LOCAL STRESS ANALYSIS OF STIFFENED SHELLS USING MSC/NASTRAN'S SHELL AND BEAM p-ELEMENTS

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Abstract

In large finite element models of aircraft structures, traditional h-elements give sufficient accuracy for most purposes, for example in vibration analysis. However, for local stress analysis of stiffened shells, h-elements may give inaccurate answers at shell-stiffener connections. The paper shows how to use p-elements at those locations where more accurate stresses are required. P-elements work with the existing h-element mesh. A few modifications of the input are necessary to convert local parts of the model into p-elements. The p-version elements improve local stresses significantly. The increase in accuracy is demonstrated on two examples of stiffened shells.

Introduction

Stiffened shells in aircraft structures are traditionally modeled with lower order shell and beam elements. In some cases, even simpler shear panel elements are used instead of shell elements. Lower order elements are used because they give sufficiently accurate answers for most purposes, for example, in vibration analysis or in determining forces in main structural members. Most finite element models of aircraft structures have a large number of degrees of freedom. Lower order elements keep the model to a manageable size.

However, there are cases where h-element models can produce local stresses which do not meet the required accuracy. For example, local stresses and forces at connections of shells and stiffeners may not be accurate enough if h-element shells and beams with offsets are used. The deficiency was reported in a paper by Z.Borowiec [1] and in several client service requests [4]. Some methods are presented in [1] to correct beam end moments.

The approximation of the displacements in lower order elements causes the inaccuracy in stresses and forces. The distribution of displacements is linear along an h-element edge. Therefore, strains, stresses and forces are constant along an edge. The constant forces within an element cause jumps between elements and lead to inaccurate local forces. There are two ways to solve the problem, refine the mesh or use higher order elements. Mesh refinements are always time consuming. In addition, it has been shown that mesh refinements with h-elements converge slowly in case of stiffened shells.

Higher order elements give the desired linear or higher distribution of forces along element edges. The mesh size does not need to be altered compared to h-element mesh refinements. There are two types of higher order elements, Lagrange elements with mid-side grids or p-version elements with vertex grids and hierarchical degrees of freedom. The change from h-elements to higher order Lagrange elements with mid-side grids is not an easy task. The model has to be remeshed, at least at the location of interest. Mesh transitions from lower to higher order elements are problematic and may cause inaccuracies.

The other type of higher order elements are the p-version elements. In certain cases, p-version elements can use the same mesh as h-elements, for example, in global-local analysis. We have a global h-element model for general purpose analysis and we want to get more accurate stresses at selected locations of the model. We use the existing h-element mesh and define a few elements to be higher order p-elements at locations where more accurate stresses are required. Further modifications of the h-element model are necessary when p-elements are used in areas with curved geometry. The piecewise linear h-element geometry must be snapped on to the true geometry. In addition, grid point oriented loads and boundary conditions must be converted to continuous loads and boundary conditions when applied to p-elements.

In this paper, we want to show how the accuracy of local stresses improves when existing h-element models are converted to p-elements. The examples are simple beams and stiffened plates. The h-element model is easy to convert to a p-element model because the geometry is not curved, boundary conditions and loads are constant.

Theoretical Background

Shell and beam p-elements have been implemented into Version 69 of MSC/NASTRAN. The shells and beams complement solid p-elements which were introduced in Version 68. All p-element types can have different p-order in different directions. The element edges and faces have a common geometric description so that they can be joined in any combination. All p-elements can also be used with traditional h-elements so that global-local stress analysis on large models is feasible. The connectivity entries and the property entries for p-elements are the same as for h-elements.

The shell p-elements are three or four noded with unique normals at the nodes. The beam p-elements are two noded. A built-in-twist can be defined for the beam p-element. Offsets can be defined in both element types which is useful to model stiffened shell structures. The geometry for the p-elements is described with linear, quadratic or cubic edges. If curved geometry is provided, MSC/NASTRAN generates cubic edges with two vertex grids and two points at the 1/3 and 2/3 parametric location of the edge. If the geometry is smooth, adjacent cubic edges are fit to have common tangents. An accurate approximation of curved geometry is important in thin curved shells.

