DISCRETE OPTIMIZATION in MSC.Nastran

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Abstract

MSC.Nastran 2001 contains the ability to apply discrete variables in the optimization process. This was done in recognition that engineers are often required to deal with discrete variables in engineering design activities. MSC developed and implemented Design of Experiments (DOE) and Conservative Discrete Design (CDD) approaches to deal with discrete variables with limited computational cost. These two approaches, together with engineering rounded-off methods, can be used to process discrete variables at any specified continuous design optimization cycle for structural design problems. Brief background and a theoretical discussion about these approaches are given in this paper. Finally, the new discrete optimization feature in SOL 200 is illustrated by some examples.

1.0 Introduction

MSC.Nastran SOL 200 has been able to support comprehensive structural design optimization for continuous design variables for over ten years. A shortcoming of this capability is that it could not address the actual design task often requires that the design variables be selected from a specified set of discrete variables. For example, structural members may have to be designed using selections available in standard sizes, member thickness may have to be selected from the commercially available ones, and composite laminates are built using an integer number of plies. Civil engineering structures, in particular, need to restrict their member sections to a set of standard properties. This shortcoming has been addressed in the latest release of MSC.Nastran by the addition of a discrete variable processing capability.

Considerable interest has been shown for discrete optimization methods since continuous optimization methods were well established in the late 1980s. In general, discrete optimization methods are aimed at solving the so-called mixed-discrete nonlinear programming problem, where the term "mixed-discrete" indicates that both discrete and continuous design variables are present. Reviews of the developments in discrete optimization methods have been presented by Arora [1] and Vanderplaats [2]. From these reviews, it can be said that no practical method has emerged as the recognized standard in this area.

This paper first defines the nature and requirements of the discrete and mixed-discrete optimization problems. Current discrete optimization methods are briefly reviewed and methods appropriate for structural design are emphasized. It is shown that finding the best discrete design becomes a challenging task if there are more than around ten discrete design variables and that most methods become intractable when there are over one hundred discrete design variables. The method selected has to be applicable to hundreds, even thousands, of discrete variables.

The simplest and least expensive engineering method for processing discrete variables is by rounding up or down the continuous solution obtained from solving a corresponding continuous optimization problem. This approach has been implemented in MSC.Nastran 2001for quick discrete design solutions. However, the design constraints can be easily violated or become over-designed by this approach. In order to produce a good discrete solution with limited computational cost for large discrete optimization problems, MSC.Nastran 2001 contains two additional approaches: a) the Conservative Discrete Design (CDD) approach and b) the Design of Experiments (DOE) approach. Both work on the explicit approximate discrete problem. The CDD approach is employed to quickly obtain a conservative discrete solution based on the continuous optimal solution and by using the sensitivity information values. The DOE approach aims to obtain a good discrete design by evaluating the approximate objective and constraints with extra but limited computational cost. These two approaches can be used for processing discrete variables at any specified continuous design optimization cycle. Brief background and a theoretical discussion about these two approaches are given in this paper. Finally, the new discrete optimization feature in SOL 200 is illustrated by some examples.

2.0 Discrete Design Variables Problem

The general formulation for a problem having discrete variables is presented here. First, let us consider a continuous optimization problem:

$$
\begin{array}{lll}
\text{minimize} & f(\mathbf{x}) & \text{objective} \\
\text{subject to} & g_j(\mathbf{x}) \le 0, \;\; j=1,2,...,m & \text{inequality constraints} \\
 & x_i^L \le x_i \le x_i^U, \;\; i=1,2,...,n & \text{side constraints}
\end{array}
\tag{1}
$$

where f and g_j are continuous real functions of n real continuous design variables and m indicates the number of inequality constraints. In a discrete optimization problem, the objective f and constraints g_j in Eq. (1) are not changed, but the components of the design variables are restricted to prescribed sets of discrete real values, i.e., the side constraints of the Eq. (1) become

$$
x_i^L \le x_i \le x_i^U, \;\; x_i \in A_i = \left\{a_i^1, \; a_i^2, \; ..., \; a_i^{ki}\right\}, \;\; i=1,2,...,n \tag{2}
$$

where A_i is the design candidate subset (called discrete value set) for the i^{th} design variable, and ki is the number of design candidates. If all of the design variables are discrete, the problem is called a discrete optimization problem. If some of the design variables are discrete and others are continuous, we have the so-called mixed-discrete optimization problem. It should be noted that many problems are actually mixed - they contain both discrete and continuous design variables.

