Automotive CAE Durability Analysis Using Random Vibration Approach

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Abstract

Dynamic road loads to automotive structures are random in nature and can be conveniently characterized as random processes in terms of the power spectral density (PSD) functions. Typical multiple dynamic loadings are modeled as a correlated PSD complex matrix. The random responses are simulated efficiently by employing MSC.Nastran frequency response module. The statistical characteristics of a random response are found through the moments of its PSD function. The total damage and durability life of a structure due to multiple road load events, corresponding to multiple road surfaces and events of a typical vehicle proving ground test, are then derived. A practical procedure for automotive CAE durability analysis using the random vibration approach is presented in this paper. Two example results of automotive structures, a rear axle assembly made of metals, and a headlamp assembly made of plastics, are given to demonstrate the approach and their applications. The results show that the random vibration approach costs less CPU time and memory.

Introduction

Computer aided engineering (CAE) provides a means of verifying design for durability of automotive structures without making hardware prototypes. CAE durability can thus reduce time and cost for product development. There are, in general, two approaches for dynamic automotive CAE durability analysis, one is time domain approach and the other frequency domain approach. The time domain durability approach is an original durability approach for structure evaluation. The input loads to and response results from an automotive structure, are all expressed in terms of their respective time histories. While in the frequency domain approach, the loads and response results are expressed in terms of their respective power spectral density (PSD) functions.

Dynamic road loads to automotive structures are random in nature and can therefore be conveniently defined as random processes in terms of the PSD functions [1,2]. Typical multiple dynamic loadings are modeled as a correlated PSD complex matrix. The random responses, such as stresses and accelerations of an automotive structure, are simulated efficiently by employing MSC.Nastran frequency response module. The statistical characteristics of a random response are found through the moments of its PSD function. The accumulated fatigue damage and durability life of a structure are then evaluated by using a simple closed form solution.

The frequency domain approach provides an alternative for CAE durability evaluation, which can reveal the frequency characteristics of a structure and require less CPU simulation resources. The frequency domain PSD method has been used in space and automotive industry for many years, for structures under simple loading conditions [4,6,7]. In this paper a systematic PSD approach for automotive CAE durability analysis, dealing with multiple correlated input loads and multiple road load events, is presented. The total damage and durability life of a structure due to multiple random road load events, corresponding to multiple road surfaces and events of a typical vehicle proving ground test, are derived and implemented.

A practical three-step procedure for automotive CAE durability analysis using the random vibration approach is illustrated, for system level structures under multiple road loading inputs. The procedure incorporates (1) the proving ground (PG) road load data process, (2) the finite element (FE) frequency response simulation, and (3) the fatigue life evaluation. The fundamentals of the CAE frequency domain durability approach are briefly reviewed in the next three sections. Two example results of automotive structures, a rear axle assembly made of metals, and a headlamp assembly made of plastics, are given to demonstrate the approach and their applications.

Random Load Description

Road surfaces traversed by ground vehicles are random in nature. It has been established that most road surface irregularities are normally distributed and may be accurately described by a stationary random process [1,2,6]. For a stationary ergodic random phenomenon, the ensemble averages are equal to the time averages. The statistical properties of a stationary ergodic process can then be computed from a single time history of sufficiently long period. The time average of a random variable *x(t)* is equal to the expected value of *x(t*), as defined as:

$$E[x(t)] = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\ dt \qquad (1)$$

The mean square value of *x(t)* is found by applying average operation to variable $x^2(t)$ over a time interval *T*.

$$E[x^2(t)] = \overline{x^2} = \lim_{T\to\infty} \frac{1}{T} \int_0^T x^2(t)\, dt \qquad (2)$$

Correlation function is a measure of the similarity between two random quantities in a time domain *t*. For a single record *x(t),* the autocorrelation *R(t)* of *x(t)* is the expected value of the product *x(t)x(t+t):*

$$R(t) = E\left[x(t)\, x(t+t)\right] = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, x(t+t)\ dt \qquad (3)$$

When *t=0*, the above definition reduces to the mean square value.

$$R(0) \quad = \overline{x^2} = \quad E[x^2(t)] \qquad (4)$$

For two random quantities *x(t)* and *y(t),* the cross correlation function is defined by the equation:

$$R_{xy}(t) = E\left[x(t)\, y(t+t)\right] = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, y(t+t)\ dt \qquad (5)$$

A random process can be described in frequency domain in terms of power spectral density (PSD) functions. It can be shown that the power spectral density functions are related to the correlation functions by Fourier transform pairs. For a single random record *x(t),* the relationship can be shown in the following equations:

$$S(w) = \int_{-\infty}^{\infty} R(t)\, e^{-iwt}\ dt \qquad (6)$$

and

$$R(t) = \frac{1}{2p} \int_{-\infty}^{\infty} S(w)\, e^{iwt}\ dw \qquad (7)$$

where ω=2πf. From equations (3) and (6) it is clear that auto-correlation *R(t)* is an even function of τ , and the auto PSD *S(w)* is real. When *t=0*, the mean square value is related to PSD *S(w)* from (4) and (7) by

$$\overline{x^2} = R(0) = \frac{1}{2p} \int_{-\infty}^{\infty} S(w)\, dw \qquad (8)$$

For two random quantities $x(t)$ and $y(t)$, the cross spectral density and cross correlation functions are defined by Fourier transforms:

$$S_{xy}(\omega) = \int_{-\infty}^{\infty} R_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau \qquad (9)$$

and

$$R_{xy}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xy}(\omega)\, e^{i\omega\tau}\, d\omega \qquad (10)$$

The cross PSD $S_{xy}(\omega)$ is generally complex and has following properties:

$$S_{xy}(\omega) = S_{yx}^{*}(\omega) = S_{yx}(-\omega) \qquad (11)$$

where * denotes the complex conjugate.