The shell and beam p-elements for all p-levels are shear flexible C^0 elements. Special provisions have been built in the elements to prevent shear and membrane locking. The out-of-plane shear terms are modified to improve bending behavior for thin shells and beams for all p-levels. Residual bending flexibility terms are introduced based on virtual work equivalence. The shell p-elements with $p<3$ have additional internal degrees-of-freedom, or bubble functions, to improve bending and membrane behavior. For the Gauss integration, a selective reduced integration scheme is used. The elements pass the patch test and are free of zero energy modes for all p-levels.

Demonstration of Concept Using A Simply Supported I-Beam

The I-beam is loaded at the mid span and stress values are observed along the cross section at the mid span. The example was sent to MSC by a client [4]. The upper flange is modeled with 10x2 shell elements and the remaining inverted T section is modeled with 10 beam elements with offsets (see Figure 1 and 2). Two models are investigated. The first model uses beam and shell h-elements. The second model uses uniform p=3 beam and shell p-elements. The finite element results are compared with the exact solution from beam theory (see Figure 3 and Table 1).

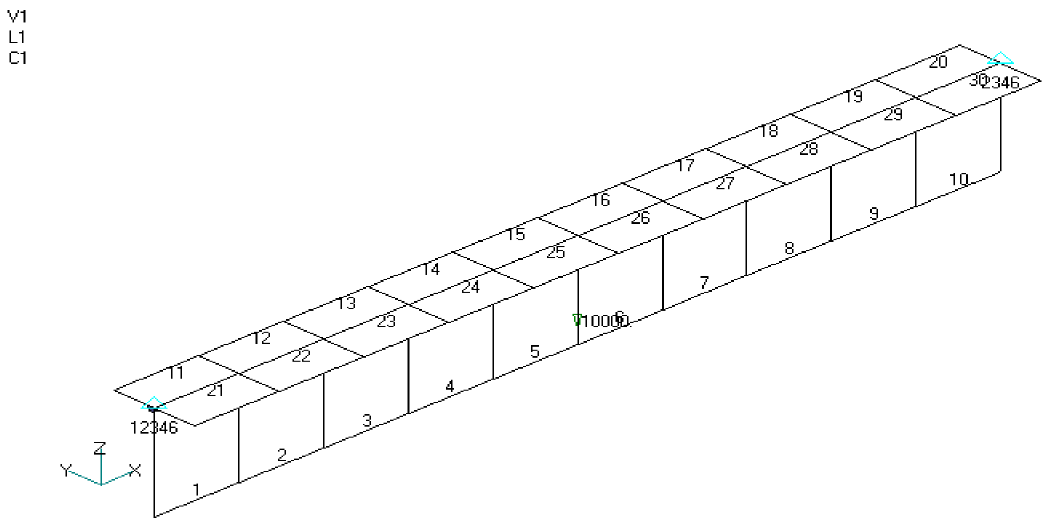

Figure 1: Beam modeled with plate and beam elements using offsets.