Initially, one might suppose that, since fewer possible solutions exist, the discrete problem may be easier to solve than the continuous problem. However, except for the most trivial cases, the discrete problem presents a great increase in difficulty. In general, the discrete design space is disjoint and nonconvex. Thus, the powerful continuous optimization methods can not be directly used to solve the discrete and mixed-discrete problems. In a very small discrete problem, one could possible test all possible combination of discrete variables and select the best. This process, called enumeration, will certainly find the optimum of the discrete problem. Consider however, a problem of 10 discrete design variable where each variable may assume one of 10 values. Ten to the 10^{th} power possible solutions exist. Even with the tremendous computing power available today, this approach is much too costly, especially if a complete finite element analysis must be executed for each combination.

Current discrete optimization methods can be classified as either deterministic or probabilistic [1]. Probabilistic methods have been applied to solve engineering optimization problems for a long time. The most well known probabilistic methods for both continuous and discrete optimization are Genetic Algorithms (GA) [4] and Simulated

Annealing (SA) [5]. One major advantage of probabilistic methods is their ability to directly deal with discrete design variables. However, these methods are extremely expensive, and impractical for the optimization of real structures using the finite element method.

On the other hand, deterministic methods such as gradient-based mathematical programming methods have been shown to be able to locate the continuous optimal design for large practical structures. However, they cannot directly deal with discrete design variables. A great various approaches have been developed to apply deterministic methods for discrete and mixed-discrete optimization problems. The simplest and least expensive method for obtaining a discrete solution is by rounding up the continuous solution to the value of the closest discrete sizes. However, this rounding up process can easily result in an overweight or violated discrete design. Therefore, the applicability of the rounding up methods to practical structural designs may be limited.

The branch and bound method (BBM) is probably the best known and most frequently used discrete optimization method. It is import to note that the BBM is guaranteed to find the global optimum if the problem is linear or convex, but not guaranteed to converge in the general nonconvex case. The method is based on the sequential analysis of a discrete tree for each variable, the computational cost grows exponentially with the number of design variables. Therefore, the method is not suitable for the analysis of problem with many design variables using finite element analysis [6,7].

Recently, various discrete rounding methods have been proposed, such as the pseudo-discrete rounding [7], dynamics rounding [8], and orthogonal array approaches [9]. The dynamics rounding method provides a more refined approach to indiscriminate rounding. Here, the value of the variable satisfying a Lagrangian selection criterion is increased to that of its discrete upper neighbor, and the resulting mixed-discrete problem is optimized using a continuous method. This process is repeated until only a single variable remains, of which the value is then also increased to that of its discrete upper neighbor. This method requires solving (n-1) continuous optimization problems, where n is the number of design variables. Obviously, this approach is much too costly for a discrete problem with large number of discrete variables.

Based on current research, it is well known that discrete optimization is more difficult to solve and requires more computational cost than continuous design optimization. To date, searching for the “best” discrete design regardless of computational cost is impractical. Instead, MSC SOL 200 is expected to produce a good discrete solution with limited computational cost for large discrete structural optimization problems.

MSC.Nastran 2001 contains the Conservative Discrete Design and Design of Experiments approaches to address explicit approximate discrete problem. These approaches aim to obtain a good discrete design with limited computational cost. The following section describes the theoretical background of the proposed approaches.

3.0 Discrete Variable Processing Methods

MSC.Nastran 2001 contains four discrete variable processing methods: a) rounded-up; b) rounded-down; c) Conservative Discrete Design (CDD); and d) Design of Experiments (DOE). The first two methods are by simply rounding up or down the continuous solution obtained from solving a corresponding continuous optimization problem. These two rounded-off methods have been implemented in MSC.Nastran 2001for quick discrete design solutions. The other two more complex methods are discussed in this section.

