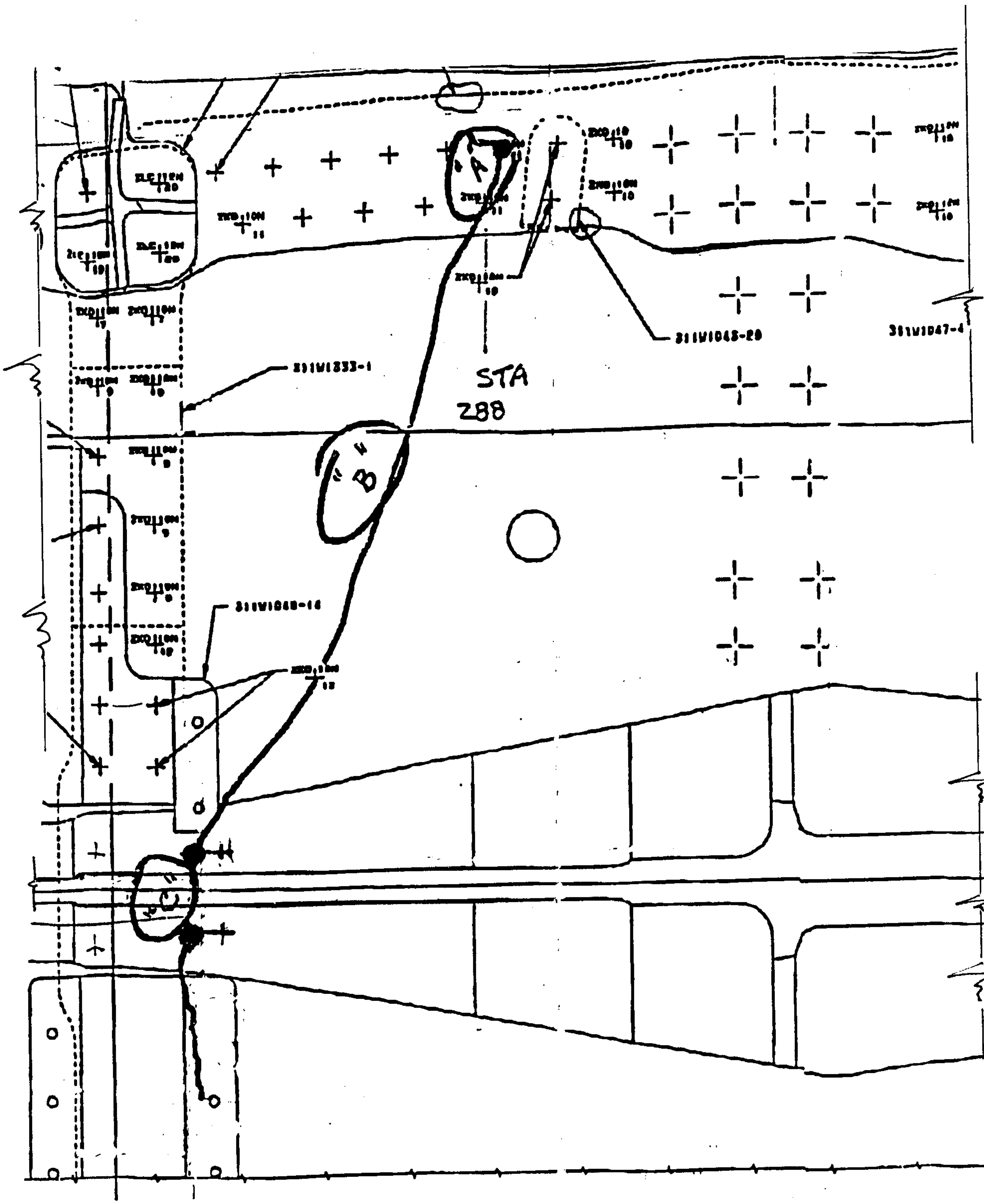

Fig 21