Predicting the Effects of Transmission Housing Flexibility and Bearing Stiffness on Gear Mesh Misalignment and Transmission Error.

O.J. Harris[1], M. Douglas, B.M. James, A.M. Woolley, L.W. Lack
Romax Technology Ltd., 67 Northgate, Newark, NG24 1HD, England

ABSTRACT

The paper describes a simulation method to predict efficiently and accurately how changes to a transmission's housing affect gear mesh misalignments. Two key elements of this analysis are described. A fully coupled non-linear algorithm to analyse the shaft/bearing/gear/housing hyperstatic system has been developed to analyze the non-linear components. Using MSC.Nastran, a reduced stiffness matrix superelement representing the transmission housing is derived.

An investigation of the effects of stiffness of each of the components is presented using a case study. The results show that the inclusion of the interaction between the housing and internal components significantly affects the predictions for bearing life and gear mesh misalignments. This affects how the gear is modified to minimise transmission error and noise.

The application of this technique at both concept and detail design stages is discussed.

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[1] Corresponding author. Tel +44 1636 614000, e-mail: owenh@romaxtech.com

INTRODUCTION

Automotive transmissions today must be lighter, quieter and cheaper to meet increasingly demanding CAFÉ and customer requirements. There is a conflict in seeking an optimum solution since lighter often means smaller sections and more flexible housings, which increases deflections that lead to misalignments at the gear mesh. Gear mesh misalignment is critical as this not only reduces its life, but also increases its transmission error, which gives rise to an increased level of noise. To compensate for the effect of misalignment, transmission engineers modify the gear tooth micro-geometry but good design minimises the deflections. The question is how to do this.

Up to now the design approach for transmission housings has made it difficult to achieve this target. A designer might make some judgement as to where ribs are required to provide stiffness, but this is based on engineering experience and Finite Element Analysis (FEA) of the stand-alone housing. Understanding how changes to the housing affect the internal components of the transmission is even harder – the deflection of a complex 3D casting under multiple loads is made even more incomprehensible by the inclusion of non-linear contact elements such as bearings.

Traditionally, calculations of the housing influence on gear mesh misalignment are performed using FEA by making a series of approximations (e.g. linear bearings, rigid housings), with corresponding reductions in accuracy. The key problem with this approach is that the analysis is not accurate enough if the bearing stiffness is not taken into account and this is an iterative calculation since it is load dependent. This is further complicated when tapered roller bearings are used since the bearing pre-load will change their stiffnesses. As one of our clients observed "when a pre-load is applied to a pair of taper roller bearings, you find that the pre-load on another pair on another shaft is reduced and so you end up chasing the pre-load around the bearings".

Full FE models of the whole transmission system have been developed. However, this requires a high level of expertise and involving significant investments in time and cost, before it can even begin to be successful. For design analysis, this approach is painfully slow. For design optimization or concept design where lots of configurations need to be considered, it is just not viable.

The purpose of this paper is to describe an approach using a numerical simulation method and a FEA derived housing stiffness model to predict efficiently and accurately how changes to a transmission's housing can affect gear mesh misalignments. A study of how the housing affects a typical five-speed transaxle transmission is presented.

GEAR NOISE AND MISALIGNMENT

It is over forty years since the concept of transmission error was first proposed and linked to the generation of noise by gears [1-2]. This subject has been studied much in the intervening years and the relationship between misalignment and transmission error developed [3-4]. The importance of gear mesh misalignment to gear noise is thus well established.

A number of analysis methods exist that take a given gear pair, the transmitted torque, gear micro-geometry and misalignment and calculate a prediction for the transmission error, for example see [5]. It is customary to modify the gear micro-geometry to accommodate misalignment and minimise transmission error.

The precise effect that the gear misalignment due to the housing stiffness has on gear noise depends as much on the gear micro-geometry as on the magnitude of the misalignment itself. Some gear designs can accommodate misalignment with little detrimental effect, for others it is more of a problem. The gear designer's challenge is to create a design to accommodate the misalignment. The method presented in this paper for predicting system deflections provides the designer with the accurate values of misalignment that are necessary for low noise design.