Frequency Domain Random Response

The equation of motion of a linear structural system, in general, is expressed in matrix format in equation (12). The system of time domain differential equations can be solved directly in the physical coordinate system, corresponding to each load-time step.

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{p(t)\} \qquad (12)$$

where $\{x(t)\}$ is a system displacement vector, $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices, respectively, $\{p(t)\}$ is an applied load vector with multiple inputs.

When the multiple input loads are random in nature, a matrix of the loading power spectral density functions, $[S_p(\omega)]$, can be generated by employing Fourier transform of load vector $\{p(t)\}$.

$$[S_p(\omega)]_{mxm} = \begin{bmatrix} S_{11}(\omega) & \cdots & S_{1i}(\omega) & \cdots & S_{1m}(\omega) \\ \vdots & \ddots & & \cdots & \vdots \\ S_{i1}(\omega) & & S_{ii}(\omega) & & S_{im}(\omega) \\ \vdots & \cdots & & \ddots & \vdots \\ S_{m1}(\omega) & \cdots & S_{mi}(\omega) & \cdots & S_{mm}(\omega) \end{bmatrix} \qquad (13)$$

Where m is the number of multiple input loads. The digonal term $S_{ii}(\omega)$ is the auto-correlation function of load $p_i(t)$, and the off-digonal term $S_{ij}(\omega)$ is the cross-correlation function between loads $p_i(t)$ and $p_j(t)$. From properties of the cross PSDs, it can be shown that the multiple input PSD matrix $[S_p(\omega)]$ is a Hermitian matrix. The system of time domain differential equation of motion of the structure in (12), is then reduced to a system of frequency domain algebra equations.

$$[S_x(w)]_{n\times n} = [H(w)]_{n\times m} [S_p(w)]_{m\times m} [H(w)]^{*T}_{m\times n} \qquad (14)$$

where n is the number of output response variables. The T denotes the transpose of a matrix. [*H(w)*] is the transfer function matrix between the input loadings and output response variables.

$$[H(w)] = (-[M]w^2 + i[C]w + [K])^{-1} \qquad (15)$$

The random response variables [$S_x(w)$], such as displacement, acceleration and stress response, in terms of power spectral density functions, are obtained by solving the system of the linear algebra equations in (14).

PSD Stress and Fatigue Analysis

In frequency domain, fatigue damage of structures is estimated based on the statistical properties of the response stress PSD function. The stress PSD functions are usually the results from finite element frequency response analysis. The statistical characteristics of the response stress PSD can be obtained through the moments of the PSD function. The nth spectral moment of the stress PSD function *S(f)*, frequency *f* in unit of Hz, is defined by the following equation:

$$m_n = \int_0^\infty f^n S(f) df \qquad (16)$$

Properties of a continuous stationary Gaussian process can be related to the above nth moments, m_n of the PSD function. The root mean square value, σ of PSD function is

$$s = (m_0)^{1/2} = \left(\int_0^\infty S(f) df \right)^{1/2} \qquad (17)$$

The average rate of zero crossing with positive slop, *E*[0], which is also called as equivalent frequency in unit time, is expressed as:

$$E[0] = \left[\frac{m_2}{m_0} \right]^{1/2} \qquad (18)$$

The average rate of peaks, *E*[*p*] in unit time, is expressed as:

$$E[p] = \left[\frac{m_4}{m_2} \right]^{1/2} \qquad (19)$$

The irregularity factor, α of PSD function, is defined as:

$$a = {E[0]} \Big/ {E[p]}, \qquad 0 \le a \le 1 \qquad (20)$$

The material fatigue properties are usually measured as S-N curve, which defines the relationship between the stress amplitude level, S_A, versus the mean cycles to failure, N. For most high cycle fatigue problems ($N \ge 10^4$), the S-N curve can be expressed as a simplified form:

$$B = N S_A^m \qquad (21)$$

where B and m are the material properties varying with loading and environment conditions, such as mean stress, surface finishing, and temperature.

