# IMPROVEMENTS TO THE DOUBLET-LATTICE METHOD IN MSC/NASTRAN

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## ABSTRACT

The Doublet-Lattice Method (DLM) is in use worldwide for flutter and dynamic response analyses of aircraft at subsonic speeds. The DLM is an aerodynamic finite element method for modeling oscillating lifting surfaces that reduces to the Vortex-Lattice Method at zero reduced frequency. The number of finite elements (boxes) required for accurate results depends on aspect ratio and reduced frequency, among other parameters. At high reduced frequency, the chordwise dimension of the boxes must be small. A new version of the DLM relaxes limitations of the previous version to permit higher box aspect ratios so that the number of spanwise divisions (strips) can be reduced significantly, leading to a reduction in the total number of boxes. The new version also improves the accuracy of predicted aerodynamic damping. The new version has been integrated into MSC/NASTRAN as a no-charge option that is selected using a NASTRAN system cell.

The present paper summarizes the chordwise and spanwise convergence criteria and presents examples illustrating new modeling guidelines, a tip correction to reduce the sensitivity to the number of spanwise strips, and the reduced computing time possible with the new version.

## INTRODUCTION

A new version of the subsonic Doublet-Lattice Method (DLM) is available in MSC/NASTRAN Version 70.5. The original version of the DLM in MSC/NASTRAN was developed for the US Air Force by the Douglas Aircraft Company in 1972 and is called N5KA. It was incorporated into MSC/NASTRAN in 1977. It accounts for interference of multiple lifting surfaces and bodies. The history of its development is described in Reference 1.

The new version is called N5KQ and contains some improvements in analysis of lifting surfaces. The treatment of bodies remains the same as in N5KA. The improvements are documented in Reference 2. The original formulation assumed a parabolic variation of the numerator of the incremental oscillatory kernel function across the span of each box bound vortex that permitted the normalwash factors to be evaluated analytically. The new formulation replaces the parabolic approximation with a more accurate quartic approximation. A secondary improvement in accuracy in N5KQ is obtained from a better approximation to the integrand that appears in an integral in the expression for the kernel function.

## REQUIREMENTS FOR CONVERGENCE

The DLM is a finite element method and, as such, needs suitable criteria to achieve sufficient accuracy. These include requirements for numbers of chordwise boxes determined by the reduced frequency, magnitude of box aspect ratios, and numbers of spanwise strips.

### Chordwise Convergence Criteria

Convergence studies are contained in Reference 2 and additional studies are presented in Reference 3. One of the findings in those papers is that the recommendation in Reference 4 of 12 chordwise boxes per wavelength should be revised to 50 boxes per wavelength. The “wavelength” referred to here is the distance $\lambda$ traveled by a particle at the freestream speed $V$ during a period of oscillation, $\lambda = \pi\bar{c} / k_r$, where $\bar{c}$ is the reference chord and $k_r$ is the maximum anticipated reduced frequency based on the reference semichord. The new box chord requirement can thus be stated as $\Delta x \le 0.02V / f$, where $V$ is the minimum freestream speed of interest and $f$ is the maximum frequency of interest, in Hz. To illustrate further, consider the unsteady lift coefficient of a rectangular wing, with $\bar{c} = 1.0$, oscillating in pitch about midchord at a reduced frequency $k_r$ of 2.0 and a Mach number M of 0.8. The number of chordwise boxes and the number of strips is kept equal, and the wing aspect ratio is varied to give different box aspect ratios. Figure 1-a and Figure 1-b present N5KA/N5KQ comparisons of the real and imaginary parts of the oscillatory lift coefficients for a box aspect ratio of 1.0. (Note the abbreviated scale.) For both versions, the variations in the coefficients decrease markedly as the number of chordwise boxes exceeds 30 or so.

For this example, the recommended 50 boxes per wavelength translates to 32 chordwise boxes. As will be shown below, the quartic approximation offers no real advantage for box aspect ratios this low, so the differences in the coefficients are attributed to the enhanced integrand approximation that was also included in N5KQ.

