BALANCED CONTINUOUS TURBULENCE GUST LOADS USING SOLUTION 146*

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ABSTRACT

This paper describes and illustrates a method to obtain balanced (time-correlated) continuous turbulence gust loads using MSC/NASTRAN Solution Sequence 146 (SOL 146). Continuous atmospheric turbulence is modeled in the frequency domain as a power spectral density (PSD) function using the von Karman gust PSD. SOL 146 is used to obtain the complex frequency response for each load quantity. Balanced loads are determined using the equations of random process theory. Maximized load quantities correspond to RMS values determined from response PSD functions. Balanced loads for each maximized load correspond to scaled cross-correlation functions. Results are presented for a typical gust load condition.

INTRODUCTION

Dynamic gust load conditions applied to aircraft consist of discrete gust and continuous turbulence (or continuous gust). These conditions are covered in the Federal Aviation Regulations FAR 25.341 (Reference 1). For discrete gust loads the atmospheric turbulence is assumed to have a 1-cosine velocity profile that can be prescribed as a function of time. Response and load time histories correspond to solutions of the airplane equations of motion in the time domain. For continuous gust loads, the atmospheric turbulence is assumed to have a Gaussian distribution of gust velocity intensities that can be specified in the frequency domain as a power spectral density (PSD) function. FAR 25.341(b) specifies the von Karman PSD function. For a linear airplane, the input and output PSD functions are described by standard random process theory equations. Response PSD functions can be integrated in the frequency domain to obtain mean square responses. NASTRAN solution 146 (Dynamic Aeroelasticity, Reference 2) calculates the root-mean-square (RMS) responses due to a gust PSD input.

For a given response (or load) quantity, the RMS value represents the maximum response. RMS responses, e.g. wing root RMS bending moment and wing root RMS torque, are not correlated in time, since in general each response quantity is maximized at a different time. Hence, RMS loads are not balanced. Balanced distributions of loads are required to determine the airplane internal stresses.

ANALYSIS METHOD

This paper describes a method to compute time-correlated (balanced) continuous turbulence gust loads using MSC/NASTRAN solution 146 *Aeroelastic Response* (Reference 1). The method is based on the approach of Reference 3. Time-correlated loads are determined using the equations of Random Process Theory (RPT). Maximized load quantities correspond to RMS values determined from response PSD functions. Balanced loads for each maximized load correspond to scaled cross-correlation functions.

Solution sequence 146 is a modal frequency domain approach. Unsteady modal aerodynamic forces and gust loads are calculated in the frequency domain. Structural stiffness, mass and damping are modeled using rigid body and elastic modes with structural damping. The resulting aeroelastic modal equations of motion (in the frequency domain) can be written in the following form (Reference 1):

$$\left[-w^{2}[M_{hh}]+(1+ig)[K_{hh}]-\tfrac{1}{2}\,rV^{2}[Q_{hh}(m,k)]\right]\{u_{h}\}=\{P(w)\} \qquad (1)$$

where

$[M_{hh}]$	= Modal Mass Matrix
$[K_{hh}]$	= Modal Stiffness Matrix
m	= Mach Number
k	= Reduced Frequency = $wc/2V$
c	= Reference Length
$[Q_{hh}(m,k)]$	= Unsteady Aerodynamic Force Matrix
w	= Circular Frequency = $2pf$
f	= Frequency in Hz.
g	= Structural Damping
r	= Air Density
V	= Airplane Velocity
$\{u_h\}$	= Modal Amplitude Vector
$\{P(w)\}$	= Applied Gust Loading

For continuous turbulence gust loads the input gust loading PSD is given by

$$G_p(f)=w_g^2\,\frac{a}{p}\,\frac{\left[1+\frac{8}{3}(1.399af)^2\right]}{\left[1+(1.399af)^2\right]^{\frac{11}{6}}} \qquad (2)$$

where

$G_p(f)$ = von Karman Atmospheric Turbulence PSD (per unit Hz.) defined over positive values of f only

a	= $2p\dfrac{L}{V}$
L	= Scale of Turbulence
w_g	= RMS Gust Velocity

Parameters for $G_p(f)$ are specified on a TABRANDG Bulk Data entry (Reference 4). The von Karman gust PSD for unit RMS gust velocity and $a = 6p$ is shown in Figure 1.

