On Calculating the Response of Aerojet Engines with Rotor Imbalance and Non-Linear Bearings

David F. Bella
Leo W. Dunne

CDH GmbH

Ingolstadt, Germany
cdh@cdh-gmbh.com

Abstract

Non-linear bearings are commonly used between the rotors and casing of aerojet engines. For rotor imbalance response calculations, these nonlinear components are often modeled as linear elements to avoid using the time consuming procedures required for nonlinear analysis. The standard procedure to determine the engine response requires several analyses in which the response of the previous run is used to determine the linear bearing properties for the new run. This iterative procedure is time consuming and neglects the engine response due to the nonlinearities of the bearings. The methodology presented allows for efficient calculation of nonlinear response due to nonlinear bearings. The procedure is more stable and less resource intensive than a direct time-domain calculation and does not require manual iteration of bearing properties.

The analysis method takes advantage of the harmonic nature of rotor imbalance excitation and resulting engine response. The engine and bearing properties are transformed to a harmonic or frequency-domain representation. The system equations are then solved using an iterative procedure to determine the bearing properties and engine response. The iterative procedure calculates the response for one rotor revolution, the fidelity of the response is controlled by the number of harmonics used in the calculation. After the iterative calculation has converged, the system harmonic response is transformed to the time domain for standard data recovery. Suggestions for further improvements are also made.

1 Introduction

Advances in the numerical capabilities of MSC/NASTRAN allow the solution of problems which were previously not possible or would have consumed large amounts of computer resources. The procedure desribed in this paper relies on the ability of the decomposition routine to efficiently decompose Hermitian matrices containing null diagonal terms. The efficiency and robustness of the sparse decomposition routine for complex unsymmetric matricies allowed this procedure to be implemented without special considerations to accomodate solution numerics.

The methodology to determine the response of an aerojet engine with nonlinear bearings to rotor imbalance uses a frequency-domain solution procedure. The system response is calculated in the frequency domain, then transformed to the time domain for data recovery. This procedure is both faster and more robust than using the nonlinear radial gap (NLRGAP) for transient response calculations or other procedure which rely on time-integration techniques. The method is implemented using a DMAP alter to the direct transient response solution sequence (SOL 109).

2 Technical Approach

The aerojet response due to rotor imbalance is dependent on the rotor bearing properties. The bearing properties, in turn, are dependent on the eccentricity of the rotor within the bearing. This dependency makes the system response a nonlinear function of the bearing eccentricity. To solve the problem, an iterative approach in the frequency domain is used. New bearing properties are calculated for each iteration until the solution has converged. The time-domain response of the aerojet rotor and casing is calculated from the frequency-domain response.

2.1 Problem Statement

The general equation of motion, including the displacement and velocity dependent bearing properties, is

$$[M]\{\ddot{u}(t)\} + \left([B] + \left[\bar{B}\left(u, \dot{u}\right)\right]\right)\{\dot{u}(t)\} + \left([K] + \left[\bar{K}\left(u, \dot{u}\right)\right]\right)\{u(t)\} = \{P(t)\} \tag{1}$$

where
$[M]$ = mass
$[B]$ = damping
$[\bar{B}]$ = displacement and velocity dependent damping
$[K]$ = stiffness
$[\bar{K}]$ = displacement and velocity dependent stiffness
$\{u\}$ = displacement

The solution to the above equation can be solved by iterating in the time domain, at each analysis time step, until the solution converges.

If we can assume that the loading is periodic, which in our situation it will be, then the above equation can also be transform to the frequency domain and solved. Solution in the frequency domain also requires an iterative procedure, but the iteration procedure addresses the complete loading and response cycle as opposed to a single time interval.