THE MODEL

Introduction to the Model

The model allows the user to carry out a static analysis model of an entire transmission by linking the shafts, bearings, gears and now housings together. Figure 1. shows the example transmission studied in this paper - a typical transverse five-speed front wheel drive transmission (transaxle).

The system model is formulated by combining the individual component stiffness matrices to assemble a single system stiffness matrix. Application of the gear and external loads, and a standard matrix inversion are used to solve for the transmission deflection. A Newton-Raphson type iterative scheme is used when non-linear components are included (see discussion about bearings below). This was carried out using the RomaxDesigner™ transmission design and analysis software system.

The resulting analysis provides a complete force/deflection model of a transmission, so that the following can be calculated as accurately as possible:

- the misalignment of bearings and its effect on bearing life
- the mesh misalignment of gears

In providing such data to the engineer, the software allows the engineer to more accurately predict the micro-geometry modifications required for quiet and durable gears, reducing the trial-and-error methods that are currently used.

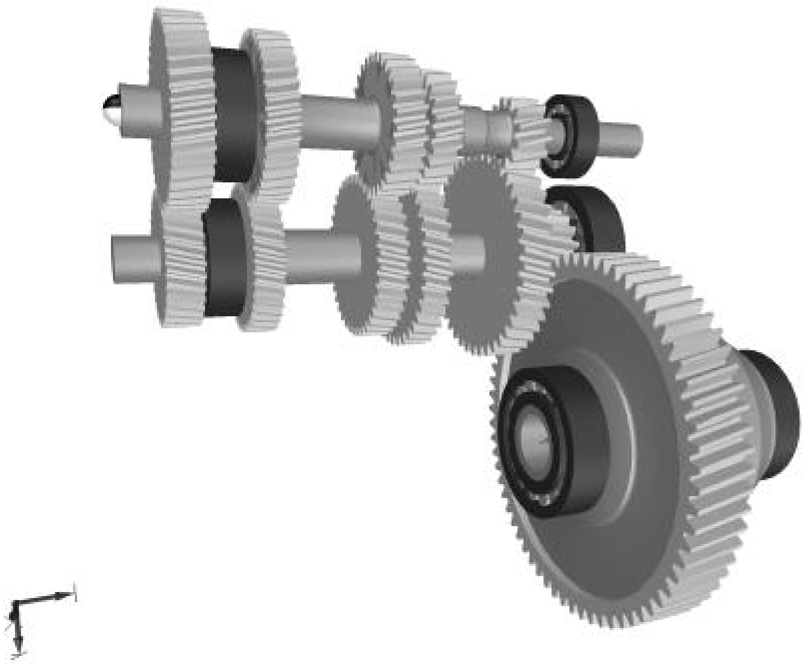

Figure 1. *The transmission studied - a typical five-speed transaxle*

Non-linear Bearings

Initially, when a user creates a model of the transmission, the non-linear stiffness of the bearings is calculated, taking into account the applied load, internal geometry and internal clearance as described in standard literature [6-7]. When compared with the classic "simply-supported beam analysis", this provides a more accurate calculation of the shaft deflection, and allows three-bearing and pocket-bearing systems to be calculated.

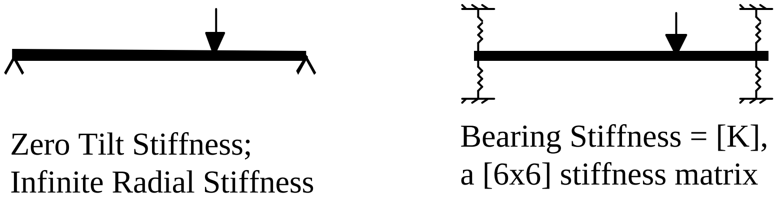

Figure 2. *A sketch showing the loading of a beam which is either simply supported or mounted via bearings.*

However, this assumes that the bearing outer ring is held in an infinitely stiff housing. Whilst this is a perfectly good approximation for concept design and development, refinement of the transmission benefits from the model being as complete as possible. This is where the housing flexibility must be included.

