DIAG2 - GEOMETRIC NON-LINEAR PARAMETERS CALCULATION FOR DIAGONAL TENSION SIMULATION USING MSC/NASTRAN

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ABSTRACT

The analysis of Diagonal Tension effects on a thin walled reinforced shell structure that buckles under compression/shear loadings is essential. DIAG2 program was written to simulate the non-linear effects due to the skin buckling (under compression/shear) of cylindrical structures with frames (ribs) and stiffeners. The original structure model is therefore changed to simulate the post-buckling behavior of the structure.

1 - INTRODUCTION

A thin walled reinforced shell structure can present some interesting geometrical non-linear behaviors under some loading conditions. One of the most common effect is the Diagonal Tension that appears when such thin walled panels buckle under compression/shear loadings.

The Fuselage Structural Analysis process is divided in two parts:

1) Linear Static Analysis;
2) Non-Linear Static Analysis (Geometric non-linearities)

For the linear static analysis a finite element model is prepared in order to obtain the displacements, strains, stresses and internal forces of the structure. To do this we use MSC/NASTRAN program (SOL 101 Linear Static Analysis, Ref. 1). Once these values have been obtained for the structure, the margin of safety could be obtained using the applicable engineering methods.

The non-linear analysis to consider diagonal tension effects is also divided in two parts:

1) Calculation of non-linear effects due to buckling of the skin in compression and/or shear without taking into account the diagonal tension effects.
2) Calculation of non-linear effects due to buckling of the skin, in order to consider the diagonal tension effects.

This paper shows the theoretical background of the first part of the non-linear structural analysis, that is executed by MSC/NASTRAN program using linear finite element model, modified by a FORTRAN program DIAG2 (Ref. 4a and 4b).

The DIAG2 program can be applied using the whole finite element model or only a set of selected ring spaces with the corrected boundary conditions simulated by applying the boundary displacements or forces.

2 - PROBLEM DEFINITION

2.1 - PRINCIPLE OF DIAGONAL TENSION

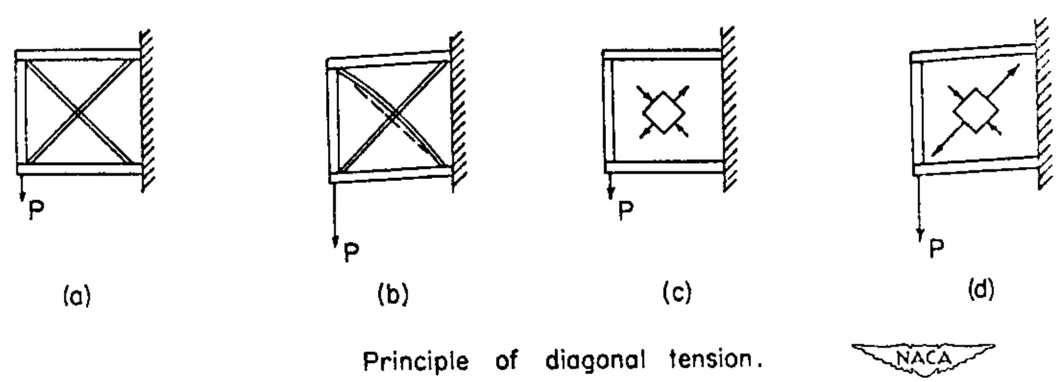

Figure 1

A thin walled reinforced shell structure under shear loading condition will buckle with diagonal wave skills. The *Figure 1* shows the Principle of diagonal tension (Ref.2). A tension strength appears at the diagonal of the frame because the distance between the lower-left corner and the upper-right corner tends to increase when the load P is applied.

2.2 - BUCKLING OF A PANEL UNDER COMPRESSIVE LOADING.

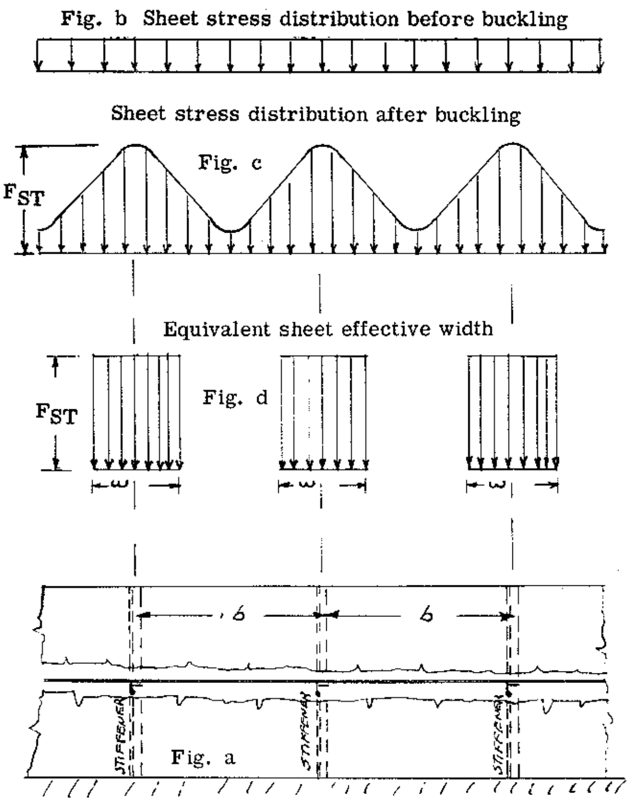

Figure 2 - Sheet-stiffener panel

The detail *Fig.a* of *Figure 2*, shows a sheet-stiffener panel subjected to a compressive loading condition. The panel stress distribution before buckling is shown in the detail *Fig.b*.