Transforming Equation (1) to the frequency domain results in the following equation of motion.

$$\begin{aligned}[M]\{\ddot{u}(\omega_n)\} + [B]\{\dot{u}(\omega_n)\} + [K]\{u(\omega_n)\} & \\ + \sum_{m=-N}^{N} \left([\hat{B}(\omega_m)]\{\dot{u}(\omega_{n-m})\} + [\hat{K}(\omega_m)]\{u(\omega_{n-m})\}\right) &= \{P(\omega_n)\} \end{aligned} \tag{2}$$

where

$$\begin{aligned} [M] &= \text{mass} \\ [B] &= \text{damping} \\ [K] &= \text{stiffness} \\ [\hat{B}] &= \text{Fourier Transform of } \left[\bar{B}(u,\dot{u})\right] \\ [\hat{K}] &= \text{Fourier Transform of } \left[\bar{K}(u,\dot{u})\right] \\ \{u\} &= \text{displacement} \\ \omega_n &= n\text{th harmonic of excitation frequency } \omega \end{aligned}$$

The summation in the above equation is the result of transforming a multiplication of two time-dependent variables to the frequency domain. A multiplication of two time-dependent functions in the time domain transforms to a convolution of their Fourier Transforms in the frequency domain. This convolution operation couples the harmonics; therefore, the frequency response equations cannot, in general, be solved separately but must be solved as a coupled system. The examination of Equation (2) shows that if $\hat{K}$ and $\hat{B}$ are nonzero for $\omega \neq 0$ (i.e. $\bar{K}$ and $\bar{B}$ are not constant), the frequency equations are coupled. This means that for a system with nonlinear stiffness or damping, excitation at one frequency will generate responses at other frequencies.

For the rotor imbalance problem, the excitation is limited to a single frequency, the rotation speed of the rotor, but because the bearing properties are nonlinear higher harmonic responses will be generated. The solution procedure used to solve Equation (2) is described below.

2.2 Solution Procedure

Equation (2) is the frequency-domain representation of the equation of motion. It can be simplified by use of the following identities for the velocity and acceleration:

$$\{\dot{u}(\omega_n)\} = in\omega\{u(\omega_n)\} \tag{3}$$

$$\{\ddot{u}(\omega_n)\} = -(n\omega)^2\{u(\omega_n)\} \tag{4}$$

Substituting the above relationships into Equation (2) results in

$$\begin{aligned}\left(-(n\omega)^2[M] + in\omega[B] + [K]\right)\{u(\omega_n)\} \\ + \sum_{m=-N}^{N}\left(i(n-m)\omega[\hat{B}(\omega_m)] + [\hat{K}(\omega_m)]\right)\{u(\omega_{n-m})\} &= \{P(\omega_n)\} \qquad (5)\end{aligned}$$

Because of the nonlinear properties of the bearing, a numerical solution requires an iteration procedure. The solution procedure is based on Newton's iteration method. Equation (5) is rewritten as follows:

$$\{F(u)\} = \{P\} \qquad (6)$$

where $\{F(u)\}$ = Reaction forces
$\{P\}$ = Applied load

The reaction forces $\{F(u)\}$ are determine from the structural impedance, displacements, and nonlinear reaction forces.

$$\{F(u)\} = [\text{Imp}]_{linear}\{u\} + \{F(u)\}_{nonlinear} \qquad (7)$$

where $[\text{Imp}]_{linear}$ = Linear impedance of structure
$\{u\}$ = Structural displacements
$\{F(u)\}_{nonlinear}$ = Reaction forces due to nonlinear componenets

The impedance, displacements, and loads contain all harmonics considered in the analysis.

For the solution procedure, the equilibrium equation, Equation (6), is rewritten as:

$$\{P\} - \{F(u)\} = \{0\} \qquad (8)$$

Where the reaction forces $\{F(u)\}$ are a nonlinear function of displacement and velocity due to the nonlinear bearing properties. To numerically solve for the equilibrium equation an iterative scheme is required.

Equation (5) is initially solved with nominal values for the nonlinear components. The calculated displacements are then used to update the values for the nonlinear components and determine the structural reaction forces. If the applied loads and reaction forces are balanced, the equilibrium equation is satisfied. If not, the difference between the applied loads and reaction forces, termed the unbalanced or residual load, is used to deterimine an incremental displacement. This incremental displacement is added to the overall displacement and the reaction forces are recalculated. If the applied loads and reaction forces are balanced, the equilibrium equation is satisfied. If not, the procedure is repeated until convergence.

