

APPLICATION OF STRUCTURAL OPTIMIZATION ON REDESIGNING THE FRENCH-BRAZILIAN MICRO SATELLITE

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ABSTRACT

The French-Brazilian Micro Satellite (FBMS) is a scientific satellite, which will be piggyback launched by the rocket Ariane 5. Its most critical design constraints are: the lower bound of 40.0 Hz on the first natural frequency, in order to avoid coupling between the rocket excitation modes and the natural vibration modes of the satellite; and the upper bound of 10.5 kg on the structural mass. The structure of the FBMS is composed of a cylindrical aluminum alloy adapter for connection with the rocket, and eight sandwich panels (each composed of three layers) that define its topology. In this paper, we show the importance of structural optimization and design sensitivity analysis in the redesign cycles of Space Structures, by presenting all the steps taken and the difficulties encountered as we tried to maximize the first natural frequency from the low value of 18.78 Hz obtained with the first trial design, while maintaining the structural mass below the predefined upper bound. All the modal and sensitivity analyses as well as the optimization steps were performed using MSC/NASTRAN. The design variable space for the structural optimization steps was composed of the thicknesses of the faces and core of the sandwich panels.

INTRODUCTION

Structural optimization seeks to find a point in a given design space (a set of design variables) so that a certain functional (objective function) is minimized or maximized. Usually, a number of constraint functionals, which depend on the design variables of the problem, and a set of lateral constraints on the design variables are imposed, delimiting the so called feasible region where the solution lies. Although structural optimization, as a field of studies, is not new [3,7,10], only in the 1980's it really start to be applied to complex structural systems[13]. This was possible due to the increase in computer power and to the advances in optimization methods such as the method of feasible directions developed in that decade by Vanderplaats[14] and that has been used extensively since then. The gradient of the functionals with respect to the design variables, needed in that method, were computed by finite difference which is very costly. Since then, advances in Design Sensitivity Analysis [4,8] formulations, that made the design gradient computations very efficient, helped further to disseminate the use of structural optimization in complex structures. Nowadays, many of the advancements in both structural optimization techniques[2,5,9] and design sensitivity analysis are implemented and, therefore, available in commercial finite element analysis software such as MSC/NASTRAN [8,12].

In this paper, we show the importance of structural optimization and design sensitivity analysis in the redesign cycles of the French-Brazilian Micro Satellite (FBMS). First, we present the problem definition and the challenge for the structural optimization study. Next, we describe the analysis strategy and the steps taken for solving the structural optimization problem. Finally we discuss the results and draw some conclusions.

PROBLEM DEFINITION

The French Brazilian Micro Satellite (FBMS) is a satellite of low terrestrial orbit, designed to carry out scientific experiments in space. It will be piggyback launched by the rocket Ariane 5. The payload consists of the equipment for the following scientific experiments:

- FIRE – “Flare Infrared Experiment” – will perform continuous measurements of solar flares from space;
- PDP – “Plasma Diagnostics Package” – consists of three different plasma diagnostics experiments, which will measure plasma parameters of the ionosphere;
- CBEMG – “Confined Boiling Experiment under Microgravity”– will allow for the study of nucleation, of nucleate boiling and of heat flux, under microgravity conditions, along four test sections, each one confined between two aluminum flat plates;
- CPL – “Capillary Pumped Loop” – will test a small scale capillary pumped loop in order to assess its performance under microgravity conditions;
- FLUXRAD – “Fluxmeter/Radiometer” – will measure the heat flux exchanged between the FBM faces directed towards the sun and those facing the cold space.

The satellite's structure is composed of a cylindrical aluminum alloy adapter for connection with the rocket, and eight sandwich panels (each composed of three layers) that define its topology (see Figure 1).

