Influence of Modeling for Orthotropic Material Properties

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Abstract

Shell type structures are usually modeled with plate elements (CQUAD, CTRIA). In case of thermal loads and orthotropic thermal expansion coefficients the analytical results do not represent the real behavior with sufficient accuracy. The reason is that changes in the thickness are not considered in the element stress/strain description which is correct for flat plates only. However, when a curvature is present, this effect leads to additional stresses and displacements.

As example a nozzle made of carbon fiber reinforced ceramics is used. For comparison an equivalent modeling with solid elements where the orthotropic properties can be fully represented, is applied. The differences in the results of the two models are shown.

1. Problem Definition

In aerospace shell type structures are widely used. Static and dynamic analyses are usually performed by modeling the shell with plate or shell elements (CQUAD, CTRIA). This approach is straightforward as long as changes in the wall thickness due to the applied loads have no significant influence on the in-plane stress distribution. This is in general the case except at the fixation of the shell to other structure elements and at load introduction points. This statement is also valid for orthotropic material properties as for fiber reinforced structures.

However, when orthotropic thermal expansion coefficients are present, thermal loads produce strains in the thickness direction that differ from those in the shell plane. For curved structures this thickness change influences the curvature and causes additional stresses and displacements. Plate and shell elements do not consider this effect and may lead to wrong results especially when high temperature differences exist.

2. Illustration of Physical Phenomenon

In Fig. 1 a circular arch with an angle φ is shown. The thermal expansion coefficient in radial direction is α_r, in tangential direction α_t. For a temperature rise ΔT the radius R of any fiber of the arch increases by

$$\Delta R = R\alpha_r \Delta T$$

The arch angle ν changes according to the formula

$$\Delta\varphi = \varphi(\alpha_t - \alpha_r)\Delta T$$

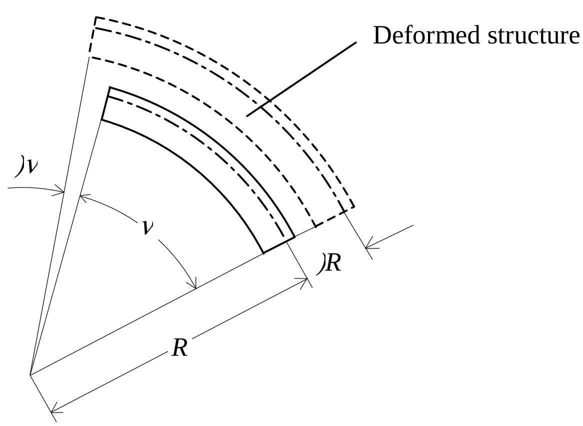

Fig. 1: Thermal Deformation of an Arch

It can be noticed that the radius is influenced by the radial expansion coefficient, whereas the angle depends on the difference between the tangential and the radial expansion coefficient. Only when both expansion coefficients are equal, the arch angle remains unchanged. In all other cases an increase or decrease of the angle is obtained. Fiber reinforced structures have usually a greater thermal expansion perpendicular to the fibers than in the fiber direction, which results in a reduction of the arch angle.

If such an arch is modeled with CQUAD elements situated at the center of the arch cross section, and a thermal coefficient corresponding to α_t is applied, then both the resulting curvature and the angle are wrong; only the arc length is calculated correctly. If the arch is for example constrained at both ends, the modeling with CQUAD elements will not reveal the correct stresses.

3. Possibility of Model Size Reduction

As plate and shell elements with orthotropic thermal properties are not able to deliver the real stress/strain state under thermal loads, adequate modeling has to be performed with elements that take into account deformations in the thickness direction. Consequently solid elements (CHEXA, CPENTA, CTETRA) have to be used also for shell type structures. For fiber structures each layer has to be modeled separately. This may results in a huge increase of model size and degrees of freedom.