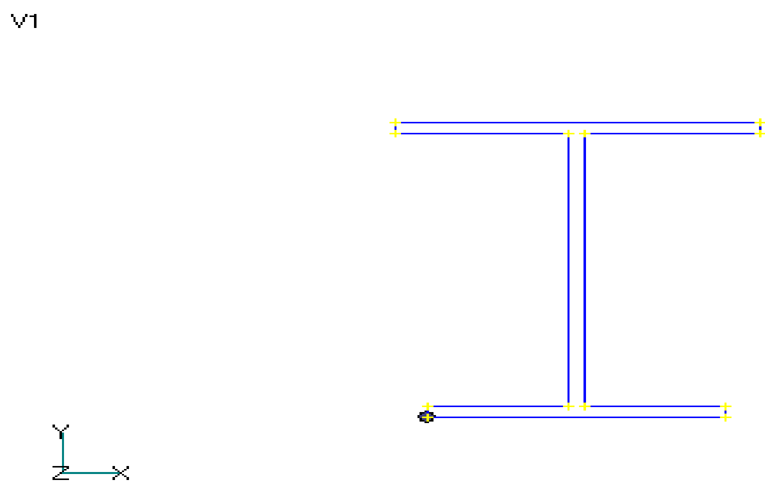

Figure 2: Beam cross section showing stress data recovery points.

It is evident from the chart that large stress discontinuities are observed at the interface where the plate and the offset beam attach . The stress values at location 3 are from the shell elements and the stress values at location 2 are from the beam elements. Compared to the exact solution, the error in stresses at location 2 is 31.6 % for h-elements and 10% for p-elements. This example demonstrates that a simple conversion of h-elements to p-elements improves the accuracy significantly. A quadratic (p=2) or cubic(p=3) p-order provides the linear or quadratic force distribution along element edges so that local stresses become more accurate.

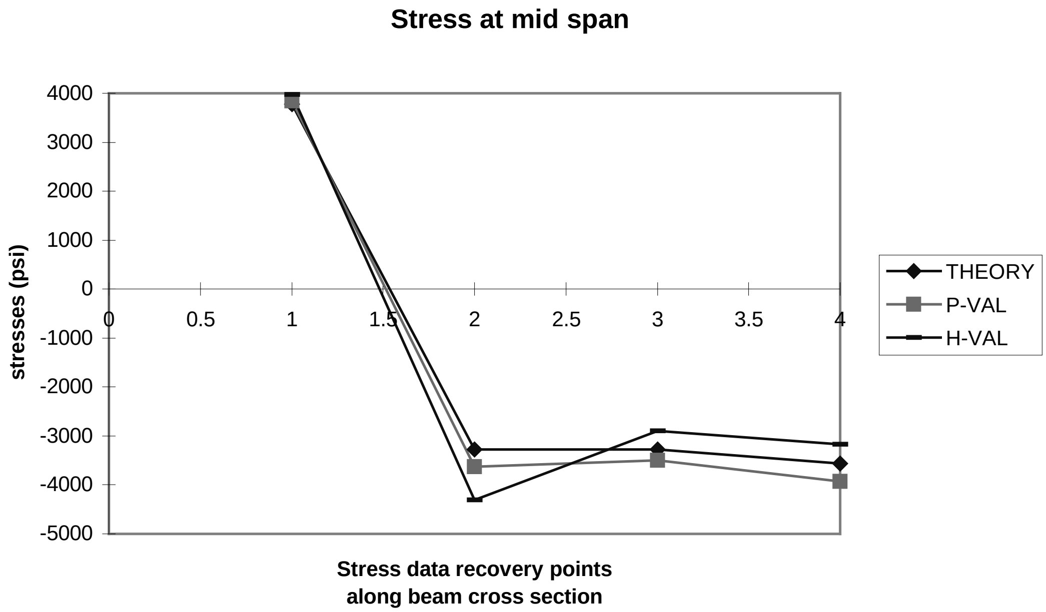

Figure 3: Stress comparison of p-element, h-element and theory.

TABLE 1

Stresses at cross section at mid span		THEORY	p=3	% diff p	h	% diff h
top of the plate at the midspan	4	-3558.00	-3934.20	10.00%	-3168.20	10.95%
bottom of the plate at the midspan	3	-3275.60	-3497.00	7.00%	-2896.20	11.60%
interface (top of the offset beam)	2	-3275.60	-3613.80	10.00%	-4311.00	31.60%
bottom of the offset beam	1	3784.00	3847.00	1.66%	3978.00	5.12%

Stress Analysis of Stiffened Plate

The example is taken from Bruhn [2], problem A21.13. A cantilevered plate is fully clamped at the left edge and loaded with five concentrated forces at the three vertical and one horizontal stiffeners (see Figure 4). The stiffened plate is modeled with shells and beams. We use h-elements with various mesh densities and compare them to p-elements with p-levels from 1 to 3. Two different types of stiffeners are investigated, stiffeners without offsets and stiffeners with offsets.