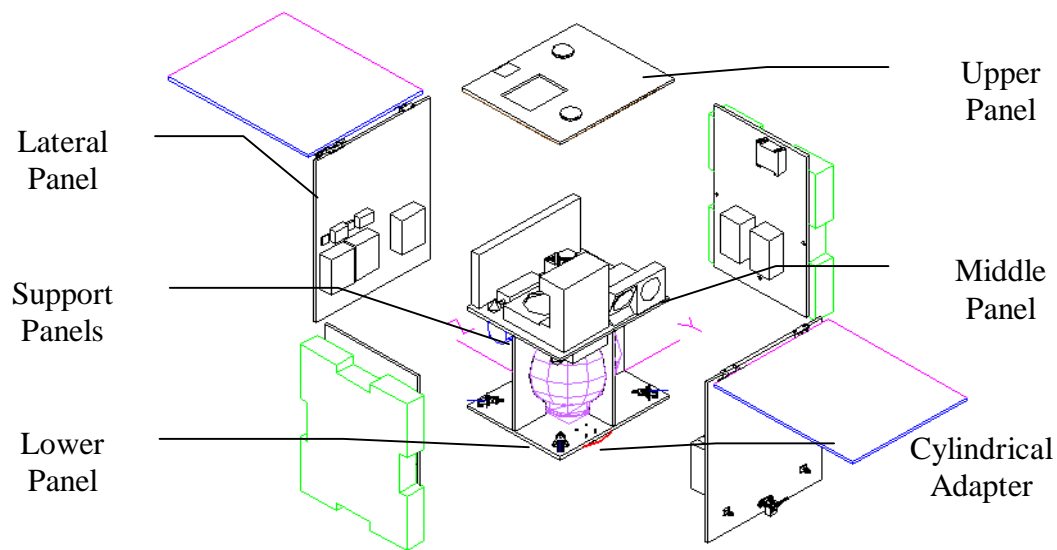


Figure 1. Exploded view of the satellite

The two external layers (faces) of the sandwich panels are made of aluminum sheets which is a much stiffer material than that of the inner layer (core) which is composed of a low-density aluminum honeycomb (see Figure 2). This type of sandwich configuration presents a high bending stiffness and a reduced weight.

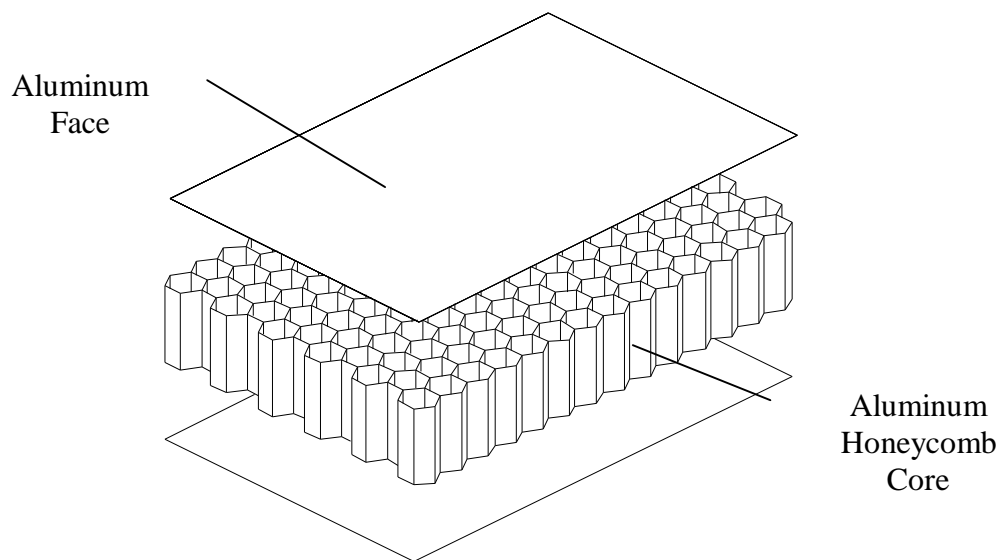


Figure 2. Sandwich Panel Representation

After defining the topology of the FBMS and the type of sandwich panel to be used, a number of design constraints was established in order to perform some preliminary