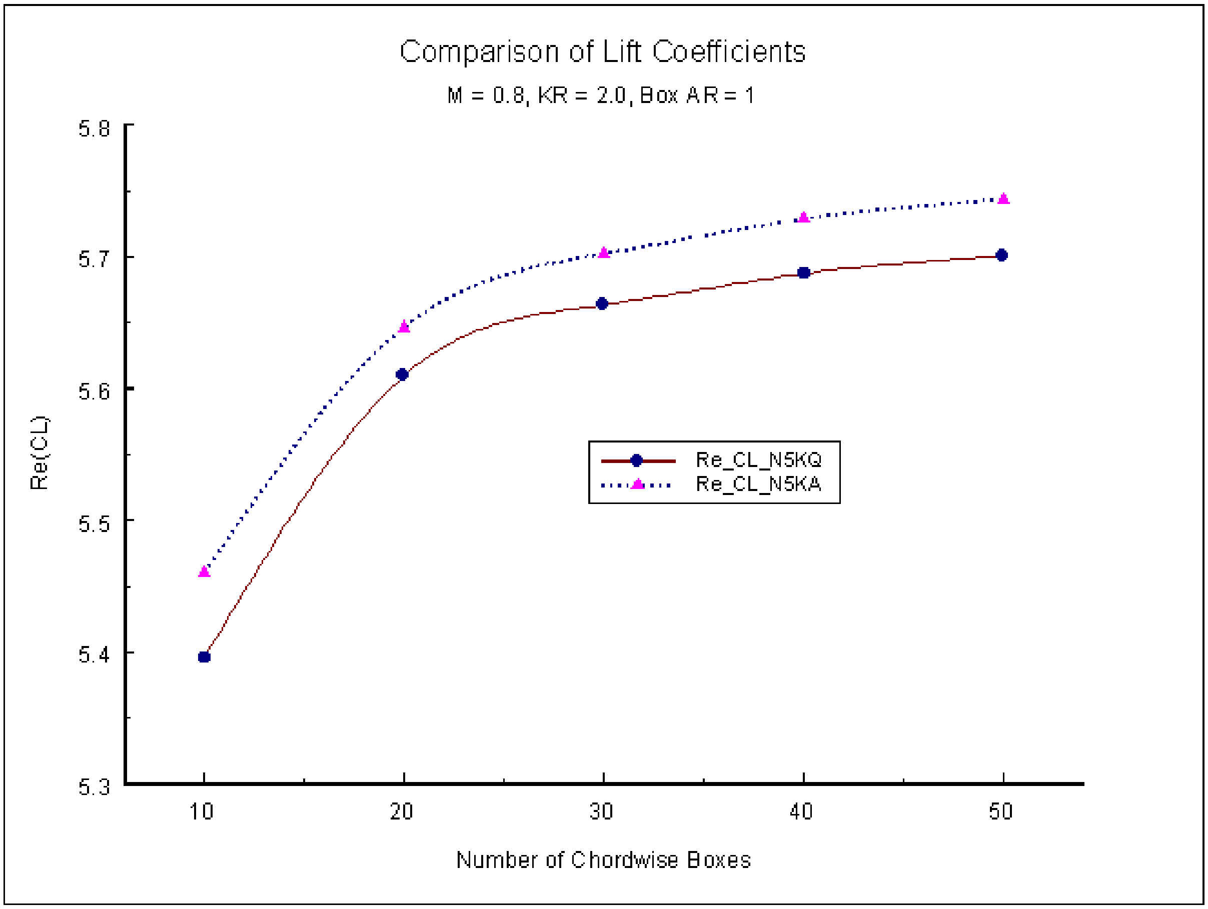


**Figure 1-a Comparison of Real Parts of Lift Coefficients - Box AR = 1.0.**

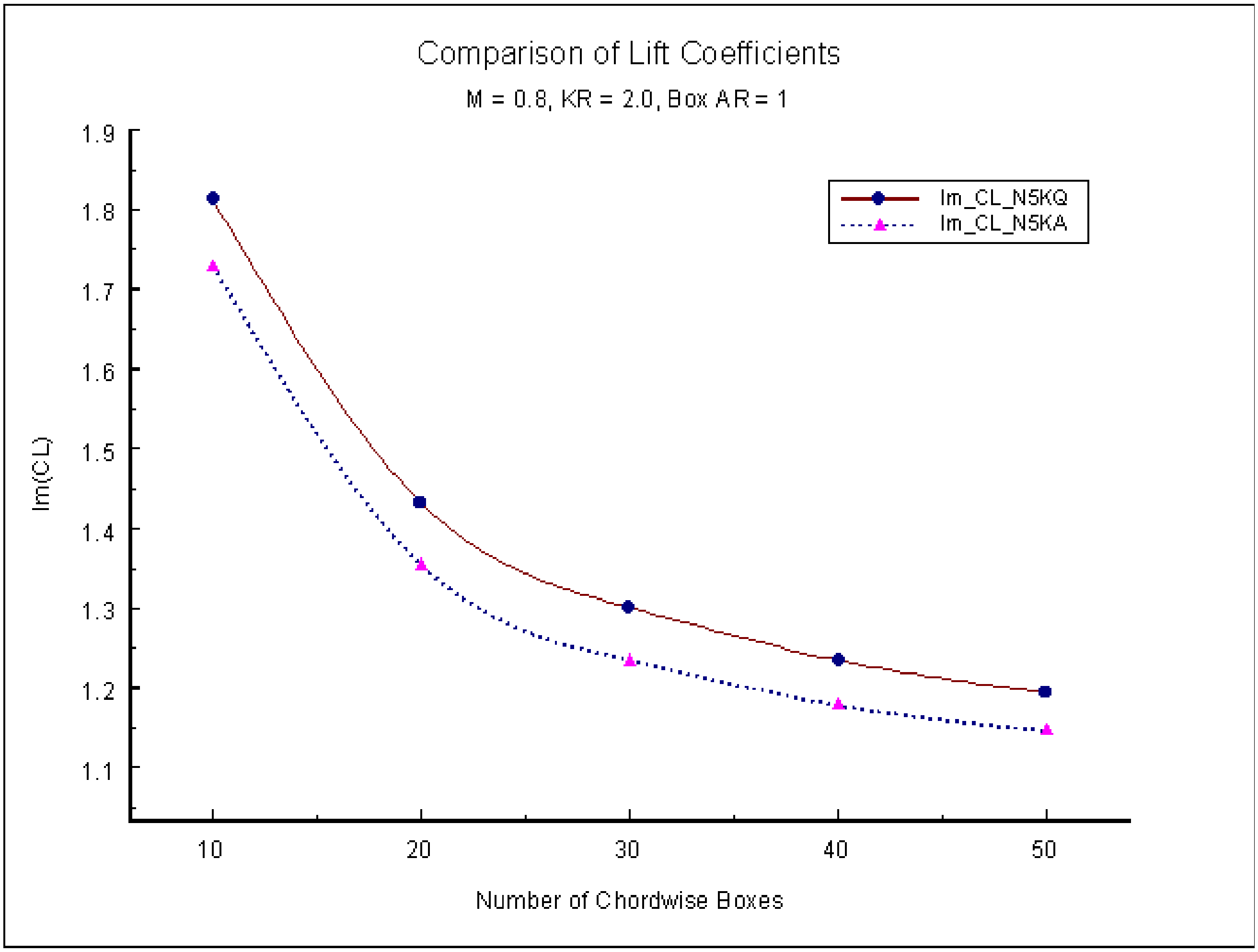


**Figure 1-b Comparison of Imaginary Parts of Lift Coefficients - Box AR = 1.0.**

This situation is quite different when the box aspect ratios are 10.0, as is illustrated in Figure 2-a and Figure 2-b. The data for N5KA suggest that even 50 boxes per wavelength might not be sufficient, and the differences between the two versions are much more pronounced, particularly for the imaginary parts. The increased degrees of freedom in the quartic approximation are more suitable for capturing the behavior of the imaginary part of the kernel across the box span and this explains the observation in Reference 3 that N5KQ offers superior damping estimates.