3.1 Discrete Post-Processing Using Design of Experiments (DOE)

DOE methodology was originally developed by R.A. Fisher in 1930's [10]. Fisher showed that a full factorial array could be reduced to a smaller, but still statistically meaningful set, by using arrays called fractional factorial designs. Since then, DOE methods and fractional factorial arrays have been developed over years by many researchers and applied in many disciplines. Taguchi utilizes fractional factorial arrays from DOE theory, called Orthogonal Array (OA), to study the parameter space with a small number of experiments as compared to a full factorial array study. A full factorial array needs 2^n experiments for a problem with n two-level factors. Use of OA can dramatically reduce the required number of experiments as the number of parameters increases. As an example, seven parameters can be studied by conducting only 8 experiments using an L8 OA, as opposed to 128 (2^7) by a full factorial array. A major advantage of the DOE is its simplicity in applications, non-gradient methodology, and ability to handle discrete variables. However, if the design problem involves expensive function evaluations such as the use of finite element methods, the DOE cannot efficiently deal with large numbers of design parameters. More detail discussion about DOE can be found in References 10 and 11.

In this paper, the DOE process together with the approximate concepts is employed to deal with discrete variables. A high level flowchart is shown in Figure 1.

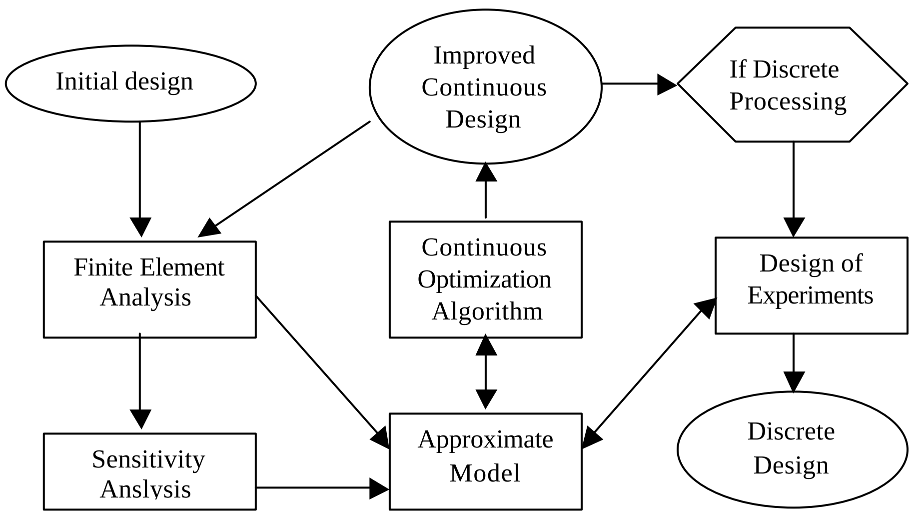


Figure 1 Discrete Processing by DOE with Approximate Concepts

DOE Design Problems

Instead of the initial design model, the continuous optimal design obtained from current SOL 200 is used as a baseline for discrete variable processing by using the DOE. Since it is assumed that the discrete optimum is close to the continuous optimum, it is expected that a discrete solution by the DOE be close to the discrete optimum due to the selected design baseline. Therefore, searching a feasible discrete design is emphasized in the DOE processing. The objective and retained constraints at the continuous optimum are used for the measurements of the DOE processing.

DOE Factor, Levels, and OA

All discrete variables are treated as factors in the DOE. For mixed-discrete problems, the continuous design variables are treated as constants in the DOE discrete variable processing and their values are the optimal solutions at the current continuous design cycle.

In the proposed approach, each discrete variable has two levels. It is recommend that the nearest upper and lower neighbors of the continuous optimum be the two levels. Thus, SOL 200 will search discrete solutions within a narrow design space around the continuous optimum. Even though this two-level approach limits the search design space, more than two levels are not recommended since the approximation method is not accurate enough to evaluate the objective and constraints in a large design space (see later discussions). Instead, users are allowed to select any existing continuous design cycle to start the discrete variable processing in order to increase the discrete design space.

The smallest size OA will automatically be the lowest resolution and lowest computational cost. On the other hand, high fractional factorial array will be high resolution and high computational cost. It is desirable for the discrete variable processing capability to utilize high fractional factorial array with allowable computational cost. It is important to note that full factorial array or high fractional factorial array could be used since the approximation method is used to evaluate the objective and constraints.