analyses. As a result of these analyses, an initial configuration was reached that satisfied all the design constraints except the one associated with the first natural frequency of the structure. The initial configuration had a structural mass of 10.5 kg which coincided with the upper bound on structural mass. However, the first natural frequency of the structure was only 18.8 Hz which was well below the minimum allowable of 40.0 Hz. This latter constraint was imposed in order to avoid coupling between the rocket excitation modes and the natural vibration modes of the satellite. These two constraints are the most critical and difficult to satisfy because it is very hard to increase the stiffness of the structure without a corresponding increase of structural mass. Therefore, the structural design challenge is to increase the first natural frequency without violating none of the constraints.

In the following sections, we discuss the importance of structural optimization and design sensitivity analysis in helping to face that challenge. We also present all the steps taken and the difficulties encountered as we tried to maximize the first natural frequency from the low value of 18.78 Hz, obtained with the first trial design, while keeping the structural mass below the predefined upper bound

ANALYSIS

In this section, we describe the finite element model, the analysis strategy and all the steps taken in order to obtain an optimized design.

The Finite Element Model

The finite element model of the satellite (Figure 3) was constructed using the pre- and post-processing package, FEMAP[16]. The cylindrical aluminum alloy adapter was modeled by shell elements made of an isotropic material (Aluminum 2024-T3) whose properties are defined on Table 1. The eight sandwich panels were modeled by laminate shell elements made of composite material which use the properties of Aluminum 2024-T3 for the faces and the properties of the aluminum honeycomb (3/8-5052-0.0015) for the core, also listed on Table 1. The total number of shell elements is 1,293, connected to a total of 1,114 nodes. The geometric and material properties of the shell elements are lumped in 9 different groups, one for each of the eight sandwich panels and one for the cylindrical adapter. The equipment were modeled as non-structural masses distributed over the sandwich panels. The spherical tank, located in the center of the satellite, is modeled as a lumped mass positioned at its center of gravity and possessing both, translational and rotational inertia properties. Interconnection of panels are modeled by rigid elements (NASTRAN's RBE2 element). Boundary conditions were simulated by single point constraints located at the attachment points of the cylindrical adapter to the rocket.

Before any stress or modal analysis was performed, a series of tests, suggested by NASA[15] to verify the correctness of the Finite Element model, was done.

The Analysis Strategy

In order to facilitate the discussion section, here, we present a description of all the steps we took in the attempt to maximize the first natural frequency of the FBMS' structure.

Table 1. Material properties

Aluminum 2024-T3	Isotropic material
Young's modulus	$E = 6.80E+10 \text{ N/m}^2$
Shear modulus	$G = 2.56E+10 \text{ N/m}^2$

Table 1. (continued)

Aluminum 2024-T3	Isotropic material
Poisson's ratio	$\nu = 0.33$
Density	$\rho = 2,700.00 \text{ N/m}^3$
Aluminum Honeycomb 3/8-5052-0.0015	Orthotropic 2D material
Young's modulus	$E_{12} = 1.000E+6 \text{ N/m}^2$
	$E_{1z} = 2.206E+8 \text{ N/m}^2$
	$E_{2z} = 2.206E+8 \text{ N/m}^2$
Poisson's ratio	$\nu = 0.33$
Density	$\rho = 36.80 \text{ N/m}^3$

Step 1: Definition of the design space

After lumping the element properties, a total of 16 design variables was defined: each face thickness (the two faces of a sandwich panel are considered of having the same thickness) and each core thickness of the 8 sandwich panels

Step 2: Preliminary optimization

Two preliminary optimizations with the same goal of reducing or redistributing structural mass and augmenting the first natural frequency, but with different problem formulations were performed. The upper and lower bounds on the design variables were arbitrarily set and a continuous variation of the design variables was allowed.