The residual load at any iteration step can be defined as

$$\{R\}_i = \{P\} - \{F(u_i)\} \qquad (9)$$

where $\{R\}_i$ is the residual load and $\{F(u_i)\}$ is evaluated using the displacements for iteration i.

$$\{F(u_i)\} = [\text{Imp}]_{linear}\{u\}_i + \{F(u_i)\}_{nonlinear} \tag{10}$$

The displacements due to the residual load are determined and added to the current displacements.

$$\{u\}_{i+1} = \{u\}_i + \{\Delta u\}_{i+1} \tag{11}$$

where $\{\Delta u\}_{i+1}$ is determined from the following:

$$[\text{Imp}]\{\Delta u\}_{i+1} = \{R\}_i \tag{12}$$

This procedure is repeated until the residual load $\{R\}_i$ and/or the incremental displacement $\{\Delta u\}_i$ is reduced to an exceptable value. Note the the impedance [Imp] in Equation (12) need not be the same linear impedance used in Equation (10). The impedance used to solve Equation (12) depends on the solution stategy, which is described below.

2.3 Solution Strategies

To determine the system response the following iteration procedure is used:

1. Calculate the bearing properties in the time domain ($\bar{K}(t)$ and $\bar{B}(t)$) based on the rotor and support displacements and velocities from the previous iteration (initial displacements and velocities will be zero).
2. Transform the time domain bearing properties ($\bar{K}(t)$ and $\bar{B}(t)$) to the frequency domain ($\hat{K}(\omega)$ and $\hat{B}(\omega)$).
3. Solve the frequency-domain response problem.
4. Check for convergence. If converged, exit. If not converged, calculate the time-domain response based on the current harmonic values and return to 1.

To assist the above procedure in converging, the following additional features are included:

Incremental Loading. The loading will be incrementally applied. The iteration procedure will be run to convergence for each increment before adding an additional incremental load. Applying the load incrementally should allow the procedure to smoothly follow the nonlinear bearing properties. This should prevent the iteration procedure from oscillating due to too large a change in bearing properties.

Load Bisecting. If the iteration solution does not converge within the number of prescribed iterations, the incremental loading will be bisected and the iteration procedure for the new load will be initiated.

Three methods are implemented to solve the iteration problem. All three differ only in the determination of the structural impedence [Imp] in Equation (12) during the iteration procedure.

- **Modified Newton's Method.**

  The Newton-Raphson Method works in conjuction with the load incrementation and bisection procedures. At the beginning of each new incremental loading, the impedance matrix is updated based on the previous displacement values. This impedance matrix is used during the iteration procedure until convergence or load bisection. If the loading is bisected, the impedance matrix is updated before the new incremental load is applied. Figure 1 illustrates the iteration method using this updating strategy.

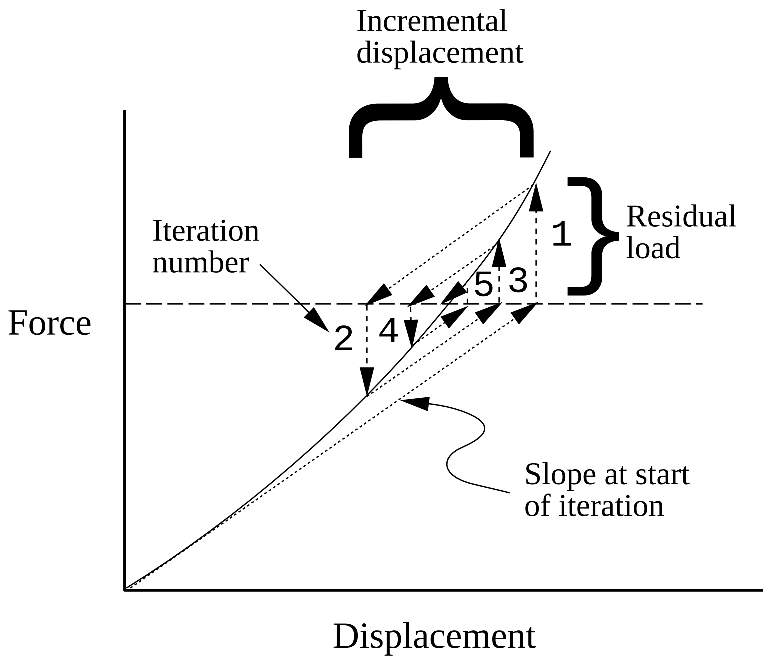

Figure 1: Iteration Strategy Using Modified Newton's Method

- **Newton-Raphson Method.**

  The Newton-Raphson Method updates the impedance matrix at every iteration. Figure 2 illustrates the iteration procedure using this updating strategy. The number of iterations required for convergence may be less than those required for the Newton-Raphson Method, but will require additional time for generation and decomposition of the impedance matrix during each iteration cycle.