The accumulated damage, E[AD], due to fatigue random loading is evaluated based on the Palmgren-Miner's rule, and expressed as:

$$E[AD] = \int_0^\infty \frac{n(S_A)}{N(S_A)} dS_A = \int_0^\infty \frac{p(S_A) N}{N(S_A)} dS_A \qquad (22)$$

where $n(S_A)$ is the number of cycles applied at stress amplitude level S_A, $p(S_A)$ is the probability density function of the stress amplitude. Substituting equation (21) into (22), a general equation of fatigue damage from random stress response is obtained.

$$E[AD] = \frac{T E[p]}{B} \int_0^\infty S_A^m \, p(S_A) dS_A \qquad (23)$$

where T is time duration of random loading. Many methods for fatigue damage estimate have been developed, based on equation (23), by using different definitions of the probability density function $p(S_A)$. The function $p(S_A)$ is usually defined as a function of statistical parameters of the response stress PSD, such as $E[0]$, $E[p]$, a and m_ns. The most popular damage estimating methods are Narrow Band method (Bendat 1964 [1]), Wirsching's Method (1980) [9], and Dirlik method (1985) [11].

Narrow Band method assumes the $p(S_A)$ is of the Rayleigh distribution which is represented by a narrowband process. That is,

$$p(S_A)_{NB} = f(m_0) = \frac{S_A}{m_0} \exp\left(-\frac{S_A^2}{2m_0} \right) \qquad (24)$$

Substituting (24) into (23) and integrating (23), the fatigue damage is then obtained as:

$$E[AD]_{NB} = \frac{T E[p]}{B} \left(\sqrt{2} s \right)^m \Gamma\left(\frac{m}{2} + 1 \right) \qquad (25)$$

where $\Gamma(.)$ is the Gamma function.

Wirsching's methods [9] is a modification on Narrow Band method with a "correction factor" for the wide band process.

$$E[AD]_W = f(m_0, m_2, m_4) = \lambda(\alpha, m) E[AD]_{NB} \qquad (26)$$

where the empirical rainflow correction factor, $\lambda(\alpha, m)$ is expressed as:

$$\lambda(\alpha, m) = (0.926 - 0.033m) + [0.033m + 0.074]\left(1 - \sqrt{1-\alpha^2}\right)^{(1.587m - 2.323)} \qquad (27)$$

Dirlik method proposes another empirical closed form expression for the probability density function $p(S_A)_D$, based on extensive computer simulation, such as Monte Carlo technique.

$$p(S_A)_D = f(m_0, m_1, m_2, m_4) \qquad (28)$$

The detailed contents of the function $p(S_A)_D$ can be found in the nCode/nSoft user manual [10] and [11].

A typical proving ground for vehicle durability tests consists of multiple types of road surfaces (events), such as Cobble Stones, Silver Creek and so on, with different distances and driving speeds [12]. A complete set of road loads has several, n, road load curves, PSD_i and corresponding driving times, T_i. The total accumulated damage, $E[AD]_T$, of an automotive structural component is thus a sum of the damages caused by each of the road events.

$$E[AD]_T = \sum_{i=1}^{n} E[AD(PSD_i, T_i)]_i \qquad (29)$$

Where $E[AD(PSD_i, T_i)]_I$ is the fatigue damage due to the ith road load event PSD_i with duration T_i. The total number of repeats, N_T, of the fatigue life for the complete set of the proving ground events is the inverse of the total damage $E[AD]_T$.

$$N_T = 1 / E[AD]_T \qquad (30)$$

The total equivalent life, T_{ET}, corresponding the complete set of road load events, can be therefore computed from the following equation.

$$T_{ET} = \sum_{i=1}^{n} T_i \Big/ E[AD]_T \qquad (31)$$

CAE Durability Analysis Procedure

A flow chart showing the procedure for Automotive CAE durability analysis, using random vibration approach, is presented in Fig. 1. The procedure consists of three major steps in the whole process: (1) the proving ground (PG) road load data process, (2) the FE model preparation and frequency response simulation, and (3) the fatigue damage and life evaluation.