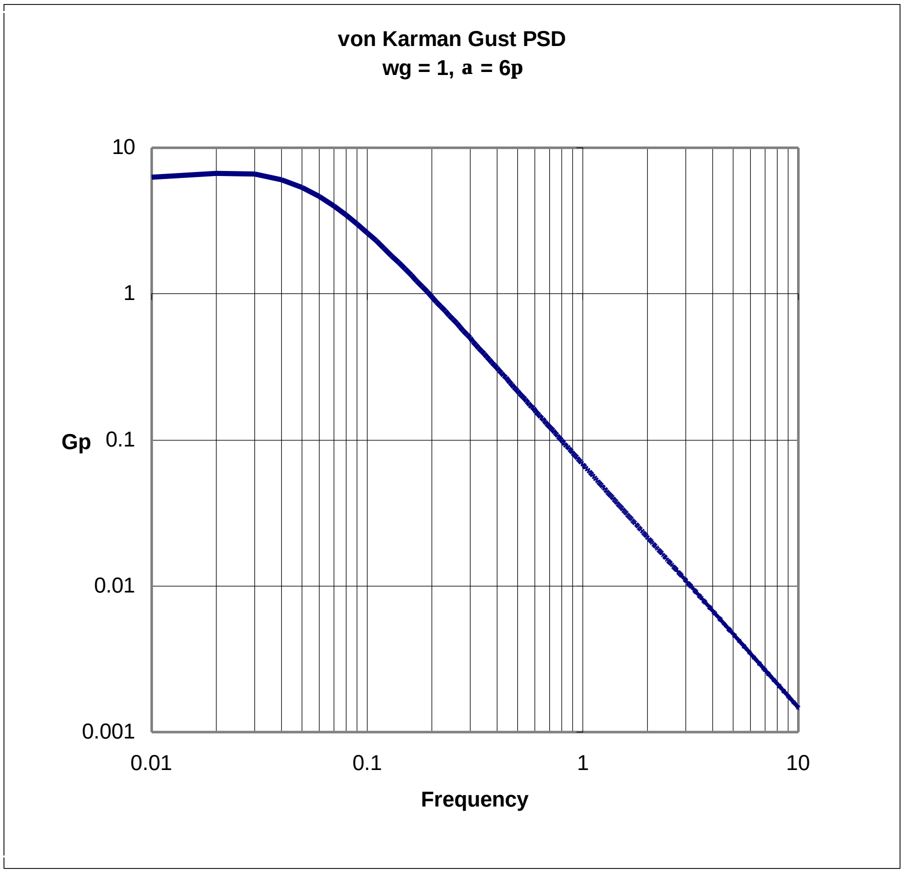


Figure 1. von Karman Gust PSD

Balanced loads for Eq. (1) can be determined using the cross-PSD function of RPT (Reference 3):

$$S_{ij}(w) = H_i(w) H_j^*(w) S_p(w) \qquad (3)$$

where

$S_p(w) = S_p(-w) = \dfrac{G_p(f)}{4p}$ = two-sided Atmospheric Turbulence PSD (per unit radian) defined over positive and negative values of w

$H_i(w) =$ complex frequency response function of load variable i

$H_j^*(w) =$ complex conjugate of $H_j(w) = H_j(-w)$

Balanced loads P_j correlated to the RMS load $\bar{A}_i$ are calculated as follows:

$$\bar{A}_i = \left[\int_{-\infty}^{\infty} S_{ii}(w)dw \right]^{\frac{1}{2}} \qquad P_j = \frac{\int_{-\infty}^{\infty} S_{ij}(w)dw}{\bar{A}_i} \qquad (4)$$

Equations (3) and (4) can be combined and written in terms of cross-correlation coefficients r_{ij} and the one-sided gust PSD $G_p(f)$:

$$\bar{A}_i = \left[\int_0^{\infty} H_i(f)H_i^*(f)G_p(f)df \right]^{\frac{1}{2}}$$

$$r_{ij} = \frac{\int_0^{\infty} \mathrm{Re}\left[H_i(f)H_j^*(f)\right]G_p(f)df}{\bar{A}_i\bar{A}_j} \qquad (5)$$

$$P_j = r_{ij}\bar{A}_j$$

Equation (5) forms the basis of the present balanced loads method. An examination of Eqs. (4) and (5) indicates that for each RMS load $\bar{A}_i$ $(i = 1 \ldots N)$, there is a corresponding set of balanced loads P_j $(j = 1 \ldots N)$, where N is the number of load quantities. Thus, for a given gust condition, the number of balanced load cases is equal to the number of load quantities. The advantage in using cross-correlation coefficients is that all values are between -1 and 1 with $r_{ii} = 1$. This provides a quick check of the results, since any cross-correlation values outside of this range indicate an error in the calculations. The sign of the coefficients indicates whether two load quantities are in phase (positive) or out of phase (negative). A given load quantity is maximized when $j = i$ with $P_i = \bar{A}_i$.

Implementation of Eqs. (5) using MSC/NASTRAN solution 146 is shown in Figure 2. A generic input file is shown in Figure 3. Since MSC/NASTRAN output for the complex frequency response functions is at discrete frequencies, the integrals of Eqs. (5) must be computed numerically. Discrete frequency output is controlled by the FREQ, FREQ1 and FREQ2 Bulk Data entries (Reference 4). The FREQ1 entry was utilized for the present method because it defines a uniform frequency increment that is well suited for numerical integration. Frequencies defined by this entry are given by (Reference 4):

$$f_k = F1 + DF(k-1) \qquad (6)$$

where

$F1$ = First Frequency in Set

DF = Frequency Increment

NDF = Number of Frequency Increments

k = Number of Frequencies = 1 to (NDF + 1)

Complex frequency response functions were read from the .op2 file using the fortran program of Reference 5. For the generic input shown in Figure 3, the .op2 file is data.f18. Reading data from the .op2 file is much faster than reading it from the .f06 file; furthermore, the .op2 file stores data using more significant figures.