- **Initial Impedance (No Impedance Update).**

  This method uses the initial impedance for the complete solution calculation. The impedance matrix is never updated after the initial calculation. The iteration strategy, shown in Figure 3, is the same as the Modified Newton's method, except that the impedance matrix is not updated at the start of each loading increment. The number of iteration required for converges may be greater than those required for the Newton-Raphson Method, but the time required for generation and decomposition of the impedance matrix will be saved.

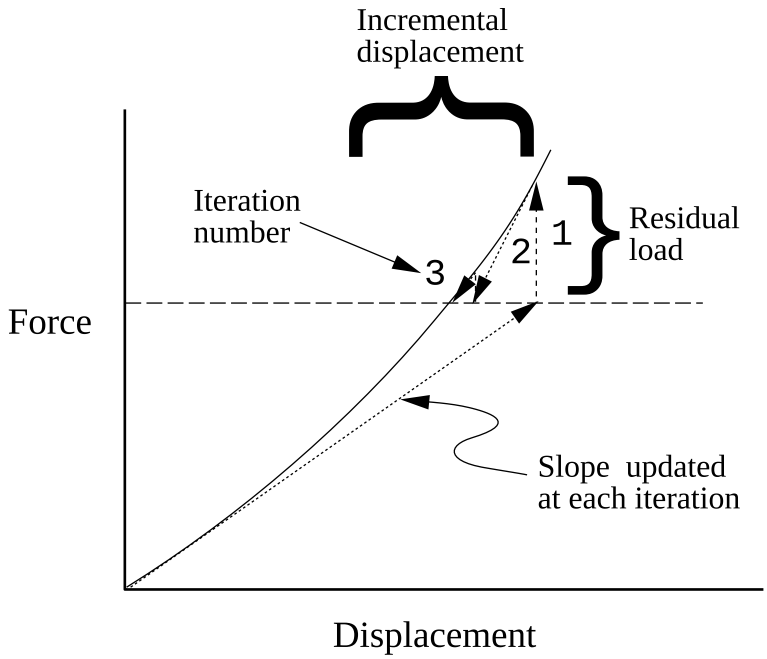

Figure 2: Iteration Strategy Using Newton-Raphson Method

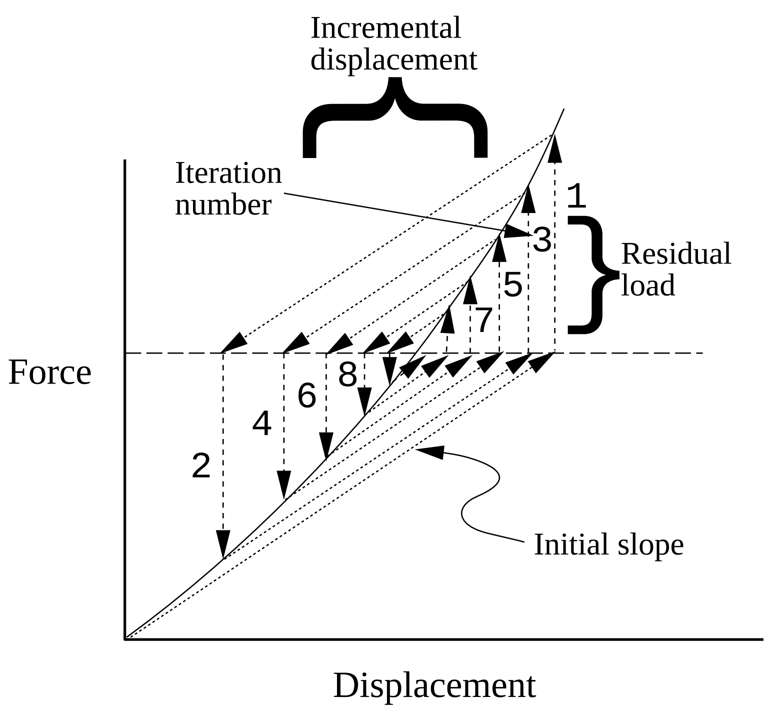

Figure 3: Iteration Strategy Using Initial Impedance

2.4 Calculation of Bearing Stiffness and Damping