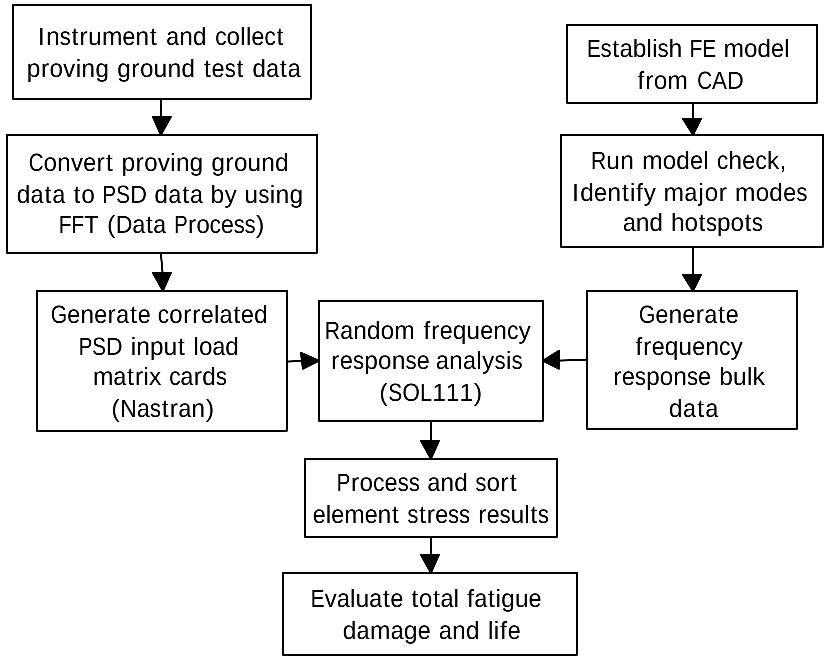

Fig. 1 A flow chart of CAE random vibration durability analysis

Application Examples

A Real Axle Assembly

The FE model of a rear axle assembly is shown in Fig. 2. The FE model consists of 10,963 elements and 12,666 nodes. The major components of the rear axle assembly include axle shaft made of forged steel; differential carrier of cast iron; pinion and ring gears of steel; diff case; rear cover; tube assemblies; track bar bracket; jounce, UCA and LCA, and shock brackets; and spring seats. The bolted joints and bearings are modeled as interface rigid elements with controlled degrees of freedom. The welded joints between the structural components are modeled in the same way as the parent materials.

The road load inputs to the axle assembly are the measured forces form SP level vehicle in the proving ground tests. The dynamic road load data include events such as, Power Hop Hill (PHH), Silver Creek (SCR), and Hard Route (HR). The forces are measured from those components which interface with the axle assembly such as control arms. The spatial orientation of each load is derived based on the average orientation of the correspondent component. The load inputs to the axle FE model are multiple force excitations. Total nine spatial orientated dynamic forces are input into the FE model.

As illustrated in the theory section, the input load matrix, $[S_p(\omega)]$, is generated by employing fast Fourier transform (FFT) of the load time histories. The diagonal term $S_{ii}(\omega)$ is auto-PSD which is a real function of frequency $\omega=2\pi f$. While the off-diagonal term $S_{ij}(\omega)$ is cross-PSD and a complex function of frequency $\omega=2\pi f$. It is known that due to the property of cross-PSD, $S_{ij}(\omega) = S^{*}_{ji}(\omega) = S_{ji}(-\omega)$, only half of the diagonal terms are needed to define the input PSD matrix. In MSC.Nastran, the cross-PSD forcing functions, expressed as real and imaginary parts, are input as two random load tables (TABRND1), respectively. Some typical correlated PSD load data, at two locations, are shown in Figs. 3 to 6.

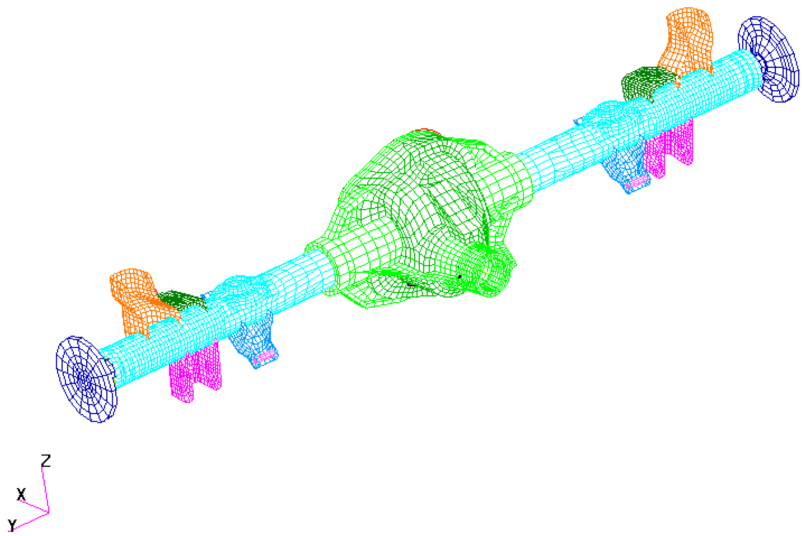

Fig.2 A FE model of rear axle assembly

The critical components and their potential high stress areas (hot spots) are determined from the strain energy density (SED) information of the normal mode analysis. From load PSD profiles it can be seen that the road load excitation has high energy components in the low frequency ranges. For comparison purpose, the time domain simulation is also implemented. The duration of the transient input segment, for Power Hop Hill (PHH), is 150 seconds. The road loading has a total of 30,720 time steps. For the frequency domain PSD simulation, the frequency range is 0.001 to 100 Hz, with unevenly spaced 150 frequency simulation points (more points clustered around natural frequencies).

Axle Results and Comparisons

The computer used for the simulation is a work station. For PHH loading simulation, the CPU time is 109 sec for modal frequency (SOL 111); and 458 sec for modal transient (SOL 112). The DBALL size is 4129 blocks for modal frequency (SOL 111); and 5934 blocks for modal transient (SOL 112). It is obvious that frequency domain method uses less CPU time and memory.

Two critical components in the axle assembly are identified: the right upper control arm (UCA) bracket (Element 53691); and the right lower control arm (LCA) bracket (Element 52853). The stress PSD profiles of the UCA bracket and LCA bracket, corresponding to the APG power hop hill (PHH) road load, are presented in Figs. 7 and 8, respectively. Some statistical properties of the PSD stress results are summarized in Table 1.