Approximation Methods

As discussed in above sections, even the smallest OA still requires n+1 experiments for n factors. Obviously, the DOE process is still too expensive for structural design problems in terms of number of finite element analysis. The approximation methods are used in MSC.Nastran SOL 200 continuous optimization capability in order to reduce the number of finite element analysis [12]. In this paper, the implicit and costly finite element method is replaced by explicit approximation methods to evaluate the objective and constraint functions of each combination of discrete values. The approximating functions used in MSC.Nastran SOL 200 are based on Taylor series expansions of the objective and constraints. For any function f(x), its linear approximation can be obtained as:

$$f(x) = f(x^0) + \frac{\partial f(x^0)}{\partial x}(x - x^0) \quad (3)$$

Where $f(x^0)$ and its derivatives are known at x^0. Therefore, if the function values and derivatives are known at the continuous optimal design, approximate function values can be evaluated at the nearby discrete designs by Equation 3.

Note that the approximation method enables SOL 200 to deal with large discrete design problems, and high fractional factorial arrays can be used to explore more numbers of combinations of discrete values. However, this approach may result in infeasible designs since the approximation method is not accurate for high nonlinear problems, especially for widely spaced discrete problems in which the nearest discrete neighbors are distanced from the continuous optimum.

Selection of Discrete Designs

A discrete solution is selected from the OA to have the minimum cost and no constraint violation. If all discrete designs yield violated constraints, then a discrete design is selected to have a minimum constraint violation.

Stop Criteria

There is no convergence check for discrete designs. The stop criteria for SOL 200 continuous optimization are still valid for discrete design problems. A finite element analysis will be carried out at each discrete variable processing cycle. The discrete designs will be checked whether they are hard or soft feasible (see Reference 12).

An Algorithm for Discrete Variable Processing Using DOE

An algorithm for the discrete variable processing at a continuous design cycle is suggested as follows:

1) The first step is performing the corresponding continuous optimization problem defined in Eq. (1). Once a continuous optimal design is obtained, proceed to discrete variable processing if a discrete variable processing is required at this continuous design cycle.
2) Perform a detailed finite element analysis for the continuous optimal design
3) Evaluate all the constraints and retain those that are critical or near critical for further consideration during the discrete variable processing cycle.
4) Perform a sensitivity analysis for the retained design responses at the continuous optimal design
5) Two-level discrete values are selected.
6) A proper OA is selected, the objective and retained constraint functions are evaluated for each design combination of the OA by the approximation method.
7) A best discrete design is selected from the Steps 6
8) A finite element analysis is carried out at the best discrete design, and soft and hard feasibility is checked

3.2 A Conservative Discrete Design

For a design problem with large number of discrete variables, it is impossible to use a full factorial OA for the DOE process. For example, a full factorial OA needs a 1.27×10^{30} (2^{100}) function evaluations for 100 discrete variables. This is impossible even for the approximation method. In this case, a smaller fractional factorial OA must be used. It means that a smaller number of discrete design combinations will be searched in the DOE process, and the opportunity of getting a feasible discrete solution becomes less as the number of discrete variables increases. As discussed in above sections, the major purpose in the discrete variable processing is to search a feasible discrete solution. In order to increase the opportunity of getting feasible discrete solutions, a conservative discrete design methodology is proposed as a complement to the DOE process. Another advantage is that the conservative discrete design is able to produce a better discrete solution than the rounded-off method with very comparable computational cost. Therefore, SOL 200 provides clients a very quick discrete variable processing means for very large discrete problems.

For a continuous optimum design $\mathbf{X}^0$, suppose that the ith discrete variable x_i has the closest discrete lower value x_i^{LC} and upper value x_i^{UC}. By the approximation method, we use the following equations to evaluate all retained constraints at $x_i = x_i^{LC}$ and $x_i = x_i^{UC}$, respectively while the other discrete variables keep constants.

$$\tilde{g}_j\left(x_i^{LC}\right) = g_j\left(\mathbf{x}^0\right) + \frac{\partial g_j\left(\mathbf{x}^0\right)}{\partial x_i}\left(x_i^{LC} - x_i^0\right) \tag{4}$$

$$\tilde{g}_j\left(x_i^{UC}\right) = g_j\left(\mathbf{x}^0\right) + \frac{\partial g_j\left(\mathbf{x}^0\right)}{\partial x_i}\left(x_i^{UC} - x_i^0\right) \tag{5}$$