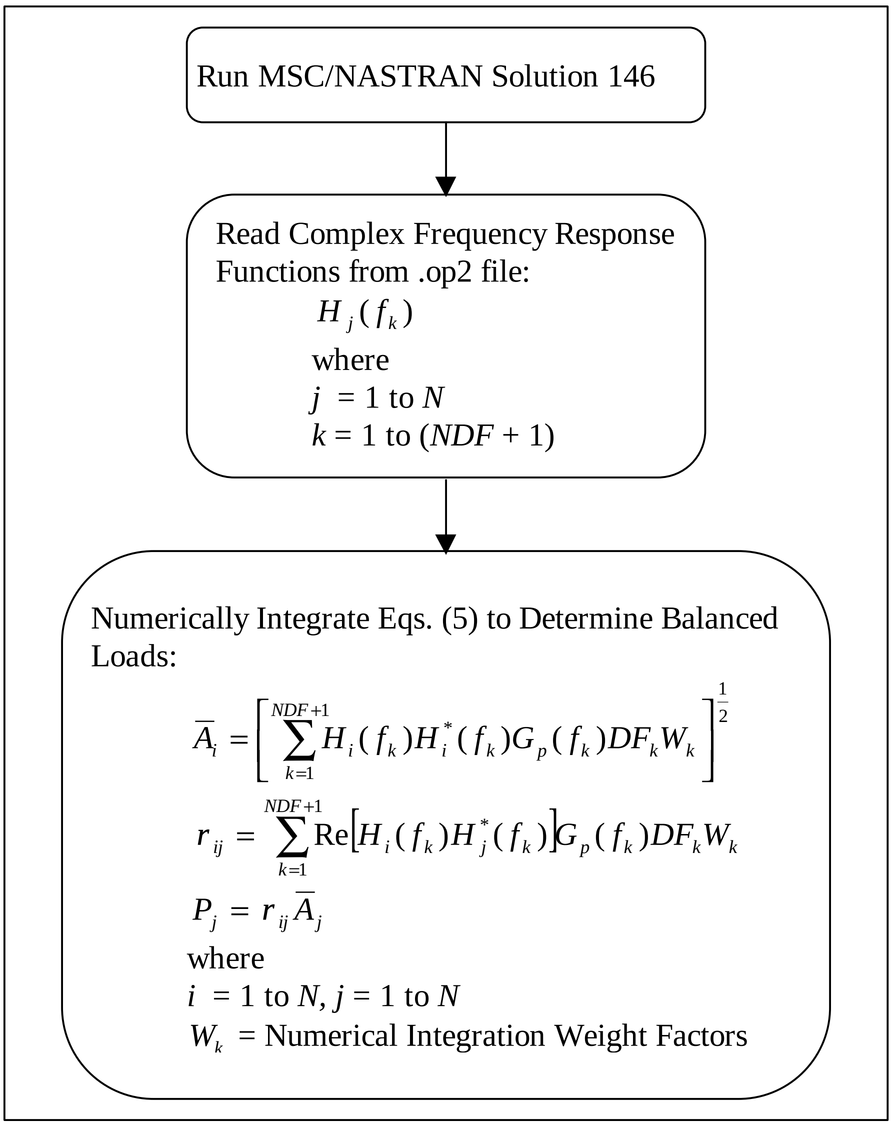


Figure 2. Balanced Loads Computation Method

```
$
$ NASTRAN bulk data file for Time Correlated PSD Gust Loads
$
$ .op2 file
assign output2='data_test.f18',unit=18,form=unformatted,status=unknown
TIME 100
SOL 146
$
$ Following Dmap will save .op2 file on unit 18
$
MALTER '(OUGV2, OES2, OEF2, ETC.)'(,-1)
output2 ougv2,oef2,oes2,, // -1/18 $
$
ECHOOFF
CEND
SEALL = ALL
SUPER = ALL
ECHO = NONE
METHOD = 20
GUST = 2000
DLOAD = 2010
FREQ = 2020
SDAMP = 2030
RANDOM = 2040
SPC = 100
$
$ Frequency Response
$ Beam Element Forces
SET 10 = 2210 THRU 2690
FORCE = 10
$ Nodal Accelerations
SET 11 = 2000 THRU 2049
ACCE = 11
$
BEGIN BULK
$
$ STRUCTURAL AND AERO MODELS
$ GEOMETRY, STIFFNESS, MASS, DAMPING AND AERO DATA
$
INCLUDE 'geom.dat'
INCLUDE 'stiff.dat'
INCLUDE 'mass.dat'
$INCLUDE 'damp.dat'
$
$  DATA FOR PSD GUST ANALYSIS
$
$.......2.......3.......4.......5.......6.......7.......8.......9.......0......
.
DAREA   2011    2000   3      1.0
FREQ1   2020    F1      DF      NDF
GUST    2000    2010    1/veloc X0      veloc
RLOAD1  2010    2011    0       0       2012
TABLED1 2012                                                            TABLED1
+ABLED1 0.0     0.0     0.001   1.0     100.0   1.0     ENDT
RANDPS  2040    1       1       1.0     0.0     2050
TABRNDG 2050    1       L/veloc wgrms
PARAM   GUSTAERO-1
$
ENDDATA
```