The fatigue analysis results for component left UCA bracket are: 119.2 hours total fatigue life from frequency domain method and 126.5 hours from time domain method. The relative error in life is only 5.8% w.r.t. the time domain method. The total fatigue life for the right LCA bracket is 383.1 hours from frequency method; 488.5 hours from time domain method. Which yields a relative error of 27.5% w.r.t. the time domain method.

A Headlamp Assembly

The headlamp assembly model is shown in Fig 9. The finite element model is built by employing 8,562 elements and 8760 notes. The assembly is used for an accelerated key life test (KLT). The major components are made of plastics, include the housing, bezel, reflector, front and side lens, back cap and hooks, and the supporting frame structure made of steel.

The accelerated load inputs are the correlated base accelerations in X, Y and Z directions. The auto-PSD acceleration loading in Z direction is shown in Fig. 10. Frequency simulation rang is set from 1.0 to 6000 Hz, with unevenly spaced 118 points. The time simulation duration of the steady state input segment is 0.32 second. The loading has total 3202 time steps.

Headlamp Results and Comparisons

All dynamic stress simulations for two methods are performed on a Cray computer. The CPU time for modal frequency (SOL 111) is 722 sec; for modal transient (SOL 112) is 5,457 sec. The DBALL size is 5,588 blocks for modal frequency (SOL 111); and 12,560 blocks for modal transient (SOL 112). It again shows that frequency domain method uses much less CPU time and memory.

Two critical components in the headlamp assembly are identified as the bezel made of pc, and the housing of pp. The stress PSD profiles of the bezel (Element 11961) and the housing (Element 16235) are shown in Figs. 11 and 12, respectively. The statistical properties of their PSD stress results are summarized in Table 2. The fatigue life results, due to the accelerated loadings, for bezel (Element 11961) are: 4.82 hours from frequency domain method and 5.09 hours from time domain method. The relative error in life is 5.3% w.r.t. time domain method. The fatigue life for the housing component (Element 16235) is 547 hours from frequency method; 601.4 hours from time domain method. The relative error is 9.05% w.r.t. time domain method.

