

# **On Direct Computation of Beam Dynamic Stiffness Coefficients using MSC.Nastran**

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## **Abstract**

The forced frequency analysis is very important in the design of automotive structures. In particular, the Frequency Response Functions (FRF) computation is an important step in determining Noise, Vibration and Harshness (NVH) of any automotive vehicle. In general, the CAE engineer must address the severity and compliance of the design limits set in the NVH environment. Usually, a CAE engineer in the automotive industry will first compute the modal characteristic of the component or full body/trim structure, and then compute the frequency response functions to aid in the determination of its ability to withstand the random load input applied on the structure. There exists a second method in MSC.Nastran to compute the frequency response functions on structures without the need to compute its modal characteristics. This direct method of FRF computation is based on the lumped or coupled mass formulation in conjunction with the classical finite element stiffness matrices.

It is to be noted that the effect of the distributed mass of the elements plays a critical role in the exact FRF computation. Such a formulation provides yet another method to compute FRF, and this method is henceforth named as the dynamic stiffness approach. To simplify the study of this method, only beam/frame structures will be considered for our purposes. In this paper, the direct numerical computation of the dynamic stiffness coefficients of beam structures using Euler-Bernoulli beam theory are discussed using the DMAP language of MSC.Nastran. There are other tools and compilers that can serve this purpose, but the future motivation of this work is in solving directly for the dynamic response of beam/frame structures using MSC.Nastran. The present dynamic stiffness analysis approach will also provide a direct method to compute the exact dynamic response on beam/frame structures using MSC.Nastran.

It is easier to change the mass distribution of structure by changing mass density in the dynamic stiffness approach to perform mass parameter effect studies on the frequency response values. Accounting for the mass distribution exactly using the mass density in the dynamics stiffness approach is certainly better than the classical lumped or coupled mass approach. As a numerical study, a comparison of the dynamic stiffness coefficients between the direct method described in this paper and the classical approach is done using a beam example. In addition, the time history mid-span displacement due to a step load is compared using the two approaches.

## Introduction

The response of an actual linear continuous structural system subjected to dynamic forces is obtained on the basis of a mathematical model representing the system and the mathematical functions representing the dynamic forces. Here, the term "continuous" is used in the sense of continuity in the distribution of mass and stiffness properties of the system. In general, the mathematical model representing actual structure is constructed by the finite element method. According to that method, the continuous structure is discretized into a number of finite elements connected to each other at their nodal points for which approximate element stiffness and mass matrices are developed. An appropriate superposition of these element matrices produces the structural stiffness and mass matrices of the system for which the equations of motion are of the form [1]:

$$[M] \{\ddot{U}(x,t)\} + [B] \{\dot{U}(x,t)\} + [K] \{U(x,t)\} = \{F(t)\} \quad (1)$$

In equation (1), [M] is the mass matrix in lumped or consistent form; [B] is the viscous damping matrix; [K] is the stiffness matrix; {F(t)} is the dynamic force vector; {U(x,t)} is the displacement amplitude vector of the structural grids at locations {X} in the basic coordinate system.

In performing structural analyses, the finite element method has proved to be very useful in the computation of response on structures, including dynamics. The commercial MSC.Nastran [2] program is a very powerful computational finite element tool that exists in the industry. This program uses the classical finite element approach to solving structural, thermal, acoustic, structure-fluid interaction and combined analysis problems. In particular, the classical finite element approach in dynamics uses the mass, stiffness and damping matrices of structures to solve for the required dynamic response and/or to compute for the modal frequencies.

Solution of equation (1) can be accomplished in MSC.Nastran by either the frequency response method or the numerical integration method, which employs various algorithms such as those of Newmark [3] and Wilson [4] to directly solve the equations. By the Frequency Response method, the transformed structural system equation (1) can be written in the frequency domain, in matrix form, for a particular frequency, as:

$$[D(x,\omega)] \{U(x,\omega)\} = \{F(\omega)\} \quad (2)$$

where  $\{U(x,t)\} = \{U(x,\omega)e^{i\omega t}\}$ ,  $\{F(t)\} = \{F(\omega)e^{i\omega t}\}$ , and

$$[D(x,\omega)] = -\omega^2 [M] + i\omega[B] + [K]. \quad (3)$$

In addition to these methods, there exists two numerical operational methods, i.e. methods employing the Fourier and Laplace Transforms. Equation (2) resembles the Fourier Transformed structural system of equations of motion (1) where  $[D(X,\omega)]$  is the system dynamic stiffness

matrix in the Fourier Domain. The Fourier Transform method of solution is available in MSC.Nastran as a solution sequence for solving aeroelastic problems. However, the importance of the numerical operational methods for solving structural systems in a "direct" manner has not been effectively addressed in commercial codes including MSC.Nastran.

There exists many mathematical and engineering reference works on the subject of operational methods, its applicability, strategies for general problems, and solutions to several specific structural problems. The Fourier series method has been applied effectively for gust, random or cyclic load on a structure. This paper is not attempting to discuss the mathematical concept of the operational methods or any of the details of Fourier series method for structural solution due to time varying load, and the subject of operational methodology to differential equations in mathematics is assumed in this paper.

