

# **Use of DSA analysis results for Ritz fast analysis of perturbed models**

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## **ABSTRACT**

Finite element analysts often have to perform analysis of perturbed models in order to validate the dimensioning through sensitivity analysis. This is also the case during Finite Element model updating process or for stochastic dynamic analysis where we want to obtain a statistical description of a solution, provided the statistical distributions of some design driving physical parameters. These analyses often lead to a huge amount of calculations if the analysis is performed using a loop inside a Finite Element software.

CNES has developed a fast dynamic reanalysis method based on a Ritz formulation using the results of a Design Sensitivity Analysis performed with MSC/NASTRAN in order to introduce the variation of some physical parameters in the analysis. This paper presents the basic developments and implementation of this method.

## 1 INTRODUCTION

Finite element analysts often have to perform analysis of perturbed models in order to validate the dimensioning through sensitivity analysis. During Finite Element model updating process, there is generally a dynamic analysis to be performed for each set of physical parameters tested as a converged solution during the process.

This can be also the case for stochastic dynamic analysis where we want to obtain a statistical description of a solution, given the statistical distributions of some design driving physical parameters. Then, for the different methods used for this kind of analysis (direct statistical analysis, Monte-Carlo or Fuzzy numbers method), this leads to a huge amount of calculations if the analysis is performed using a loop inside a Finite Element software.

Working with a fast dynamic reanalysis method based on a Ritz formulation using the results of a Design Sensitivity Analysis performed with MSC/NASTRAN in order to introduce the variation of some physical parameters in the analysis can significantly reduce the data manipulation and the analysis time.

The paper will present the formulation of the method and its use for FEM updating and for a stochastic analysis performed to study the stability of the line of sight of an earth observation satellite. Performance will be discussed through results presentation.

## 2 NOTATIONS

$[M]$	Mass matrix of the system
$[C]$	Damping matrix of the system
$[K]$	Stiffness matrix of the system
$F$	Vector of the applied loads
$\ddot{x}, \dot{x}, x$	Vectors of respectively dof accelerations, velocity and displacement
$[\varphi]$	Eigen vector of the system
$\omega, f$	Eigen pulsation and frequency
$\eta$	Modal coordinate
$\Delta K, \Delta M$	Matrices of sensibility versus perturbed parameter $p$ associated to stiffness and mass
	$\Delta K = \frac{\partial[K]}{\partial p}$ , $\Delta M = \frac{\partial[M]}{\partial p}$
$p$	Perturbed parameters (Young modulus, mass density, CBAR area or inertia...)
$[I]$	Identity matrix

## 3 BASIC DEVELOPMENT

The basic idea of the presented formulation is to write all the equations in the modal space

$$[M]\ddot{x} + [C]\dot{x} + [K]x = F \quad 1)$$

which becomes by projection

$$[\varphi]^T [M][\varphi]\dot{\eta} + [\varphi]^T [C][\varphi]\dot{\eta} + [\varphi]^T [K][\varphi]\eta = [\varphi]^T F \quad 2)$$

That means that the motion of a structure can be represented as the sum of the different contributions of each of its eigen-mode [(see figure (1))].

Considered this equation in the frequency domain without damping and external load, it can be re-written as :

$$([\varphi]^T [K][\varphi] - \lambda.[\varphi]^T [M][\varphi])\eta = 0 \quad 3)$$

which allows the computation of the eigenvalues [ $\lambda = \omega^2 = (2\pi f)^2$ ] and eigenvectors [ $\eta$  is the unity matrix in this case].