$$\begin{aligned}
 \text{PO1: } & \begin{cases} \text{Minimize } W \\ \text{subject to} \\ W \leq 10.5 \text{ kg} \\ I_1 \geq 20\text{Hz} \\ 12.7 \times 10^{-5} \text{ m} \leq t_{\text{face}} \leq 1.0\text{m} \\ 1.0 \times 10^{-3} \text{ m} \leq t_{\text{core}} \leq 1.0\text{m} \end{cases} & \text{PO2: } & \begin{cases} \text{Maximize } I_1 \\ \text{subject to} \\ I_1 \geq 20\text{Hz} \\ W \leq 10.5\text{kg} \\ 12.7 \times 10^{-5} \text{ m} \leq t_{\text{face}} \leq 1.0\text{m} \\ 1.0 \times 10^{-3} \text{ m} \leq t_{\text{core}} \leq 1.0\text{m} \end{cases}
 \end{aligned}$$

where W is the structural weight, λ_1 is the first natural frequency in Hz, t_{face} are all the face thicknesses and t_{core} are all the core thicknesses.

Step 3: Preliminary optimization with commercial lower bound on t_{face}

Similar to Step 2 except that the lower bound on the face thicknesses were set to the minimum aluminum plate thickness available in the market,

1.524E-4 m. Notice that the honeycomb core is milling machined upon demand with the required thickness. We will refer to these optimization runs as POLB1 and POLB2.

Step 4: Optimization cycle with commercial thickness correction I

In this step, an optimization cycle based on POLB2 is performed, according to the following algorithm:

- i) Perform optimization POLB2 with the current set of design variables;
- ii) At the new location in the design space, look for the design variable which is closest to a commercial value;
- iii) Set the value of the design variable found in ii) to that commercial value and remove that variable from the design space;
- iv) Repeat i) to iii) until all the design variables are set to commercial values.

We will refer to this optimization cycles as OCI_i , where the index i ranges from 1 to the number of face thickness design variables.

Step 5: Design space reduction

In this step, a design sensitivity analysis is performed, prior to any further optimization step, in order to identify the design variables that, for a small increase in their original values, there is a significant increase in the first natural frequency of the structure (positive signs of the sensitivity array). In principle, the other design variables should be eliminated from the design space (not allowed to vary) since they do not have much influence on the first natural frequency of the satellite structure. Notice that, for the problem at hand, the design variables corresponding to negative sensitivity terms have the most beneficial effect, since they increase the first natural frequency with a corresponding reduction of the structural mass.

Step 6: Optimization cycle with commercial thickness correction II

This cycle, referred to as $OCII_i$, is identical to OCI_i except that in $OCII_i$ we look for the design variables with smallest sensitivity terms.

Step 7: Changes in the lower panel

The design sensitivity analysis of Step 5 indicated that the design variables of the lower panel were the most effective in changing the first natural frequency. In this step, the optimized model of Step 6 was used as the basis for two new models that differ only on the honeycomb topology or on the material type of the faces of the lower panel. In order to construct the first model (model M1HC), seven types of honeycomb cores were tested (HCLP_i). The best honeycomb configuration was adopted in the second model. To complete the second model, a four layer carbon fiber composite laminate was used as faces for the lower panel. A final optimization run was performed on model 2, here referred as M2CF.