After the buckling of the sheet, the stress distribution of the panel is as shown in the detail *Fig.c*, the panel does not resist effectively any additional compression, redistributing the loading to the adjacent stiffeners that can resist much more loading than the sheet of the panel. The effective amount of the sheet that resists to the compressive load can be represented as an effective width shown in the detail *Fig.d*.

2.3 - MODELING TECHNIQUE

2.3.1 PANEL INFORMATIONS

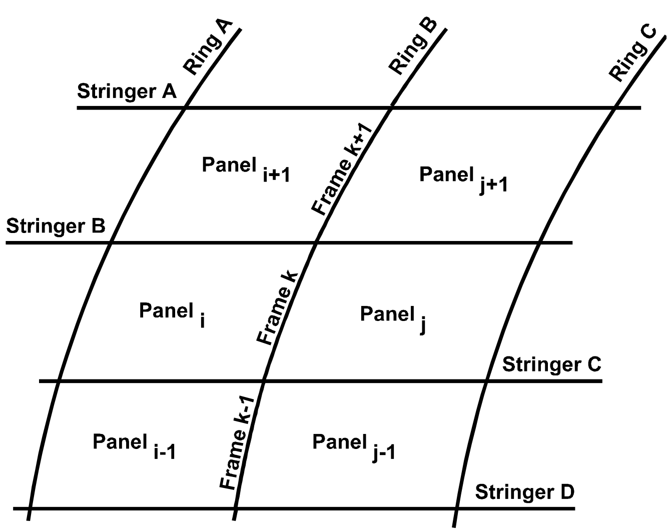

Figure 3 - Panel representation.

The original structure can be represented as shown in the *Figure 3*. Each panel is defined by its skin (sheet) and its surrounding frames and stringers.

Let´s take a closer look to, p.ex. *Panel i,* and see how the panel is represented with the FEM model. The sheet of the panel could be modeled with QUAD4 and/or TRIA3 elements.

2.3.2 - PANEL MODELING WITH QUAD4 ELEMENTS

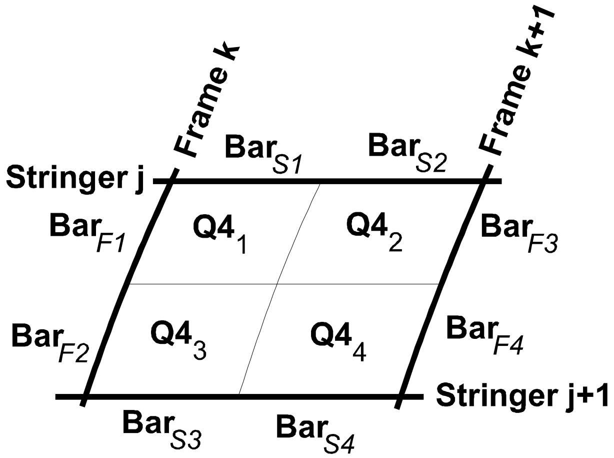

Figure 4 - Panel model with QUAD4 elements.

When a panel is modeled with QUAD4 elements, it could be represented as shown in *Figure 4*. In the *Figure 4,* the panel is defined by:

- four QUAD4 shell elements representing the sheet,
- two BAR elements representing the upper *Stringer j,*
- two BAR elements representing the lower *Stringer j+1*,
- two BAR elements representing the left *Frame k*, and
- two BAR elements representing the right *Frame k+1*.

2.3.3 - PANEL MODELING WITH TRIA3 ELEMENTS

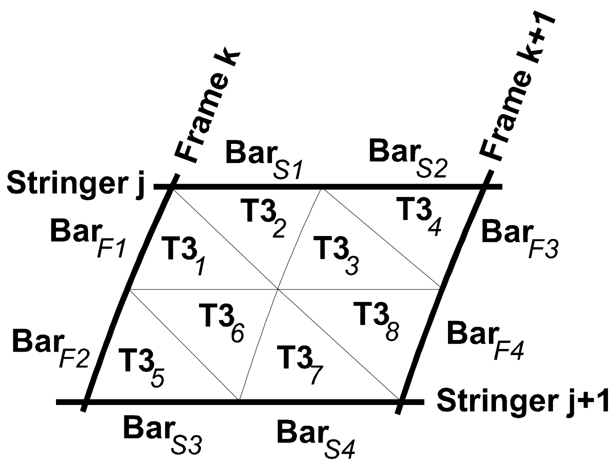

Figure 5 - Panel model with TRIA3 elements.