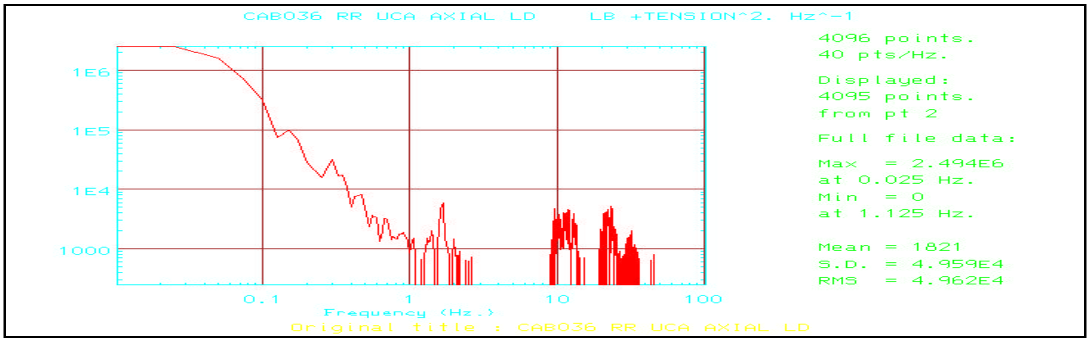

Fig.3 Auto PSD of load Ch#36 for PHH, on UCA bracket

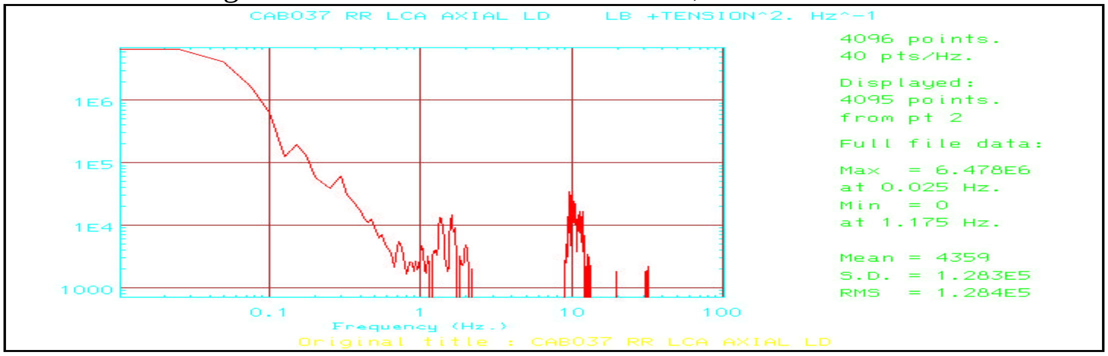

Fig.4 Auto PSD of load Ch#37 for PHH, on LCA bracket

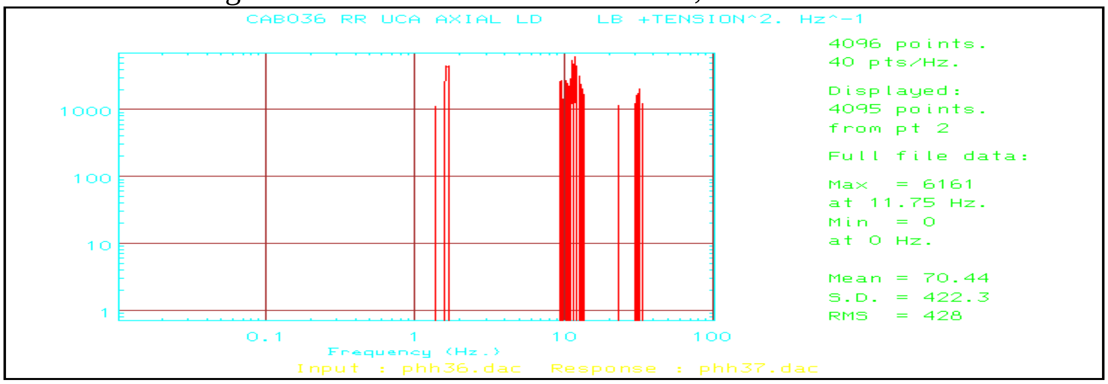

Fig. 5 Cross PSD amplitude, between load Ch# 36 and 37, PHH

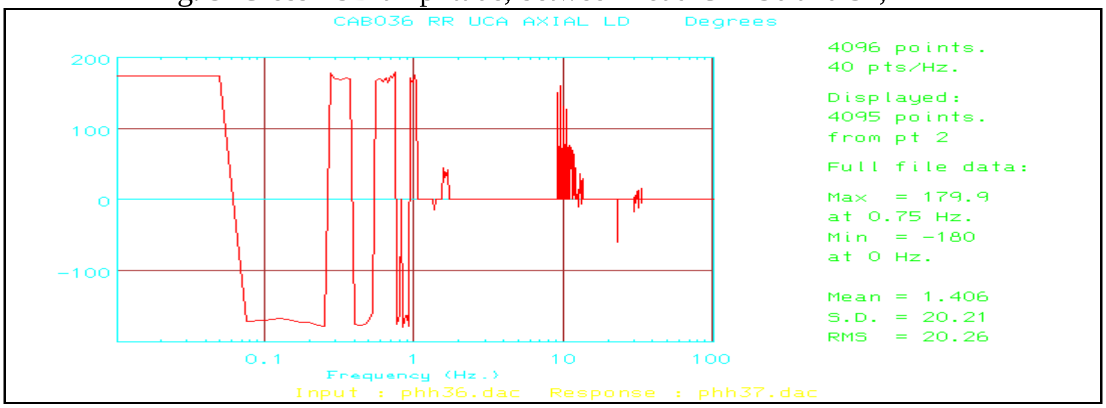

Fig. 6 Cross PSD phase, between load Ch# 36 and 37, PHH

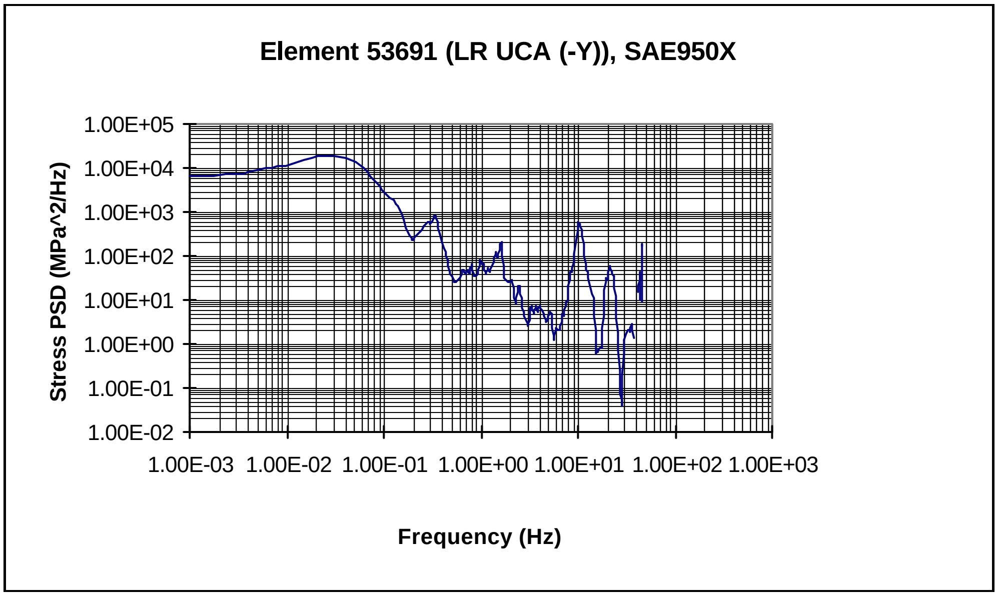

Fig.7 Stress PSD Profile of Element 53691 (UCA Bracket)

