Brake Analysis and NVH Optimization Using MSC.NASTRAN

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ABSTRACT

Brake Analysis and NVH (Noise, Vibration and Harshness) Optimization have become critically important areas of application in the Automotive Industry. Brake Noise and Vibration costs approximately $1Billion/year in warranty work in Detroit alone. NVH optimization is now increasingly being used to predict the vehicle tactile and acoustic responses in relation to the established targets for design considerations. Structural optimization coupled with frequency response analysis is instrumental in driving the design process so that the design targets are met in a timely fashion. Usual design targets include minimization of vehicle weight, the adjustment of fundamental eigenmodes and the minimization of acoustic pressure or vibration at selected vehicle locations.

Both, Brake Analysis and NVH Optimization are computationally expensive analyses involving eigenvalue calculations. From a computational sense and the viewpoint of MSC.Nastran, brake analysis exercises the CEAD (Complex Eigenvalue Analysis Dmap) module, while NVH optimization invokes the DSADJ (Design Sensitivity using ADJoint method DMAP) module. In this paper, two automotive applications are presented to demonstrate the performance improvements of the CEAD and DSADJ modules on NEC vector-parallel supercomputers. Dramatic improvements in the DSADJ module resulting in approx. 8-9 fold performance improvement as compared to MSC.Nastran V70 were observed for NVH optimization. Also, brake simulations and experiences at General Motors will be presented. This analysis method has been successfully applied to 4 different programs at GM and the simulation results were consistent with laboratory experiments on test vehicles.

1. Introduction

Brake analysis and NVH (Noise, Vibration and Harshness) optimization are key applications in the automotive industry [1,2,3,6,7]. MSC.Nastran is now increasingly being used to stabilize the brake system modes to eliminate brake squeal and to perform NVH optimization for predicting the vehicle tactile and acoustic responses in relation to the established targets for design considerations. The first section of this paper focuses on brake squeal analyses. The next section deals with NVH optimization and the improvements introduced by NEC in MSC.Nastran V70.7 for design optimization using the adjoint method. Performance improvements of 80-90% have been observed in the DSADJ module resulting in a reduction of the overall user cpu time by almost 50%. Finally, results from brake simulation at General Motors are presented for typical automotive applications with an emphasis on the performance of MSC.Nastran V70.7. A full vehicle structural simulation is also presented that demonstrates the performance improvements in MSC.Nastran V70.7 on NEC SX-4 and SX-5 systems for NVH optimization.

2. Brake Analysis

Friction induced vibrations of the brake assembly is a topic of ongoing research. Coupled vibration of the brake rotor and pad generates an uncomfortable noise. Depending upon the frequency of vibrations, the resulting vehicle response may be classified as follows:

Brake Grunt occurs at a slow creeping speed less than 2 mph. Usually this occurs below 300Hz. The audible result is a loud low frequency grinding sound.

Brake Moan occurs at vehicle speed higher than 2 mph and produces a low frequency vibration in the 150-300 Hz range. The brake system and suspension vibrate strongly and a loud noise is radiated.

Brake Squeal occurs when the frequency falls in the 2,000-20,000 Hz range and it results in a high pitched squeaky noise. Brake squeal is caused by large amplitude nonlinear vibration. The discussions and results in this paper are limited to brake squeal analysis.

2.1 Brake Squeal Analysis

Studies have identified dynamic instabilities as the cause of the vibration and noise produced during brake squeal. Dynamic stability analysis requires that a stable reference motion be established. A dynamic perturbation is then given to the reference motion and the resulting new motion is analyzed for stability [1,2].

The study in this paper employs the linear stability method and assumes a harmonic motion so a linearized complex modes method can be used. In MSC.Nastran a friction stiffness matrix is used to model the contact between the pad and rotor. The normal springs at the contact surface are adjusted to agree with experimental measurements. Next, linearized complex modes are found. In the linearized complex modes approach, the interface is set at slip. An averaged friction coefficient is measured and used in the brake squeal analysis. The value used is approximately $\mu \approx .5$, and it accounts for an approximate friction law. This produces the maximum friction work out of the system.

