# VALIDATION AND UPDATING OF AN AERO ENGINE FINITE ELEMENT MODEL BASED ON STATIC STIFFNESS TESTING

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## ABSTRACT

Today's aerospace industry uses finite element analysis in a huge variety of applications in order to optimize structures and processes before hardware is procured. Efficiencies can be enhanced and margins are reduced because external loads and the structural properties are identified with higher confidence. The accuracy of finite element analysis predictions therefore becomes more and more important and directly influences the competitiveness of the product on the market.

In particular, accurate shell element models are difficult to generate because of the inability to account for fillet radii and problems with coupling of the in-plane rotational stiffness. Another common uncertainty is the accuracy of modeling shell structure bolted flanges and especially their behaviour under large loads. These features are widely used for structural finite element modeling within the aeronautic industry.

Although modal testing has proved to provide valuable reference data for FE model validation and updating, static stiffness tests have the advantage to allow the application of large forces. In addition, static deformations are independent of the mass parameter.

To process static test data and MSC/NASTRAN analysis results and perform correlation analysis and FE model updating, appropriate methods were developed and implemented into the existing FEMtools software. This paper describes the approach and gives reference to a successful application.

## INTRODUCTION

Finite element models are becoming more and more complex and used in a widening range of applications. Design, meshing, analysis and postprocessing are highly integrated and automated. However, this trend holds some danger as it will be increasingly difficult to satisfy the demand for functionality, performance, reliability, and ease of use.

By systematically comparing the results from analytical and experimental analysis techniques, FE models can be validated and improved so that they can be used with more confidence in further analysis. Making use of different types of test data, a recommended procedure is to use a sequence of analysis in which mass, stiffness, damping and external loading are validated, and if necessary updated. This is illustrated in Figure 1.

In the field of structural dynamics, the Experimental Modal Analysis (EMA) method offers a well-accepted solution to derive the modal parameters of a structure from the Frequency Response Functions (FRF) measurements. FE model updating using modal or FRF data is becoming a routinely used part of the product design and analysis cycle. It has the advantage that in one analysis, information on stiffness, mass and damping is included. The drawback is that it is difficult to separate modelling errors due to stiffness-related variables from errors due to mass-related variables. Damping, in most cases, remains a difficult phenomenon to model.
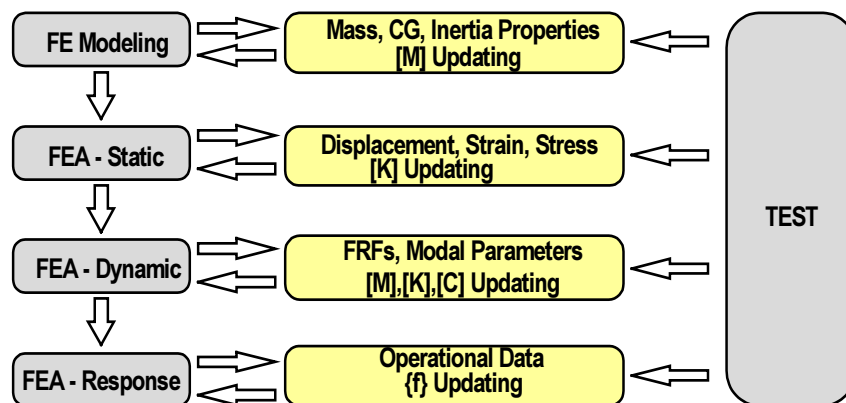


**Figure. 1    Finite Element Model Analysis and Validation Phases**

Using static data simplifies the choice of updating parameters because only stiffness is concerned. Furthermore, numerical processing is simplified because real static deformations are used whereas eigenmodes may be complex. Matching of calculated and measured eigenmodes, often not straightforward because of incomplete test data, is altogether avoided when using static load cases. In addition, static loads can be chosen to stress areas of the structure that are of particular interest and that are difficult to excite or measure using modal testing. The force level that can be applied in general is much higher in static testing than that which is acceptable for dynamic excitation.

The resources required for thorough static testing in general surpass those required for modal testing. Control of loading forces and installation of pickups are limiting factors in the precision of the measurement. Mounting of the test structure on a rigid support is required as a reference for the measured absolute displacements. For the work described in this paper, this requirement, which in practice is extremely difficult to satisfy, was circumvented by using specially developed updating software that accepts relative displacements as updating targets and uses MSC/NASTRAN for static analysis.