This paper is the first in a planned series of piecewise work for developing, computing directly the dynamic stiffness influence coefficients of beam elements in MSC.Nastran, and hence, solving for the time or frequency response of beams and frames due to time varying loads in MSC.Nastran. In doing so, appropriate previous work is described here for better continuity and understanding of the subject. The concentration of this paper is on the derivation of dynamic stiffness influence coefficients of beam element in the Laplace Domain and its numerical computation using the developed special purpose DMAP in MSC.Nastran. It is shown that the Laplace Transform method to formulate the dynamic stiffness influence coefficients for the Euler-Bernoulli beams from its equation of motion directly is mathematically exact.

In subsequent papers, such computed dynamic stiffness element matrices will be used in beams and frame structures to assemble and form the system dynamic stiffness matrix,  $[\bar{D}(x,s)]$ , in the Laplace Transform domain. Subsequently, an equation similar to (2) will be solved to obtain the frequency response of the structure,  $[\bar{U}(x,s)]$  without requiring the knowledge of the natural frequencies or the mode shape of the structure. It should be noted that  $[\bar{D}(x,s)]$  is the system dynamic stiffness matrix in the Laplace Transform domain. The Laplace Transform parameter 's' is related to the frequency ' $\omega$ ' as in ' $s=c+i\omega$ '.

## **Derivation of Dynamic Stiffness Matrix for Uniform Beam**

The concept of deriving the stiffness matrix of a finite element that satisfies the equation of equilibrium in a static state is well known and understood, and is used in many commercial codes in the world, including MSC.Nastran. This same concept is carried over in this paper for a uniform beam finite element that satisfies the equation of motion in a dynamic state of that element. The derived stiffness matrix for the uniform beam in vibration is termed here as the 'Dynamic Stiffness Matrix' for this beam. The individual terms of the dynamic stiffness matrix are the influence coefficients of the associated degrees of freedom of this beam element. For simplicity, only the Euler bending

of beam is considered. The bending degrees of freedom of a beam element, the transverse displacement 'u' and rotation 'q' in the plane of bending, are the two degrees of freedom of beam considered for this paper. The details of derivation are given in this section.

Consider a uniform beam element of length 'L', bending rigidity 'EI' and mass per unit length 'm' as shown in Figure 1. For simplicity, no damping is considered in this derivation. The equation of free, undamped motion of this bar, using the Euler-Bernoulli theory, is:

$$EI u''''(x,t) + m \ddot{u}(x,t) = 0 \quad (4)$$

where prime denotes differentiation with respect to the distance x along the length of the beam from end A, and dots are differentiation with respect to time.

By definition, the Laplace Transform  $\bar{f}(s)$  of a function of time  $f(t)$  is defined by:

$$L[f(s)] = \bar{f}(s) = \int_0^{\infty} e^{-st} f(t) dt \quad (5)$$

where 's' stands for the Laplace transform parameter. Since the time function that is applied on structures starts at zero time, the Fourier Transform  $\bar{f}(\omega)$  can be obtained from the Laplace Transform by substituting for 's' parameter in terms of  $\omega$  as  $s=i\omega$ .

For non-zero initial conditions, it can be shown that:

$$L[\dot{f}(t)] = s\bar{f}(s) - f(t=0) \quad (6-a)$$

$$L[\ddot{f}(t)] = s^2 \bar{f}(s) - s f(t=0) - \dot{f}(t=0). \quad (6-b)$$

Application of Laplace Transform on equation (4) with respect to time, and under zero initial conditions, results in:

$$\bar{u}''''(x,s) + 4k^4 \bar{u}(x,s) = 0 \quad (7)$$

where  $\bar{u}(x,s)$  is the Laplace Transform of  $u(x,t)$  and  $4k^4 = \frac{ms^2}{EI}$ .

Notice that equation (7) is an ordinary differential equation of fourth order, similar to the static beam on elastic foundation, except that this equation is in the Laplace domain. Hence, the concept of stiffness derivation is applied on this transformed equation in the Laplace domain.

The solution of equation (7), which resembles the equation of a static bar on elastic foundation, is:

$$\bar{u}(x,s) = e^{kx}(A \cos kx + B \sin kx) + e^{-kx}(C \cos kx + D \sin kx) \quad (8)$$

The bending moment,  $M(x,t)$  and the shear force,  $V(x,t)$ , with mechanics sign convention, are:

$$M(x,t) = -EIu''(x,t) \quad (9-a)$$

$$V(x,t) = -EIu'''(x,t). \quad (9-b)$$

The Laplace Transform application with respect to time on (9) leads to the transformed bending moment,  $\bar{M}(x,s)$  and the transformed shear force,  $\bar{V}(x,s)$  of the form:

$$\bar{M}(x,s) = -EIu''(x,s) \quad (10-a)$$

$$\bar{V}(x,s) = -EIu'''(x,s). \quad (10-b)$$

Hence, by using the solution (8) for  $\bar{u}(x,s)$  in (10), the transformed forces at the nodes A and B of the beam element of Fig. 1 are:

$$\bar{M}(x=0,s) = -2EI k^2 (B-D) \quad (11-a)$$

$$\bar{V}(x=0,s) = -2EI k^3 (B-A+C+D) \quad (11-b)$$

$$\bar{M}(x=L,s) = -2EI k^2 [e^{kL} (B \cos kL - A \sin kL) + e^{-kL} (-D \cos kL + C \sin kL)] \quad (11-c)$$

$$\bar{V}(x=L,s) = -2EI k^3 [e^{kL} \{(B-A) \cos kL - (A+B) \sin kL\} + e^{-kL} \{(C+D) \cos kL + (D-C) \sin kL\}] \quad (11-d)$$