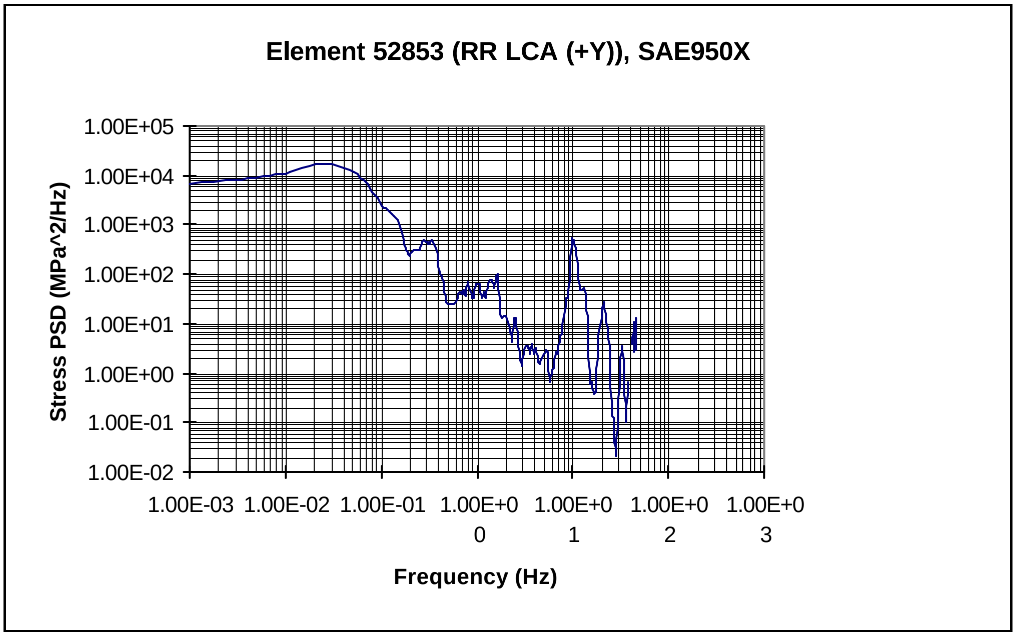

Fig.8 Stress PSD Profile of Element 52853 (LCA Bracket)

Table 1: Statistical Properties of PSD Stress Results (Axle)

Element No.	Effective Frequency (Hz)	Irregularity Factor (α)	Max. PSD (MPa^2/Hz)	RMS (MPa)
53691	15.5	0.4117	1.965E4	54.86
52853	9.6	0.3724	1.735E4	49.1

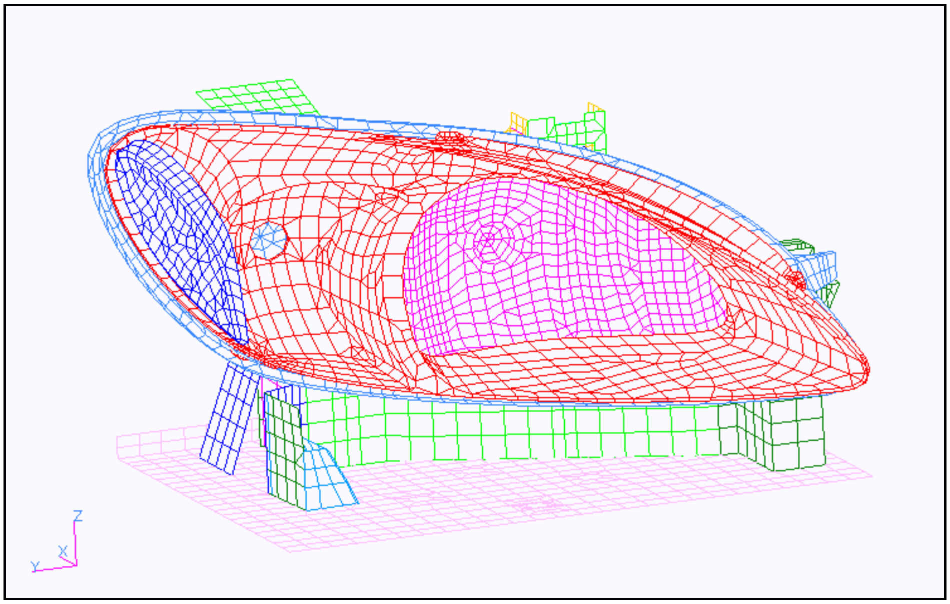

Fig. 9 Headlamp assembly FEM without front lens

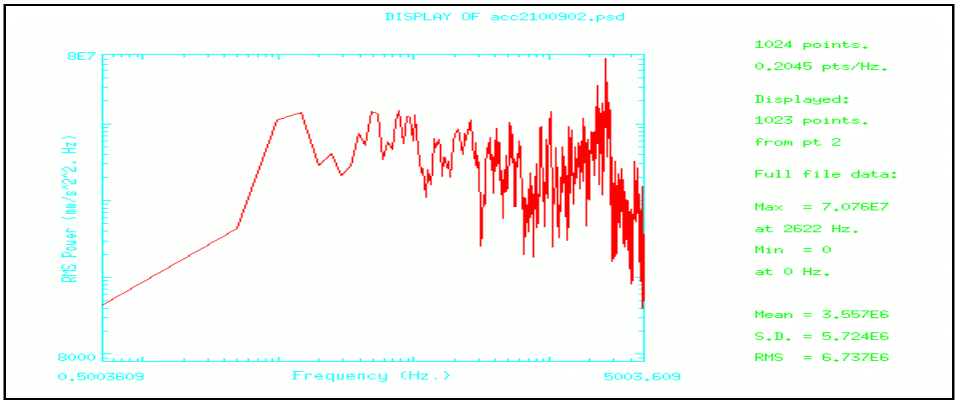

Fig. 10 PSD profile of base acceleration input (Z)