When a panel is modeled with TRIA3 elements, it could be represented as shown in *Figure 5*. In the *Figure 5,* the panel is defined by:

- eight TRIA3 shell elements representing the sheet,
- two BAR elements representing the upper *Stringer j,*
- two BAR elements representing the lower *Stringer j+1*,
- two BAR elements representing the left *Frame k*, and
- two BAR elements representing the right *Frame k+1*.

2.3.4 - PANEL MODELING WITH QUAD4 AND TRIA3 ELEMENTS

The same could be made mixing QUAD4 and TRIA3 elements.

3 - ANALYSIS

3.1 - FLOW DIAGRAM OF THE METHOD

This method is iterative in order to achieve the convergence at the effective width of the skin. The steps from B to D are performed by the DIAG2 program:

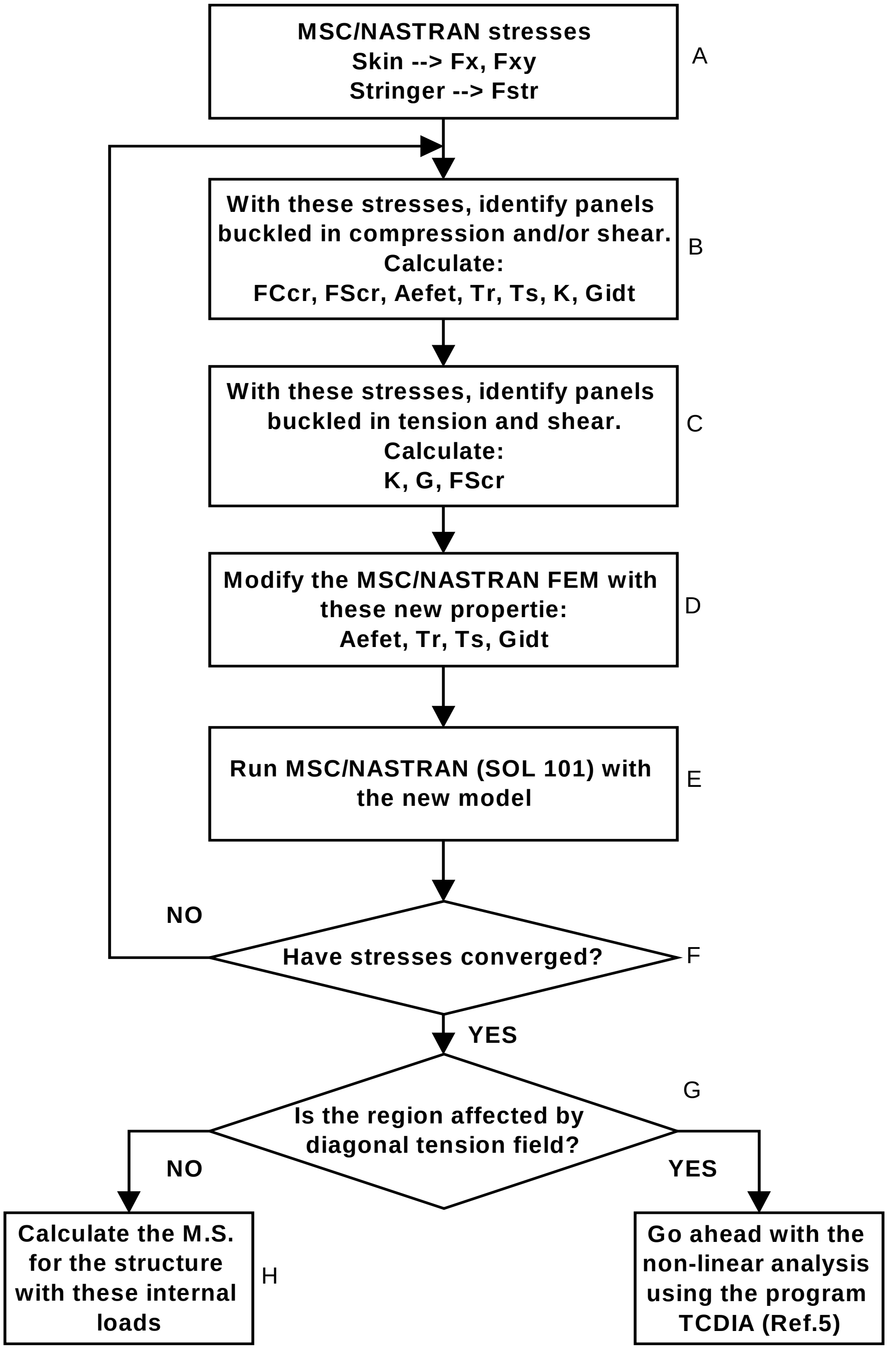

Figure 6

3.2 - LINEAR STATIC ANALYSIS RESULTS.

The analysis method described here is based on the usage of MSC/NASTRAN (SOL101 linear static analysis) with a finite element model where the properties of the buckled panels and their adjacent stringers were modified by the DIAG2 program. This program gets the stress results from MSC/NASTRAN linear analysis punch file (.pch) and calculates the modifications of the stringer areas, thickness and material properties of the sheet elements, affected by the buckling of the panels, and also calculates the radial loads at the frames of the buckled panels. The program also generates the MSC/NASTRAN cards with these modifications and calculates some data to be used at the second part of the non-linear analysis (Ref. 5).