Figure 3. Input File for Time-Correlated PSD Gust Loads

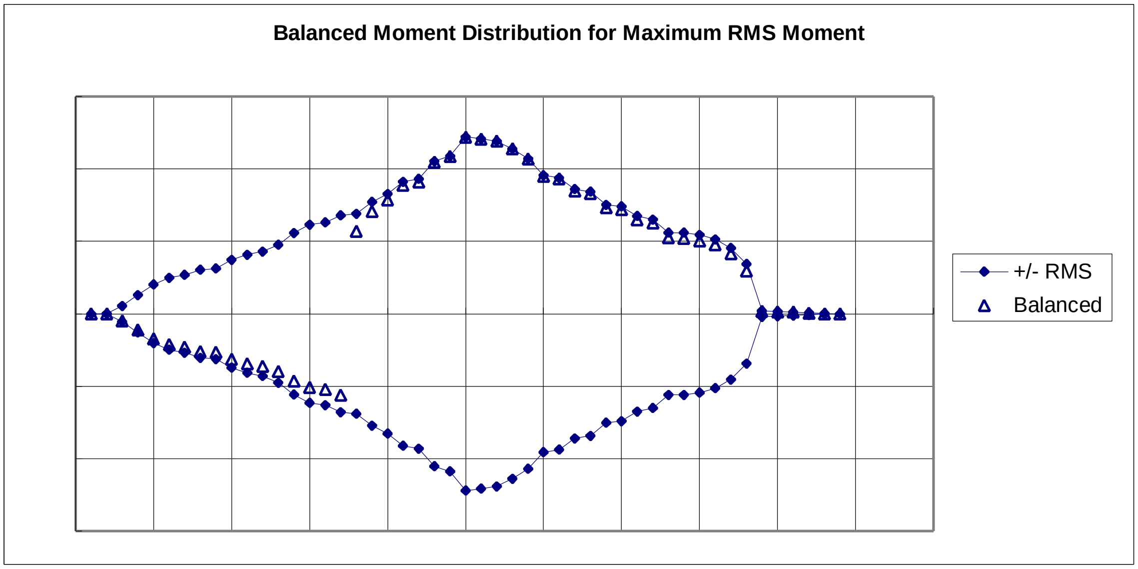
NUMERICAL RESULTS

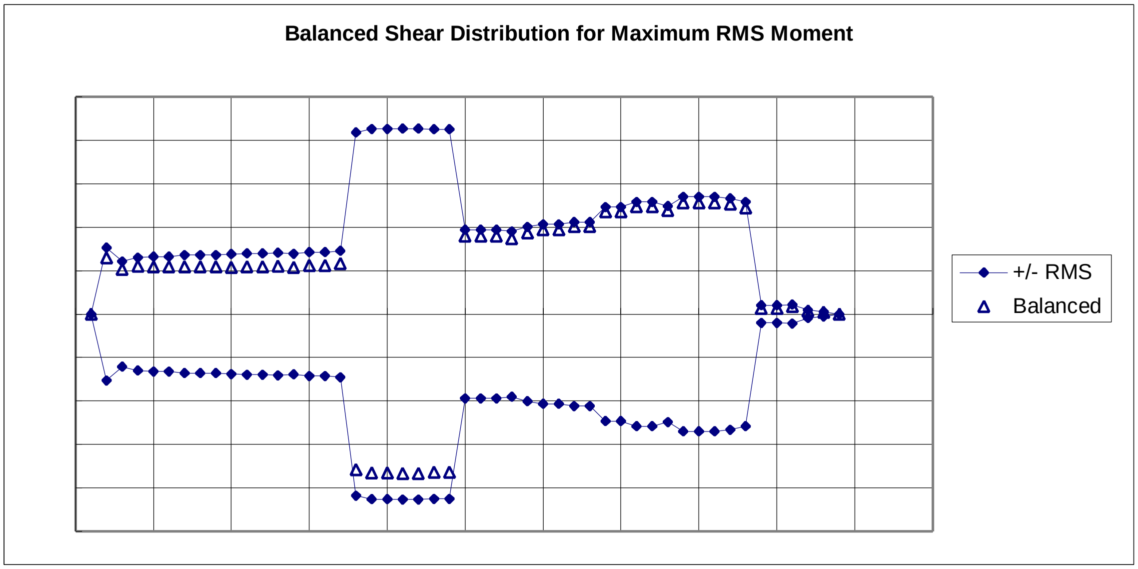
Results are presented for an arbitrary gust load condition. All load quantities were computed using MSC/NASTRAN Solution Sequence 146 in conjunction with the Balanced Loads Computation Method shown in Figure 2.

Gust load distributions of shear, moment, and torsion are shown in Figures 4 through 6. All results are for the same condition but correspond to a different maximized RMS load quantity. The Figure 4 balanced results are time-correlated with the maximum RMS moment. That is, the balanced load distributions shown in Figure 4 represent the loads at a point in time when the peak moment achieves a value equal to the maximum RMS moment. RMS values are also shown for comparison. The results indicate that all loads are close to the RMS values at locations in the vicinity of the maximum RMS moment. At locations farther removed from the maximum moment location, balanced load values decrease compared to the RMS values. In some cases the sign of the load also changes, indicating that the given load is out of phase with the maximum RMS moment. Phasing information is especially important when two or more load quantities need to be combined. A common example is the transformation of loads from one coordinate system to another. Satisfactory results can not be obtained without the proper sign and magnitude.