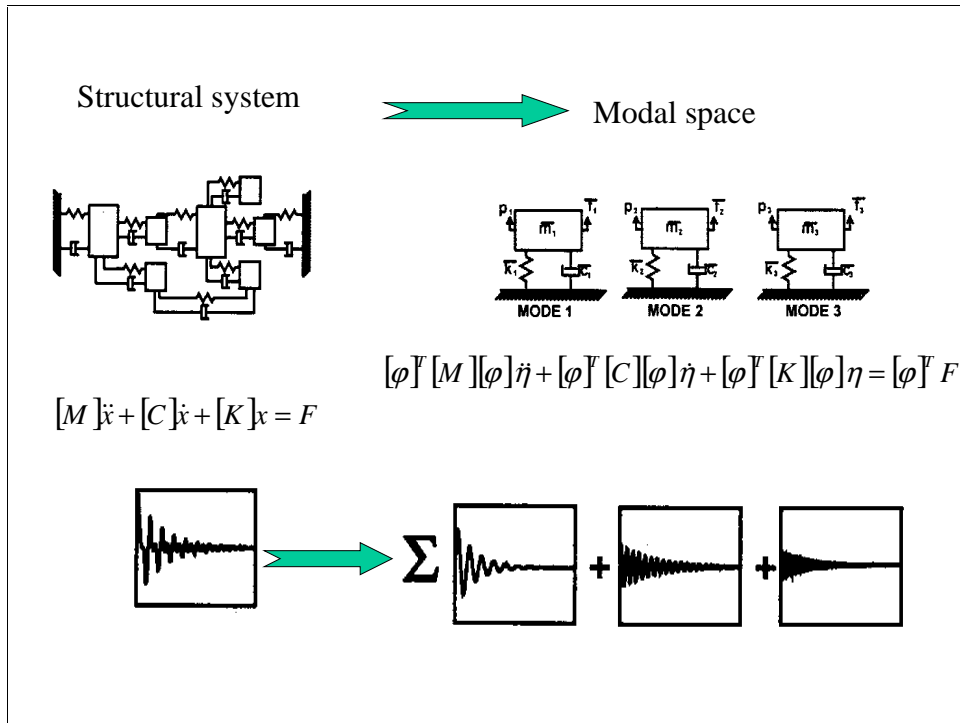


Figure (1) - Physical system versus Modal space system

The basic idea of our development is to use an approximation of the eigenvalues equation of the perturbed model (versus  $p$  parameters) through its projection into the modal space associated to the original model. Equation 3) becomes :

$$[[\varphi]^T [K(p) - \lambda.M(p)][\varphi]]\eta = [[\varphi]^T [K(0) + p.\Delta K [\varphi] - \lambda[\varphi]^T [M(0) + p.\Delta M [\varphi]]]\eta = 0 \quad 4)$$

or

$$[[\varphi]^T [K(0)[\varphi] + p.[\varphi]^T [\Delta K [\varphi]] - \lambda[\varphi]^T [M(0)[\varphi] + p.[\varphi]^T [\Delta M [\varphi]]]\eta = 0 \quad 5)$$

$$[[diag(\omega^2) + p.[\varphi]^T [\Delta K [\varphi]] - \lambda[I] + p.[\varphi]^T [\Delta M [\varphi]]]\eta = 0 \quad 6)$$

This equation can easily be solved with little effort considering the calculations needs.  $\lambda$  gives the perturbed model eigenfrequencies and the new eigenshapes are derived from :

$$[\varphi_{new}] = [\varphi] \eta \quad 7)$$

All the information associated to the original model and to the sensitivity analysis can be obtained through a DSA analysis with a very simple DMAP script which is provided in annex :

- $\omega^2$  LAMBDA
- $[\varphi]$  PHG
- $[\varphi]^T [\Delta K] [\varphi]$  SK
- $[\varphi]^T [\Delta M] [\varphi]$  SM

All the matrices are output in OUTPUT4 ASCII format in order to enable their transfer and manipulation on other machines than the one on which MSC/NASTRAN is used.