Inclusion of The Housing Flexibility

The next stage is to take the model of the transmission and apply a single, multi-dimensional stiffness matrix to the locations where the bearing outer rings are attached to the housing. This is a fully coupled stiffness matrix, so that the effect of the loading and deformation of the different bearings on one another (via the housing) is included.

Figure 3. below shows two shafts supported by bearings in a housing. The dotted line shows the undeformed shape. As a single point-load is applied to Shaft 1, Bearing A deflects and the housing of Bearing A also deflects. At the other end of the shaft, Bearing B and its housing also deflect. Additionally, there is an influence on Bearings C and D, which are affected by the applied force via the transmission housing. Shaft 2 displaces despite there being no load on it.

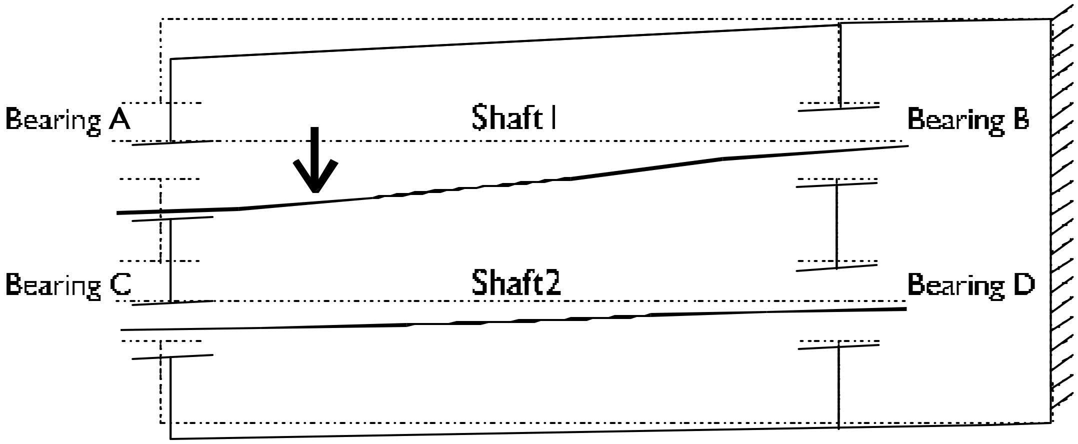

Figure 3. *The effect of cross coupling between the bearing housings*

In addition to dealing with the transfer of loads from one shaft to another, the fully coupled model is capable of dealing with the pre-loading of bearings and the effect of external loads on the transmission housing.

Derivation of the Housing Stiffness Matrix using FEA

The housing stiffness matrix that is required to complete this analysis was derived using a MSC.Nastran model of the housing. A single node needs to be defined for each bearing-to-housing connection. This was achieved using RBE2 elements at each bearing position. These connect the bearing seating to a point at the centre of the bearing assuming an infinite stiffness. The number of nodes around the circumference of the seating determines the number of nodes at the seating end of this element. At the other end of the element is a common node at the centre of the bearing. It is the 6 degrees-of-freedom of this latter node that define the behaviour of the housing for that bearing.

For a transmission with N bearing housings, each with 6 degrees-of-freedom, MSC.Nastran will yield a [6N x 6N] stiffness matrix. Other nodes may be included to represent loading points on a housing, e.g. suspension loads on motorsport or tractor transmissions.

Once all these new elements are defined, a static condensation is carried out to derive the stiffness matrix describing the properties of the housing. This can be achieved either by using an ALTER (see Appendix A) or more directly with the "PARAM EXTOUT DMIGPCH" card (see Appendix B). A description of the method of static condensation can be found in standard FE textbooks [8].

It is not necessary to create a complete, stress-quality model of the housing since only the stiffness is required, leading to a much-reduced lead-time for the creation of the FE model. Once the housing stiffness matrix is output from the FEA, it is then assembled with the stiffness matrix of the internal components to make a transmission system stiffness matrix.