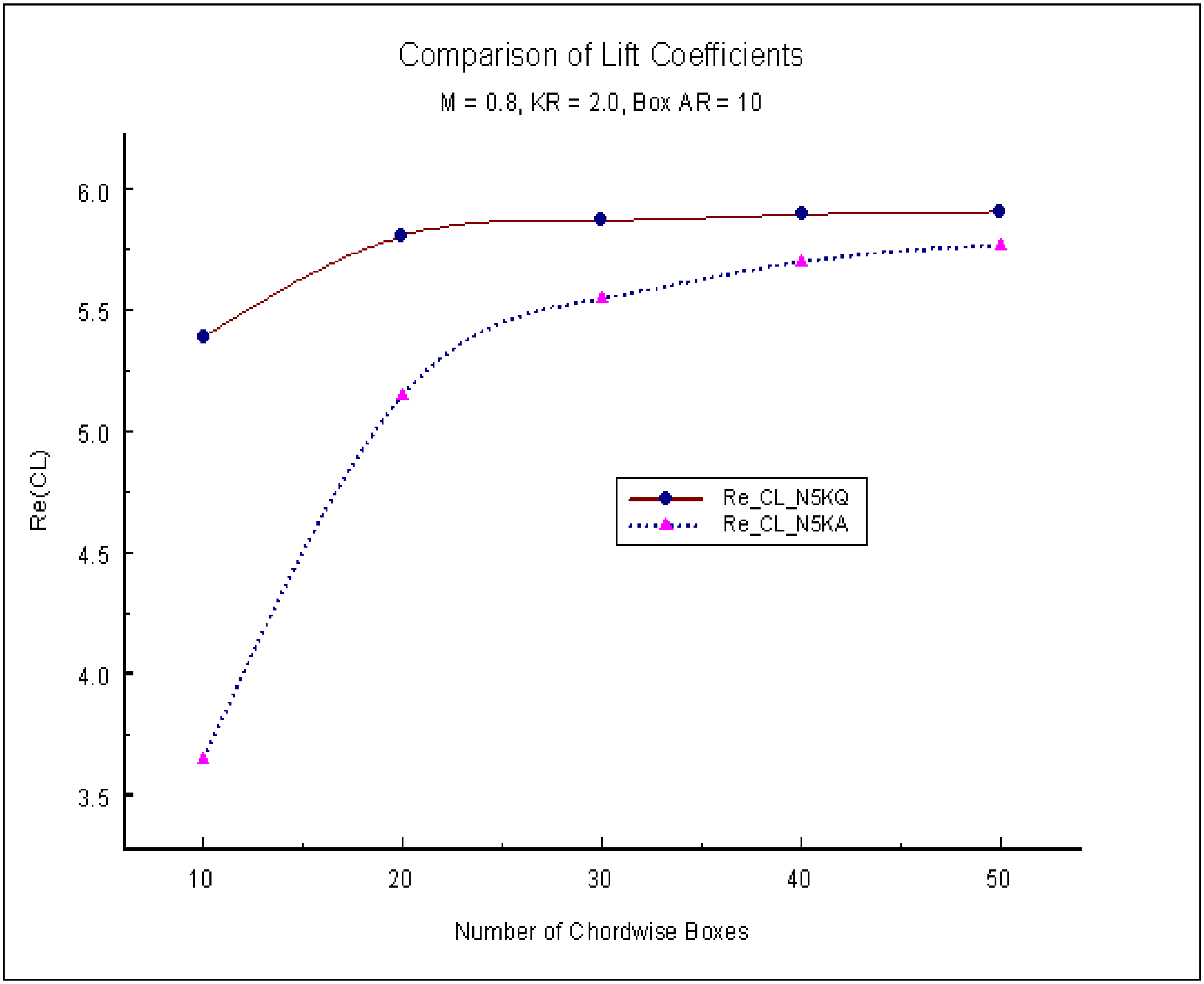


**Figure 2-a Comparison of Real Parts of Lift Coefficients - Box AR = 10.0.**

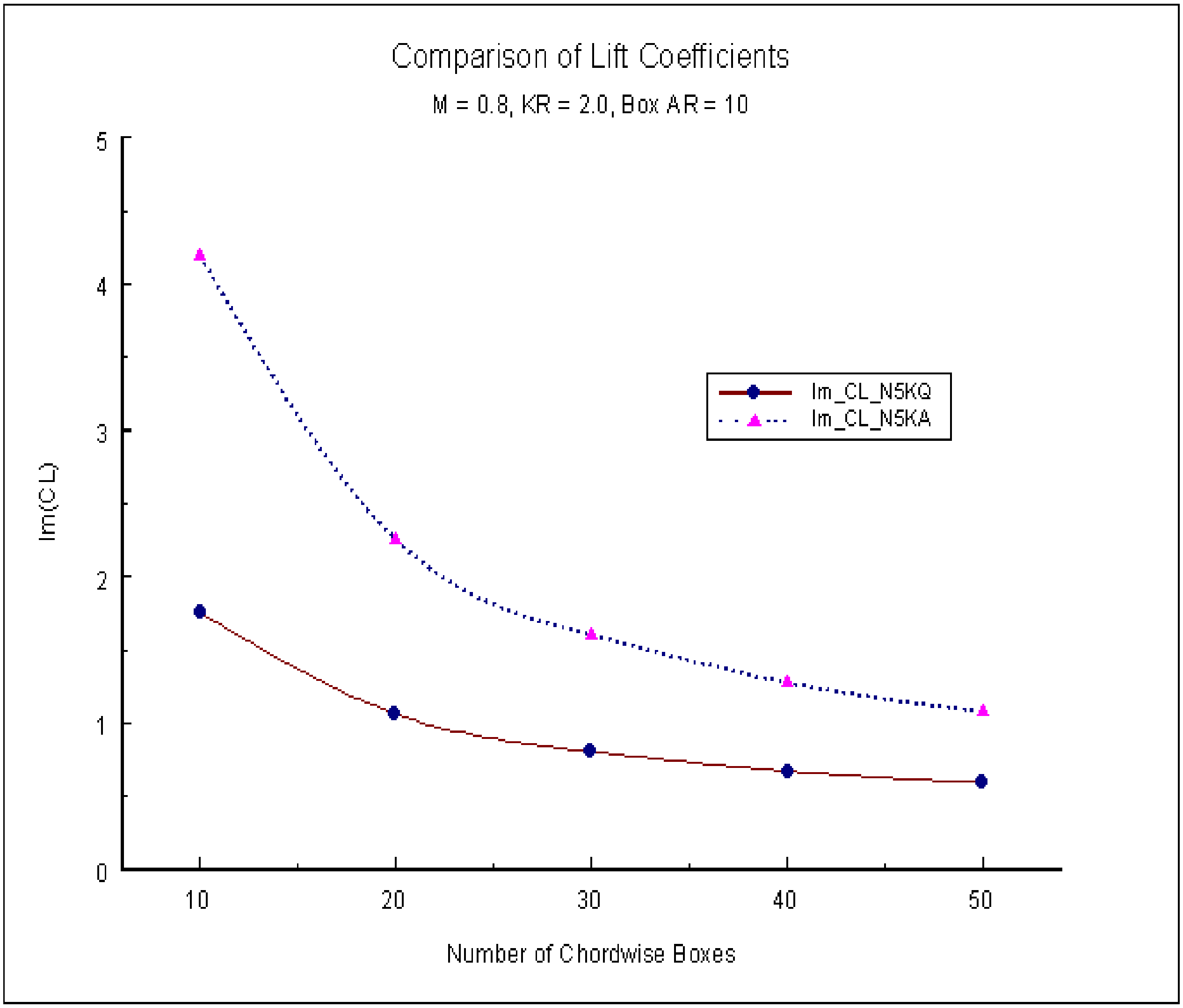


**Figure 2-b Comparison of Imaginary Parts of Lift Coefficients - Box AR = 10.0.**

## Increased Box Aspect Ratios

Previously published guidelines limited the box aspect ratios for N5KA to 3.0. In Reference 3, some evidence was found that box aspect ratios as high as 10.0 could be used for N5KQ. To examine this question further, we compare incremental oscillatory kernel functions for a 5x5 grid of boxes. The box chords are fixed at 0.2 units, and the box spans are varied to produce different box aspect ratios. The reference chord $\bar{c}$ is varied to provide the desired number of chordwise boxes. The layout is illustrated in Figure 3; note that the sending box is taken to be the central one, identified by the location parameters $IX$ = 3, $IY$ = 3. To meet the criterion of 50 boxes per wavelength for $k_r = 2.0$, a reference chord of 7.0 is chosen, which gives 35 chordwise boxes (NC = 35). Figure 4-a and Figure 4-b present comparisons of the real and imaginary parts of the planar kernel increment for identical sending and receiving boxes. The box aspect ratios, equal to the spanwise grid width *S*, are 5.0. The Mach number is 0.8. The quartic approximation is superior to the parabolic approximation, in particular for the imaginary part. Since the doublet-lattice solution involves integrating the kernel functions over the box spans, the areas under the curves are a truer measure of approximation errors, and it is seen that the quartic errors tend to be self-cancelling. Data for receiving boxes directly in front of and behind the sending box -- i.e., in the IY = 3 column -- yield similar conclusions. As the distance between receiving and sending boxes increases, both parabolic and quartic approximations approach the exact value. The situation for box aspect ratios of 10.0 is illustrated in Figure 5-a and Figure 5-b. Here the superiority of the quartic approximation is more pronounced. These results suggest that the previous aspect ratio limit of 3.0 for N5KA is reasonable, and that box aspect ratios higher than 10.0 should be avoided with N5KQ. It is strongly recommended, however, that some convergence studies be made for each user's configuration.

To examine the effects of sweep, the rectangular grid in Figure 3 was transformed to a parallelogram, swept 30 deg, and parabolic and quartic approximations were compared with exact data. Although the curves are shaped differently - for example, the curves in the IY=3 column are no longer symmetric about midspan - the superiority of the quartic approximation is still apparent, particularly for the receiving boxes nearest the sending box.