In order to verify if a panel has buckled or not, only the axial stress parallel to the flight direction and the shear stress in the element local coordinate system will be used (therefore, the shell elements must be oriented with the local X axis aligned to the flight direction).

At the first iteration, when the panel is composed only by QUAD4 and/or TRIA3 elements, the working stress used will be the average of the stresses of MSC/NASTRAN elements that compose the panel.

For the second and next iterations, when the panel has been modified and SHEAR elements and TRIA3 elements were introduced to simulate the post buckling shear strength, and the original QUAD4 and TRIA3 elements have their material and geometric properties modified to simulate the residual axial strength of the panel (we will show further on, how this modifications are done), the working stress used will be a weighted average based on the thickness of the MSC/NASTRAN shell elements that compose the panel.

3.3 - BUCKLING CRITERIA AND ALLOWABLE STRESSES FOR SKIN PANELS SUBJECTED TO COMBINED LOADS

3.3.1 - PANEL UNDER COMPRESSION AND SHEAR (REF. 2)

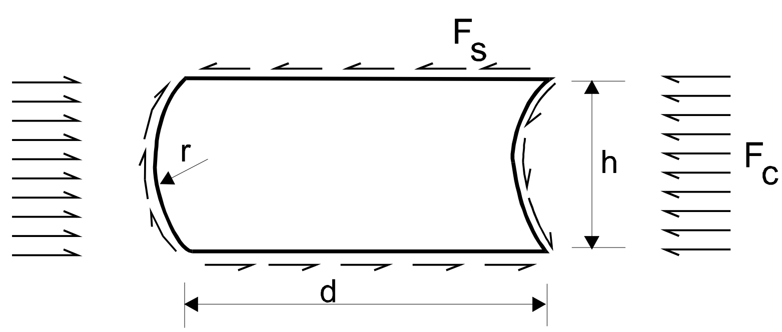

Figure 7 - Curved panel subject to combined compression-shear loading

$$\left(\frac{F_C}{F_{C_{cr0}}}\right)+\left(\frac{F_S}{F_{S_{cr0}}}\right)^2=B$$

if $B \geq 1$ the panel is buckling.

Now for any particular panel ($F_{C_{cr0}}$, $F_{S_{cr0}}$ from Ref. 6)

$$\frac{F_{S_{cr0}}}{F_{C_{cr0}}}=const=A \quad \Rightarrow \quad F_{S_{cr0}}=A\times F_{C_{cr0}}$$

The stresses F_S and F_C will bear a constant ratio to each other until buckling occurs, after which the compression stress no longer increases, thus we can write:

$$\frac{F_S}{F_C} = const = B$$

$$F_S = B \times F_C$$

For convenience of notation, there are also the iteration factors

$$R_C = \frac{F_{C_{cr}}}{F_{C_{cr0}}} \quad \text{and} \quad R_T = \frac{F_{S_{cr}}}{F_{S_{cr0}}}$$

Now using the buckling equation and the relations above, the iteration factor can be written in the form:

$$R_T = \frac{-A}{2B} + \sqrt{\frac{A^2}{4B^2} + 1} \quad \text{and} \quad R_C = \frac{A}{B} \times R_T$$

So the buckling stress for each panel subject to combined compression and shear will be:

$$F_{C_{cr}} = R_C \times F_{C_{cr0}} \quad \text{and} \quad F_{S_{cr}} = R_T \times F_{S_{cr0}}$$

3.3.2 - PANEL UNDER TENSION AND SHEAR (REF. 2)

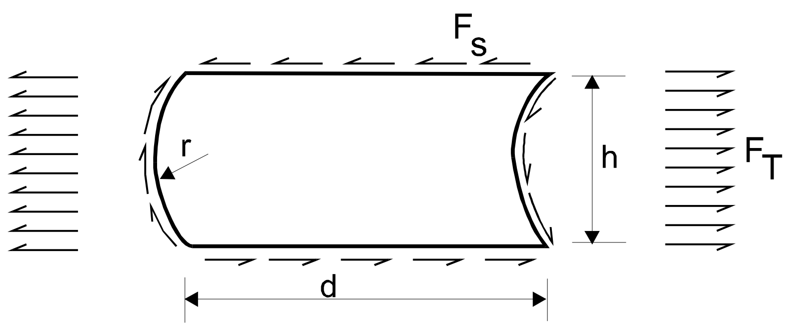

Figure 8 - Curved panel subject to combined tension-shear loading

$$\frac{F_S}{F_{S_{cr0}}} - \frac{1}{2} \times \frac{F_T}{F_{C_{cr0}}} = B$$

if $B \geq 1$ the panel is buckling.