The results shown in Figure 5 are time-correlated with the maximum RMS shear. Figure 6 results are time-correlated with the maximum RMS torsion. Both sets of results closely resemble those shown in Figure 4. Deviations from RMS values are small in the vicinity of the maximized RMS load quantity. However, large deviations can occur at other locations. This is clearly demonstrated in the Figure 5 distributions.

An alternative way to examine the results is to plot the cross-correlation coefficients, see Eq. (5). For a given condition these represent the ratio of the balanced load to the RMS load. A plot of cross-correlation coefficient distributions is shown in Figure 7. A value near 1 indicates that the balanced load is approximately equal to the RMS load; –1 indicates the same magnitude with opposite sign (or out of phase). The term out of phase means that the balanced load is out of phase with the maximized RMS load. The results shown in Figure 7 indicate that many balanced load quantities are either out of phase or small compared to the RMS value.





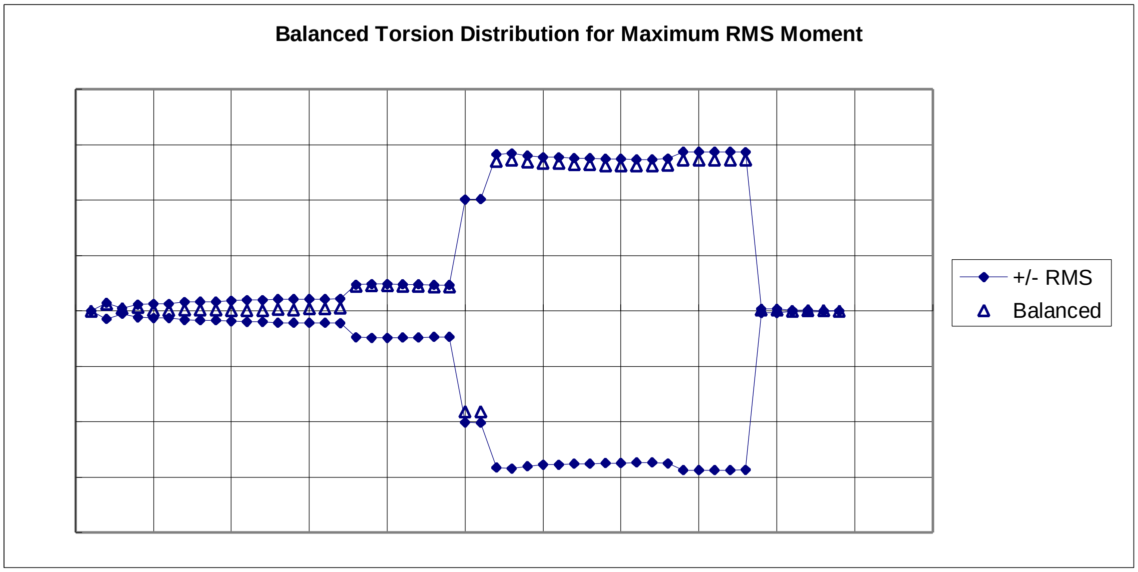
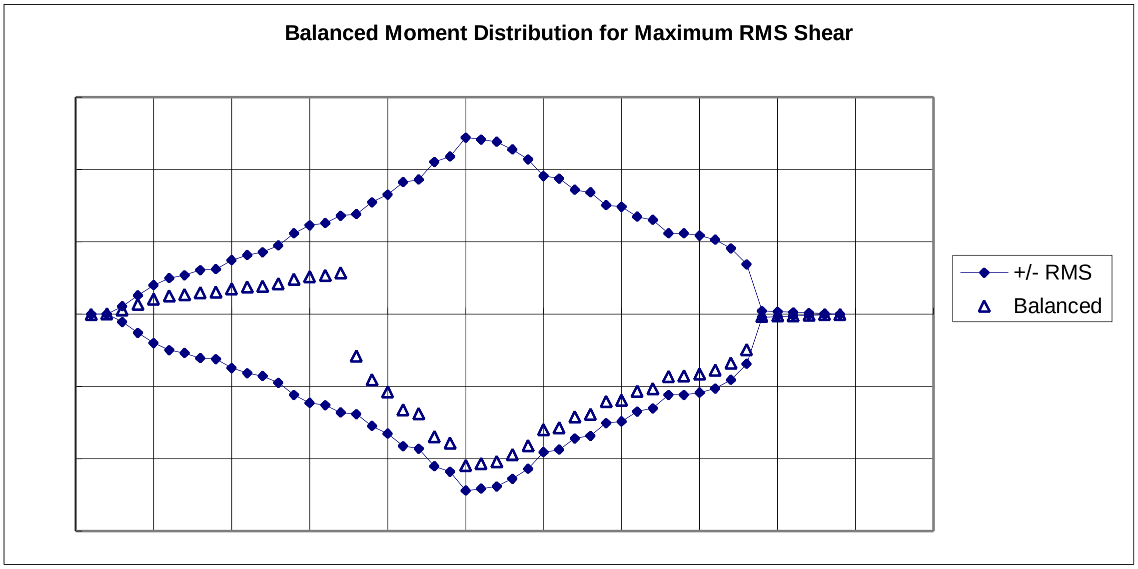
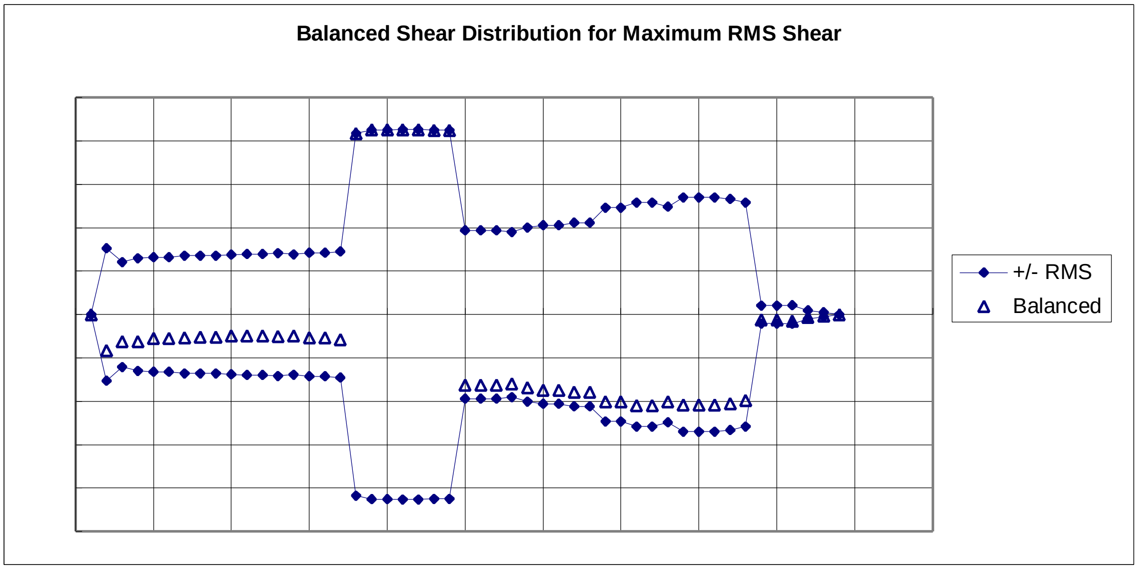