The two degrees of freedom for this beam, being  $\bar{u}(x,s)$  and  $\bar{q}(x,s) = \bar{u}'(x,s)$ , contributes to two sets of stiffness influence coefficients at each node, a total of 4 sets for this uniform beam. Any one of the set of dynamic influence coefficients can be obtained by computing the forces and moments in the transformed domain required at nodes to maintain a unit value of transformed displacement at a particular degree of freedom while the other three are fixed. Thus, the four sets of influence coefficients can be obtained by solving for parameters 'A', 'B', 'C', 'D' for the four states, given below. The substitution of A, B, C and D from each of these states in (11) generate the associated forces in the transform domain which are the required dynamic stiffness influence coefficients for that degree of freedom of motion.

The four sets of motion configuration states for the four degrees of freedom at beam nodes are:

$$1) \quad \bar{u}(x=0,s)=1, \quad \bar{u}'(x=0,s)=\bar{u}(x=L,s)=\bar{u}'(x=L,s)=0; \quad (12-a)$$

$$2) \quad \bar{u}'(x=0,s)=1, \quad \bar{u}(x=0,s)=\bar{u}(x=L,s)=\bar{u}'(x=L,s)=0; \quad (12-b)$$

$$3) \quad \bar{u}(x=L,s)=1, \quad \bar{u}(x=0,s)=\bar{u}'(x=0,s)=\bar{u}'(x=L,s)=0; \quad (12-c)$$

$$4) \quad \bar{u}'(x=L,s)=1, \quad \bar{u}(x=0,s)=\bar{u}'(x=0,s)=\bar{u}(x=L,s)=0; \quad (12-d)$$

The details of obtaining these parameters 'A', 'B', 'C' and 'D' for each of these above states symbolically and then substituting for obtaining the associated forces to determine the influence coefficients are tedious. Such a process of obtaining the dynamic stiffness influence coefficients yield 'exact' analytical stiffness terms for the uniform beam. These coefficients have been obtained in Ref.[5].

But it is done here again with the help of the Maple V symbolic software [6] to verify the manual derivation of the dynamic stiffness terms obtained in Ref.[5]. Figure 2 show the associated Maple V input lines to compute the dynamic stiffness influence coefficients,  $\bar{D}_{11}, \bar{D}_{12}, \bar{D}_{13}, \bar{D}_{14}$ , for state 1. Here, the bar symbol represents that the dynamic stiffness coefficients are in the Laplace domain. Similarly, the other coefficients can be computed symbolically by using the Maple V software. Table 1 shows the dynamic stiffness matrix  $[\bar{D}]$  containing all these 16 influence coefficients for a uniform beam element.

## DMAP Program for Computing Beam Dynamic Stiffness Matrix

A special purpose MSC.Nastran DMAP program is written for this paper to compute the dynamic stiffness influence coefficients for uniform beam, given in Table 1, at one or several frequencies. The Maple V program is also used to optimize the number of multiplication and additions needed to compute the numerical values of the dynamic stiffness matrix. The optimized FORTRAN statements from the Maple V software are translated directly into a submap in MSC.Nastran. Figure 3 shows the MSC.Nastran DMAP program listing used to compute the dynamic stiffness matrix for the uniform beam element.

Many symbolic and numerical checks, validations and verifications have been done to assure that the translation of these statements into MSC.Nastran DMAP statements are done correctly. Two of them are worth mentioning here. The first useful check is the comparison of dynamic stiffness matrix values as computed by this special DMAP with that obtained using the Maple V software for one particular frequency. As an example, the computed numerical dynamic stiffness matrix  $[\bar{D}(s)]$  for a uniform beam is displayed as Table 2. These numerical values are obtained by the MSC.Nastran DMAP program, given in Fig. 3, with single precision for a frequency of 100 radians, i.e.  $s=i\mathbf{W}$ . The numerical values are sufficiently accurate except for the truncation errors, and they compare very well with that obtained using MAPLE V software.

Secondly, the numerical values of  $D_{ij}$  terms given in Table 1 can be checked with plots for various frequencies in the range (0.25,4000). On close examination of the symbolic dynamic stiffness matrix terms, it is observed that the values of  $D_{11}$ ,  $D_{12}$ ,  $D_{13}$  and  $D_{34}$  increase with frequency parameter,  $s$ . Instead of listing numerous large numbers, it is more useful to use  $1/D_{11}$ ,  $1/D_{12}$ ,  $1/D_{22}$  and  $1/D_{34}$  for plotting and visual check purposes. These plots for these inverted terms are not

included in this paper due to limitation of space. Similarly, additional plots can be studied for D13, D14, D23 and D24.