So,

$$F_{S_{cr}} = \left(1 + \frac{1}{2} \times \frac{F_T}{F_{C_{cr0}}}\right) \times F_{S_{cr0}}$$

3.4 - FEM MODEL CHANGES

3.4.1 - EFFECTIVE AREAS TO BE ADDED TO THE STRINGER ADJACENT TO THE PANELS, BUCKLED IN COMPRESSION AND/OR SHEAR. (REF. 2)

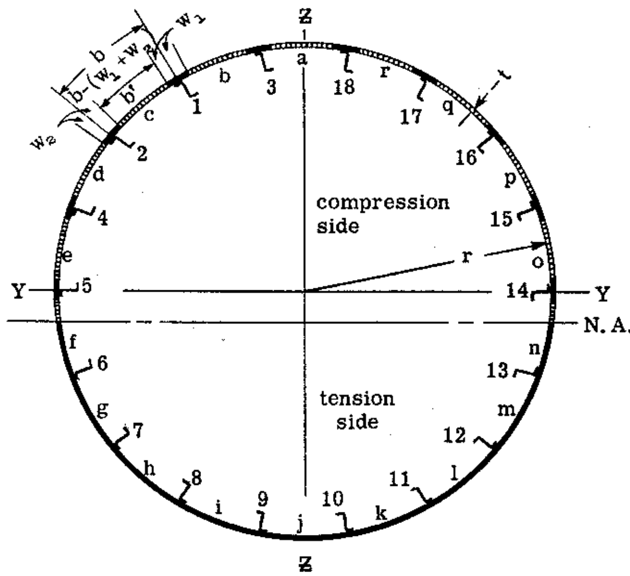

Figure 9 - Fuselage cross section with effective width representation.

When we verify the cross section of a fuselage (*Figure 9*), the same concept previously shown by *Figure 2*, can be applied.

$$A_{efet} = h \times t \times \eta$$

$$\eta = 0.894 \times R_C \times \sqrt{\frac{F_{C_{cr}}}{F_{Str}}} \qquad 0 \le \eta \le 1$$

Note that if $F_S = 0$, we have $R_C = 1$, and $F_{C_{cr}} = F_{C_{cr0}}$, and for a flat panel

$$F_{C_{cr0}} = \frac{K_C \times \pi^2 \times E}{12(1-\nu^2)} \times \left(\frac{t}{h}\right)^2$$

using $K_C = 4$ (simply supported plate, $h/d \ge 2$) and $\nu = 0.3$ we have

$$F_C = 3.61 \times E \times \left(\frac{t}{h}\right)^2$$

So,

$$\eta = 0.894 \times 1.90 \times \sqrt{\frac{E \times t^2}{F_{Str} \times h^2}} \quad \Rightarrow \quad \eta = 1.7 \times \frac{t}{h}\sqrt{\frac{E}{F_{Str}}}$$

and, $$A_{efet} = h \times t \times 1.7 \times \frac{t}{h} \times \sqrt{\frac{E}{F_{Str}}} \quad \Rightarrow \quad A_{efet} = 1.7 \times t^2 \times \sqrt{\frac{E}{F_{Str}}}$$

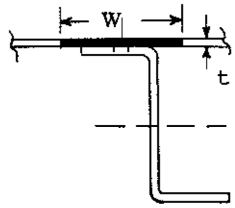

Figure 10 - Effective width representation

$$A_{efet} = w \times t$$

where

$$w = 1.7 \times t \times \sqrt{\frac{E}{F_{Str}}}$$

as we can see this is the expression used for effective width calculation of flat panel in compression in Ref. 3.

One half of the effective area of each panel must be added to each one adjacent stringers of the panel.

3.4.1 - RESIDUAL THICKNESS AND SHEAR THICKNESS

When the panel has buckled, the original properties of the shell elements must be modified. The original QUAD4 or TRIA3 elements have their compressive effectiveness reduced due to the buckling, therefore their thicknesses are changed to a calculated residual thickness.

Another element to simulate the shear strength is added for each QUAD4 and TRIA3, a SHEAR and a TRIA3 with modified material property, respectively.

Figure 11 shows the case for quadrilateral elements and *Figure 12* shows the case for triangular elements.

Until Version 69 of MSC/NASTRAN the QUAD4 and the SHEAR elements could have the same identification number. However, starting with Version 70, the identification of the elements must be unique.