The bearing properties are determined from the relative displacements of the rotor and casing. The bearing radial and tangential properties are converted into equivalent stiffness and damping in two axes perpendicular to the rotation axis (e.g. x and y for rotation about the z-axis), as shown in Figure 4. The stiffness and damping histories ($\bar{K}(t)$ and $\bar{B}(t)$) are used to determine the frequency-domain stiffness and damping ($\hat{K}(\omega)$ and $\hat{B}(\omega)$) and/or bearing forces ($\hat{P}_K(\omega)$ and $\hat{P}_B(\omega)$).

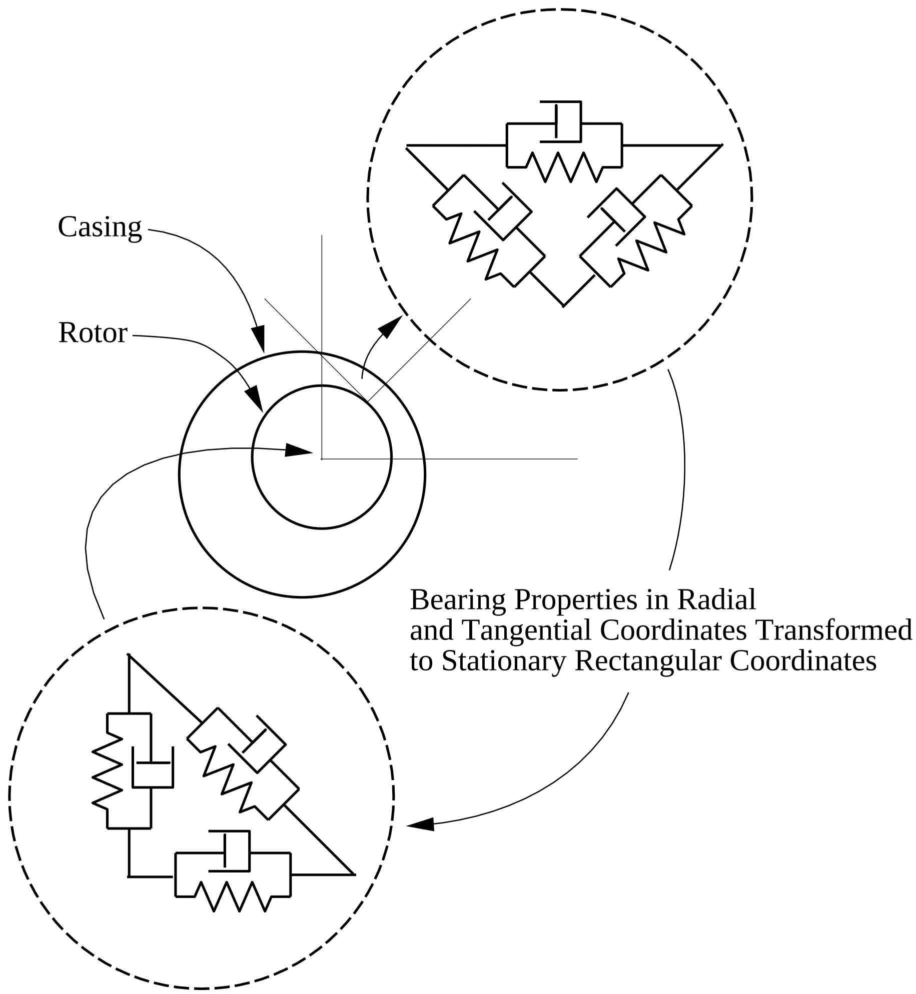

Figure 4: Transformation of Bearing Stiffness and Damping Properties from Radial Coordinate System to Stationary Rectangular System

The bearing property stiffness and damping properties are input using DTI bulk data entries. This information is input as function of relative displacement and velocity of the rotor to casing. The relative displacements and velocities are in the radial direction from the rotor to casing. Transformation to the required coordinate systems is performed within MSC/NASTRAN.