On close examination, four items of interest about the dynamic matrix are observed:

- a) The dynamic stiffness matrix is symmetric as seen in Table 1 or 2;
- b) The imaginary part of the  $D_{ij}$  terms of this dynamic matrix must be very small in the Fourier domain, and the Table 2 shows these imaginary values are sufficiently large due to the truncation errors of using the single precision. The DMAP program in Fig. 3, modified to accommodate double precision accuracy, produce the imaginary part of  $D_{ij}$  terms with a very small value. This has been verified successfully.
- c) The mechanics sign convention used for deriving the dynamic influence coefficients produce opposite sign for D12 and D34, i.e.  $D_{34} = -D_{12}$ .
- d) In the basic coordinate system convention of MSC.Nastran, the moment at left end and the shear at right end of beam, seen in Fig. 1, are opposite to that used here. This will change the signs of D12, D13, D23 and D34 if used with the MSC.Nastran convention. This is indicated here for future purposes towards MSC.Nastran DMAP alter development for frame structures.

## Beam Example to Validate Numbers

An undamped simply supported uniform beam of length  $L$ , mass  $m$ , and modulus of rigidity  $EI$ , as shown in Fig.4, is used to validate the numerical values of the dynamic stiffness influence coefficients of the uniform beam given in Table 1. The frequency or time history of the vertical displacement of the beam at its mid-span subjected to a suddenly applied load  $F(t)$  is of interest here.

Because of symmetry, only even modes contribute to the mid-span vertical deflection, and thus, the rotation at mid-span is taken to be zero. Hence, formulating the dynamic stiffness matrix for one-half of the beam loaded by ' $F(t)/2$ ' at mid-span and applying the boundary condition  $\bar{u}_A = \bar{q}_B = 0$ , the following relation is established in the Laplace transform domain:

$$2 \left[ \bar{D}_{11} - \frac{\bar{D}_{14}^2}{\bar{D}_{22}} \right] \bar{u}_B(s) = \bar{F}(s) \quad (13)$$

where the coefficients  $\bar{D}_{ij}$  are given in Table 1 and  $\bar{F}(s) = F_0/s$ . Thus,  $\bar{u}_B(s)$  can be written as

$$\bar{u}_B(s) = \frac{F_0}{8EI m^3} \bar{f}(s) \quad (14)$$

where  $\bar{f}(s)$  is given by the following expression:

$$\bar{f}(s) = \frac{(e^{4kL} - 2e^{2kL} \sin(2kL) - 1)(e^{4kL} - 2e^{2kL} (1 + \sin^2(kL)) + 1)s^{-2.5}}{(e^{4kL} + 2e^{2kL} \sin(2kL) - 1)(e^{4kL} - 2e^{2kL} \sin(2kL) - 1) - 8e^{2kL} (1 - e^{2kL})^2 \sin^2(kL)}$$

$$\text{with } k = \mathbf{m} s^{1/2} \quad \text{and } \mathbf{m} = \frac{m^{1/4}}{(4EI)^{1/4}} .$$

Here,  $\bar{u}_B(s)$  is the transformed displacement response of this simply supported beam at its mid-span in the Laplace domain. The frequency response is obtained by using  $s = i\omega$  in (14). Figure 5-a and 5-b shows the forced frequency response of the beam using the dynamic stiffness matrix as computed by the special DMAP program given in Fig. 3. An associated MSC.Nastran frequency response analysis is possible by using the classical direct frequency method available as SOL 108.

MSC.Nastran uses the discrete consistent mass approach, and the Laplace Transform method uses the exact solution of the equation of motion in the transform domain to compute the dynamic stiffness of uniform beam. Figure 5 shows the singularity at the first natural bending mode frequency of this beam. The numerical values of the frequency response of beam are of the same order of magnitude between MSC.Nastran by SOL108 and the Laplace Transform method using the special DMAP program. Table 3 shows the frequency displacement response at mid-span at several frequencies.

Even though the frequency response values are compared, it is fruitful to compare the time history results of vertical deflection of the beam. The MSC.Nastran input data for obtaining the time result of mid-span displacement using modal approach is given in Fig. 6. Figure 7 shows the modal time response plot of the mid-span deflection from MSC.Nastran by using SOL 112. A numerical inverse Laplace transform is used in Ref.[7] to obtain the mid-span deflections at selected times, and these are shown on this same plot for comparison and validation purposes. The function  $\bar{f}(s)$  is transformed back onto the time domain by means of the numerical inverse Laplace transform, as computed with the aid of Fast Fourier Transform(FFT). The details of the numerical inverse Laplace transform is outside the scope of this paper, and is discussed in detail in Ref.[5,8]. The comparison shows that the dynamic stiffness matrix values for the beam can be used to obtain the time response of the structure.

## Remarks

- 1) The dynamic stiffness matrix of Euler beam is very useful in obtaining the 'exact' frequency response of beam or frame structures without requiring the knowledge of natural frequencies;
- 2) The powerful MSC.Nastran solver and data management capability can be used for solving large frame structures using transform methods if the beam dynamic stiffness matrix can be available as a DMAP program to aid in developing an alter to SOL 108. The development of the alter to SOL 108 is left for future work, and this paper

provides the special DMAP program to compute the dynamic stiffness matrix for Euler beam;