For structures with rotational symmetry and axial symmetric loads there is the possibility to analyze only a sector with one element in the circumferential direction and apply appropriate boundary conditions. Fig. 2 shows such a sector. The standard boundary conditions for rotational symmetry cannot be applied because for layers with fibers inclined to the shell meridian (corresponds to an inclined material coordinate system) the displacement in circumferential direction has to be allowed. However, all grids at constant radius and the same axial distance must show equal displacements in all directions. Therefore, instead of single point constraints (SPC) multipoint constraints (MPC) have to be introduced. Explicitly the MPC's for equivalent grids of solid elements read as follows:

$$u_{j,left\,boundary} - u_{j,right\,boundary} = 0;\ \ 1 \le j \le 3;\ \ u_j = \text{displacement in direction j}$$

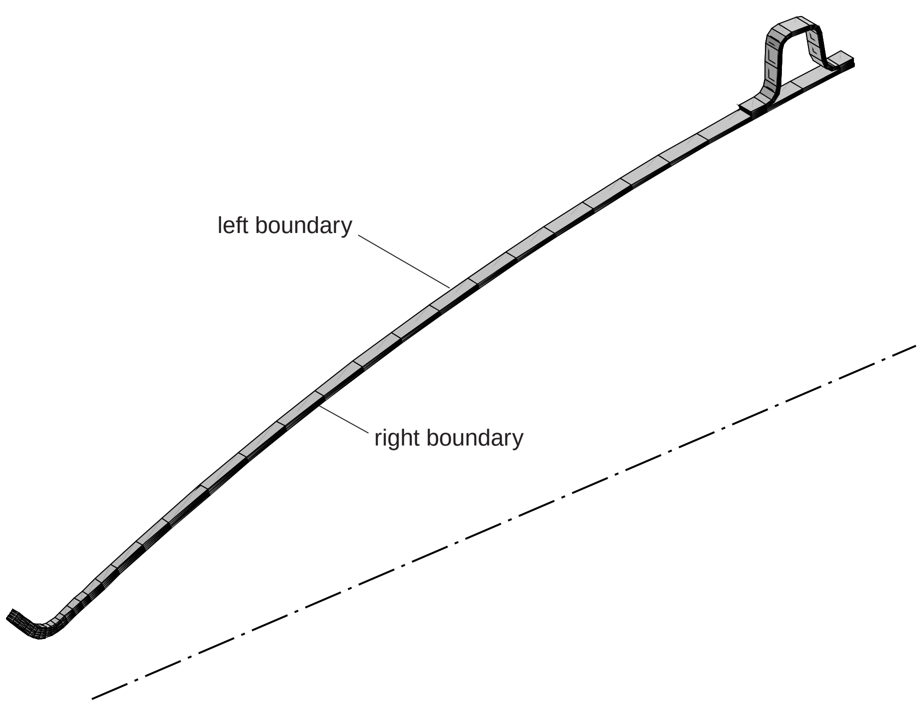

Fig. 2: Sector of a Shell of Revolution (Nozzle Modeled in Solid Elements)

The same relationship is possible also for plate and shell elements to be applied only when thermal loads are not present. For the remaining rotational degrees of freedom the MPC conditions for a cylindrical displacement coordinate system with the z-axis parallel to the shell axis are:

$$u_{4,\,6} = 0$$

$$u_{5,\,left\ boundary} - u_{5,\,right\ boundary} = 0$$

4. Analyzed Example

To demonstrate the thermal effect for real hardware properties a nozzle of a rocket engine has been analyzed both with plate (s. Fig. 3) and solid elements (s. Fig. 2). The element size is identical in both cases. The nozzle is made of carbon fiber ceramics, the ply stack is symmetric but the ply angle alternates from ply to ply. The material properties of each ply are highly orthotropic:

Modulus of elasticity in fiber direction: 150 000 N/mm^2
Modulus of elasticity perpendicular to fiber direction: 14 000 N/mm^2
Thermal expansion coefficient in fiber direction: 1.6(10^{-6} 1/K
Thermal expansion coefficient perpendicular to fiber direction: 6.3(10^{-6} 1/K

The properties normal to the shell plane are directly those perpendicular to the fiber direction. At one end the nozzle has a flange to allow a clamping connection with the thrust chamber. At the jet exhaust there is a reinforcement ring to comply with stiffness requirements.

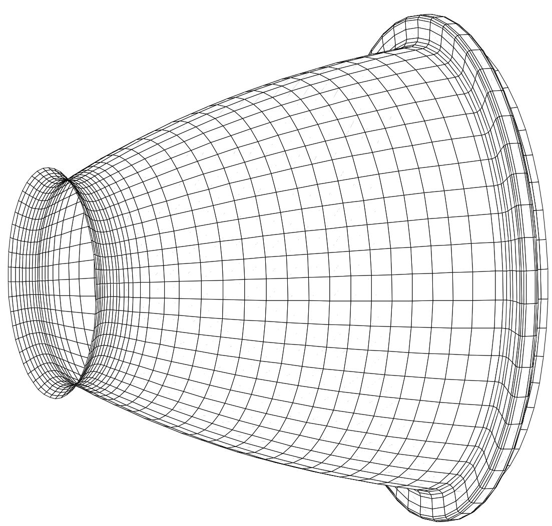

Fig. 3: Nozzle Modeled in Plate Elements

As thermal load, the cooling from processing temperature down to environmental temperature has been selected. At the processing temperature the internal stress state is considered to be zero. The total temperature difference is 1 600 K. The nozzle is completely free to deform.