THE CASE STUDY

The Transmission

Figure 1. shows the 3-dimensional view of a standard, 5-speed transaxle transmission. This transmission is based on a number of different designs and has been developed for testing, validation and demonstration purposes. It has a simple, 5-speed duty cycle applied to it, and it allows the effect of including the housing stiffness to be studied.

The housing stiffness matrix was also derived from real data but modified to maintain confidentiality. Two alternative housings were considered during the design optimization. For the sake of brevity, the full details of the transmission dimensions, loads cases and housing stiffness (a 36x36 matrix) are not reproduced here.

Running the Analysis

The analysis was carried out both with the full model (including the housing) and with the simplified model in which the housing flexibility is neglected (i.e. assuming an infinite housing stiffness). This allows the investigation of the effect of including a flexible housing.

The total analysis time for running the static analysis on all five load cases was under 1 minute on a Windows NT workstation.

Results – Bearing Misalignment and Life Prediction

The results showed that there are substantial changes to the predicted misalignment of the bearings under load. Further, and of more direct practical interest, the software used calculates the bearing percentage damage over the duty cycle.

For each loadcase a predicted bearing life is evaluated based on an ISO life calculation (4), and with an adjustment to take into account of misalignment and radial internal clearance. The total percentage damage is then calculated by summing the ratios of the predicted life and loadcase duration.

These bearing duty cycle results for configuration A are shown in Table 1. below. Quite significant changes are predicted. Note that it is not possible to make any general statement as to whether predicted bearing lives will be increased or decreased by including the flexibility of the housing. This is due to the complex effects of cross coupling between shafts.

	Bearing Duty Cycle Results (Percentage Damage)		
Bearing	Infinite housing stiffness	FE housing stiffness (Design A)	Change
Input Shaft Left	100 %	104 %	+4 %
Input Shaft Right	12.9 %	13.0%	+0.0 %
Lay Shaft Left	56.6 %	57.3 %	+1.2 %
Lay Shaft Right	5.86 %	5.14 %	-12%
Differential Left	3.20 %	2.05 %	-36 %
Differential Right	8.15 %	2.86 %	-65%

Table 1. *The effect of housing stiffness on bearing duty cycle damage*

Results – Gear Mesh Misalignment

This transmission contains five gear pairs that are each loaded in a single loadcase. Each has one gear that is mounted on a synchroniser, and the tilt of these gears is affected by the needle roller bearing underneath the synchronised gear. This can be modelled with the software, but has not been done so in this model.

The final drive gear pair is more challenging as it is loaded in all loadcases, with the layshaft deflecting in different ways according to the position of the applied loads. Hence, it is vital to know both the magnitude of the misalignment and the range across which it varies from loadcase to loadcase. This can be seen in Table 2. (for housing configuration A):

	Mesh misalignment values (FBetaX)		
Loadcase	Infinite housing stiffness	FE housing stiffness (Design A)	Change
1st Speed	-100 um	-162 um	+62 %
2nd Speed	-56 um	-111 um	+98 %
3rd Speed	-35 um	-73 um	+109 %
4th Speed	-32 um	-60 um	+88 %
5th Speed	-33 um	-54 um	+64 %
Range:	*67 um*	*108 um*	

Table 2. *The effect of housing stiffness on final drive gear pair mesh misalignment (Housing Design A)*

Thus it can be seen that not only do the individual values of mesh misalignment change, but the range of mesh misalignments with which the gear pair has to cope changes from 67 um to 108 um, an increase of 60%. This may substantially alter the way that micro-geometry modifications are applied to the gear pair.

A second analysis run was carried out to investigate the effect of an alternative housing (Design B). The results for this housing are shown in Table 3. below.