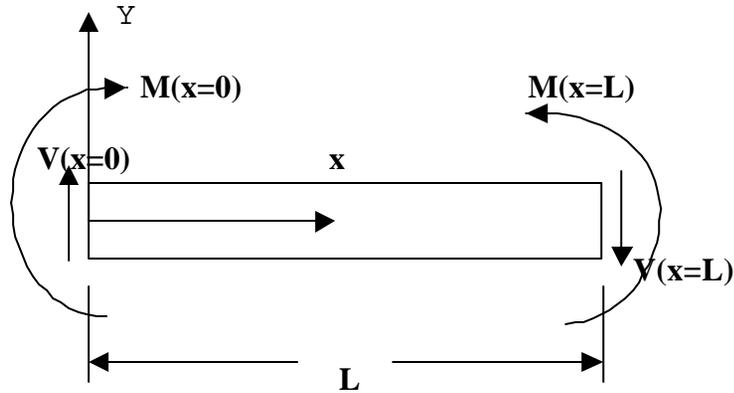
- 3) The dynamic stiffness matrix of a uniform beam can be used only for the vibrating beam or frame structures, and the values are singular at zero frequency, as shown in Fig. 5.

## Future Work

This paper concentrated on the development of the special purpose DMAP using MSC.Nastran for a uniform beam. The dynamic stiffness matrix of uniform beam will be used in the next study to write additional DMAP modules to solve for the response of frame structures. Such a study will enable to write a general DMAP module for computing frame structural dynamic matrices. The incorporation of the dynamic stiffness matrix of beam and frame structures as an alter to direct frequency response solution, SOL 108, in MSC.Nastran will be planned in a much later study.

## References

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**Figure 1 – Uniform Beam Element**

```

> with(linalg):
> y:=(x)-
> exp(k*x)*(A*cos(k*x)+B*sin(k*x))+exp(k*x)*(C*cos(k*x)+Dc*sin(
> k*x));
> yd:=(x)->diff(y(x),x):
> yd(x):
> eqn1:=y(0)=1;
> eqn2:=eval(subs(x=0,yd(x)))=0;
> eqn3:=y(L)=0;
> eqn4:=subs(x=L,yd(x))=0;
> solve({eqn1,eqn2,eqn3,eqn4},{A,B,C,Dc});
> assign("):
> A:
> factor(expand(simplify(subs(tan(k*L)=sin(k*L)/cos(k*L),""))):
> A:=factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> B:
> factor(expand(simplify(subs(tan(k*L)=sin(k*L)/cos(k*L),""))):
> B:=factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> C:
> factor(expand(simplify(subs(tan(k*L)=sin(k*L)/cos(k*L),""))):
> C:=factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> Dc:
> factor(expand(simplify(subs(tan(k*L)=sin(k*L)/cos(k*L),""))):
> Dc:=factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> VF:=(x)->-EI*diff(y(x),x$3):
> MF:=(x)->-EI*diff(y(x),x$2):
> eval(subs(x=0,simplify(VF(x)))):
> D11:=-factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> eval(subs(x=0,simplify(MF(x)))):
> D21:=factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> simplify(eval(subs(x=L,VF(x)))):
> D31:=factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));
> simplify(eval(subs(x=L,MF(x)))):
> D41:=-factor(expand(subs(cos(k*L)^2=1-sin(k*L)^2,")));