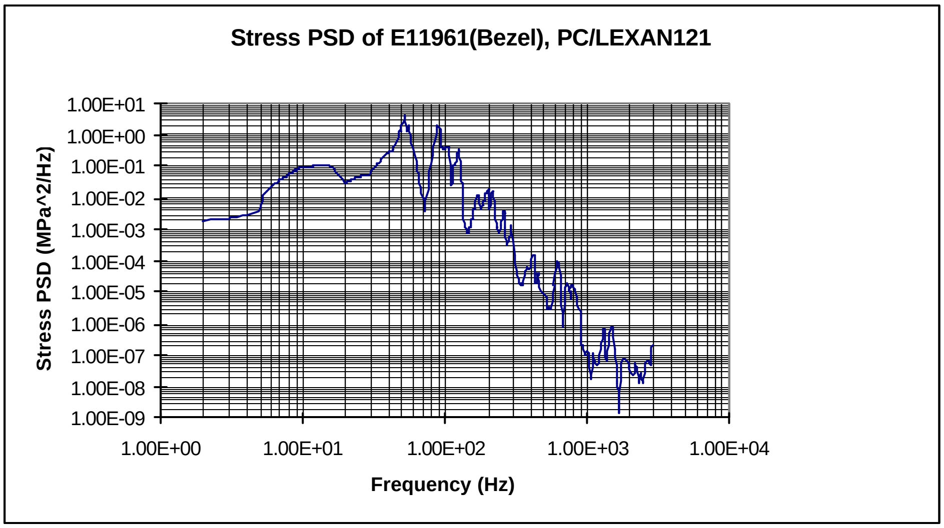

Fig. 11 Stress PSD profile of Element 11961 (Bezel)

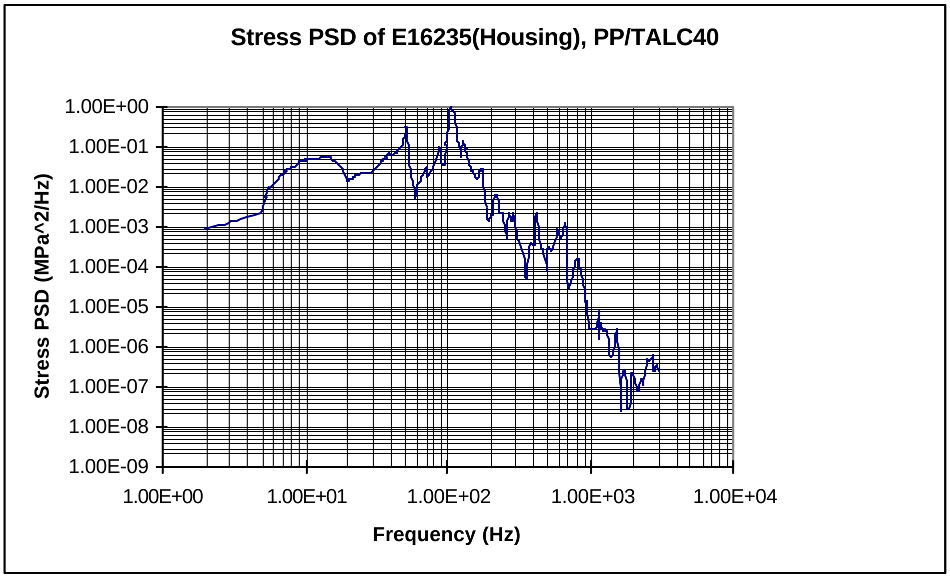

Fig. 12 Stress PSD profile of Element 16235 (Housing)

Table 2: Statistical Properties of PSD Stress Results (Headlamp)

Element No.	Effective Frequency (Hz)	Irregularity Factor (α)	Max. PSD (MPa^2/Hz)	RMS (MPa)
11961	264.1	0.7115	3.967	7.28
16235	754.6	0.4096	0.938	4.14

Conclusion

A practical procedure for automotive CAE durability analysis using the random vibration approach is presented in this paper. Two examples of automotive structures, a rear axle assembly made of metals, and a headlamp assembly made of plastics, are presented to illustrate the analysis approach and their applications. The example results show that the random vibration approach costs less CPU time and memory. The two examples also reveals that the results from both frequency and time domains are comparable, in terms of stress levels and fatigue life prediction. In addition, the frequency domain method can improve our understanding of system dynamic behaviors, in terms of frequency characteristics of both structures and loads, and their couplings. More work is underway to make automatic PSD load matrix generation, and to smooth link all three steps in the evaluation.

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