The linearized approach is correct for steady sliding up to the bifurcation point. Beyond that point, the complex modes approach is more conservative than a nonlinear transient solution.

The state equations are developed with friction. Friction provides work out of the system and is a nonconservative force. The nonlinear state equations for stick slip vibrations are:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K + K_f(q)]\{q\} = \{0\}$$

where, M is the mass matrix, C is the viscous damping matrix, K is the stiffness matrix and $K_f(q)$ is the friction stiffness matrix. The steady sliding equilibrium position is found by experimental measurements at the contact surface. The contact springs in the friction stiffness matrix are adjusted so the pressure obtained in the finite element model matches the experimental measurements. The steady sliding position is q_0 it is where the entire friction interface is at slip. A harmonic solution is assumed,

$$\{q\} = \{\Phi\}e^{\lambda t}$$

The characteristic equation that results is,

$$\left[\lambda^2 M + \lambda C + K + K_f(q_0)\right]\{\Phi\} = \{0\}$$

where, $K_f(q_0)$ is the linearized tangent friction stiffness about the steady sliding position.

For complex modes analysis, it is assumed that the entire interface is set at slip and the friction coefficient is that due to measurements during squeal. This matches the conditions for steady sliding. The maximum work out of the system results when friction is set at slip, so this is a conservative approach. Another option is to do a nonlinear transient analysis, which assumes that the interface is in slip stick vibration. For this case, it is only during slip that work is extracted from the system. However, when the interface is assumed to be set in slip, the complex modes approach is used and is more conservative beyond the bifurcation point.

The complex roots are λ and the complex mode shapes are Φ. The real part of the root is damping and the imaginary part is stiffness. If the damping of a mode is less than zero, unstable solutions are generated. If there are no roots with unstable damping, the design is complete.

2.2 MSC.Nastran Implementation of Brake Squeal

Brake squeal analysis involves two steps: a preprocessor uses the statically measured pressure distribution and the normal springs in the model are adjusted to match the data. Then a complex modes solution (SOL 107) is done to assess the dynamic stability. Material damping is applied in the form of structural damping to the brake system. The interface springs are all required to be in compression at steady sliding. Moreover, the contact springs are adjusted to agree with experimental pressure measurements. If springs show tension or if the forces are small then they must be removed. Contact between the pad and rotor is modeled by the friction stiffness method using DMIG cards. An unsymmetric tangent stiffness is found to prevent penetration and to provide friction on the contact surface. Unsymmetric matrices that occur in nonconservative systems are a requirement for a bifurcation.

In the runs, the friction coefficient was used as the average measurement at squeal, $\mu \approx .5$. Bifurcation could occur at lower friction coefficients, but better friction laws are needed in the analysis to capture this effect.