```

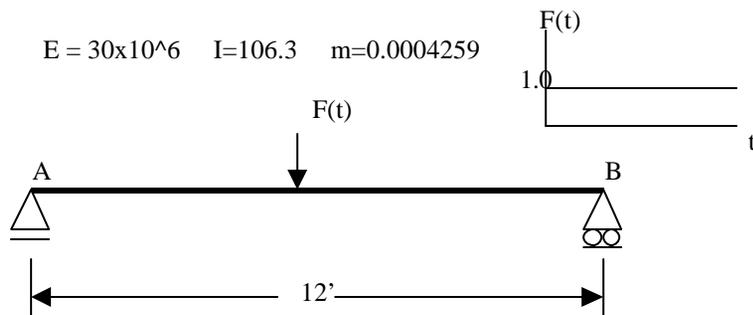
**Figure 2 – Maple V input for State 1**

```

$ MSC.Nastran DMAP Program to compute
$ Beam Dynamic Stiffness Matrix
$ using Single Precision
time 5
diag 8
sol 100
compile userdmap,souin=mscsou $
alter 2 $
type parm,,cs,n,t1,t2,t3,t4,t5,t6 $
type parm,,cs,n,t7,t8,t9,t10 $
type parm,,cs,n,t11,t13,t15,t16 $
type parm,,cs,n,t18,t21,t22,t25 $
type parm,,cs,n,t29,t30,t32 $
type parm,,cs,n,t37,t40,t45,t47 $
type parm,,cs,n,t48,t52,t55,t60,b,s $
type parm,,rs,y,L=144.0 $
type parm,,cs,y,emod=30.0E+6,imom=106.3 $
type parm,,rs,y,mass=0.004259,EI,meu$
type parm,,i,y,knt=1 $
type parm,,cs,n,DSM11,DSM12,DSM13,DSM14 $
type parm,,cs,n,DSM21,DSM22,DSM23,DSM24 $
type parm,,cs,n,DSM31,DSM32,DSM33,DSM34 $
type parm,,cs,n,DSM41,DSM42,DSM43,DSM44 $
EI = emod*imom $
meu=(mass/4.0/EI)**0.25 $
s = cmplx(0.0,0.0)
$ do while(knt < 4) $
s=s + cmplx(0.0,knt)*cmplx(100.) $
message //'For Frequency value s = '/s $
b=cmplx(meu)*sqrt(s) $
t1=b*b $
t2 = t1*b $
t3 = t2*cmplx(EI) $
t4 = b*cmplx(L) $
t5 = exp(t4) $
t6=t5*t5 $
t7 = sin(t4) $
t8 = t6*t7 $
t9 = cos(t4) $
t10 = t8*t9 $
t11=t6*t6 $
t13 = t5*t7 $
t15 = cmplx(1.0)/ (t6+cmplx(2.0)*t13
-cmplx(1.0)) $
t16 = (cmplx(-1.0)+
cmplx(4.0)*t10+t11)*t15 $
t18 = cmplx(1.0)/(t6-cmplx(2.0)*t13
-cmplx(1.0)) $
t21 = t1*cmplx(EI) $
t22 = t7*t7 $
t25 = (cmplx(1.0)-cmplx(2.0)*t6+
cmplx(4.0)*t6*t22+t11)*t15 $
t29 = t5*t2*cmplx(EI) $
t30 = t9*t6 $
t32 = (t30 - t9 + t8 + t7)*t15 $
t37 = t5*t1*cmplx(EI)*t7 $
t40 = (t5-cmplx(1.0))*(t5+cmplx(1.0)) $
t45 = cmplx(1.0)/(-t6+
cmplx(2.0)*t13+cmplx(1.0)) $
t47 = t21*t25*t45 $
t48 = b*cmplx(EI) $
t52 = t48*(cmplx(1.0)+cmplx(4.0)*t10-
t11)*t15*t45 $
t55 = t37*t40*t15*t45 $
t60 = t48*t5*(-t8+t30-t9-t7)*t15*t45 $
$ DSM matrix calculation part
DSM11 = cmplx(4.0)*t3*t16*t18 $
DSM12 = cmplx(2.0)*t21*t25*t18 $
DSM13 = cmplx(-8.0)*t29*t32*t18 $
DSM14 = cmplx(8.0)*t37*t40*t15*t18 $
message //'DSM Matrix Printout below' $
message //'DSM11,DSM12,DSM13,DSM14 = ' $
message //'DSM11/DSM12/DSM13/DSM14 $
DSM21 = cmplx(-2.0)*t47 $
DSM22 = cmplx(2.0)*t52 $
DSM23 = cmplx(8.0)*t55 $
DSM24 = cmplx(4.0)*t60 $
message //'DSM21,DSM22,DSM23,DSM24 = ' $
message //'DSM21/DSM22/DSM23/DSM24 $
DSM31 = cmplx(8.0)*t29*t32*t45 $
DSM32 = cmplx(8.0)*t55 $
DSM33 = cmplx(-4.0)*t3*t16*t45 $
DSM34 = cmplx(2.0)*t47 $
message //'DSM31,DSM32,DSM33,DSM34 = ' $
message //'DSM31/DSM32/DSM33/DSM34 $
DSM41 = cmplx(-8.0)*t55 $
DSM42 = cmplx(4.0)*t60 $
DSM43 = cmplx(2.0)*t47 $
DSM44 = cmplx(2.0)*t52 $
message //'DSM41,DSM42,DSM43,DSM44 = ' $
message //'DSM41/DSM42/DSM43/DSM44 $
knt=knt+1 $
enddo $
cend
title = dmap DBEAM stiffness CALC
subtitle = For Uniform Beam
begin bulk
enddata

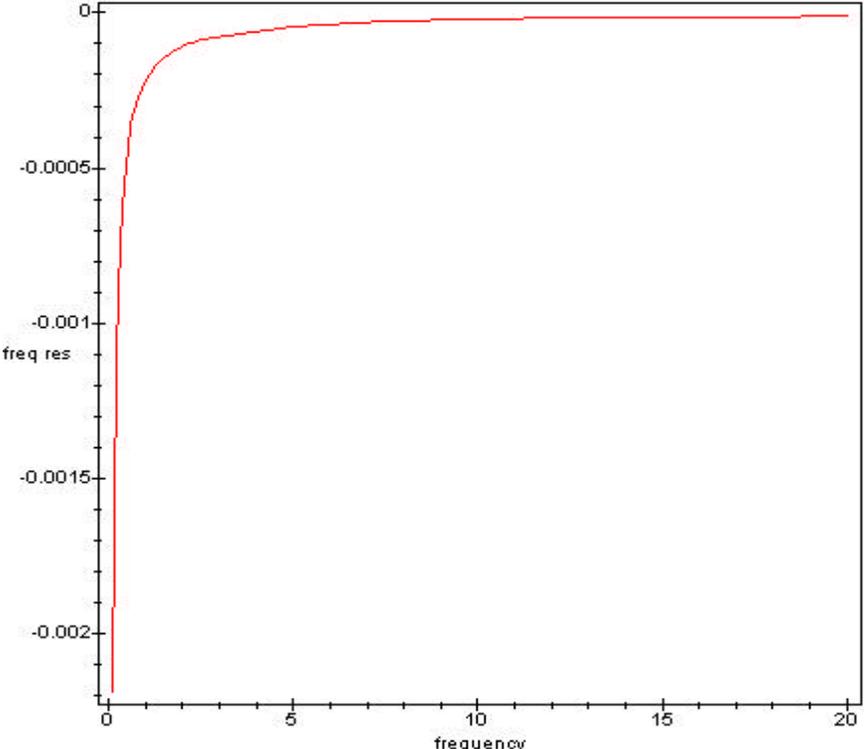
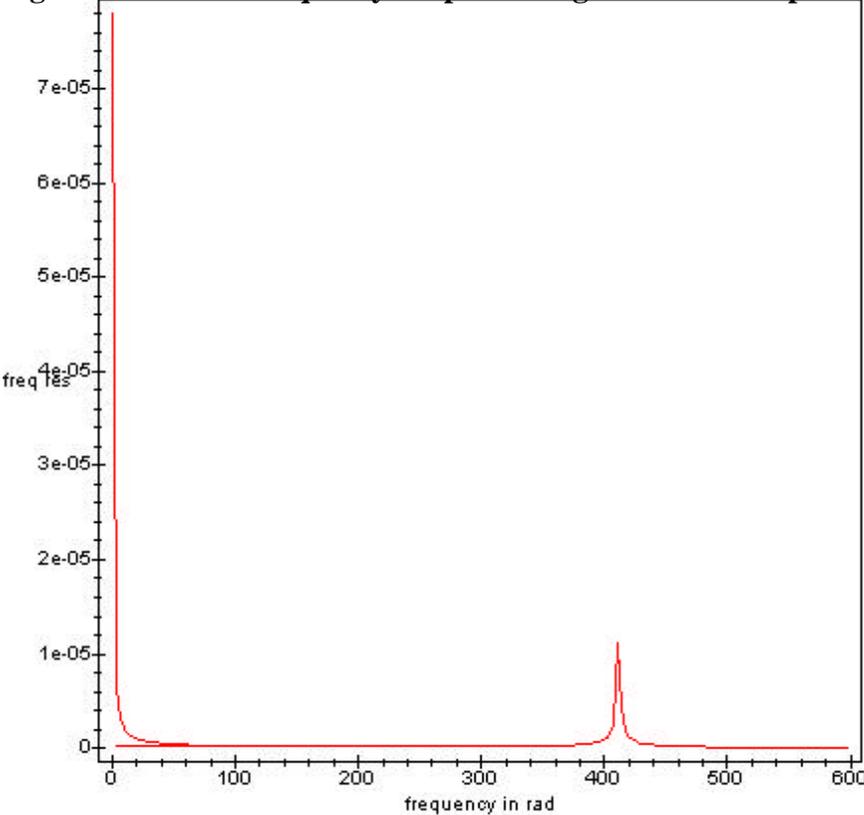
```