Where j = 1, .., NC, and NC is the number of retained constraints. To obtain a conservative discrete design, the discrete variable x_i is selected from x_i^{LC} and x_i^{UC} to have the minimum maximum constraint. This procedure repeats for all discrete variables. Therefore, a conservative design is obtained. Since the number of retained constraints is limited, the cost for evaluating the approximated constraints is not high. It should be noted that this approach cannot guarantee to produce a "true conservative design" since Equations 4 and 5 neglect the interaction of discrete variables and the approximations may not be accurate enough. However, this approach is expected to increase the possibility of getting a feasible discrete design for a problem with a large number of discrete variables.

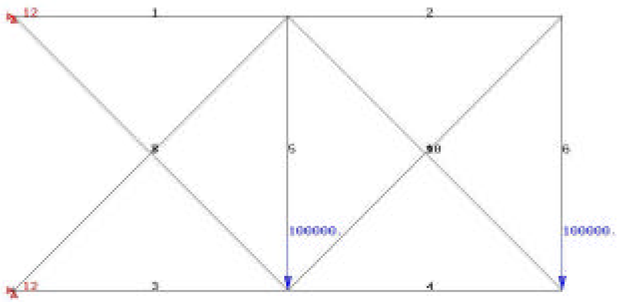
The algorithm for discrete variable processing using CDD is very similar to the algorithm using DOE. The difference is in the Step 6 in the above section while the remained steps are the same. For the CDD, a conservative discrete solution is chosen by evaluating the approximate objective and retained constraint functions in the Step 6.

4 Numerical Examples

The results of several test problems are presented to demonstrate the discrete variable optimization capability of the methods discussed. It should be noted that the examples are not intended to present the scope of application of the method. Some reference solutions exist for the 25- bar truss examples, a comparison is presented even though it is difficult because of the importance of internal parameters used in numerical optimization approaches. In numerical optimization methods, feasibility is determined by a constraint tolerance. The constraint tolerance is the maximum value allowed for a constraint to be considered unviolated. This tolerance can affect results. In this paper, the MSC SOL200 default tolerance of 0.005 is used. The maximum constraint value is listed in the result tables for each case. In discrete optimization result tables, weight or volume indicates the objective, brackets indicate infeasible solutions, NFE indicates number of finite element analyses.

<u>10-Bar Truss</u>

The first example is the classical 10-bar truss. The configuration, material, and load information can be found in Figure 2. The design variables are the cross-section area of the 10 bars. Each member is allowed a maximum stress of $\pm$25,000 psi and a minimum area of 0.1 in. The vertical displacement of all joints is limited to 2.0 in. The available discrete values of the variables are {0.1, 0.5, 1.0, 1.5, … }.



Material:	E=10^7 psi and $\rho = 0.1\,lbm/in^3$
Stress limit:	25000.00 psi
Displacement limit:	2.0 in

Figure 2. 10-bar truss

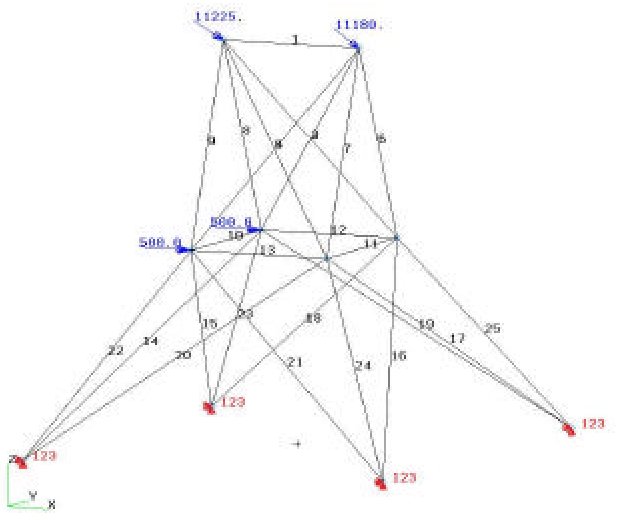
Table 1 shows the results for continuous optimization, rounded-off solutions, and discrete solutions for SOL 200 DOE and CDD approaches. SOL 200 DOE and CDD approaches succeeded in producing a feasible discrete solution with lower weight. The rounded-down produces an infeasible discrete solution and the rounded-up gives an overweight solution. It is noted that DOE produces the best discrete solution and CDD produces the most conservative discrete solution that is lighter than the rounded-up solution.