	Mesh misalignment values (FBetaX)		
Loadcase	Infinite housing stiffness	FE housing stiffness (Design B)	Change
1st Speed	-100 um	-122 um	+22 %
2nd Speed	-56 um	-101 um	+80 %
3rd Speed	-35 um	-53 um	+51 %
4th Speed	-32 um	-40 um	+25 %
5th Speed	-33 um	-34 um	+3 %
Range:	*67 um*	*88 um*	

Table 3. *The effect of housing stiffness on final drive gear pair mesh misalignment (Housing Design B)*

In this case the incremental change in magnitude and range of mesh-misalignments is reduced. This can be correlated to extra stiffening ribs aligned to the direction of bending load which were added based on the load vectors calculated during the system analysis. The procedure was as follows: apply ribs to the FE model and thicken plate elements, solve to extract the reduced stiffness matrix superelement, assemble the system stiffness matrix and re-run the system analysis. All this was completed in a timescale compatible with the current prototype and tooling procurement schedule.

Effect of Misalignment on Transmission Error

As discussed in earlier, the literature provides a number of methods for predicting the effects of gear misalignment on transmission error. Figure 4. shows transmission error results obtained using the Ohio State University Load Distribution Program (5). For the sake of brevity the final drive gear pair in first speed (the heaviest load) only is shown.

The results are for the unmodified gears without any micro geometry modification and show the effects of including the misalignment predicted with and without including the housing stiffness. One can clearly see that the inclusion of the housing flexibility when predicting the gear mesh misalignment gives rise to significantly different predictions of the transmission error. The magnitude of the peak-to-peak change that the gear designer must now attempt to compensate for is increased by approximately 100%.

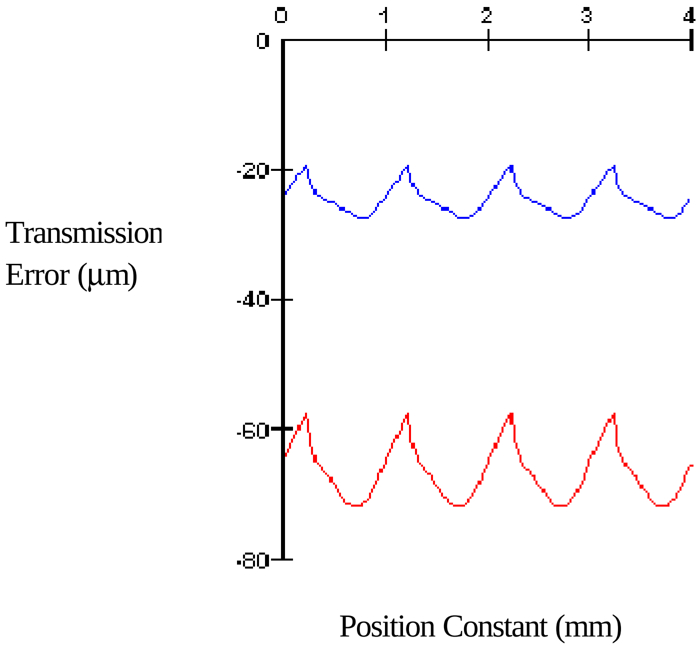

Figure 4. *The predicted transmission error for the final drive gear pair in first speed using gear misalignment predictions corresponding to an infinitely stiff housing (upper line) and including the housing stiffness (lower line)*

USING THIS APPROACH IN DESIGN

At the detail design stage, FE models usually exist of the housing for packaging and detail stressing and life analysis. Consequently this approach fits in easily with the timescales and data available. NVH work including gear micro-geometry design can be carried out with much more confidence using the full system stiffness model.

In other studies we found that the housing stiffness made little difference to the gear mesh misalignments. This makes it even more vital to assess whether the housing is sufficiently stiff prior to committing to the expensive process of "prototype manufacture – test – re-design – prototype manufacture – test" etc.

However, many of the decisions that shape the design and its cost basis are already made at this detail design stage. It is perhaps even more desirable to look at the total system stiffness during the early concept design stage since alternative concept layouts may provide a better solution.