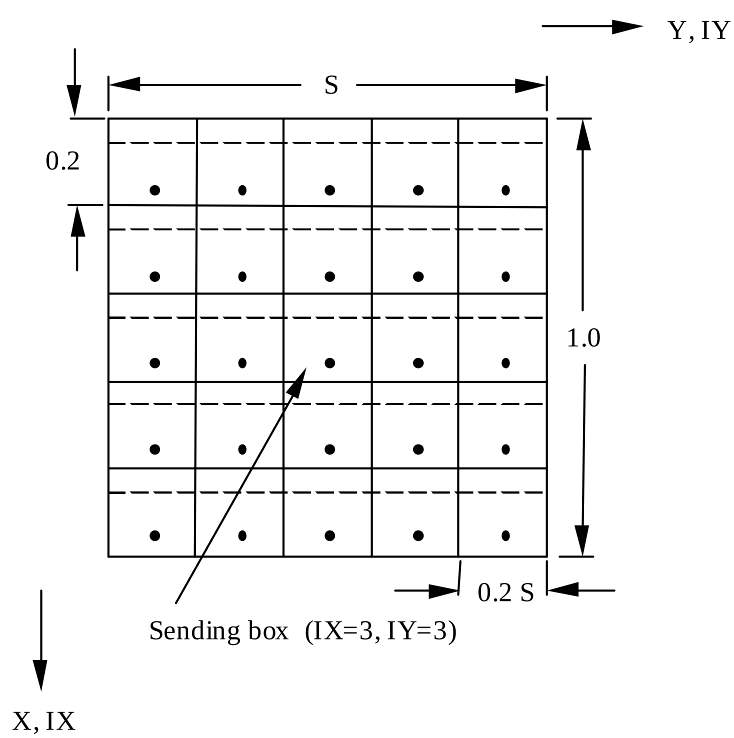


**Figure 3. Layout Used for Box Aspect Ratio Comparisons.**

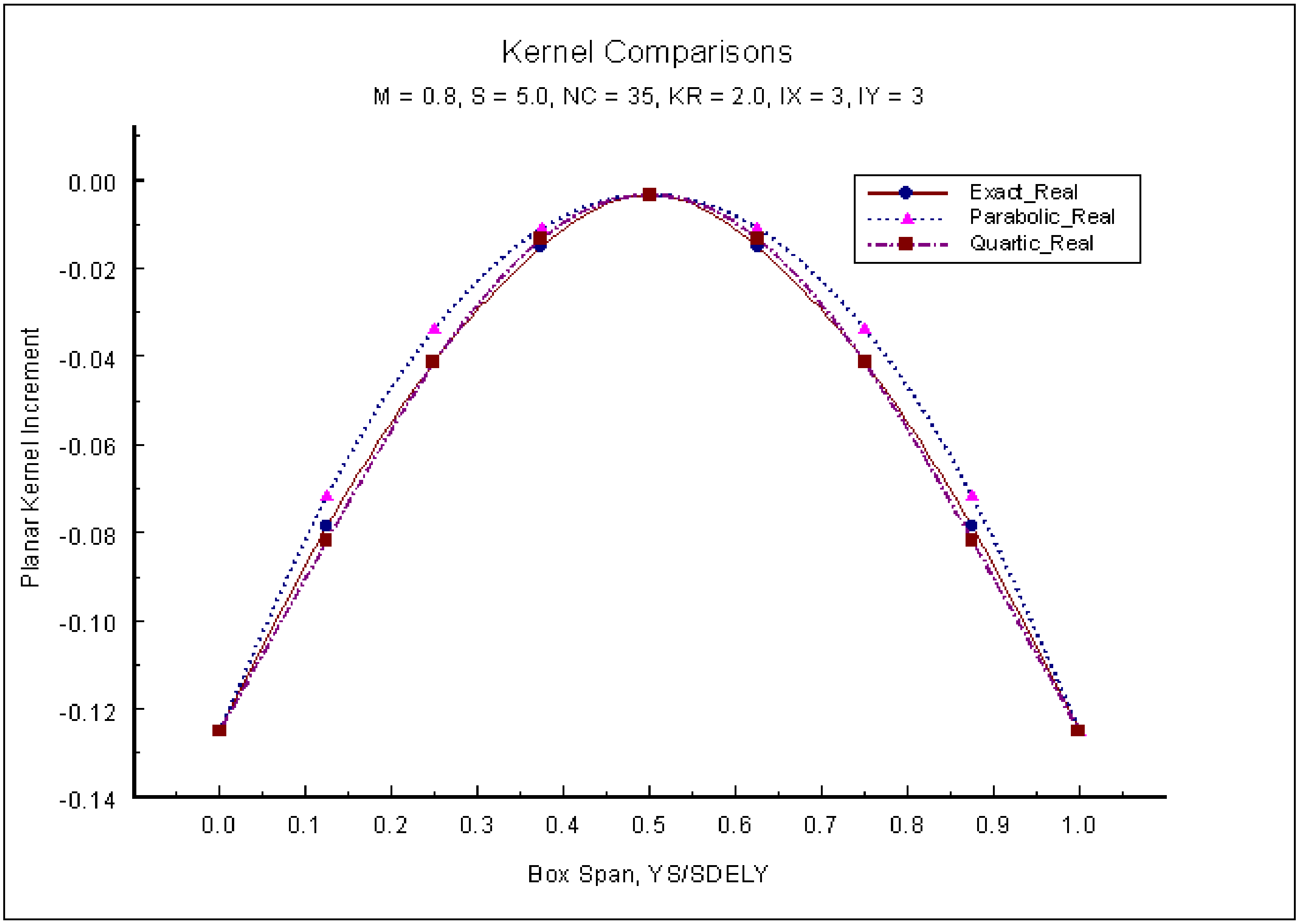


**Figure 4-a Comparison of Real Parts of Kernels - Box AR = 5.0.**

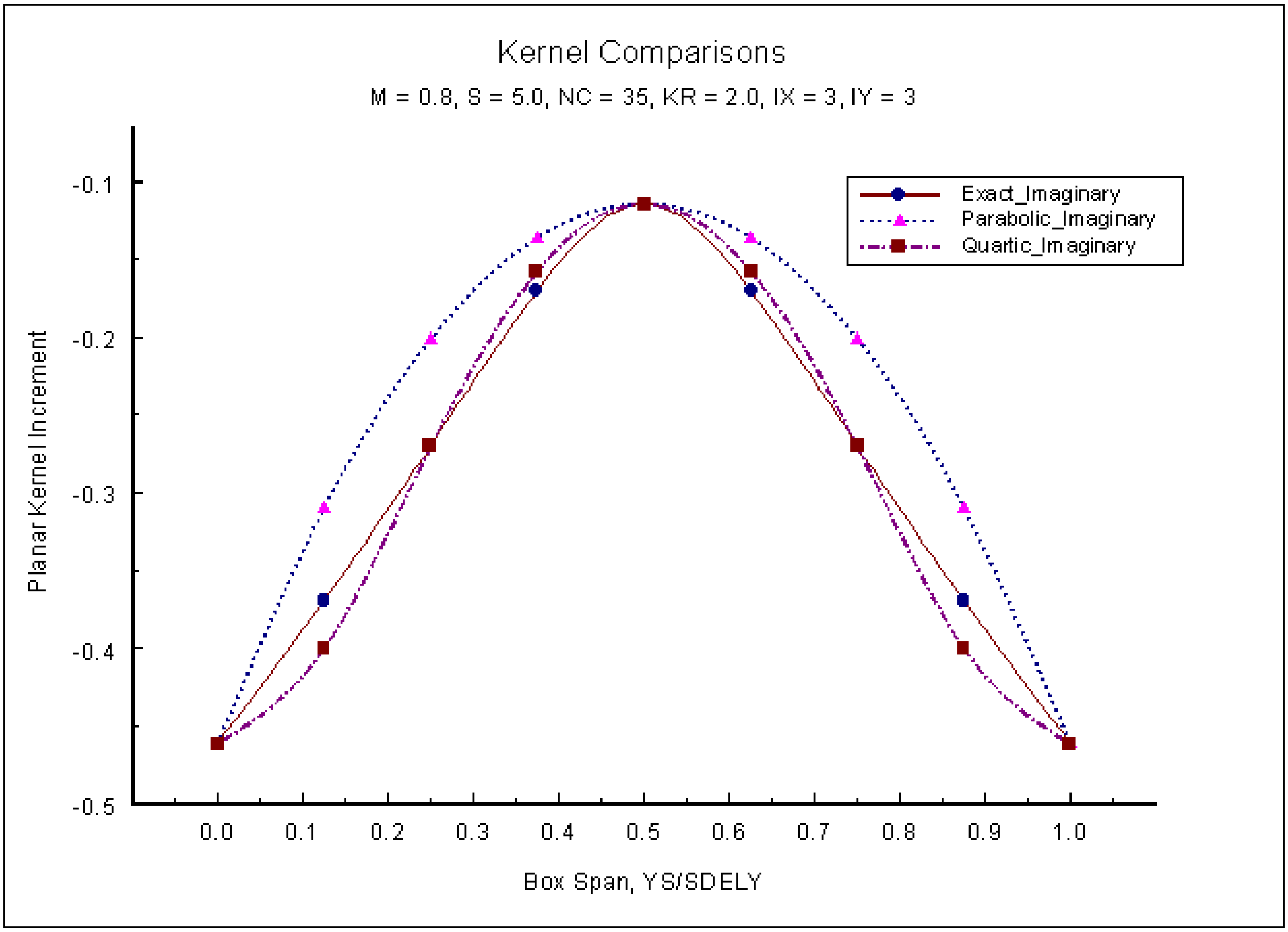


**Figure 4-b Comparison of Imaginary Parts of Kernels - Box AR = 5.0.**

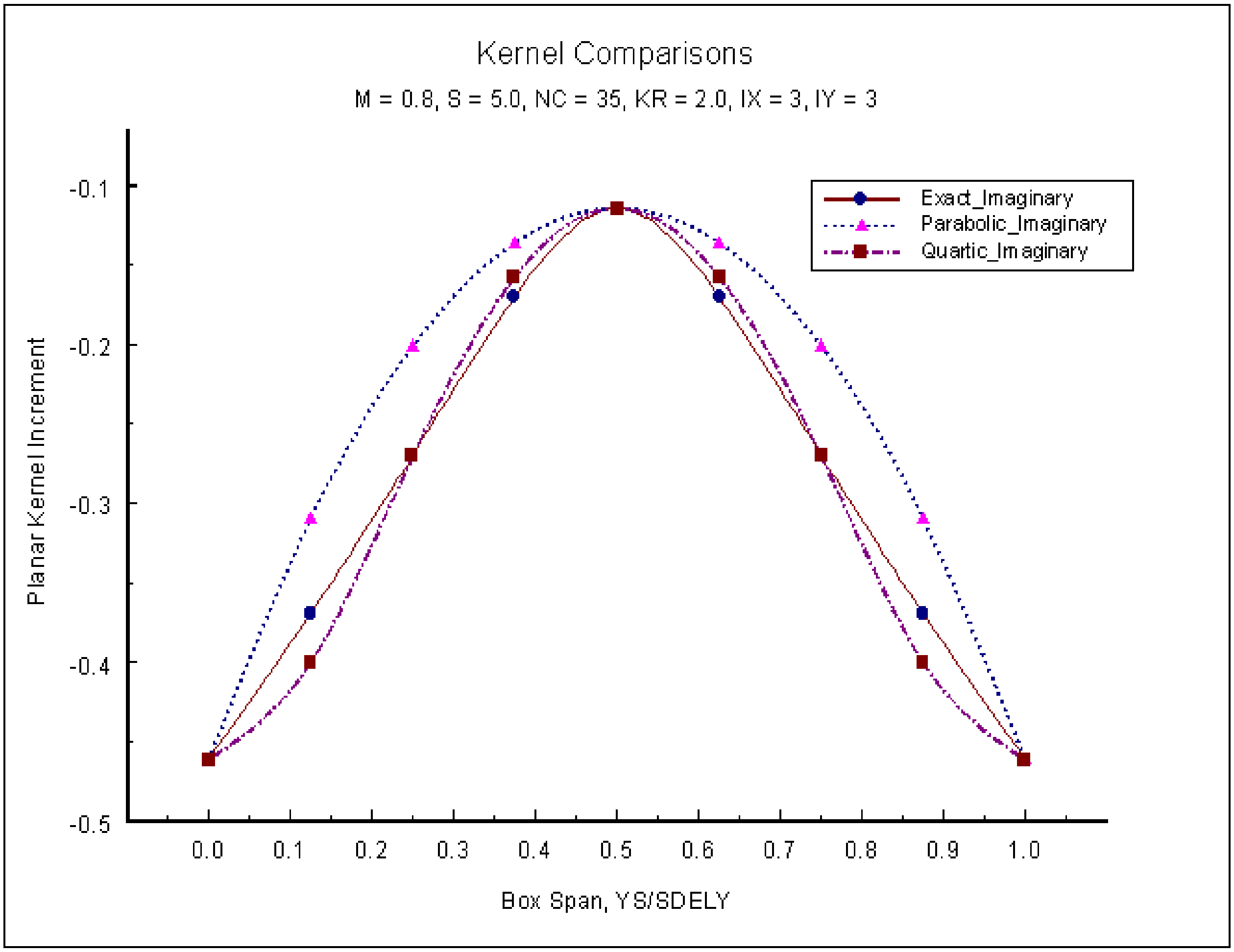


**Figure 5-a Comparison of Real Parts of Kernels - Box AR = 10.0.**

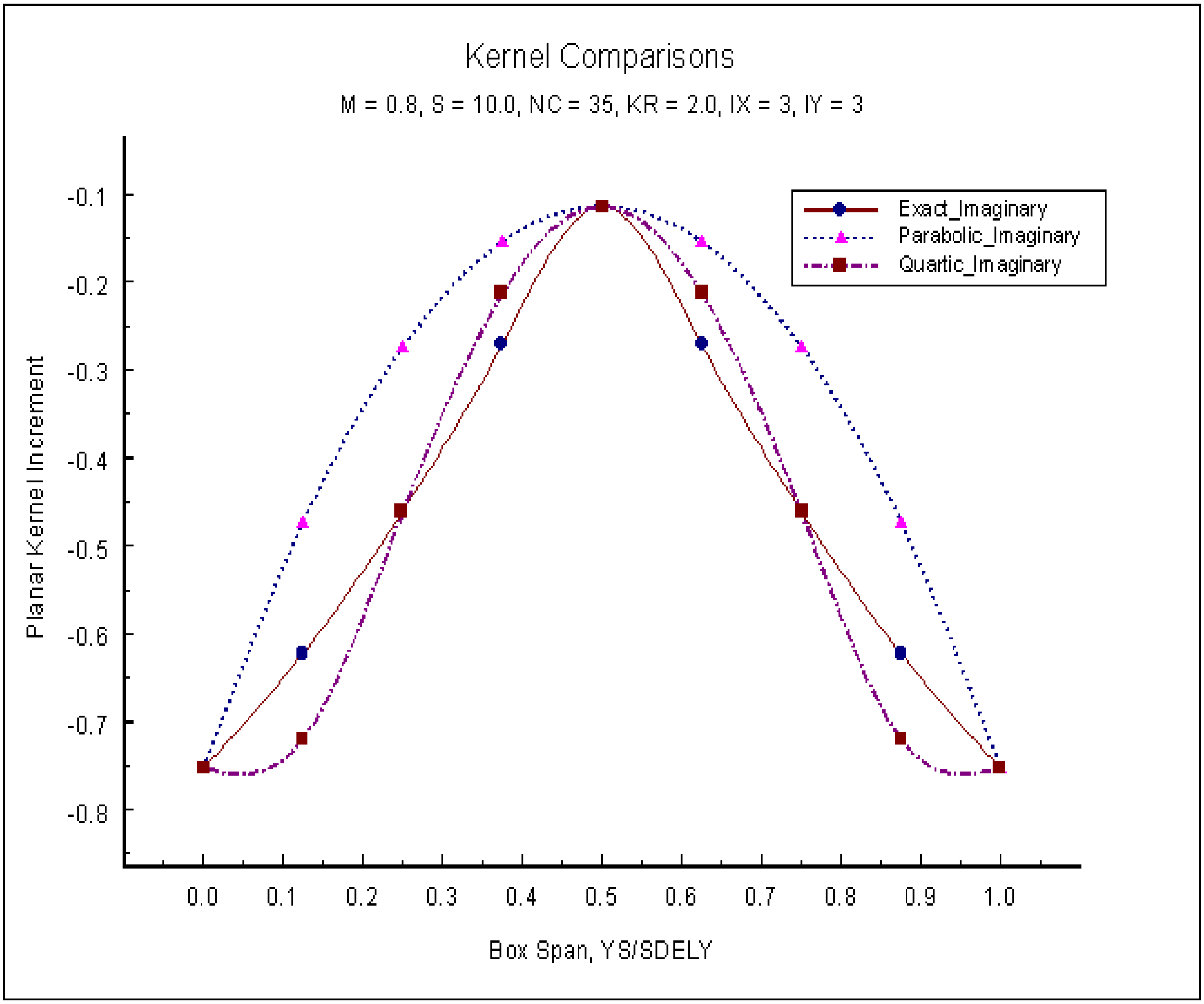