Table 1 Results for 10-Bar Truss

	Initial design	Contin. Optim.	Discrete DOE	Discrete CDD	Rounded up	Rounded down
Weight	8393.9	5067.6	5081.4	5146.7	5195.9	5002.0
Max. cons.	-1.5%	0.2596%	-0.063%	-1.189%	0.414%	(1.780%)
A1	20.0	30.804	31.0	31.0	31.0	30.5
A2	20.0	0.1164	0.1	0.1	0.5	0.1
A3	20.0	23.527	24.0	24.0	24.0	23.5
A4	20.0	14.702	15.0	15.0	15.0	14.5
A5	20.0	0.1000	0.1	0.1	0.5	0.1
A6	20.0	0.1484	0.1	0.5	0.5	0.1
A7	20.0	8.4151	8.5	8.5	8.5	8.0
A8	20.0	20.948	20.5	21.0	21.0	20.5
A9	20.0	21.003	21.0	21.5	21.5	21.0
A10	20.0	0.1000	0.1	0.1	0.5	0.1
NFE		13	13+1	13+1	13+1	13+1

25-Bar Truss

The second example is another classical optimization test case – the 25-bar truss (Figure 3) that is frequently used to test optimization algorithms for both continuous and discrete structural optimization problems. Due to symmetry of the 25-bar truss, only eight different member sizes are allowed, and hence eight independent design variables are selected by linking various member sizes. The prescribed discrete value set is {0.1, 0.4, 0.7, 1.1, 1.5, 1.8, 2.0, 2.5, 3.0, 5.4, 4.5}.



Material: E=10^7 psi and $\rho = 0.1$ lbm/in ^{3}

Stress limit: 40000.00 psi

Displacement limit: 0.35in

Figure 3. 10-bar truss

Numerical results presented in Table 2 gives an indication of the relative computational cost of various discrete optimization algorithms. It is observed that the proposed discrete variable post-processing method in SOL 200 can produce a very comparable discrete solution with significantly less computation cost in terms of required number of finite element analyses.

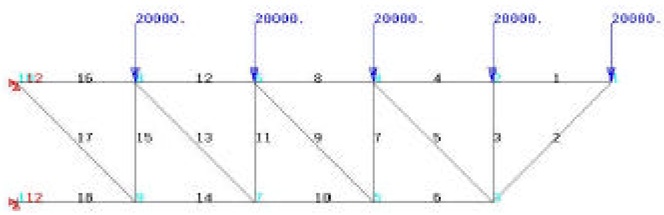
Table 2 Efficiency & Accuracy Comparison of Discrete Optimization Methods for 25-Bar Truss

	MSC Cont. Opt.	MSC Discrete DOE	* Branch & Bound	* Pseudo Rounding	*dynamic Rounding	* Neigh-bourhood Search	*Simulated Annealing	*Genetic Algorithm
Weight	534.875	543.998	543.998	543.999	544.831	543.999	544.831	544.831
Max. Cons.	0.2380%	-0.881%	N/A	N/A	N/A	N/A	N/A	N/A
NFE	11	11+1	516	12+77	164	3449	42512	8315

* Reference 7

<u>18-Bar Truss</u>

As discussed in the introduction section, many design problems are actually mixed – they contain both discrete and continuous design variables. The 18-bar truss example (shown in Figure 4a is a classical shape optimization test case. In this paper, this example is employed to demonstrate the capability of the proposed discrete variable processing method to deal with mixed-discrete problems and discrete shape design variables.