To compare the quality of the solid model with respect to the plate model a static load case consisting of the internal operating pressure has been applied. The nozzle is elastically supported at the whole length of the attachment flange.

5. Analysis Results

The results of the static load case are almost identical for plate and solid model. The maximum displacement is 0.0494 mm for the plate model and 0.0489 mm for the solid model, i. e. a difference of only 1 %. The greatest deviations in stresses occur in the reinforcement ring. The extreme values are listed in Fig. 4. The maximum differences are about 10 %. The reason for this relatively high difference lies in the different geometrical modeling. The solid model considers the offset of the flange of the reinforcement ring relative to the middle plane of the shell whereas in the plate model, flange and shell coincide. But nevertheless it can be stated that both mathematical representations lead to conforming results within the accuracy to be expected.

Model	**Stress in Fiber Direction**	**Stress Perpendicular to Fiber Dir.**	**Stress in Thickness Direction**	**In-Plane Shear**	**Out-of-Plane Shear in Fiber Direction**	**Out-of-Plane Shear Perp. to Fiber Dir.**
	N/mm^2	**N/mm^2**	**N/mm^2**	**N/mm^2**	**N/mm^2**	**N/mm^2**
Plate	3.71	-3.04	not available	0.90	0.00	0.41
Solid	4.11	-2.85	-1.3	0.78	0.10	0.36

Fig. 4: Extreme Stresses in Reinforcement Ring due to Operational Pressure

In case of the temperature load case however, the deviations in the results are significant. The greatest differences occur in the stress distribution (s. Fig. 5). The out-of-plane shear stress is not shown because the values are below 1 N/mm^2 and are therefore not important for comparison. The maximum displacement differs by 3 %. The influence of the orthotropic material properties on the stresses appears to be more distinct than on the displacements.

Portion	**Model**	**Stress in Fiber Direction**		**Stress Perpendicular to Fiber Dir.**		**Stress in Thickness Direction**		**In-Plane Shear**	
		N/mm^2		**N/mm^2**		**N/mm^2**		**N/mm^2**	
		Max.	**Min.**	**Max.**	**Min.**	**Max.**	**Min.**	**Max.**	**Min.**
Mid Shell	Plate	-17.3	-26.6	-22.6	17.6	not available		36.6	-36.5
	Solid	-14.7	-29.9	22.5	21.2	0.3	-0.3	36.3	-36.5
Attachment Flange	Plate	0.7	-50.6	40.2	30.4	not available		42.8	-42.9
	Solid	1.6	-68.1	49.5	26.6	2.1	-0.3	44.5	-46.7
Reinforcement Ring	Plate	-1.6	-18.2	17.7	4.6	not available		27.6	-27.5
	Solid	84.4	-91.0	17.4	14.2	8.4	-2.2	26.8	-27.7

Fig. 5: Extreme Stresses in Selected Portions of the Nozzle due to Temperature Load

Typical stress distributions for the plate and solid model at the most sensitive portion, the reinforcement ring, are shown in Fig. 6.