**Figure 5-b Comparison of Imaginary Parts of Kernels - Box AR = 10.0.**

### Spanwise Convergence Criteria

An investigation of the steady case, i.e., the Vortex-Lattice Method (VLM) to which the DLM reduces at zero frequency, showed slow convergence as the number of strips is increased. This has been observed in the past by Hough (Refs. 5 and 6) who showed improved convergence by following a suggestion of Rubbert (Ref. 7) that equally spaced lattices on a surface should be inset from the tip by a fraction $d$ of the lattice span $(0 \le d < 1)$(see Figure 6). Hough observed that $d = 1/4$ dramatically improves the convergence.

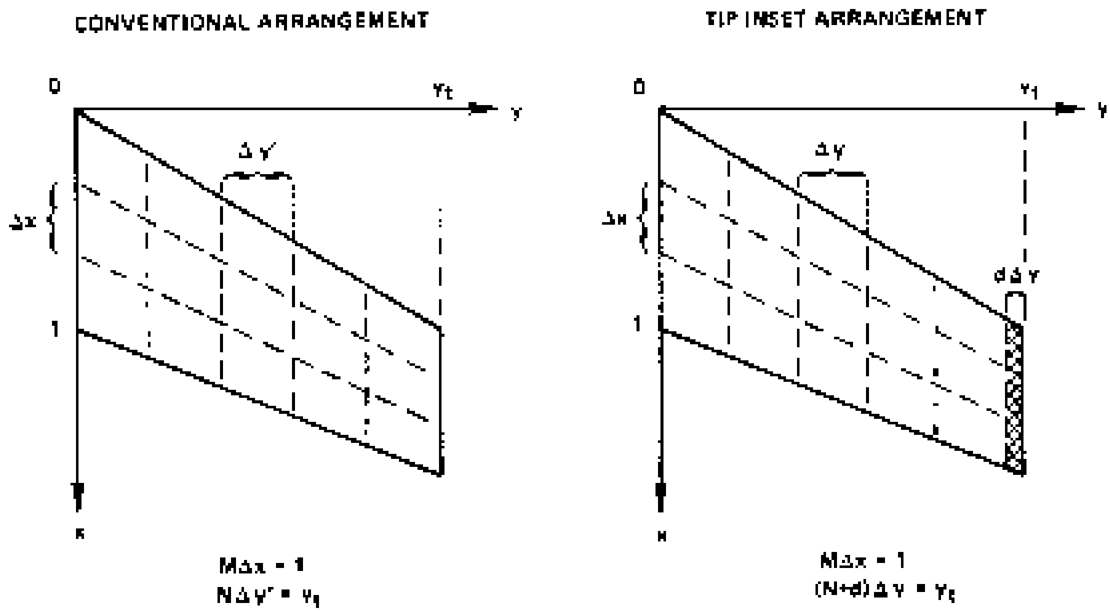


**Figure 6. Vortex-Lattice Layout (from Ref. 6)**

With a reasonable number of strips on a surface, say 20 to 30 the error is not large, perhaps 1 to 2% depending on the configuration (see Figure 7), and the tip correction may not be worth the effort unless the number of strips is small, say less than 20. However, requirements for loads analyses may dictate a larger number of strips and a tip correction may not be needed.

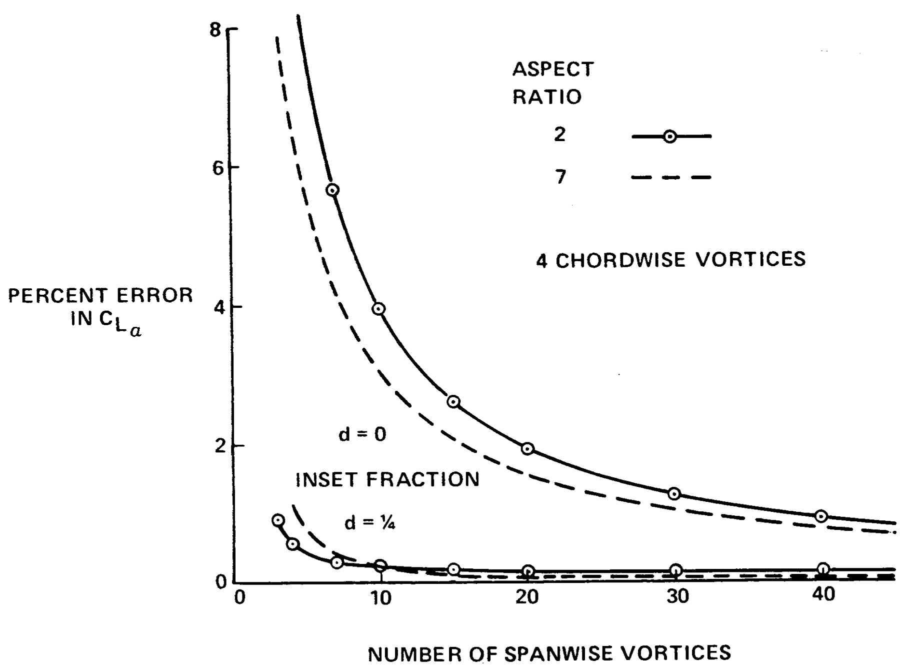


**Figure 7. Lift-Curve Slope Convergence for Rectangular Wing (from Ref. 6).**

It should be noted that Hough's argument leading to d = 1/4 is based on a symmetrical (elliptical) loading in the steady case. Extensive calculations (Ref. 8) indicate that it is also valid in the oscillatory case and for antisymmetrical (in roll) loading, so the tip correction is recommended in general, although users are advised to perform some convergence checks, with and without the tip correction, on their specific configuration.