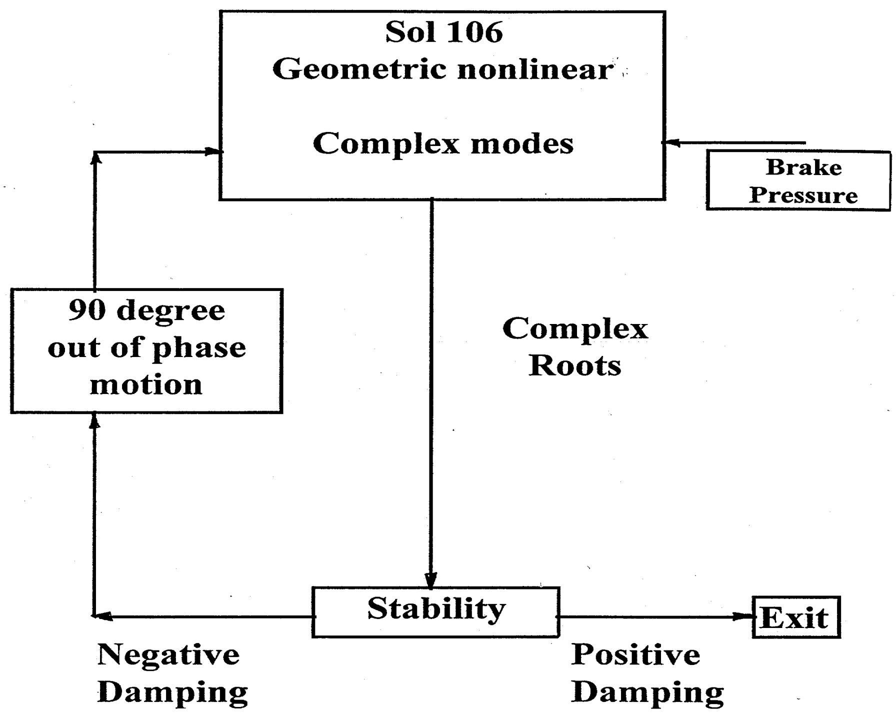


Figure 1: MSC.Nastran Solution Procedure

Modes are then found with the block bi-orthogonal complex Lanczos method. If there are no roots with unstable damping, the design is complete. However if unstable damping results, then the design is reanalyzed until it is stabilized. When negative damping occurred, two motions couple within a system mode. Interference motions (90 degree out of phase) occur at the lining rotor interface. When these motions are decoupled, the complex mode becomes stable. These interference motions can be verified experimentally with a LMS/Polytech Pi laser vibrometer system. The MSC.Nastran solution is summarized in figure 1.

2.3 Computational Analysis Using the CEAD Module of MSC.Nastran

A robust and efficient adaptive block bi-orthogonal Lanczos method has been implemented in the CEAD module of MSC.Nastran. Details of this algorithm and computational aspects of the quadratic complex eigenvalue problem can be found in [3]. The numerical robustness of the method relies on the adaptive block approach. A variable block size is especially advantageous in the case of clustered eigenvalues or to avoid the break-down of the Lanczos process. The computational efficiency of this method is mainly due to the block methodology (I/O efficiency), block BLAS kernels and the implicit execution of the operator multiplication (avoids factorization).

In Version 70.7 of MSC.Nastran, the block Lanczos method has been enhanced to improve its performance, reliability and robustness. The block Lanczos method is now the default complex Lanczos (CLAN) method. Furthermore, many errors in complex eigensolution analysis have been fixed by modernizing the Hessenberg (HESS) method. The HESS and CLAN methods are regarded as the modern methods in MSC.Nastran. The Hessenberg method now has three options for its solution method. The default method uses the new QZ algorithm (the QZ option for HESS). It is by intent an implementation for problems that fit in memory. While computationally efficient, speed of computation is sacrificed when reliability requires more computation. It is, therefore, more robust than any of the other methods. It will return defensible answers whenever the input matrices allow a unique solution. The CEAD module now provides a technique for the Singular Value Decomposition (SVD).

The block Lanczos method in V70.7 includes several enhancements that may greatly improve the performance of the method on large problems:

- The default values of the parameters IBLKSZ, MBLKSZ, and KSTEPS have been tuned to improve performance.
- Triple loop kernels (level 3 BLAS) are used throughout the code wherever possible.
- Whenever the input matrices are symmetric, symmetric decomposition and FBS routines are used.
- Whenever the input matrices are real, the reduction phase and residual calculation are executed in real arithmetic.
- The HESS method will solve any problem for which a non-null stiffness and non-null damping and/or mass matrices are available. There are no limitations involving singularity, positive-definiteness, or other mathematical properties of the input matrices.
- Both modern complex eigensolutions automatically provide a residual calculation, similar to those provided by the complex Lanczos method in V70.5. They also provide the same types of output blocks, including left hand eigenvectors.

- Both modern methods can solve problems of the type:

  $(K + p^*B) * \Phi = 0.$

- The QZ HESS method provides unique, orthogonal eigenvectors for repeated roots. The older HESS methods tend to provide the same eigenvector for repeated roots.
- The ability of QZ HESS to remove the requirements for mass at all d-set DOFs simplifies the rules for the use of scalar and extra points. The old rules and modeling techniques that avoid singular mass matrices are rather complicated. The short explanation is that any modeling that is valid in frequency response and transient response analysis is now also valid in complex eigenanalysis. There is no need to ensure that every member of the d-set has a mass term. Aeroelastic stability (flutter) analysis does not yet take full advantage of the QZ HESS method. This capability will be delivered in a future version.
- The SVD method is not implemented for production use as of yet in the standard solution sequences, but is available for research via DMAP alters.