## AUTOMATED FE MODEL UPDATING USING STATIC DATA

Updating stiffness modelling using static displacement tests involves minimising the following error function [1] (see list of symbols at the end for an explanation of terms):

$$E = \Delta\varepsilon^t C_R \Delta\varepsilon + \Delta p^t C_p \Delta p \tag{1}$$

with

$$\Delta\varepsilon = \frac{\partial\{U^j\}}{\partial p_k} \Delta p = S_{jk} \Delta p = [S] \Delta p \tag{2}$$

In equation 2, $\Delta\varepsilon$ is the difference between experimental (index e) and analytical (index a) static displacements, at the measured DOFs i, for a number of static load cases j:

$$\Delta\varepsilon = \{U_{ei}^j\} - \{U_{ai}^j\} \tag{3}$$

$\Delta p$ is the difference between updated (index u) and originally estimated (index o) parameter values:

$$\Delta p = p_{ku} - p_{ko} \tag{4}$$

From equations 1-4, the updated parameter values $p_{ku}$ are obtained as

$$\Delta p = \left(2[C_P] + [S]^t [C_R ][S]\right)^{-1} [S]^t [C_R] \Delta\varepsilon \tag{5}$$

$C_R$ and $C_p$ are respectively diagonal weighting matrices for the selected updating targets (static displacements) values and for the updating parameter values. Each weighting value is a measure of confidence in the experimental reference value, respectively in the original parameter estimation.

To compute the displacement sensitivity coefficients $S_{jk}$, the equation of static equilibrium is derived with respect to the updating parameters $p_k$:

$$[K]\{U\} = \{F\} \tag{6}$$

$$[K]\frac{\partial\{U\}}{\partial p_k} = \frac{\partial\{F\}}{\partial p_k} - \frac{\partial[K]}{\partial p_k}\{U\} \tag{7}$$

where

$\frac{\partial\{U\}}{\partial p_k}$ are the unknown displacement sensitivity coefficients.

$\frac{\partial\{F\}}{\partial p_k} = 0$ if the external loads are not dependent on structural properties.

$\frac{\partial[K]}{\partial p_k}$ is the derivative of the system stiffness matrix with respect to the parameter $p_k$. This

derivative can be obtained via a differential or finite difference formulation with small parameter perturbation:

$$\frac{\partial[K]}{\partial p_k} \approx \frac{[K(p_k + \Delta p_k)] - [K(p_k)]}{\Delta p_k} \tag{8}$$

To compute the displacement sensitivities, equation (7) needs to be solved. This solution is computationally identical to solving equation (6), and can be done in MSC/NASTRAN [2] by supplying $-\dfrac{\partial [K]}{\partial p_k}\{U\}$ as the pseudo-load vector. The advantage of this formulation is that sensitivity coefficients are obtained for all DOFs of the FE model and can easily be reduced to only include the measurement DOFs.

## DISCUSSION

The force derivative term in equation 7 is shown for completeness but is assumed to be zero. Except for geometry depending loading, this is a good approximation.

Another important property of equation 7 is that it is assumed that $[\Delta K]\{\Delta U\}$ can be neglected. Indeed, since the force is invariable,

$$([K] + [\Delta K])(U_a + \Delta U) = F = [K]\{U_a\} \tag{9}$$

and thus

$$[K]\{\Delta U\} + [\Delta K](U_a + \Delta U) = 0 \tag{10}$$

Dividing by $\Delta P$ leads to the exact formulation of the computed displacement sensitivities:

$$\frac{[K]\{\Delta U\}}{\Delta P} = -\frac{[\Delta K](U_a + \Delta U)}{\Delta P} \tag{11}$$

Note that $U_a + \Delta U = U_e$ so that the exact displacement sensitivities obtained from equation 11 are in fact pseudo-analytical sensitivities. Unfortunately, the full experimental displacement shapes are usually not available. If $[\Delta K]\{\Delta U\}$ can be neglected, it follows that equation 7 is a first order approximation to the exact solution. As a result, in order to justify the use of this approximation, the differences between the analytical and experimental displacements must be small or that the required modification of the stiffness matrix is small, which means, only small updating parameter modifications per iteration step are allowed.