## 4 DISPLACEMENT FREQUENCY TRANSFER FUNCTIONS IN THE MODAL SPACE

Re-starting from equation 2) and using Basile's hypotheses on dynamic equations non-coupling, we obtain :

$$\begin{aligned} [\varphi]^T [M] [\varphi] &= I_N \\ [\varphi]^T [C] [\varphi] &= \text{diag}(2\xi_k \omega_k) \quad 8) \\ [\varphi]^T [K] [\varphi] &= \text{diag}(\omega_k^2) \end{aligned}$$

and for the equation associated to mode k :

$$\ddot{\eta} + 2 \xi_k \omega_k \dot{\eta} + \omega_k^2 \eta = \varphi^{kT} F \quad 9)$$

or, in the frequency domain :

$$\eta_k = \frac{1}{[-\omega^2 + 2i \xi_k \omega_k \omega + \omega_k^2]} \varphi_j^k F_j \quad 10)$$

This relation provides then the generalized answer at the degree of freedom (i) associated to the excitation at (j) (Displacement Frequency Response Function)

$$x_i = \sum_k \frac{1}{[-\omega^2 + 2i \xi_k \omega_k \omega + \omega_k^2]} \varphi_i^k \varphi_j^k F_j \quad 11)$$

And for the FRF :

$$H_{ij}(\omega) = \sum_k \frac{1}{[-\omega^2 + 2i \xi_k \omega_k \omega + \omega_k^2]} \varphi_i^k \varphi_j^k \quad (12)$$

All the information extracted as explained in the previous chapter is used to determine this FRF. The « set » list is printed during the MSC/NASTRAN analysis (PARAM,USETPRT,11) in order to find the right degrees of freedom to compute the transfer. Special care must be given to the local coordinate systems to use the good orientations.

## 5 USE OF THE SIMPLIFIED CALCULATIONS

This simplified formulation has been successfully used at CNES for two kinds of analysis : correction loops during Finite Element Model Updating as explained in [ref 1 and 2] and stochastic dynamic analysis during stability budget calculation for the line of sight of an earth observation satellite.

### 5.1 FINITE ELEMENT MODEL UPDATING

A first application of the simplified perturbed model analysis has been presented in [ref. 1]. This process was included in a Finite Element Model-updating scheme that requires lots of re-analysis during the correction phase as shown in figure (2).