Material: $E=10^7$ psi and $\rho = 0.1\,\text{lbm/in}^3$

Stress limit: 20000.00 psi

Figure 4a. 18-bar truss

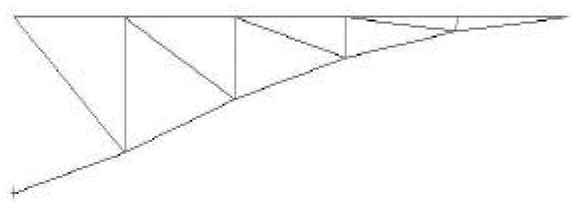


Figure 4b. 18-bar truss optimum shape

The 18-bar truss member sizes are linked to have 4 independent sizing design variables as follows: A1 = A4 = A8 = A12; A2 = A6 = A10 = A14= A18; A3 = A7 = A11 = A15; A5 = A9 = A8 = A13 = A7. The shape variables are X and Y coordinates of node 3, 5, 7, and 9 (X3, Y3, X5, Y5, X7, Y7, X9 and Y9). In design Case I, It is assumed that the sizing variables are chosen from the discrete value set {1.0, 2.0, 3.0, … } while the shape design variables can vary continuously. In design Case II, both sizing and shape variables are treated as discrete variables. The prescribed discrete value set for the eight shape variables is {-5.0, 0.0, 5.0, 10.0, 15.0, 20.0, … }.

Figure 4b shows the optimum shape of the 18-bar. Tables 3-4 show the discrete variable processing results by the proposed algorithm. It is observed that the proposed approaches produce a good discrete solution for both design cases. The rounded-up method is able to generate a good discrete solution for Case I since it is meaningful to round up truss member's cross-sectional areas for stress constrains. However, both rounded-up and down approaches cannot produce a feasible discrete solution for Case II since it is not meaningful to round up or down shape variables.

Table 3 Results for 18-Bar Truss Case I

	Initial design	Contin. Optim.	Discrete DOE	Discrete CDD	Rounded up	Rounded down
Weight	6430.39	391.99	399.23	399.23	399.23	379.20
Max. cons.	0.151%	0.105%	-0.08%	-0.08%	-0.08%	(3.99%)
A1	10.00	12.998	13.0	13.0	13.0	12.5
A2	21.65	15.392	15.5	15.5	15.5	15.0
A3	12.50	1.302	1.5	1.5	1.5	1.0
A4	7.07	2.537	3.0	3.0	3.0	2.5
X3	1.0	-0.829	-0.829	-0.829	-0.829	-0.829
Y3	1.0	231.48	231.48	231.48	231.48	231.48
X5	1.0	-0.672	-0.672	-0.672	-0.672	-0.672
Y5	1.0	192.91	192.91	192.91	192.91	192.91
X7	1.0	-0.668	-0.668	-0.668	-0.668	-0.668
Y7	1.0	135.08	135.08	135.08	135.08	135.08
X9	1.0	0.104	0.104	0.104	0.104	0.104
Y9	1.0	58.331	58.331	58.331	58.331	58.331
NFE		18	18+1	18+1	18+1	18+1

Table 4 Results for 18-Bar Truss Case II

	Initial design	Contin. Optim.	Discrete DOE	Discrete CDD	Rounded up	Rounded down
Weight	6430.39	391.99	399.13	399.26	399.24	379.20
Max. cons.	0.151%	0.105%	-0.121%	-0.89%	(21.15%)	(12.98%)
A1	10.00	12.998	13.0	13.0	13.0	12.5
A2	21.65	15.392	15.5	15.5	15.5	15.0
A3	12.50	1.302	1.5	1.5	1.5	1.0
A4	7.07	2.537	3.0	3.0	3.0	2.5
X3	1.0	-0.829	-5.0	0.0	0.0	-5.0
Y3	1.0	231.48	230.0	230.0	235.0	230.0
X5	1.0	-0.672	-5.0	0.0	0.0	-5.0
Y5	1.0	192.91	190.0	190.0	195.0	190.0
X7	1.0	-0.668	0.0	0.0	0.0	-5.0
Y7	1.0	135.08	135.0	135.0	140.0	135.0
X9	1.0	0.104	5.0	0.0	5.0	0.0
Y9	1.0	58.331	60.0	60.0	60.0	55.0
NFE		18	18+1	18+1	18+1	18+1

Intermediate Complexity Wing

The above three examples are small design problems. As mentioned in the introduction section, the proposed discrete variable processing approaches are aimed to support realistic structural designs. The last example, an Intermediate Complexity Wing (ICW) design, is employed to demonstrate this capability. The ICW structural model, shown in Figure 5, uses 62 quadrilateral and 2 triangular membrane elements to model the composite wing skins and 55 shear panel elements to model the substructure. Thirty-nine rod elements are used as posts to complete the interconnection of the upper and lower surface. The analysis model uses a cantilevered boundary condition at the root. The substructure material is modeled as aluminum, while the wing skins are made of a graphite/epoxy composite. Table 5 shows the material properties, gauge limits and stress allowable for the two materials.