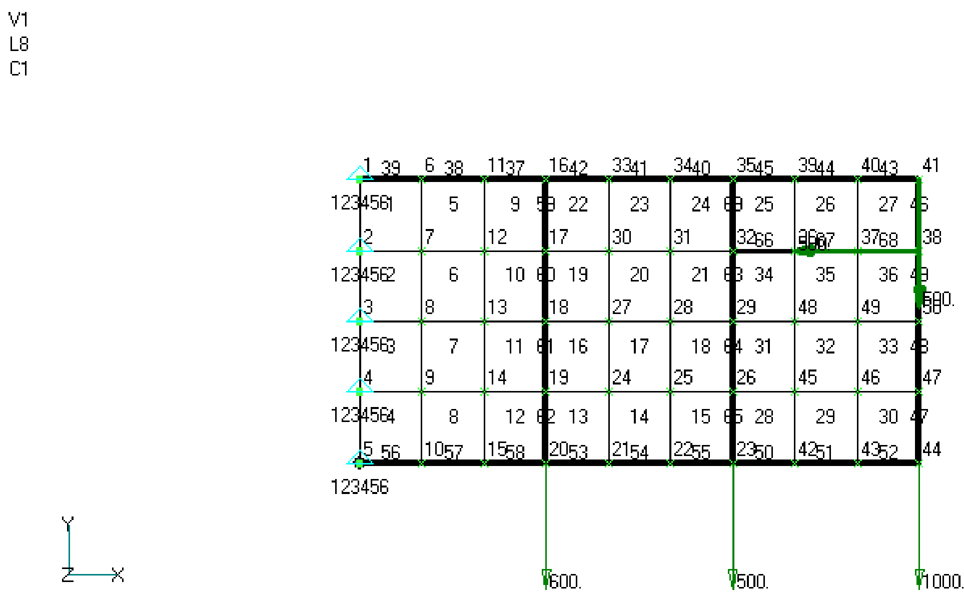

Figure 4: FE mesh for inplane shear and moment in stiffened plate

(E=1.E7 psi, ν=0.34).

The moments along the bottom edge (beam elements connecting grid 5 thru 44) of the stiffened plate are plotted below for two cases, without offsets (Figure 5) and with the offsets (Figure 6). Since there is no exact solution for this example, we refined the mesh for the h-elements to see how the results converge. The results of the finer h-element meshes converge towards the results of the p-elements with p-value of 3. We assume that the p=3 results represent the converged solution.

For the plate without offsets, we compare the results of h-elements with the results of p-elements for p=3 using the mesh shown in Figure 4. For the h-elements, the end moments in the bottom beam are off by about 28% compared to p-elements, see Figure 5, Station 0, Grid 5.

For the plates with offsets, the h-elements exhibit additional inaccuracies. The beam moments jump between elements, see Figure 6, Station 2, Grid 15. The beam moments of the p-elements for p=3 are smooth. Figure 5 and 6 show clearly that the h-element model produces inaccurate local forces.