Figure 4. Balanced Load Distributions for Maximum RMS Moment





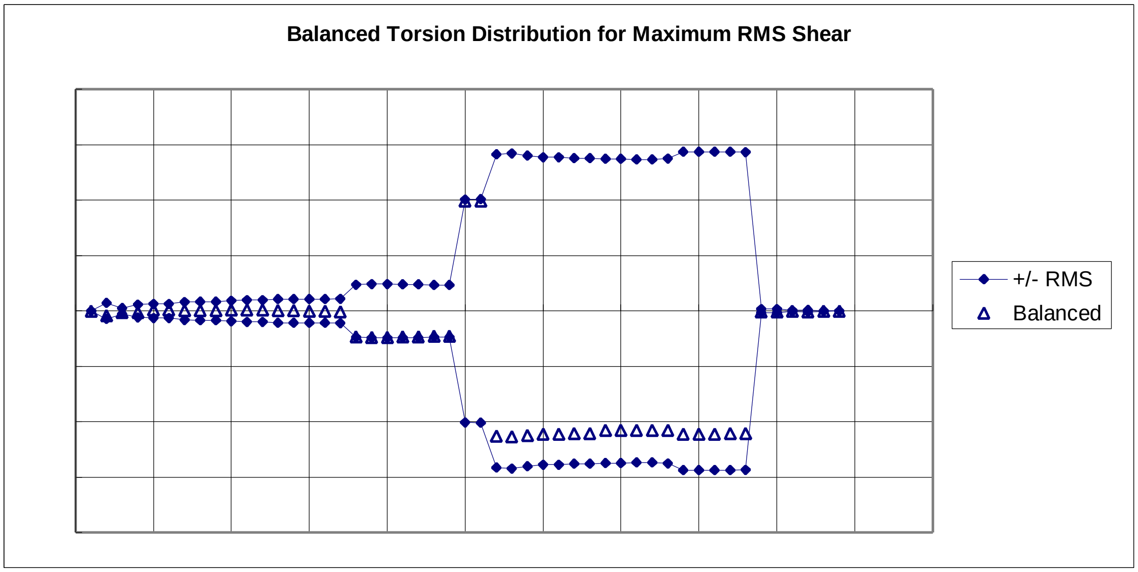
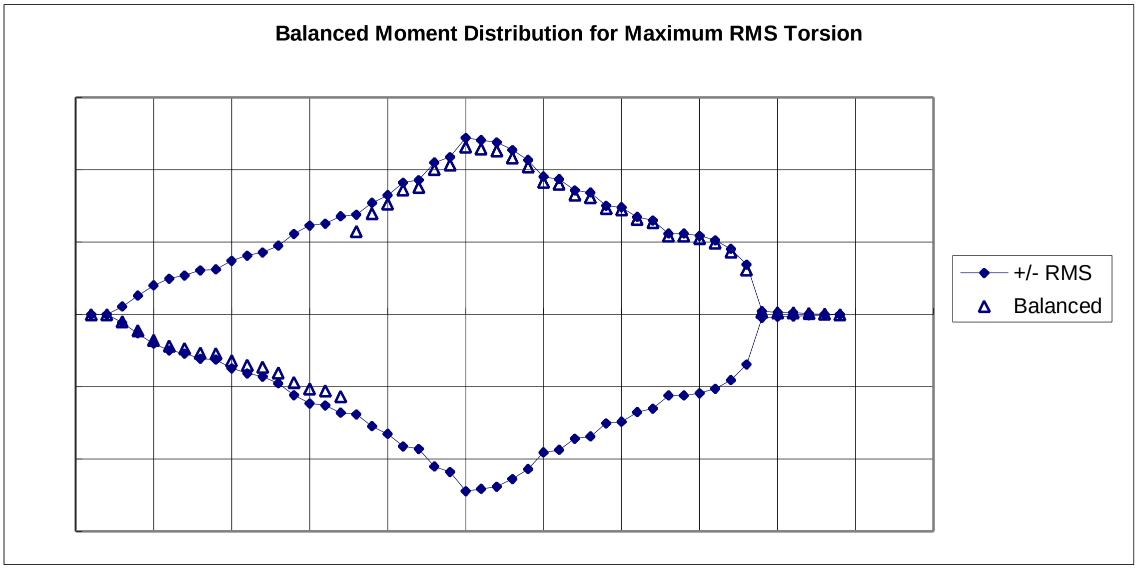
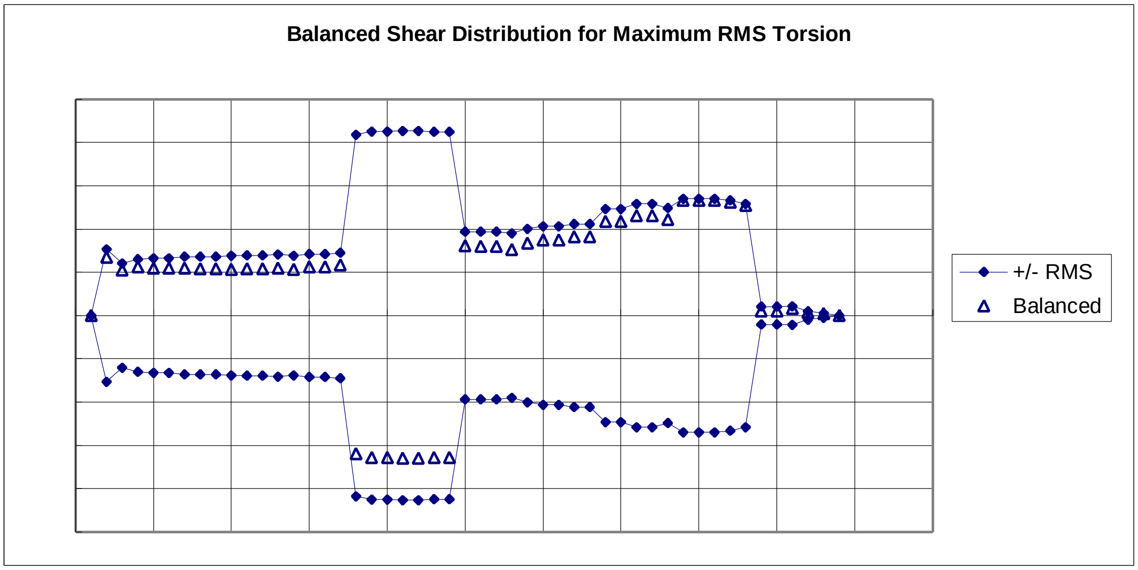