3 Implementation

The capability to add nonlinear bearing properties is implemented in the direct time response solution of MSC/NASTRAN Version 70.0 (SOL 109) using standard MSC/NASTRAN DMAP. All required data for the DMAP procedure is input using DTI and PARAM bulk data entries.

Output of the engine response is for one revolution of the reference rotor. The output format will be the same format used by MSC/NASTRAN for time-domain analyses output.

3.1 Bearing Property Input

The bearing properties are input using DTI bulk data entries. The rotor and casing grids, in addition to the bearing properties are specified on the following DTI entry.

DTI	SFD	0						
DTI	SFD	i	Rotor Gridid	Casing Gridid	Stiffness Table for K_{RR}	Stiffness Table for K_{TT}	Stiffness Table for K_{TR}	Stiffness Table for K_{RT}
	Damping Table for B_{RR}	Damping Table for B_{TT}	Damping Table for B_{TR}	Damping Table for B_{RT}				

where			
	i	=	Non-linear bearing identification number
	Rotor Gridid	=	Rotor grid identification number
	Casing Gridid	=	Casing grid identification number
	Stiffness Table for K_{RR}	=	KRR01 - KRR10
	Stiffness Table for K_{TT}	=	KTT01 - KTT10
	Stiffness Table for K_{TR}	=	KTR01 - KTR10
	Stiffness Table for K_{RT}	=	KRT01 - KRT10
	Damping Table for B_{RR}	=	BRR01 - BRR10
	Damping Table for B_{TT}	=	BTT01 - BTT10
	Damping Table for B_{TR}	=	BTR01 - BTR10
	Damping Table for B_{RT}	=	BRT01 - BRT10

The bearing properties are also specified using DTI entries. The names of these DTI entries
s[(Stiffn)]berr00(Taotor)-310v(en)]Tfl969990TDflelo0(b)-30cit(en)]Tfl24500TDfly0(-)-310e(d[(Damp).R10)]

3.2 Bearing Misalignment Input

If the rotor is not perfectly centered within the bearing, the misalignment can be specified. The misalignment will be included in the calculation of the bearing properties. The misalignment is given using DTI entries in the bulk data. The format of these entries is shown below.

DTI	MISALIGN	0						
DTI	MISALIGN	i	δ_1	δ_2				

where i = Non-linear bearing identification number
δ_1 = Relative offset of rotor in direction 1
δ_2 = Relative offset of rotor in direction 2

Directions 1 and 2 are dependent on the rotation axis as follows:

Rotation Direction	Direction 1	Direction 2
X	Y	Z
Y	Z	X
Z	X	Y

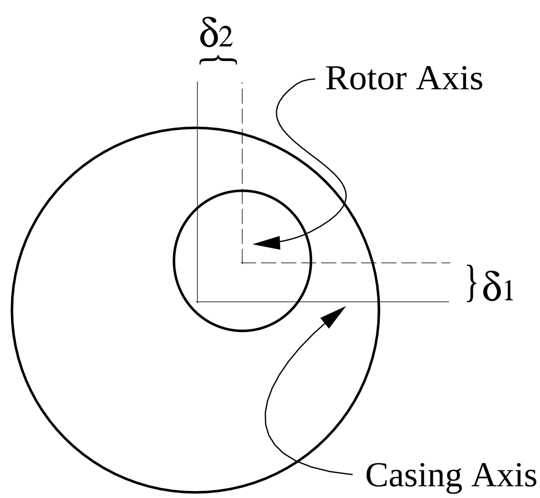

Figure 5: Relative Rotor Misalignment

3.3 Iteration Control

The iteration procedure is controlled using parameter entires in either the case control or bulk data sections. The parameters, their function, and default values are given below. The default values are set for robust operation of the iteration procedure.

SFD	Selects whether the solution will be calculated with nonlinear bearing properties (YES/NO), default= NO.
RAXIS	Specifies the axis of rotation (X/Y/Z), default= X.
REFRPT	Rotation speed for the reference rotor in revolutions per unit time (normally revolutions/second).
ITYPE	Selects Impedance update option (NEWTRAPH/MODNEWT/ NOUPDATE), default= NEWTRAPH.
NLOADINC	Number of increments for applying the specified load, default= 5.
MAXITER	Maximum number of solution iterations before incremental loading in bisected, default= 15.
MAXBISC	Maximum number of times incremental loading will be bisected, default= 3.
NHARM	Number of harmonics to include in solution procedure, default= 4.
NCOUPL	Order of harmonic coupling in solution procedure, default= 3.