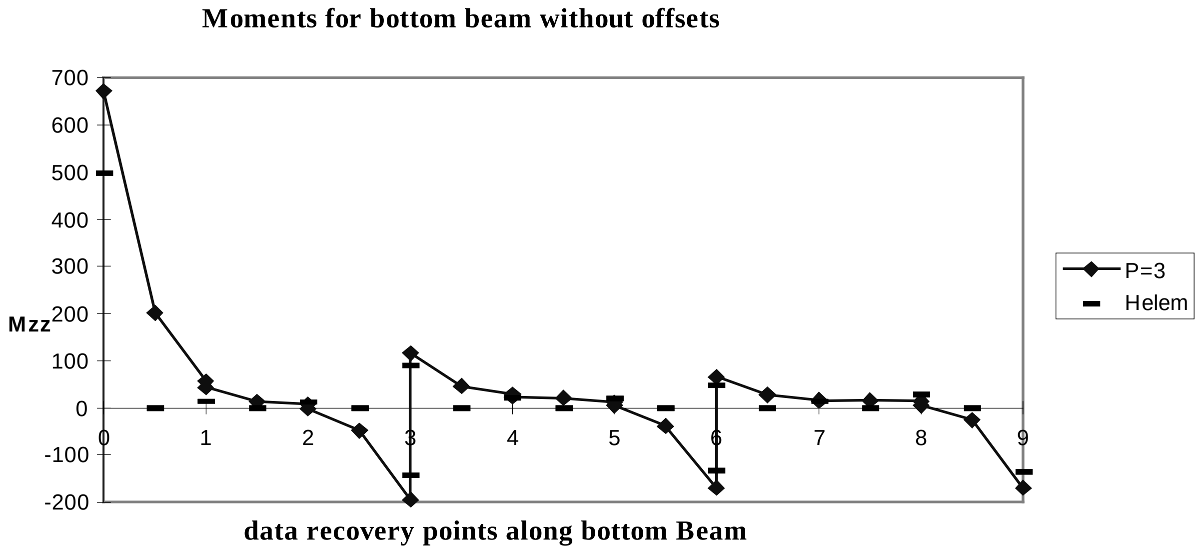

Figure 5: Bottom beam moments without the offsets

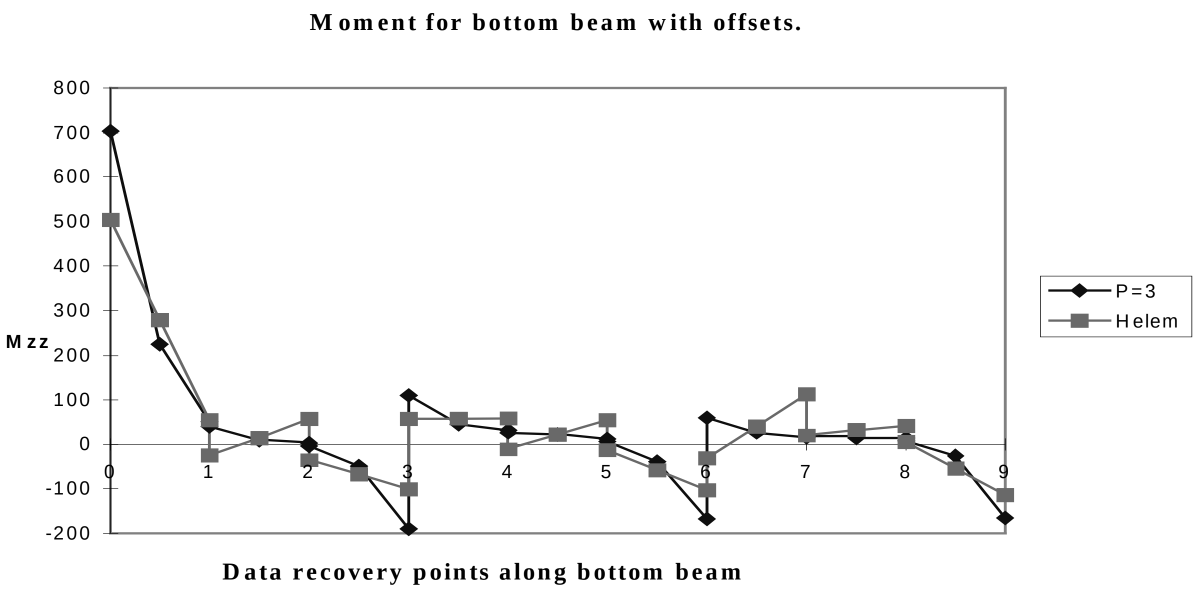

Figure 6: Bottom beam moments with offsets

For the plate without offsets, the inplane shear stresses at section A-A are compared in Figure 7. We use the mesh in Figure 4 for the coarse h-element model and for the p-element model with p=3. The mesh is refined once to show the convergence of the h-element model. The maximum shear stress is about the same for all the models. The p-element model shows the most accurate shear stress distribution. The result of the h-elements converges slowly towards the results of the p-elements.