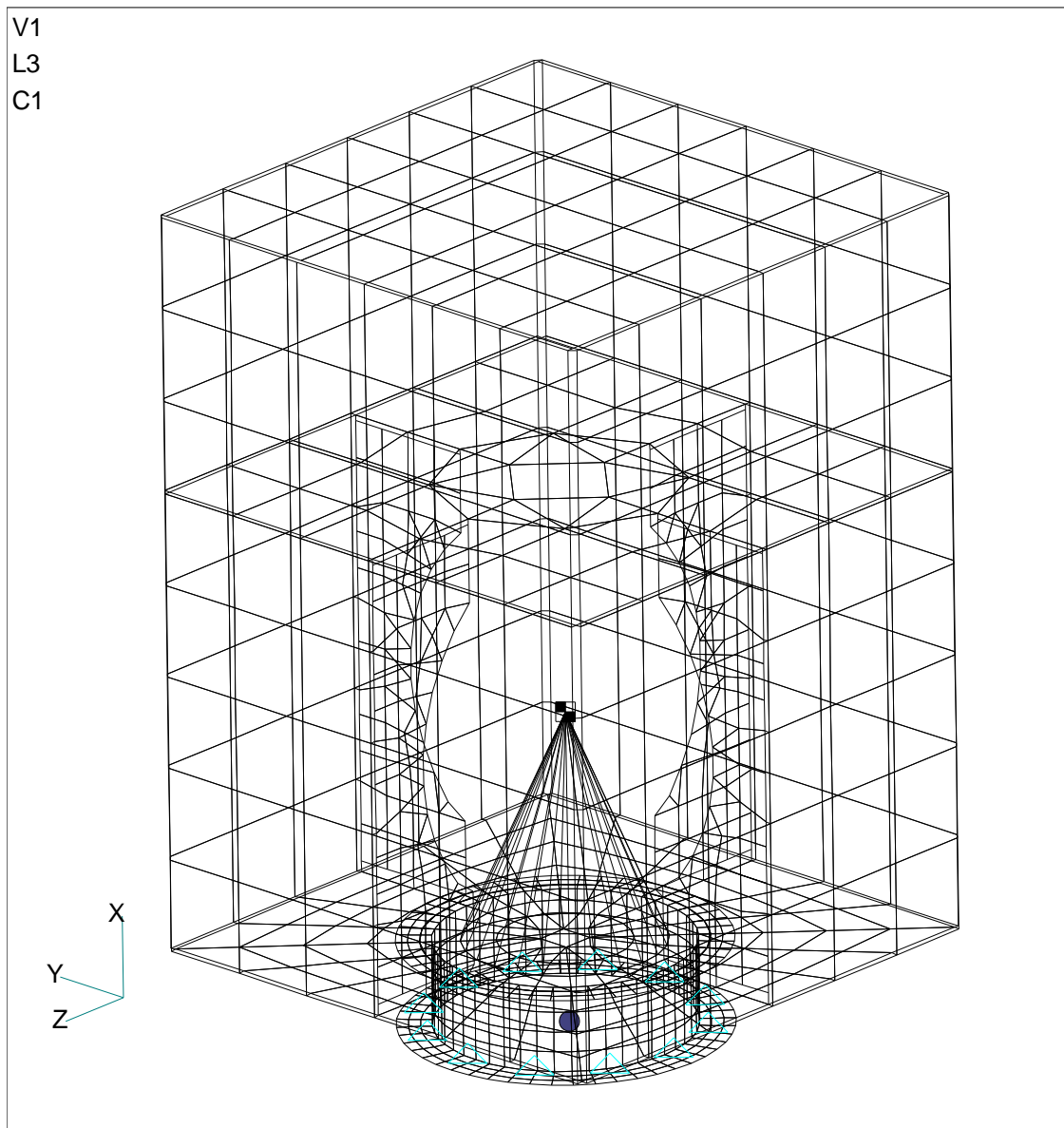


Figure 3. Finite Element model of the FBMS

DISCUSSIONS

In this section, we present and discuss the results of each analysis strategy step described in the previous section. In Table 2, we show the results of the preliminary optimizations PO1 and PO2 described in Step 2. PO1 presented a 35% reduction of the structural mass and a 6.4% increase in the first natural frequency, while PO2 presented no reduction on the structural mass but achieved a 10.2% increase in the first natural frequency.