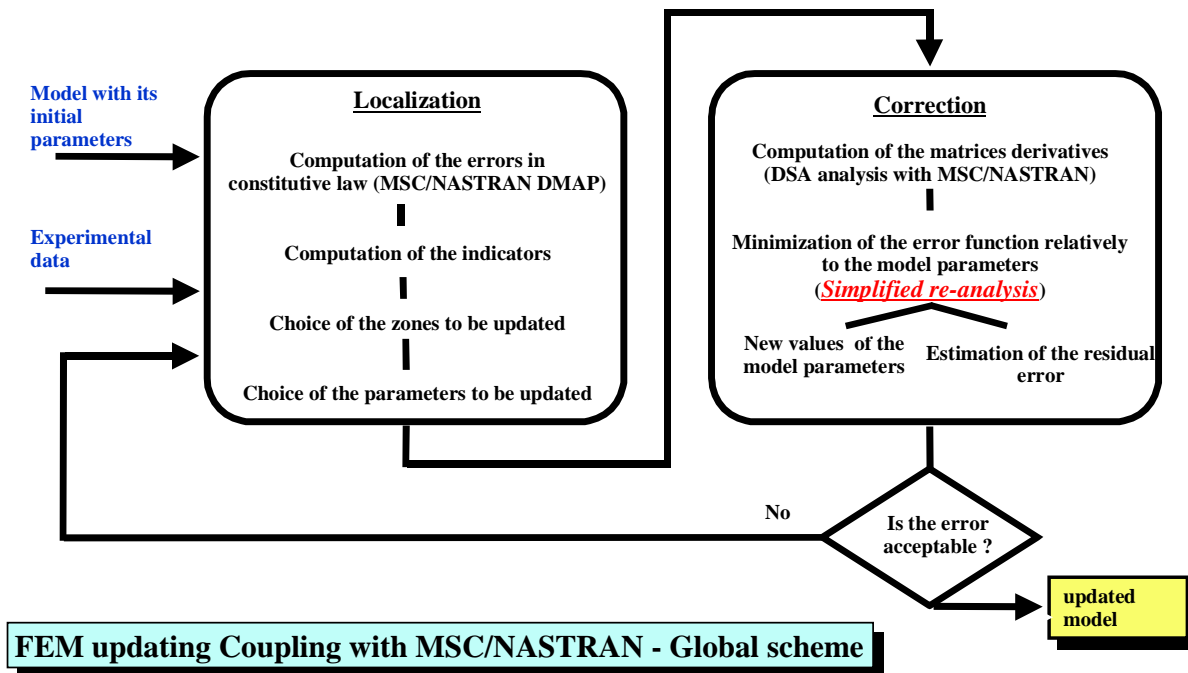


Figure (2) - FEM updating global scheme

## 5.2 STOCHASTIC DYNAMIC ANALYSIS

Stochastic dynamic analysis is often used to manage the dimensioning margins or to compute realistic envelopes for specific behavior characteristics (line of sight oscillations...).

One solution to perform the stochastic analysis is to choose sets of physical parameters following a probabilistic law and to perform a MSC/NASTRAN analysis for each set. This process is very time and computing resources consuming.

Another solution is the one-presented hereafter in paragraph 6.2. Main information concerning the model is extracted during a DSA analysis and is then used outside MSC/NASTRAN to compute the behavior of the perturbed models as explained in paragraphs 3 and 4. Other FRF than the force/displacement responses can also be computed using similar processes. This procedure is valid if the perturbed parameters are not too far from the original ones and if the variations can still be considered as linear.

## 6 EXAMPLES

### 6.1 MAQSAT 3

All the process described here before can be illustrated with calculations performed on the finite element model of the dummy satellite « MAQSAT 3 » that has been used for ARIANE 5 third qualification flight.



Figure (3) - MAQSAT 3 satellite

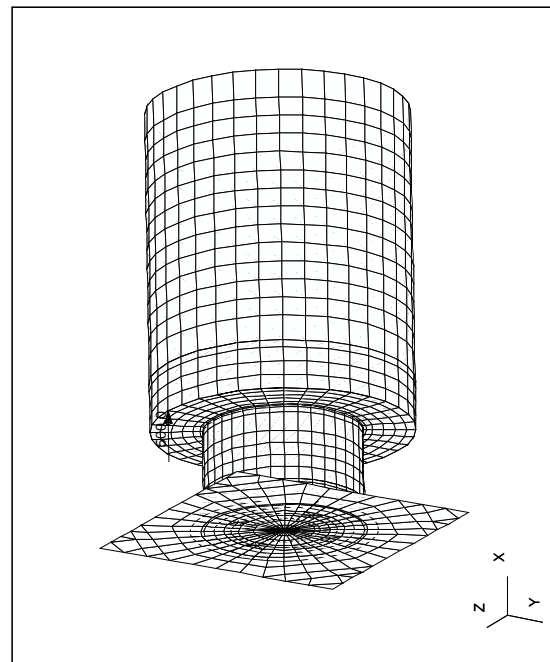


Figure (4) - MAQSAT 3 FEM

The model comprises 2972 elements and 13471 degrees of freedom (A-set).

The perturbation in the model has been introduced in the PSHELL property card associated to the lower annular plate which has been passed from a thickness of 0.034 m to 0.04 m (increase of 17.65%).

Table (1) shows the 30 first eigenfrequencies that have been successively computed with MSC/NASTRAN and the full model (fmaq2) and then with the simplified tool (fmaq2 simplified).

Figure (5) shows the FRF between an excitation point located on the annular plate at the basis of MAQSAT 3 (point 202) and a point located at the top of the structure (point 1419) and the auto FRF at point 202. The FRF computed with the simplified method and during a MSC/NASTRAN harmonic response are compared. No significant differences can be noticed. A comparison with the FRF computed during a direct response analysis shows some discrepancies due to the truncation of the modal basis.