3.4 Output Control

The solution output from the interative procedure will be standard MSC/NASTRAN time-domain data recovery. All standard time-domain data recovery is available. To support the data recovery procedure the user must include a TSTEP command in the case control section for the residual structure (superelement 0) and a corresponding TSTEP bulk data entry. The entries on the TSTEP bulk data entry are only used for identifying the output for one revolution and are not used in the actual displacement, velocity, and acceleration calculations. Output will be calculated at evenly spaced intervals of one revolution. For example, the problem may be run with a rotor rotation speed of 100 revolutions per second, but the TSTEP entry may specify 'time' output from 0.0 to 0.9, with a time increment of 0.1 (ten 'time' points in total). The results will be output for every 1/10th of a revolution.

3.5 Limitations

The following limitations apply to analyses using the nonlinear bearing procedure:

1. The bearing properties must change smoothly, there should not exist any abrupt changes in the bearing properties such as bearing gap closure.
2. The bearing rotor and casing degrees of freedom must be in the analysis set (D-set).

4 Suggestions for Further Improvements

The methodology described in this paper provides a relatively efficient method of including the effect of nonlinear bearings in the calculation of aerojet engine response to rotor imbalance. The following items are suggested to improve the efficiency and capability.

- Externalize or add a new module for the nonlinear interation procedure. This will be more efficient than the curent DMAP implementation.
- Externalize the calculation of the bearing properties. This will allow more exact property calculations based on actual rotor orbits within the bearing.
- Add coupling to a rotating coordinate system. This will allow the use of rotors which do not have isotropic properties about the rotation axis, such as two bladed propellers, symmetric rotors with unsymmetric temperature distributions, and also allow oscillating loads in the rotating system such as gravity or unequal flow loads.

5 Conclusions

The method presented in this paper offers an alternative to standard time-integration of problems with non-linear elements and cyclic loading. By addressing the complete response cycle instead of individual time intervals, the procedure is more robust and converges faster. Additionally, the response is much more 'smooth' due to the use of sine and cosine functions as the basis for response calculations. Further improvements previously stated could make the procedure more efficient and address problems which woud be very cumbersome using time-integration techniques.

6 Acknowledgments

The authors would like to acknowledge BMW Rolls-Royce GmbH and, in particular, Dr. Adrian Rienaecker for support in developing this procedure.

A Example Problem

A simple problem was devised to test the nonlinear iteration procedure. The test problem consists of a single lumped mass connected to ground by a bearing with nonlinear stiffness properties. A cyclic loading was applied to simulate a rotating load imbalance. The problem specifics are:

mass	0.01
ω	10 rps (62.83 rad/sec)
Force	1000
Bearing Stiffness	Shown in Figure 6

The equation of motion for this problem is

$$M\ddot{u} + K(u)u = F \tag{14}$$

or

$$(-\omega^2 M + K(u))u = F \tag{15}$$

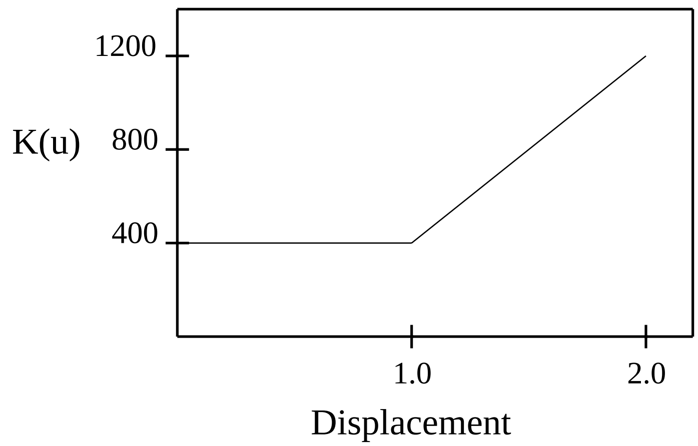

Figure 6: Nonlinear Bearing Stiffness Properties

Solving the above equation results in a radial displacement of 1.892 units. The input deck and results for the mass grid (rotation about z-axis) are given below. Comparison shows less than a 0.1% difference in the results.