Table 2. Results of preliminary optimizations PO1 and PO2

	Initial value	PO1	PO2
Non structural mass (kg)	87.05	87.05	87.05
Structural mass (kg)	10.50	6.83	10.50
Total mass(kg)	97.55	93.88	97.55
Frequency (Hz)	18.78	19.98	20.77

In Table 3, we display the results of the preliminary optimizations POLB1 and POLB2 described in Step 3. POLB1 presented a 33.6% reduction of the structural mass and a 6.5% increase in the first natural frequency, while POLB2 presented no reduction on the structural mass and achieved a 14.4% increase in the first natural frequency.

Table 3. Results of preliminary optimizations POLB1 and POLB2

	Initial value	POLB1	POLB2
Non structural mass (kg)	87.05	87.05	87.05
Structural mass (kg)	10.50	6.97	10.50
Total mass (kg)	97.55	94.02	97.55
Frequency (Hz)	18.78	20.00	21.48

In Table 4, we display the results of the optimization cycles described by the algorithm of Step 4 (OCI). The line where it reads cycle 0 refers to the results of the preliminary optimization PO2 of Step 2 and is displayed for comparison purpose. The core thicknesses displayed in column 5 are rounded to millimeter. At the end of the eighth cycle, all the face thicknesses are set to commercial values and the optimum first natural frequency is 22.83 Hz, which represents an increase of 21.6% in relation to the first trial design and approximately 10% in relation to the optimized value of PO2.

Table 4. Optimization cycle with commercial thickness correction - OCI

Cycle	Panel	Design variable	Optimized value (m)	Closest commercial value (m)	Total mass (kg)	Frequency (Hz)
0					97.55	20.77
1	Lateral [+y]	Face	3.08E-4	3.05E-4	97.55	20.72
		Core	1.50E-2	1.50E-2		

Table 4. (continued)

Cycle	Panel	Design variable	Optimized value (m)	Closest commercial value (m)	Total mass (kg)	Frequency (Hz)
2	Lateral [-y]	Face	3.01E-4	3.05E-4	97.55	22.31
		Core	1.50E-2	1.50E-2		
3	Middle	Face	1.68E-4	1.52E-4	97.55	22.93
		Core	1.54E-2	1.50E-2		
4	Lower	Face	1.56E-3	1.60E-3	97.55	22.85
		Core	3.39E-2	3.40E-2		
5	Upper	Face	1.27E-4	1.52E-4	97.55	22.80
		Core	1.50E-2	1.50E-2		
6	Lateral [-z]	Face	2.45E-4	3.05E-4	97.55	22.67
		Core	1.50E-2	1.50E-2		
7	Lateral [+z]	Face	2.40E-4	3.05E-4	97.55	22.78
		Core	5.00E-3	5.00E-3		
8	Support	Face	1.27E-4	1.52E-4	97.55	22.83
		Core	1.87E-2	1.90E-2		

The design sensitivity analysis of Step 5 computed the sensitivities of the first natural frequency of the satellite's structure with respect to changes in the design variables. The results are presented in Table 5. Notice that the larger the absolute value the more sensitive the first natural frequency is to small changes in the corresponding design variable. For the problem at hand, design sensitivity variables with negative sensitivity terms have a beneficial effect. This is so because the negative sign implies that a thickness reduction (which corresponds to mass reduction) will increase the first natural frequency and this is precisely what we want. If we impose some very tight lateral constraints on the design variables with positive sensitivities, we may still obtain an increase on the first natural frequency at the expense of a modest increase of mass. From the sensitivity values displayed on the table, we can see that the design variables associated with the Lower Panel are the most effective in increasing the first natural frequency. The sensitivity information is useful in selectively reducing the design space as was done in Step 6.