The implementation of RESIDUAL VECTORS complement (param,resvec,yes) using 9 vectors during the DSA analysis allows the vanishing of this problem as shown on figure (6) where the FRF computed with the simplified method are compared with the same FRF computed during a direct response analysis.

f maq2 (Hz)	18,29	18,31	51,47	64,71	64,71	65,11	65,34	76,01	93,59	93,59
f maq2 (hz) simplified	18,18	18,20	51,34	64,69	64,69	65,03	65,26	76,03	93,59	93,59
$\Delta f/f$	0,62%	0,62%	0,25%	0,02%	0,02%	0,13%	0,13%	-0,03%	0,00%	0,00%

f maq2 (Hz)	96,32	96,32	101,26	101,26	109,84	109,84	117,01	117,01	129,39	129,39
f maq2 (hz) simplified	96,32	96,32	101,26	101,26	109,84	109,84	117,01	117,01	129,38	129,38
$\Delta f/f$	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

f maq2 (Hz)	129,84	129,84	138,53	138,54	152,70	157,51	161,75	161,75	161,81	161,81
f maq2 (hz) simplified	129,82	129,82	138,53	138,54	152,74	157,49	161,76	161,76	161,81	161,81
$\Delta f/f$	0,02%	0,02%	0,00%	0,00%	-0,03%	0,01%	0,00%	0,00%	0,00%	0,00%

Table (1) - perturbed MAQSAT 3 satellite 30 first eigenfrequencies

A brief comparison can be made for the CPU time for each type of analysis for the FRF computation :

- Direct frequency response analysis : 3789.4 sec.
- Harmonic frequency response analysis : 83.4 sec.
- Simplified analysis : ~2 sec.

Modified MAQSAT - Harmonic response under FX202=500. N

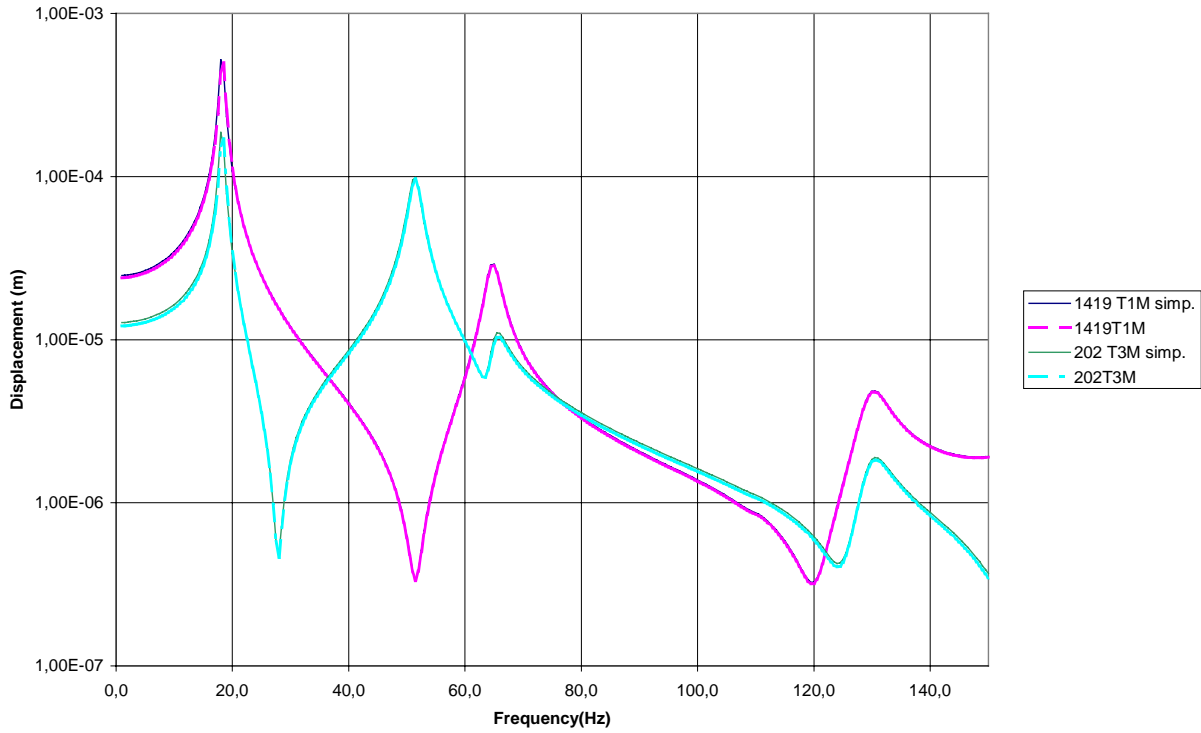


Figure (5) - MAQSAT 3 perturbed model FRF

MAQSAT modified - Direct response and simplified simulated response with residual vectors