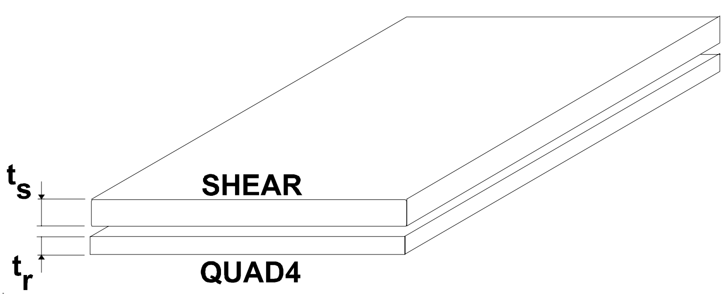

Figure 11 - SHEAR and QUAD4 elements

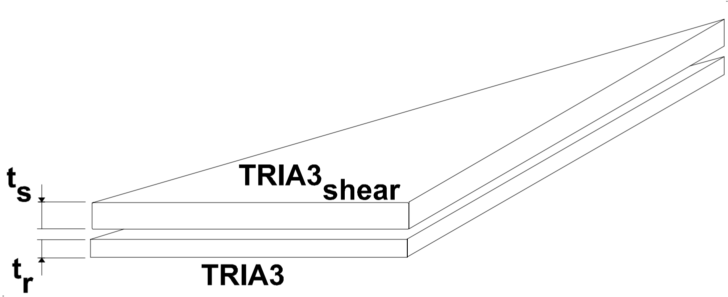

Figure 12 - TRIA3(shear) and TRIA3 elements

3.4.1.1 - THE RESIDUAL THICKNESS

Residual thickness is the thickness that must be supplied for the elements QUAD4 and/or TRIA3 of each buckled panel in place of the thickness that they had. The purpose of this is to simulate at the model, the presence of the buckled panel in compression. So this thickness multiplied by the panel width and by the axial stress that it sustain, must give us approximately the compression load necessary to maintain the panel buckled.

So,

$$t_R = t \times \left(\frac{F_{Cr}}{F_C} \right) \times \left(\frac{h - \frac{A_{efet}}{t}}{h} \right)$$

3.4.1.2 - THE SHEAR THICKNESS

The shear thickness is the thickness that must be supplied for the SHEAR elements that must be added to the panel, in addition to the QUAD4 and/or TRIA3 elements already existing, in order to simulate at the model, the actual shear thickness of the panel. So, the shear thickness must be the complement of the residual thickness to the actual thickness.

The stiffness of the buckled panel in shear will be simulated by the modification of the shear modulus as explained at the item 3.4.3. So,

$$t_S = t - t_R$$

Note: where we have TRIA3 element and SHEAR must be added, the simulation of this SHEAR must be made adding a TRIA3 element with a Poisson Ratio of -0.95 without the Modulus of Elasticity (E) and supplying the Shear Modulus as defined at the item 3.4.3, so this element will have the following Modulus of Elasticity calculated by MSC/NASTRAN program:

$$E = G \times 2(1 + \nu) = G \times 2(1 - 0.95)$$

$$E = 0.1 \times G$$

3.4.2 - THE DIAGONAL TENSION FACTOR (REF. 2)

This factors give us the feeling of the intensity of Diagonal Tension Field.

So, if $K = 0$, the panel didn´t buckle, and if $K = 1$, the panel is in pure Diagonal Tension Field.

$$K = \tanh\left[\left(0.5 + 300 \times \frac{t \times d}{r \times h} \right) \times \log_{10} \left| \frac{F_S}{F_{S_{cr}}} \right| \right], \text{ for } d > h$$

if $d < h \Rightarrow$ use $\frac{h}{d}$, instead of $\frac{d}{h}$

if $\frac{h}{d}$ or $\frac{d}{h} > 2 \Rightarrow$ use: $\frac{h}{d} = 2$, or $\frac{d}{h} = 2$

3.4.3 - THE REDUCED SHEAR STIFFNESS MODULUS (REF. 2)

We have seen that if a panel buckles in compression and/or shear, it does not sustain more compression load, so its effective area is added to its adjacent stringers and a residual thickness is maintained at the QUAD4 and/or TRIA3 elements that compose the panel in order to sustain the compressive buckling load that maintain the panel buckled. But the panel still resists the increase of the shear loads with the same thickness (complementary thickness is given by SHEAR elements that must be added to the buckled panels), but with a reduced shear modulus (G_{idt}) to simulate the stiffness of the buckled panel. If the panel buckles in tension and shear, it still supports an increase of the tension and the shear loads with the same thickness, but with a reduced shear modulus.

So, when the panel buckles it has its stiffness reduced. The stiffness reduction that affect the axial stress was simulated by using the concept of effective width as explained at the item 3.4.1, but this axial area modification don't reduce the shear stiffness since as we see at item 3.4.1.2 the shear thickness is maintained. So to consider this reduction in shear stiffness, the Shear Modulus of the buckled panel must be reduced using the expression bellow:

$$\frac{1}{G_{IDT}} = \frac{1-K}{G} + \frac{K}{G_{DT}}$$

$$\frac{E}{G_{DT}} = \frac{4}{sin^2 2\alpha} + \frac{\tan^2 \alpha}{\frac{A_{Fr}}{(d \times t)} + 0.5 \times (1-K)} + \frac{\cot an^2 \alpha}{\frac{A_{Str}}{(h \times t)} + 0.5 \times (1-K)}$$

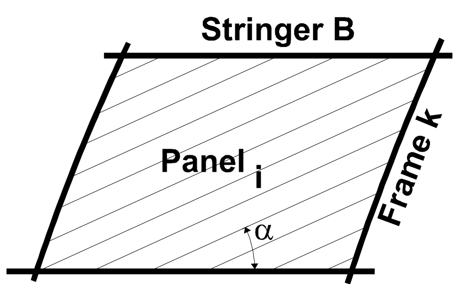

Figure 13 - Diagonal tension field angle

The diagonal tension field angle is assumed equal to 30° (*Figure 13*).

Note: The reduced stiffness will be calculated at Ref. 5 using the actual angle value, and can be corrected if necessary.