A.1 Example Problem Input Deck

```
ID EXAMPLE, PROBLEM
SOL 109
TIME 5000
INCLUDE 'SFD.V70'
CEND
$
$ REQUEST OUTPUT
$
DISP(PHASE)= ALL
VELO= ALL
ACCEL= ALL
$
$ TIME OUTPUT REQUEST
$
TSTEP= 100
$
SPC= 1
$
BEGIN BULK
$
$ SPECIFY TIME DOMAIN OUTPUT (EVERY 16TH OF A REVOLUTION)
$
TSTEP, 100, 16, .0625, 1
$
$ SFD BEARING PROPERTIES
$
DTI, SFD, 0
DTI, SFD, 1, 1, 2, KRR01, KTT01, ENDREC
$
DTI, KRR01, 0
DTI, KRR01, 1, 0., 400., 1., 400., 2., 1200.,
, ENDREC
$
DTI, KTT01, 0
DTI, KTT01, 1, 0., 400., 1., 400., 2., 1200.,
,ENDREC
$
```

```
$ ROTATING IMBALANCE LOAD
$
DMIG, LOAD, 0, 9, 3, 0, 1, , 1
DMIG, LOAD, 1, , , 1, 2, 1000., 0.,
, 1, 3, 1000., -90.
$
$ PARAMETERS FOR SFD ITERATION CONTROL
$
PARAM, SFD, YES
PARAM, NLOADINC, 2
PARAM, ITYPE, NEWTRAPH
PARAM, MAXITER, 15
PARAM, MAXBISC, 2
$
$ PARAMETERS FOR DEFINING NUMBER OF HARMONICS
$ AND COUPLING HARMONICS
$
PARAM, NHARM, 1
PARAM, NCOUPL, 0
$
$ PARAMETERS FOR DEFINING ROTOR ROTATION INFORMATION
$
PARAM, REFRPT, 10.0
PARAM, RAXIS, X
$
$ SET AUTOSPC TO NO TO PREVENT ROTOR GRID
$ FROM BEING CONSTRAINED
$
PARAM, AUTOSPC, NO
$
$ MODEL INPUT
$
GRID, 1, , 0., 0., 0.
GRID, 2, , 0., 0., 0.
GRID, 3, , 0., 0., 0.
$
CONM2, 10, 1, 0, .01
$
$ STIFF GROUND SPRINGS
$
CELAS2, 201, 1.0E+6, 2, 2, 3, 2
CELAS2, 202, 1.0E+6, 2, 3, 3, 3
$
SPC1, 1, 123456, 3
SPC1, 1, 1456, 1, 2
$
ENDDATA
```

A.2 Example Problem Results for Mass Grid Point

D I S P L A C E M E N T   V E C T O R

TIME	TYPE	T1	T2	T3
0.0	G	0.0	1.892767E+00	5.694976E-13
6.250000E-02	G	0.0	1.748689E+00	-7.243307E-01
1.250000E-01	G	0.0	1.338389E+00	-1.338389E+00
1.875000E-01	G	0.0	7.243307E-01	-1.748689E+00
2.500000E-01	G	0.0	-1.020892E-15	-1.892767E+00
3.125000E-01	G	0.0	-7.243307E-01	-1.748689E+00
3.750000E-01	G	0.0	-1.338389E+00	-1.338389E+00
4.375000E-01	G	0.0	-1.748689E+00	-7.243307E-01
5.000000E-01	G	0.0	-1.892767E+00	-5.957601E-13
5.625000E-01	G	0.0	-1.748689E+00	7.243307E-01
6.250000E-01	G	0.0	-1.338389E+00	1.338389E+00
6.875000E-01	G	0.0	-7.243307E-01	1.748689E+00
7.500000E-01	G	0.0	-2.019549E-14	1.892767E+00
8.125000E-01	G	0.0	7.243307E-01	1.748689E+00
8.750000E-01	G	0.0	1.338389E+00	1.338389E+00
9.375000E-01	G	0.0	1.748689E+00	7.243307E-01