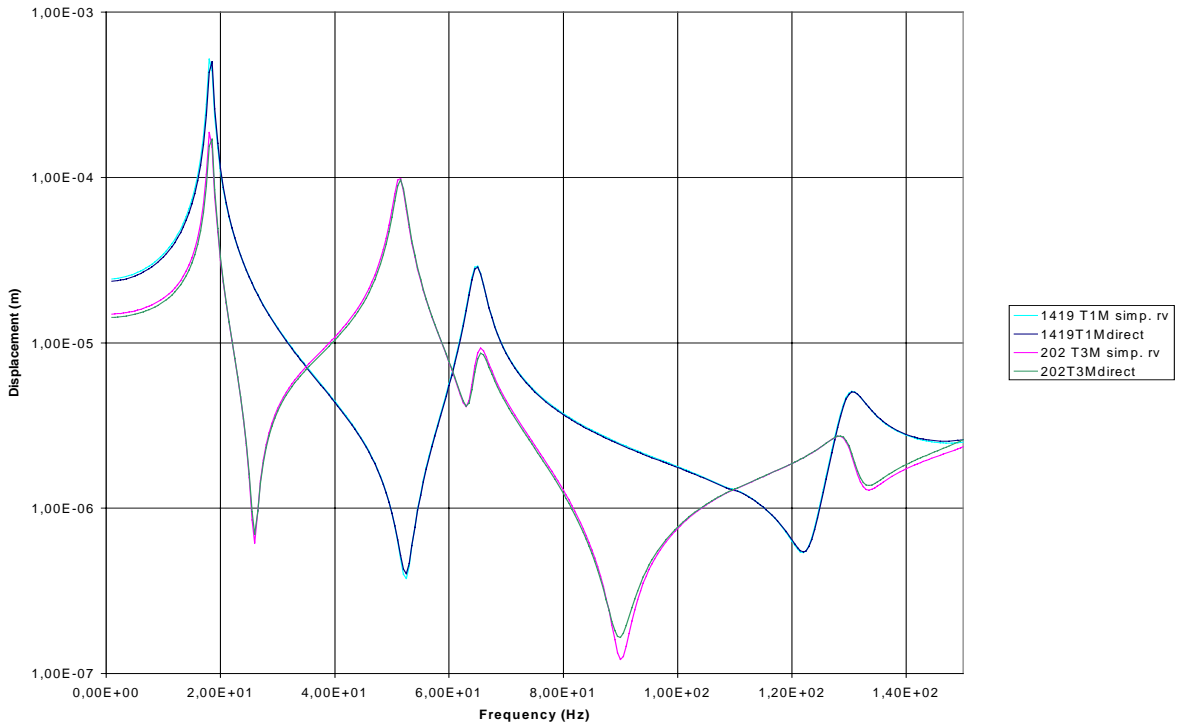


Figure (6) - MAQSAT 3 perturbed model FRF computed with 9 residual vectors

## 6.2 LINE OF SIGHT STABILITY ANALYSIS



Figure (7) - One of the various configurations studied for 3S satellite

The stability analysis of the line of sight of the earth observation satellite during operation requires the computation of the transfer function between the on-board perturbators (reaction wheels, rotation mechanisms...) and the line of sight rotation angle.

In order to establish a realistic perturbation budget for image quality analysis, a stochastic analysis can be performed using the most uncertain parameters values acting on the finite element model behavior.