**Figure 3 – MSC.Nastran DMAP to compute Beam Dynamic Stiffness Matrix**



**FIGURE 4 – Simply Supported Uniform Beam Example**

**Figure 5-a Forced Frequency Response Magnitude at mid-span of SS beam**



**Figure 5-b Forced Frequency Response of SS Beam for 0-20 rad**

```

ID ssbeamtr,msc
SOL 112
TIME 600
CEND
TITLE = Simply Supported Beam Example
ECHO = NONE
MAXLINES = 999999999
  SUBTITLE= Modal transient Response Solution
  SPC = 2
  LOADSET = 1
  DLOAD = 2
  TSTEP=40
  METHOD=10
  DISPLACEMENT(SORT1,PUNCH,PLOT,PRINT,REAL)=ALL
$ VELOCITY(SORT1,PLOT,REAL)=ALL
$ ACCE(SORT1,PLOT,REAL)=ALL
OUTPUT(XYOUT)
XGRID=YES
YGRID=YES
XTITLE=Time (sec)
YTITLE=DISP RESPONSE AT GRID 2
XYPLOT DISP RESPONSE 1/2(T2)
BEGIN BULK
PARAM AUTOSPC YES
PARAM COUPMASS 1
PARAM K6ROT 0.
PARAM WTMASS 0.00258
PARAM,NOCOMPS,-1
PARAM PRTMAXIM YES
EIGRL,10,,4000.,2
TSTEP,40,70,0.0005,1
PBAR,1,1,1.0,106.3,1.0,9.6
CBAR 1 1 1 2 0. 1. 0.
CBAR 2 1 2 3 0. 1. 0.
$2345678$2345678$2345678$2345678$2345678$2345678$2345678
MAT1 1 3.+7 .3 1.645678
GRID 1 0. 0. 0.
GRID 2 72. 0. 0.
GRID 3 144. 0. 0.
SPCADD 2 1 4
SPC1 1 12 1 3
SPC1 4 345 1 2 3
TLOAD1 4 5 1
LSEQ 1 5 3
DLOAD 2 11.2057 1. 4
FORCE 3 2 0 1. 0. -1. 0.
TABLED1 1
+ B
+ B 0. 1. 250. 1. ENDT
ENDDATA