In case the above conditions are not met, the computed sensitivities will be underestimated or overestimated depending on which is larger: the predicted analytical or experimental displacements. Consequently, the parameter modifications obtained with equation 5 from the overestimated sensitivities will be underestimated, and those from the underestimated sensitivities will be overestimated. This is summarized in the following table.

| Situation | Effect on Differential Sensitivities | Effect on Updated Parameters |
|---|---|---|
| $\{U_a\} >>> \{U_e\}$ | Pseudo-loads (eq.7) underestimated<br>Sensitivities underestimated | Updated parameters<br>overestimated |
| $\{U_a\} <<< \{U_e\}$ | Pseudo-loads (eq.7) overestimated<br>Sensitivities overestimated | Updated parameters<br>underestimated |

## PRACTICAL APPLICATION

The FEMtools software [3] was adapted to support updating using static displacements based on the methods described in the previous section. This required development of data interfaces, correlation analysis tools, an MSC/NASTRAN driver to perform 2 static analysis per iteration (one to compute displacement sensitivities and another to compute displacements after updating the FE model), and the parameter estimation algorithm (equation 5). The procedure is illustrated in figure 2.
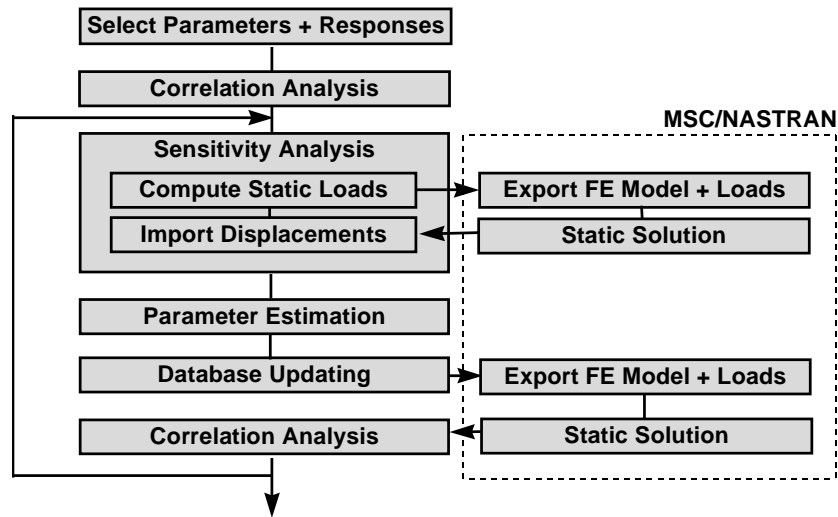
**Figure 2:   Model updating procedure**

FEMtools acts like a pre- and postprocessor to MSC/NASTRAN to perform all database management and analysis except the static solution phase. The user benefits from a dedicated interactive graphical environment to support him in the task of selecting updating targets and parameters which relies mainly on diagnostic tools, engineering judgement and insight in the assumptions and approximations during the modelling phase.

To quantify the correlation between predicted analytical results (FEA) and test, the following criterion is used:

$$\text{DAC}(U_a, U_e) = \frac{\left|\left(\{U_a\}^t\{U_e\}\right)\right|^2}{\left(\{U_a\}^t\{U_a\}\right)\left(\{U_e\}^t\{U_e\}\right)} \tag{12}$$

The Displacement Assurance Criterion (DAC), based on a similar formulation used to correlate mode shapes [4], scales and relates displacement shapes to yield values between 0 and 1. A DAC-value of 1 corresponds with two displacement shapes that are completely identical.

Because first-order sensitivities are used in equation 7, solution of equation 2 is not a one-step operation but requires an iterative procedure like shown in figure 2. For reasons, explained above, it is required to keep parameter changes small with each iteration in order to prevent the error function from oscillating or diverging.

Although displacement sensitivity analysis is available in MSC/NASTRAN using SOL 200, the general sensitivity analysis and updating procedure implemented in FEMtools offers the advantage of flexible selection of updating variables and updating targets. Updating variables like material properties, boundary conditions, spring stiffness and element geometry are selected at the element level or are assigned to groups of elements. Having the possibility to select relative displacements between reference points as updating targets is an important feature because measuring relative displacements is much easier than measuring absolute ones.

## APPLICATION ON A WHOLE ENGINE MODEL (WEM)

The optimisation of the aircraft installed turbomachinery's structural behaviour has a considerable influence on the performance of the whole aircraft. At all modern aeroengine manufacturers the mechanical simulation of this complex system within its flight and landing envelopes is performed using the WEM, a finite element model.

These simulations are significant for the determination of internal load and deformation distributions under static and dynamic loading conditions. Quasi-static loads are applied to simulate e.g. thrust, maneuver and landing conditions. Dynamic loads cover nonlinear transient conditions like bird impact and blade failures. Engine dynamics under windmilling, determination of imbalance induced carcass vibrations and critical speeds are part of the rotordynamic analyses required in an engine certification process.