Figure 5. Balanced Load Distributions for Maximum RMS Shear





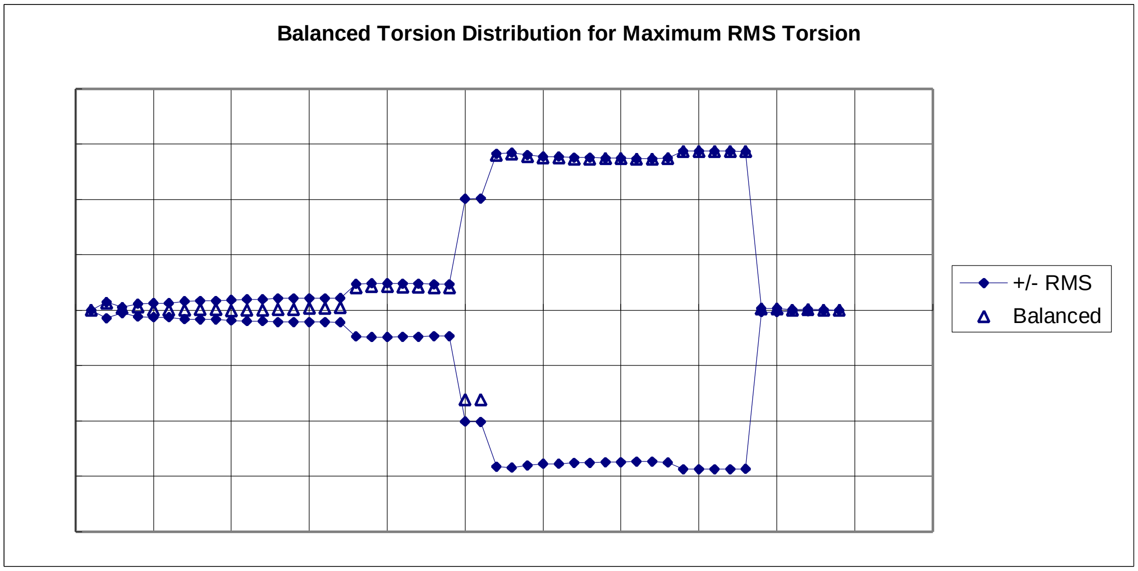


Figure 6. Balanced Load Distributions for Maximum RMS Torsion

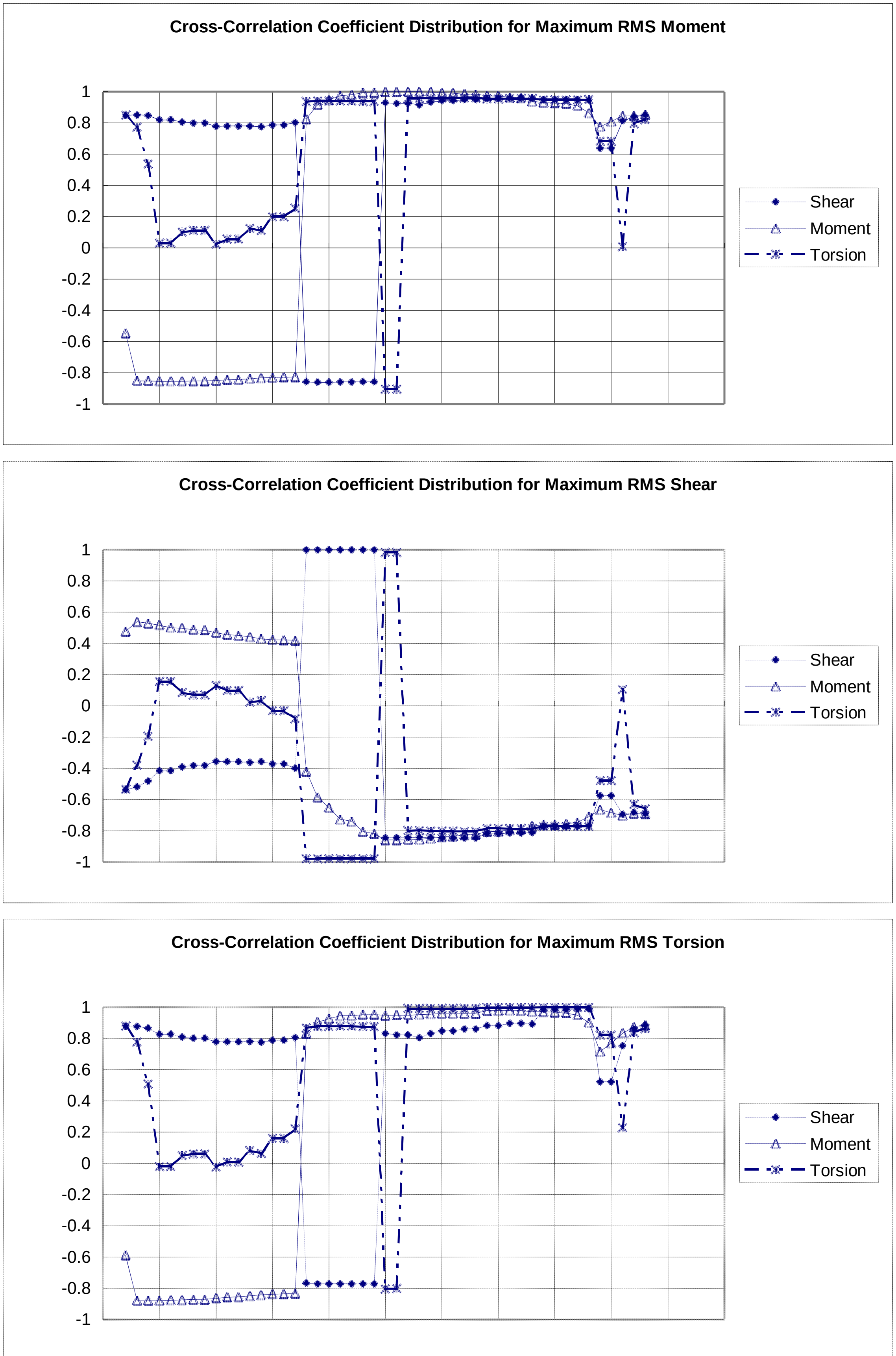


Figure 7. Cross-Correlation Coefficient Distributions

CONCLUSIONS

Turbulence loads on aircraft must be determined for certification. MSC/NASTRAN currently has the capability to model the external loading and determine RMS response quantities. This paper adds to the existing capability. An approach is presented to determine balanced (time-correlated) loads for each RMS load quantity. Example numerical results for an arbitrary gust loading condition indicate that balanced loads can differ substantially from the RMS values.

ACKNOWLEDGEMENTS

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TRADEMARK ACKNOWLEDGEMENTS

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