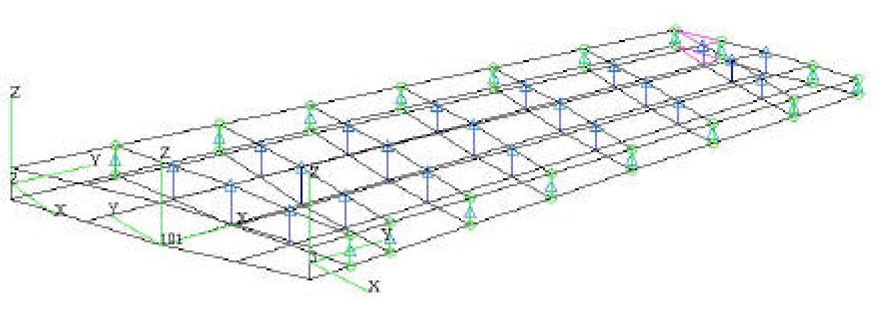


Figure 5. Intermediate Complexity Wing

Table 5 Material Properties for the Intermediate Complexity Wing

Isotropic Material	Orthotropic Material
E= 1.05x10^7 psi	E_1 = 1.85x10^7 psi
$\rho = 0.1\,lb/in^3$	E_2 = 1.6x10^6 psi
$\nu = 0.3$	$\rho = 0.055\,lb/in^3$
$\sigma_T \le 67ksi$	$\nu_{12} = 0.25$
$\sigma_C \le 57ksi$	$G_{12} = 0.65\times10^6$ psi
$\sigma_{xy} \le 39ksi$	$\lvert\sigma_x\rvert \le 115ksi$
	$\lvert\sigma_y\rvert \le 115ksi$

In this design example, the objective is minimization of the composite wing volume while the composite wing is subjected to stress, failure index, and frequency limits. The thickness of 55 shear panel elements are linked to have 24 independent discrete variables, the cross-sectional areas of 39 rod elements are linked to have one independent discrete variable. The prescribed discrete value set for the shear panel and rod elements is {0.2, 0.25, 0.30, …, 10.0}. Each composite quadrilateral and triangular element contains 4 plies that are treated as 4 independent discrete variables. The number of independent ply thickness variables is reduced to 128 through SOL200 linking capability. The ply thickness variables can be chosen from discrete value set {1.0, 2.0, 3.0, …, 40.0}. There are 153 independent discrete variables in this example so that it is hardly possible to solve by traditional discrete optimization methods.

Table 6 shows the discrete variable processing results. It is noted that the initial design is seriously violated. Both SOL 200 DOE and CDD approaches produce a feasible solution with less structural volume compared to the rounded-off methods. In this case, the CDD produces a better discrete solution than the DOE since DOE cannot explore all possible combination of discrete values due to limited computational cost.

Table 6 Results for the Intermediate Complexity Wing

	Initial design	Contin. Optim.	Discrete DOE	Discrete CDD	Rounded up	Rounded down
Volume	118.25	564.25	602.22	583,83	607.77	526.57
Max. cons.	58429.4%	0.212%	-0.650%	0.12%	-1.01%	(75.8%)
NFE		21	21+1	21+1	21+1	21+1

4 Summary and Conclusions

The results of this study show that the proposed DOE and CDD approaches provide a practical means to process discrete variables with limited computational cost. The proposed method succeeded in finding a good discrete solution that could not be anticipated with conventional rounded-off methods. The intermediate complexity wing example demonstrates that the proposed DOE and CDD approaches can be used to solve realistic structural design problems with hundreds of discrete design variables. More realistic engineering design applications are required to further improve the discrete variable processing methods.

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