Additionally the WEM is used for optimisation of design parameters like e.g. tip clearances, which directly influence the efficiency of the turbomachinery. The WEM generally is split into the following subcomponents: engine carcass, HP-rotor, LP-rotor and engine mountings, usually consisting of over 0.5 million degrees of freedom. [5]

It is obvious that the accuracy of the model predictions is essential for the ambitious tasks the finite element model has to fulfill in this design optimisation process. Additionally, in order to reduce the amount of destructive testing, modern aeroengine manufacturers intend to supersede such long lead time and costly approaches by using simulations with validated numerical models.

## EXPERIMENTAL VALIDATION OF ASSEMBLED CASING MODELS

In order to minimise computational requirements, idealisations have to be made during the modeling process. These idealisations, depending on the application, can include neglecting friction and linearized force-deflection material properties as well as linear deflection behaviour of the structure, also at casing flanges, i.e. flanges deform under tension loads similarly to compression loads under a linear static analysis.

Such an idealised analysis model must be calibrated by suitable validation methods in order to generate results with sufficient accuracy. On an application of this approach to validate a bolted aeroengine casing assembly finite element model a static stiffness test was set up aiming at examining the total flexibility of the significant load carrying casing assembly in the loadpath between rotor bearings and engine-to-aircraft attachment points. [6]

The test structure was statically determinately fixed to a very stiff mounting plate and loaded via hydraulic jacks. At the rotor bearings and midspan of the core casings, between HP-compressor and combustion chamber outer casing, radial loads were applied in several directions with large magnitudes around 60kN. A stiff unloaded reference tube was attached via a slackless spherical bearing at the intermediate casing and a knife edged simple support at the rear bearing support structure against which casing deformations were measured using inductive displacement transducers (LVDT).

## UPDATING RESULTS

Typically the updating process requires several re-analysis and the results depend on the chosen updating parameters. Large parameter changes may be required and justified physically but may also be the result of the static displacement being insensitive to this parameter. The updating software guides the user with a number of possible options where changing the model would be beneficial, but the experience and engineering judgement of the user is an indispensable condition to successful model validation [7].

Validation of individual components prior to updating of the assembly is required [8]. This approach considerably narrows the number of uncertain parameters. The main individual casings of the bolted casing assembly model were analysed and updated based on free-free modal testing. In all cases this showed that the FE model of components were too flexible compared to what was obtained with test. This phenomenon is often experienced with FE models consisting largely of QUAD4 shell elements.

On the other hand, the assembled engine carcass model was too stiff compared to all static tests. Because components were updated separately before, this information focused the attention on the modelling of the connections and flanges between components.

The updating method allows for updating of physical parameters like Young's moduli, serving as a stiffness indicator and shell thickness, acting on stiffness and mass simultaneously. An updating approach was chosen to select the global Young's modulus of a limited number of element groups as updating parameters. Then, after evaluating the updating results, only the most effective parameters were kept and the element groups were split up in smaller ones. By repeating this process several times, it was possible to narrow down to a number of components, mainly flanges that required further investigation. This confirmed the conclusions from analysing the correlation of individual components with modal test data and of the assembled engine carcass with the static test data.

## CONCLUSIONS

Updating based on static stiffness testing is costly compared to the modal testing approach, but it proved to be very useful and worth the effort, because model inaccuracies could be identified by this approach. Successful updating of a bolted casing assembly finite element model was achieved thanks to specially developed software which allowed for flexible choice of updating parameters and targets and which also took care of the data management.

The initial correlation between predictions and test data could be improved to limits set by the measurement technique, because the validation process pointed to particular items that required special attention when modeled which in turn led to a refinement of modeling practice. In particular the influence of the bolted flanges on the static stiffness under large static loads could be identified. Improvements to single component models, resulting from experimental modal analysis, could also be verified by the static stiffness validation of the assembled models.

The validation process fully confirmed the quality of the MSC/NASTRAN finite element bolted casing assembly model.

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# ABBREVATIONS

| | |
|---|---|
| WEM | whole engine mechanical FE model |
| LP-rotor | low pressure rotor |
| HP-rotor | high pressure rotor |
| LVDT | linear variable displacement transducer |
| TDC | top dead center |

# SYMBOLS AND INDICES

| | |
|---|---|
| $[C_P]$ | Weighting matrix for the parameters |
| $[C_R]$ | Weighting matrix for the responses |
| E | Error function |
| {F} | Static force vector |
| [K] | System stiffness matrix |
| P | Updating parameter |
| R | Updating target (response) |
| [S] | Sensitivity matrix |
| {U} | Static displacement vector |
| | |
| $\Delta$ | Finite difference |
| $\varepsilon$ | Convergence margin; Tolerance margin |
| $[\ ]^t$ | Transposed matrix |
| $[\ ]^{-1}$ | Inverse of a matrix |