After the choice of these parameters, a Monte-Carlo trial is performed and for each set of parameters, the FRF is computed as written in equation 12). The obtained envelope is then used to determine the stability budget.

Figure (8) presents the FRF envelope that has been computed for a variation of 4 parameters (instrument truss section, interface bar section and thickness of reaction wheel support plate).

The advantage of this technique is that only one MSC/NASTRAN run is mandatory instead of one run for each set of parameters. That means a non negligible gain of time for stochastic analysis.

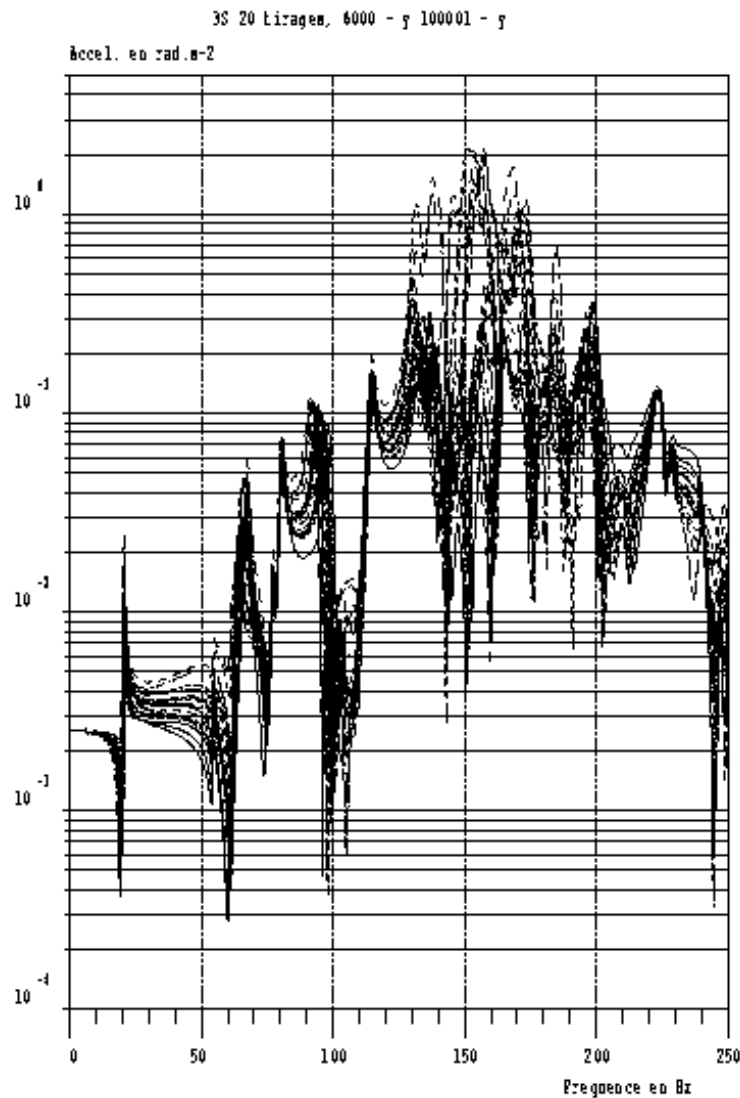


Figure (8) - 3S satellite reaction wheel/line of sight FRF envelope

## 7 CONCLUSION

This paper has presented a simplified approached technique for a fast re-analysis of perturbed model using information obtained from a DSA analysis during a MSC/NASTRAN run.

This process has been used for model updating and stochastic dynamic analysis as presented in paragraph 6.2 for 3S satellite line of sight oscillations. It allows a non-negligible improvement concerning computation time as presented in paragraph 6.1 for MAQSAT 3 analysis.

## 8 REFERENCES

- (1) Bricout J.N., « A Finite Element Model Updating Method used with MSC/NASTRAN », Proceedings of MSC european users'conference, Vienna (A), September 1993
- (2) Feuarent V., « Amélioration des modèles par recalage : Application aux structures spatiales », Ph. D thesis, Ecole Normale de Cachan / Université de Paris VI (F), 1997.