Table 5. Design sensitivity analysis of first natural frequency

Panel	Description	Sensitivities	
		w.r.t. face thickness	w.r.t. core thickness
1	Lower panel	7.1251 E+6	4.1293 E+5
2	Support panels	1.7865 E+6	2.4744 E+3
3	Middle panel	1.1914 E+6	4.7347 E+4
4	Lateral panel [-z]	4.7906 E+5	-2.0546 E+3
5	Lateral panel [+y]	1.6249 E+6	2.8084 E+4
6	Lateral panel [+z]	4.8121 E+5	-1.8696 E+3
7	Lateral panel [-y]	1.5855 E+6	2.5029 E+4

Table 5. (continued)

Panel	Description	Sensitivities	
		w.r.t. face thickness	w.r.t. core thickness
8	Upper panel	-7.5264 E+5	-4.2330 E+3

In Table 6, we display the results of the optimization cycles described in Step 6 (OCII). The line where it reads cycle 0 refers to the results of an optimization step where the eight design variables of the lateral panels were lumped in two design variables: the face thickness (considered the same for all the four panels) and the core thickness (also common to these panels). Notice that this was responsible for an increase of 30.2% in the frequency relative to the frequency of the first trial design. The core thicknesses displayed in column 5, like was done on Table 4, are rounded to millimeter. The order of the cycles follow the order of elimination of the design variables with smaller sensitivity values from the design space. At the end of the fifth cycle, all the face thicknesses are set to commercial values and the optimum first natural frequency is 25.30 Hz, which represents an increase of 34.7% in relation to the first trial design.

Table 6. Optimization cycle with commercial thickness correction - OCII

Cycle	Panel	Design variable	Optimized value (m)	Closest commercial value (m)	Total mass (kg)	Frequency (Hz)
0					97.55	24.57
1	Upper	Face	1.27E-4	1.52E-4	97.55	23.37
		Core	1.50E-2	1.50E-2		
2	Middle	Face	1.27E-4	1.52E-4	97.55	25.65
		Core	1.57E-2	1.60E-2		
3	Lateral	Face	2.68E-4	3.05E-4	97.55	25.60
		Core	1.50E-2	1.50E-2		
4	Support	Face	1.94E-4	1.52E-4	97.55	25.57
		Core	2.50E-2	2.50E-2		
5	Lower	Face	1.34E-3	1.27E-3	97.42	25.30
		Core	5.0E-2	5.0E-2		

In Table 7, we display the properties of the seven honeycomb types that were used as core material of the lower panel in the construction of the seven models M1HC of Step 7. In Table 8, we show the results of the seven optimizations performed with each of the models. Notice that the best result was obtained with model M1HC7 whose first natural

frequency is 27.7 Hz, which represents an increase of 47.5% relative to 18.78 Hz, the first natural frequency of the original model.

Table 7. Properties of honeycomb types for the Lower Panel of model M1HC

ID	Specification	Density (kg/m ³)	G _{1z} (MPa)	G _{2z} (MPa)
HCLP1	3/8-5052-0.0015	36.80	220.6	111.7
HCLP 2	1/4-5052-0.0040	126.53	896.3	364.0
HCLP 3	1/4-5052-0.0020	68.87	455.1	205.5
HCLP 4	1/4-5052-0.0015	54.46	344.7	165.5
HCLP 5	1/4-5052-0.0010	49.65	220.6	111.7
HCLP 6	3/8-5052-0.0025	59.27	379.2	179.3
HCLP 7	3/8-5052-0.0040	86.50	592.9	253.7

Table 8. Optimized frequencies for models M1HC's

Model ID	Honeycomb ID	Frequency in Hz
M1HC1	HCLP1	25.3
M1HC2	HCLP2	26.7
M1HC3	HCLP3	27.5
M1HC4	HCLP4	26.9
M1HC5	HCLP5	25.3
M1HC6	HCLP6	27.2
M1HC7	HCLP7	27.7

To construct the last model (model M2CF) of Step 7, we replaced the aluminum faces of the lower panel with four layer carbon fiber laminates. The properties of carbon fibers are shown in Table 9. The lay up of the fibers were 0°, 90°, 45° and -45° in order to the laminate to have quasi-isotropic mechanical properties. The last optimization step was performed with this model, considering as design variables only the thicknesses of the layers and the thickness of the honeycomb core of the lower panel. The results of this optimization are displayed in Table 10 along with those of model M1HC7, for comparison. The first natural frequency was increased by approximately 6% in relation to that of model M1HC7 which represents a total increase of 56.1% relative to the first natural frequency of the original model.