3.4.4 - THE RADIAL LOAD AT THE FRAMES (REF. 3)

As the fuselage panels are curved, when they work in diagonal tension, there is a trend of the panels to transform in a flat panel, and this is prevented by the frames that support the radial loads. This loads must be calculated by the following expression:

$$P_{RG} = \frac{F_S \times d \times t \times K \times \tan \alpha}{2 \times r}$$

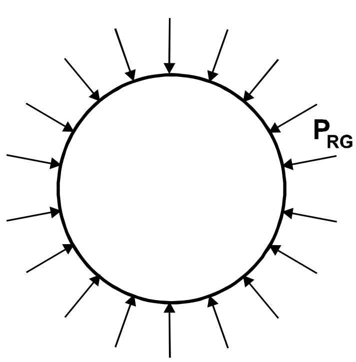

Figure 14 - Frame radial load

The diagonal tension field angle is assumed equal to 30° (*Figure 13*).

This expression is valid only for the configuration of the frame attached to the skin. For floating frames Ref. 3 must be checked.

Note: This load will be calculated at Ref. 5 using the actual angle value, and can be corrected if necessary.

4 - DISCUSSIONS

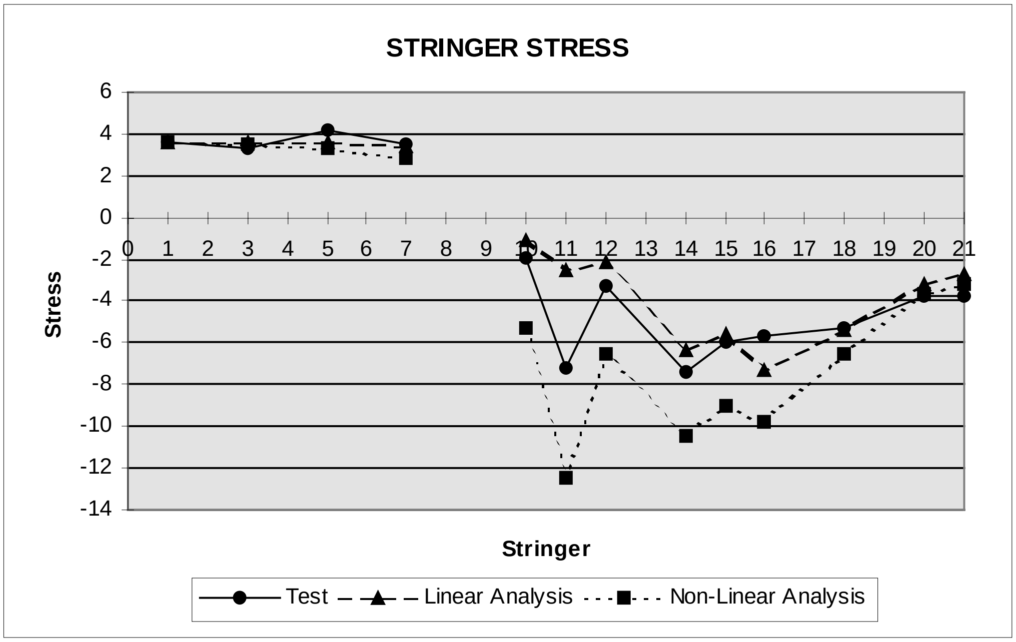

Figure 15

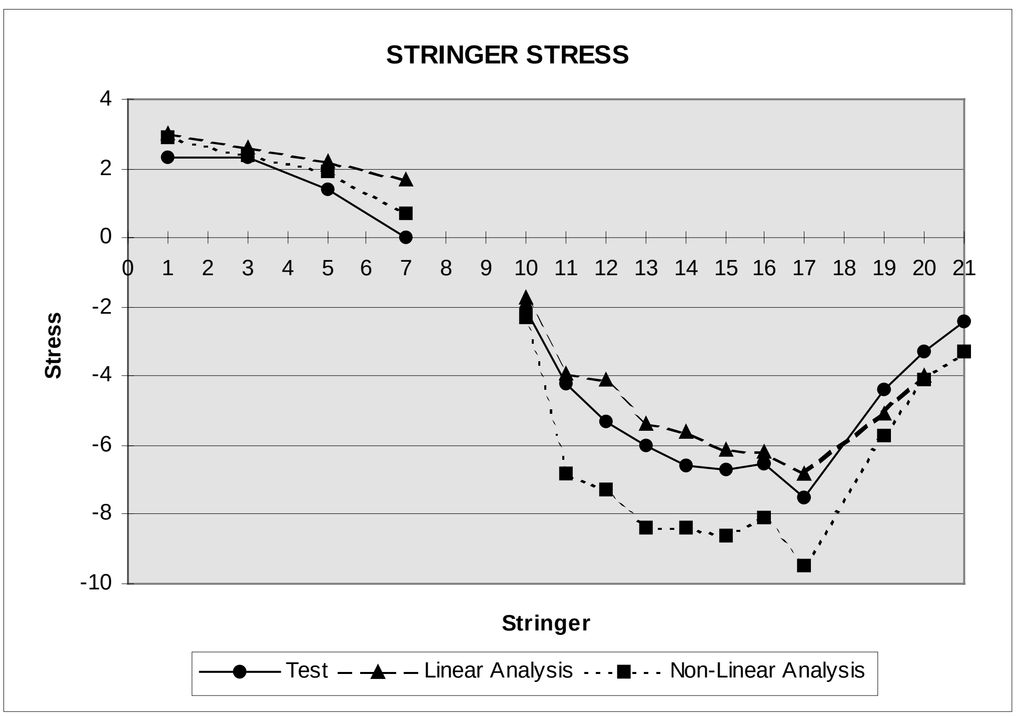

Figure 16

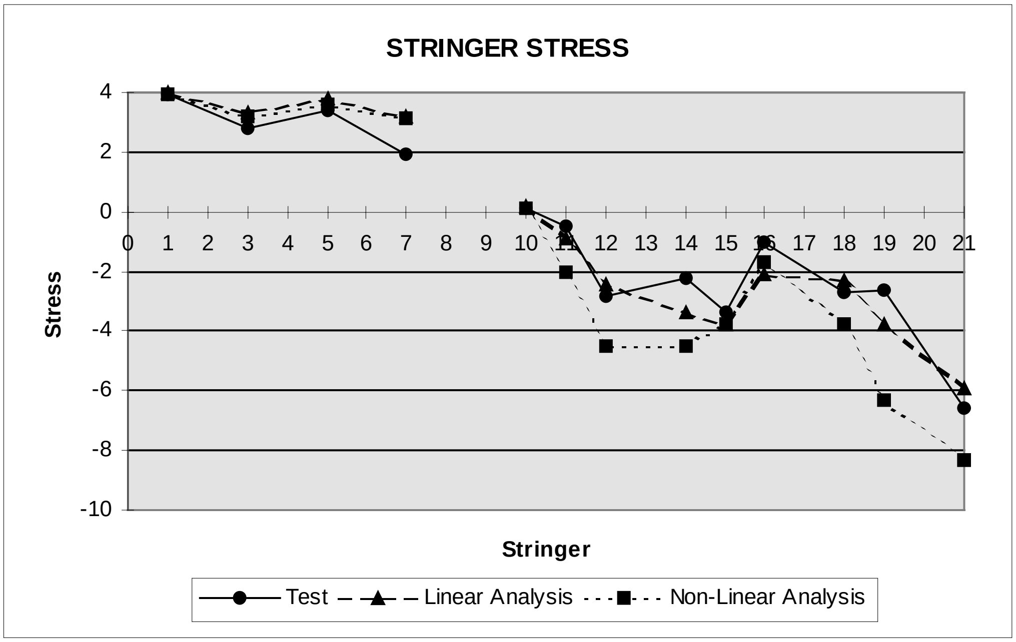

Figure 17

Figures 15 to 17 show some actual results, comparing the axial stress on the stringers across the section of the fuselage. The solid line curve is for the test stresses obtained from strain gage measurements. The long dashed curve is obtained from the linear static analysis model run of SOL 101 of MSC/NASTRAN. The short dashed curve is obtained also running SOL 101 and using the method described herein to consider the non-linear effects.

As we can see from the comparison between the non-linear analysis and the test results, the analysis has a good correlation with the test results and is basically conservative. It is important to note that the behavior between the non-linear analysis and the test is the same even when the compressive stress is reduced by buckling as we can see on *Figure 17* for the stringer 16.

5 FUTURE ENHANCEMENTS AND DEVELOPMENTS