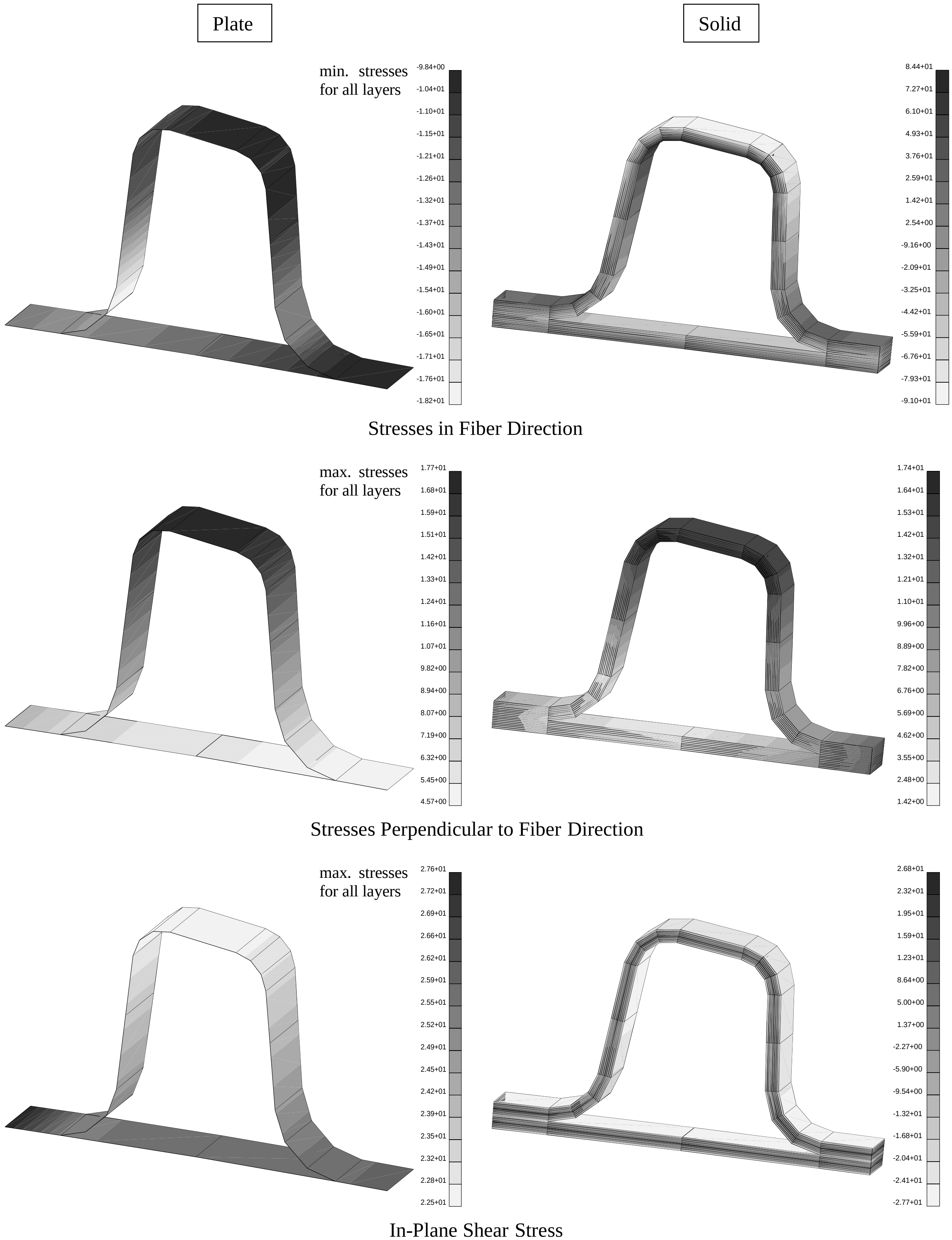

Fig.6: Stress Distribution in the Reinforcement Ring for Plate and Solid Elements

The stresses in the middle portion of the shell undisturbed by local reinforcements differ only by 10 %. The reason is that the fiber angle relative to the meridian is small and therefore the effective thermal expansion coefficient in circumferential direction is very close to that in the radial direction; that means the radial expansion has almost no influence on the displacements.

The situation is completely different at the attachment flange and especially at the hat-shaped reinforcement ring. At the same fiber angle there is effectively a certain fiber content in the radial direction because the attachment flange is inclined by about 45° and the side walls of the hat profile are almost radial. Consequently a great difference between radial and circumferential thermal expansion coefficients results. In addition in the corners of the hat profile and between the flange and shell there are radii. With reference to these radii, radial (in the thickness direction) and tangential (in the meridian direction) expansion coefficients are noticeably different. Since the curvature effects of the local radii and the shell of revolution superpose, much higher stresses in the solid model compared to those of the plate model are caused.

The stresses in thickness direction are neglected in plate elements. The solid element model shows that these stresses may rise up to 10 % of the maximum in-plane stress. The in-plane shear results of both models seem to be very close to each other, however, the distribution over the wall thickness is completely different. The reason is certainly the different mathematical formulations of plate and solid elements.

6. Conclusion

When orthotropic thermal expansion coefficients are present in shell type structures, the modeling with shell or plate elements leads to incorrect results. Only solid elements describe correctly the physical behavior. In case of layered structures the number of elements may increase significantly. For shells of revolution with axially symmetric loads a sector with only one element in circumferential direction can be used provided the appropriate multipoint constraints are applied.