Table 9. Properties of Carbon Fibers

Property	Value
Material type	Orthotropic 2D
Modulus of elasticity in the longitudinal direction	200.0E+9N/m ²
Modulus of elasticity in the lateral direction	14.5E+9 N/m ²

Shear modulus	$G_{12} = 4.9\text{E}+9 \text{ N/m}^2$ $G_{1z} = 4.9\text{E}+9 \text{ N/m}^2$ $G_{2z} = 4.9\text{E}+9 \text{ N/m}^2$
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Table 9. (continued)

Property	Value
Poisson's ratio	$\nu=0.3$
Density	$\gamma=1650.\text{N/m}^3$

Table 10 Optimization results of models M1HC7 and M2CF

Description	Model	
	M1HC7	M2CF
Non structural mass (kg)	87.05	87.05
Structural mass (kg)	10.50	10.50
Total mass (kg)	97.55	97.55
Frequency (Hz)	27.70	29.32

CONCLUSIONS

The structure of the French-Brazilian Micro Satellite has two very critical design constraints: the lower bound of 40.0 Hz for the first natural frequency, in order to avoid coupling between the rocket excitation modes and the natural vibration modes of the satellite; and the upper bound of 10.5 kg for the structural mass. We have shown the importance of structural optimization and design sensitivity analysis in the redesign cycles of Space Structures, by presenting all the steps taken and the difficulties encountered as we tried to maximize the first natural frequency from the low value of 18.78 Hz obtained with the first trial design, while maintaining the structural mass below the predefined upper bound. All the modal and sensitivity analyses as well as the optimization steps were performed using MSC/NASTRAN. The Method of Feasible Directions was used in all the optimization runs. The design variable space for the structural optimization steps was composed of the thicknesses of the faces and core of the sandwich panels.

After five optimization refinements on the initial model, we were able to increase the frequency to 22.83 Hz, which represented an improvement of 21.6% in relation to the initial design. After the design sensitivity analysis, we were able to make some judgment and improve the model further, performing an optimization step with selective reduction of the design space, which resulted in a 34.7% increase on the first natural frequency relative to the initial design. At that stage, still based on the sensitivity information, we constructed two new models: one, by changing the honeycomb properties of the lower panel's core; and the other, by using a different material (carbon fiber laminate) for the faces of the same panel. Two optimizations were performed with these models and we reached at a frequency of 29.32 Hz which corresponded to 56.1% improvement relative to the first natural frequency of the original model.

From these studies, we concluded that Structural Optimization and Design Sensitivity Analysis are tools indispensable to the redesign process of Space Structures. In the case of the French Brazilian Micro Satellite, the first natural frequency increased 56.1% relative to its value at the beginning of the study, without any violation on its mass constraint. If structural optimization were not used, it would be impossible to achieve such result without adding mass to the model. The design probably would be unfeasible, due to the high costs involved when excessive addition of mass occurs in space structures.

Despite the 56.1% increase in the first natural frequency obtained through structural optimization, where the design space consisted solely of face and core thicknesses, we find that only thickness optimization is not enough and, without any fundamental topology change in the original structural design, it is very difficult to achieve the 113% improvement required to satisfy the lower bound constraint of 40 Hz on that frequency. We suggest that, after the thickness optimization is exhausted, some stiffeners be added to the Lower Panel to increase the rigidity of the model and that shape optimization steps be performed in order to reach to a feasible optimum design.

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