We need to make:

1) An enhancement due to the restriction to use elements with the same identification number introduced with Version 70;

2) The development of a graphical interface to generate de panel identification for the DIAG2 program (Ref.4b) and TCDIA program (Ref.5);

3) The development of a graphical interface to analyze the DIAG2 program results and the results of the TCDIA program.

6 - LIST OF SYMBOLS

A_{Fr}	Area of the frame	t_S	Shear thickness of the skin that resists shear load
A_{Str}	Area of the stringer	w	Effective width of a buckled panel under compression
A_{efet}	Effective area of a buckled panel under compression	F_C	Compression stress at the skin
E	Modulus of elasticity	F_S	Shear stress at the skin
G	Modulus of rigidity (shear modulus)	F_T	Tension Stress at the skin
h	Length of the panel	F_{Str}	Axial stress at the stringer
d	Width of the panel	F_x	Axial stress at the skin parallel to the element local axis
r	Radius of the panel	F_{xy}	Shear stress at the skin parallel to the element local axis
t	Actual skin thickness	$F_{C_{cr0}}$	Compressive buckling stress for pure compression
t_R	Residual thickness of the skin that still resists compression and shear loads	$F_{S_{cr0}}$	Shear buckling stress for pure shear

7 - REFERENCES

MSC PRODUCTS

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GENERAL REPORTS

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(4b) Klaus Otto Schwarzmeier, “Program to Calculate parameters for Geometric Non-llinear Analysis from MSC/NASTRAN Results”. User´s Manual Vol II/II - EMBRAER Technical report 000-SO-019,

(5) L. F. D. Freitas, “Program to Calculate the Diagonal Tension effects on Stringers, Frames and Skins” Users´ Manual - EMBRAER Technical report 000-SO-022 Vol II/II

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