```

**Figure 6 – MSC.Nastran Input for Modal Transient Response**

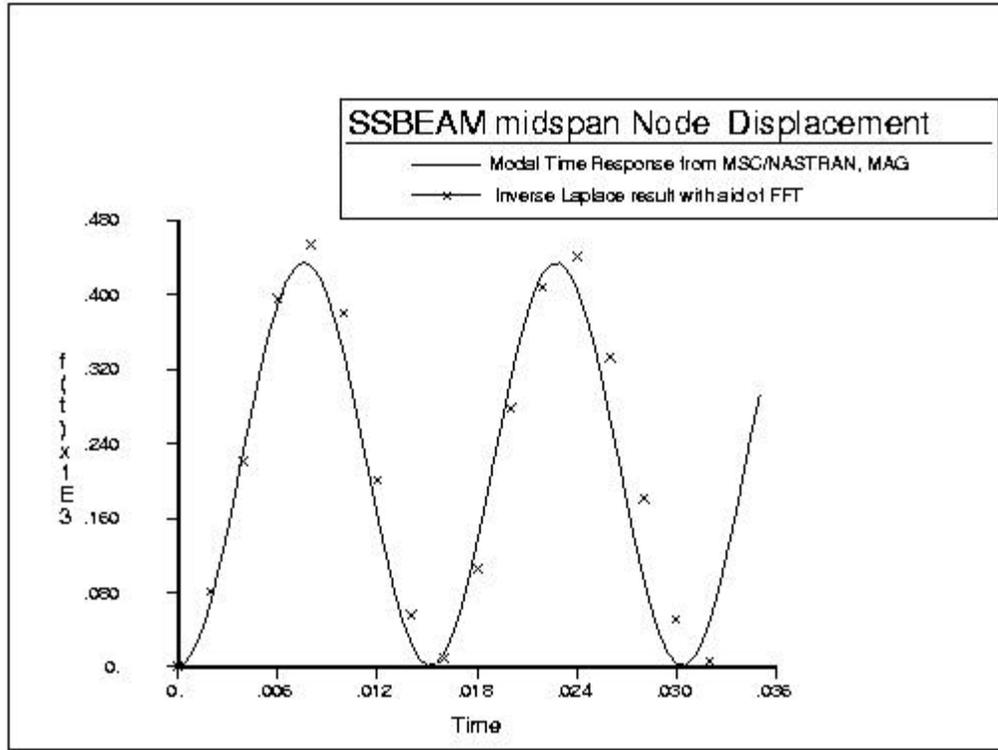


Figure 7 – Comparison of Transient Response of Simple Supported Beam

Table 1 – Dynamic stiffness Matrix for Beam Bending

$$EI \begin{bmatrix} \frac{k^3(4e^2sc-1+e^4)}{\delta\gamma} & 2\%1 & -8\frac{ek^3(ce^2-c+e^2s+s)}{\delta\gamma} & 8\frac{ek^2s(-1+e)(e+1)}{\delta\gamma} \\ 2\%1 & 2\frac{k(-1-4e^2sc+e^4)}{\delta\gamma} & -8\frac{ek^2s(-1+e)(e+1)}{\delta\gamma} & 4\frac{ke(e^2s-ce^2+c+s)}{\delta\gamma} \\ -8\frac{ek^3(ce^2-c+e^2s+s)}{\delta\gamma} & -8\frac{ek^2s(-1+e)(e+1)}{\delta\gamma} & -4\frac{k^3(4e^2sc-1+e^4)}{\delta\gamma} & -2\%1 \\ 8\frac{ek^2s(-1+e)(e+1)}{\delta\gamma} & 4\frac{ke(e^2s-ce^2+c+s)}{\delta\gamma} & -2\%1 & -2\frac{k(-e^4+4e^2sc+1)}{\delta\gamma} \end{bmatrix}$$

$$\%1 := \frac{k^2(1-2e^2+4e^2s^2+e^4)}{\delta\gamma}$$

$$\mathbf{d} = e^2 + 2es - 1; \quad \mathbf{g} = e^2 - 2es - 1;$$

$$e = \exp(kL); \quad c = \cos(kL); \quad s = \sin(kL);$$

**Table 2 – Dynamic Stiffness Matrix as computed by DMAP**

**At Frequency = 100 rad**

(1.052492E+04, I -2.382353E-03)	(8.760900E+05, I -1.501403E-01)	(-1.361614E+04, I 2.241085E-03)	(9.504490E+05, I -8.395985E-02)
(8.760900E+05, I -1.501403E-01)	(8.736014E+07, I -1.054628E+01)	(-9.504490E+05, I 8.395985E-02)	(4.521165E+07, I -2.613870E+00)
(-1.361614E+04, I 2.241085E-03)	(-9.504490E+05, I 8.395985E-02)	(1.052492E+04, I -2.382353E-03)	(-8.760900E+05, I 1.501403E-01)
(9.504490E+05, I -8.395985E-02)	(4.521165E+07, I -2.613870E+00)	(-8.760900E+05, I 1.501403E-01)	(8.736014E+07, I -1.054628E+01)

**Table 3 – Frequency Displacement Response values at mid-span  
for Example Beam**

Frequency(rad)	0.25	3	6	9	12	15	30
FR value(in)	7.80E-05	6.54E-07	3.25E-06	2.17E-06	1.63E-06	1.30E-06	6.54E-07
Frequency(rad)	60	120	180	240	300	390	396
FR value(in)	3.32E-07	1.77E-07	1.34E-07	1.22E-07	1.37E-07	4.78E-07	6.44E-07
Frequency(rad)	402	408	420	480	540	600	
FR value(in)	1.01E-06	2.53E-06	1.15E-06	1.11E-07	4.90E-08	2.88E-08	