If users wish to make the tip correction they can reduce the span of the most outboard panel on each surface on the corresponding CAERO1 continuation entry by the factor NSPAN/(NSPAN + *d*) where NSPAN is the number of strips on the panel and *d* is the inset fraction (*d* = 0.25 is recommended).

No other guidelines on spanwise convergence are suggested besides the limits on box aspect ratio discussed above.

**EXAMPLES**

Several examples illustrate new modeling guidelines, the tip correction, and relative computing time.

We illustrate the modeling guidelines by considering a rectangular wing with an aspect ratio AR = 7.0 cruising at M = 0.8 and pitching about its midchord at a reduced frequency of $k_r = 2.0$. The chord is $\bar{c} = 1.0$ and the semispan is $s = 3.5$. We consider two models for N5KA, one with a box aspect ratio of $AR = 2$ and the other with $AR = 3$. We also consider two models for N5KQ, the first with box aspect ratio $AR = 5$ and the second with $AR = 8$. The N5KQ method is activated in MSC/NASTRAN V70.5 using the 'NASTRAN QUARTIC DLM=ON' Keyword on the NASTRAN Command.

At $k_r = 2.0$ the wavelength is $\lambda = \pi\bar{c} / k_r = 1.571$ and the box chord should be $\Delta x < \lambda / 50 = 0.0314$ or the number of chordwise boxes becomes $NC = \bar{c} / \Delta x = 31.8$ which is rounded up to 32 and gives $\Delta x = 0.03125$. The first case for N5KA has a box $AR = 2$ so $\Delta y = 2\Delta x = 0.0625$ and the number of chordwise strips is $NS = s / \Delta y = 56$ . The span is adjusted for the tip correction to $S \bullet NS / (NS + 0.25) = 3.484444$ . With $NC = 32$ and $NS = 56$ the problem size is 1792 boxes. The steady lift curve slope is found at $k_r = 0.001$ to be $C_{L_\alpha} = 6.147$ and its oscillatory value is $C_{L_\alpha} = 5.817 + i0.6801$. The computing times at $k_r = 2.0$ in AMG is 370.9 CPU time units and in DECOMP 271.3 units with a total time of 642.2 units.

The maximum box AR for N5KA is 3 so the strip widths are $\Delta y = 3\Delta x = 0.09375$ and the number of spanwise strips becomes $NS = s / \Delta y = 37.3$ which is rounded up to $NS = 38$. So $\Delta y = s / 38 = 0.09210$ and the box AR becomes 2.95. The span is adjusted for the tip correction to be $s \bullet NS / (NS + 0.25) = 3.477124$ . The total number of boxes is $NC \bullet NS = 1216$. The steady lift curve slope is found at $k_r 0.001$ to be $C_{L_\alpha} = 6.147$ and the oscillatory lift curve slope at $k_r = 2.0$ is $5.789 + i0.7861$. The computing times at $k_r = 2.0$ are: AMG 169.6 CPU time units and in DECOMP 90.7 units with a total time of 260.3 units which is 40.5% of the time for the first model.

The two models for the analysis by N5KQ each require $NC = 32$ at $k_r = 2.0$ as before. The first will use a box AR = 5 and the second will use AR = 8. For the first model $\Delta y = 5\Delta x = 0.15625$ and $s/\Delta y = 22.4$ which is rounded up to $NS = 23$ strips. Thus $\Delta y = s/23 = 0.152174$ and the box AR becomes 4.87. The span is adjusted for the tip correction with the reduced span $3.5 \bullet 23/23.25 = 3.462366$. The total number of boxes is 736. The steady lift curve slope is $C_{L_\alpha} = 6.146$ and the oscillatory value at $k_r = 2.0$ is $5.837 + i06895$ in agreement with the first N5KA model. The computing times at $k_r = 2.0$ are: in AMG 97.0 and in DECOMP 21.1 units and a total of 118.1. The relative total computing times for this N5KQ model are 18.4% of the first N5KA model and 45.4% of the second N5KA model.

The second N5KQ model uses a box AR of 8 so $\Delta y = 8\Delta x = 0.250$ and $NS = 14$ strips. The adjusted span is 3.438596 and the total number of boxes is 448. The steady lift curve slope is $C_{L_\alpha} = 6.143$ in excellent agreement with the preceding three values and shows the tip correction to minimize the dependence on the number of spanwise strips. The oscillatory lift curve slope is $5.807 + i0.7975$ in agreement with the second N5KA model. The computing times at $k_r = 2.0$ are 36.7 units in AMG and 5.5 units in DECOMP and a total of 42.2 which is 6.6% of the first N5KA time, 16.2% of the second N5KA time, and 35.7% of the first N5KQ time.

The relative computing times for these examples are summarized in Table 1.

Table 1 - Comparative CPU Times Between N5KA and N5KQ for Consistent Aerodynamic Idealizations of a Rectangular Wing with AR=7 at M=0.8 and $k_r = 2.0$.

| | **N5KA** | | **N5KQ** | |
|---|---|---|---|---|
| Module | Box AR=2 | Box AR=3 | Box AR=5 | Box AR=8 |
| AMG | 370.9 | 169.6 | 97.0 | 36.7 |
| DECOMP | 271.3 | 90.7 | 21.1 | 5.5 |
| Σ | 642.2 | 260.3 | 118.1 | 42.2 |

In view of the improved approximation to the integrand that appears in the expression for the kernel function in N5KQ, it cannot be expected that N5KQ and N5KA would necessarily produce the same converged oscillatory lift coefficient in the example. It is seen, however, that when the recommended guidelines are followed for each model, the computational times for N5KQ are roughly 1/5 those for N5KA. This is a direct result of the ability of N5KQ to utilize higher box aspect ratios.

## CONCLUDING REMARKS

The recent refinement to the DLM as integrated into MSC/NASTRAN has been summarized. The quartic approximation in the kernel function allows either an increase in accuracy for current box schemes, or a reduction in the number of boxes (and subsequently the storage requirements and execution times) for the same accuracy. Present recommendations include 50 chordwise boxes per wavelength with a maximum box aspect ratio of 10. A tip correction reduces the sensitivity of results to the number of spanwise strips.

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