There is a dilemma for the transmission design team. They want as much detail design done as early as possible to help them make the best choices. There may not even be time to develop a full FE model of a new housing. One idea that is being considered is to take a "similar" transmission housing FE model and use that to provide an estimate of housing stiffness. Another approach is to develop a simple shell element housing model as a baseline model. Then ribs can be added and local thicknesses increased as an understanding of the deflections is gained during the design process. A different strategy might be to use an "automatic" housing model generator based on some physical hard points such as bearing positions and packaging constraints. All these approaches relax the "infinitely stiff" assumption when no housing stiffness matrix is coupled with the internal stiffness matrix and should provide better information about the system behaviour.

CONCLUSIONS

A fully coupled non-linear algorithm to analyse the shaft/bearing/gear/housing hyperstatic system has been developed. A reduced stiffness matrix generated using MSC.Nastran is used to very efficiently model the housing. For example, for the case study considering the analysis of a five-speed transmission including the housing stiffness has a run time of under a minute on a Windows NT™ computer.

The analysis allows the important interaction between the transmission housing and internal components to be investigated. A case study has been presented, showing that the effects of including the stiffness of a reduced weight housing can significantly change the predicted gear mesh misalignments. This should be used to better define the way that micro-geometry modifications are applied to the gears to minimise noise.

In addition the analysis provides valuable information on the interaction between the housing stiffness and bearing pre-loads, bearing misalignments and lives, and the derivation of accurate inputs to the structural (stress) analysis of transmission housing

The authors are in the process of extending this work to study the effects of planet carrier and differential cage stiffnesses on transmission performance using FEA generated stiffness data.

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APPENDIX A : ALTER Sequence to Derive Stiffness Matrix

NOTE: The underlined lines are inserted for the Guyan reduction of the stiffness.

```
NASTRAN SYSTEM(146)=4,SYSTEM(189)=0.8,T3SKEW=1
NASTRAN SPARSE=25 $ make full use of sparse matrix methods
ASSIGN OUTPUT2='LC_assy_org_fix_guyan.f12' UNIT=12
INIT MASTER(S) $ delete database files after run
INIT DBALL LOGICAL=(DBALL(30000)) $ set database file size
$-- EXECUTIVE CONTROL ------------------
ECHOOFF
TIME 20000 $ max run time in minutes
COMPILER NOLIST,NOREF
SOL 103
COMPILE PHASE1DR NOLIST
ALTER 'END OF SUPERELEMENT GENERATION LOOP' $
MATPCH KAA,MAA,USET//$
OUTPUT2 KAA,MAA,USET,,//0/12//OMAXR $
OUTPUT2 EQEXINS,,,,//0/12//OMAXR $
OUTPUT2 //-9/12 $
EXIT $
ENDALTER
CEND
$-- CASE CONTROL -----------------------
TITLE = M5GF1
ECHO = SORT(FORCE,MOMENT,MPC,PLOAD2,PLOAD4,SPC,TEMP)
MAXLINES = 20000000 $ max print file size
DISP(PLOT) = ALL
$ESE(PLOT) = ALL
SPC = 3
METHOD = 1
$-- BULK DATA --------------------------
BEGIN BULK
ECHOOFF
PARAM,BAILOUT,-1 $ dont stop on error
PARAM,GRDPNT,0 $ weight output
PARAM,AUTOSPC,YES $ fix singularities
PARAM,PRGPST,NO $ don't report them
PARAM,POST,-2 $ IDEAS format o/p
PARAM,NEWSEQ,-1 $ best sequence option for SPARSE methods
PARAM,OUGCORD,BASIC $ displacements in global coords
PARAM,OUMU,NO $ suppress kinetic energy output
$ NORMAL MODES BY LANCZOS METHOD
EIGRL,50,,,1
ASET1,123456,100001,THRU,100006
$
INCLUDE 'LC_assy_org_fix_guyan.bulk'
$
ENDDATA
```

APPENDIX B : Alternative Approach to Derive Stiffness Matrix

```
$
$ Punch stiffness matrix
$
PARAM   EXTOUT   DMIGPCH
$
$ gearbox shafts
$
ASET1   123456   540834
ASET1   123456   540857
ASET1   123456   540863
ASET1   123456   540865
ASET1   123456   540866
ASET1   123456   540867
$
$
```