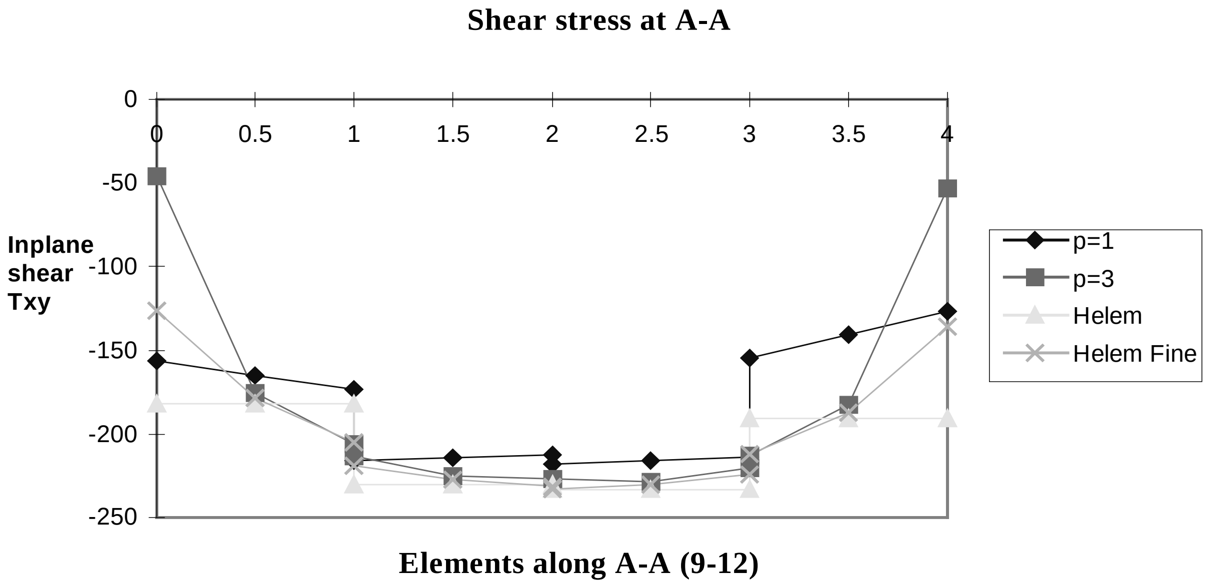

Figure 7: Shear stress distribution at Section A-A for stiffened plate without offsets.

Conclusion

The paper shows that p-elements significantly improve accuracy of local stresses in stiffened shells. The use of p-elements has two advantages. Under certain restrictions, p-elements can use the same mesh as h-elements. One does not need to remesh or create special elements for mesh transitions. Furthermore, MSC/NASTRAN's p-elements allow global-local analysis. Higher p-order can be restricted to local areas in the model and the computational effort of local stress analysis can be minimized.

The transition from h to p-elements is not always as easy as in the two examples we have shown in this paper. In case of curved geometry, additional geometric information must be provided to achieve higher accuracy with p-elements. In addition, loads and boundary conditions may have to be changed from discrete to continuous conditions. The transition from h to p-elements is not automated and the user needs to perform some manual changes to the input. We have not shown a fully p-adaptive analysis in this paper. The presented examples need additional changes in the definition of the loads and boundary conditions if more accurate results from p-adaptive analysis are required.

References

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