3. NVH Optimization

Nowadays, NVH Analysis involves frequency response analysis of detailed full vehicle structural-acoustic models. The full vehicle assembly includes tires, suspension, powertrain, body and the acoustic cavity. For typical automotive NVH analyses, modes are computed for the entire vehicle structure (including chassis and powertrain) as well as for the acoustic cavity. Both, tactile and acoustic responses to excitation are computed at the areas of interest. Tactile responses include vibrations in the seat track, toe pan and steering column, while acoustic responses include sound levels at specific locations in the acoustic cavity [6,7]. Typical full vehicle NVH simulations involve forces that may be external or internal to the vehicle. External forces include road induced shake/noise and aerodynamic forces due to contact with the surrounding air. Internal forces include powertrain combustion reaction forces, powertrain unbalance forces, tire/wheel unbalance forces, driveline unbalance forces (axle etc.) and brake induced forces.

The need for high quality design, quicker time to market and lower production costs have resulted in a great demand for NVH Optimization in the transportation industry. NVH Optimization tightly couples the modal frequency response analysis with structural design optimization. The fundamental bending and torsional modes are generally a major concern because of resonance. For this reason, better bending and torsional rigidity values are desirable for NVH, ride and handling performance. However, the task of controlling the transmission/amplification of low frequency vibrations along the structure (and air cavity) path often conflicts with the desire to minimize vehicle weight (for better fuel efficiency and lower material cost reasons). NVH Optimization is very effective for meeting the design targets using global optimization of full vehicle analysis. It automatically arrives at an optimal design that satisfies the user specified constraints and minimizes the Objective function (vehicle weight, RMS response etc.). Since NVH Optimization involves a solution strategy as well as an optimization, it is instrumental in automating full vehicle simulation and driving the design process.

3.1 Computational Demands for NVH Optimization

NVH Optimization analyses involves an eigenvalue analysis for the frequency response calculations and a sensitivity coefficients calculation during optimization phase. Usually, multiple design cycles are involved, implying that the eigenvalue analysis and sensitivity calculations are repeated for each design cycle. In the context of MSC.Nastran, the adjoint sensitivity calculations are done inside the DSADJ module [4]. The DSADJ module is computationally intensive because of the model size and the large number of design variables and excitation frequencies involved. NVH optimization, therefore, has huge computational requirements in terms of cputime, I/O as well as memory requirements. Detailed full vehicle models involving 2,500,000 dynamic degrees of freedom are routinely being analyzed and eigenvalue analysis typically involves 8,000 modes. Frequency ranges up to 500Hz are now becoming quite common. This type of analyses requires extremely powerful computers – NEC SX-4 and SX-5 Series Supercomputers are capable of meeting these computational requirements.

3.2 NEC Exclusive DSADJ Enhancements

The DSADJ module computes the design sensitivity coefficients using the Adjoint method [4]. These computations involve a series of matrix-vector products, vector updates and coordinate transformations that are very computationally intensive for NVH optimization tasks. The performance of the DSADJ module is, therefore, crucial for NVH Optimizations where extremely large models are subjected to a wide range of excitation frequencies.

NEC initiated the DSADJ Performance Enhancement Project to improve the performance of MSC.Nastran for NVH optimization analyses. This was a major undertaking effort on the part of NEC in collaboration with the MSC Software Corporation. This project involved a rigorous QA procedure and took almost 10 man-months before final integration into MSC.Nastran V70.7. The DSADJ module was totally reengineered to improve vectorization and to exploit the vector architecture of NEC SX-4 and SX-5 Series Supercomputers.

Modest DSADJ enhancements were first introduced in MSC.Nastran V70.5 on NEC SX-4. The DSADJ performance enhancement project was completed in V70.7 resulting in approximately ten-fold performance improvement in the DSADJ module as compared with V70. Some of the major enhancements in V70.7 are enumerated below:

- Perform element level operations on strings rather than on a term-by-term basis.
- Exploit the sparsity pattern of elemental matrices.
- Handle multiple solution vectors simultaneously.
- Use lumped mass formulation, if possible.

- Perform extremely efficient coordinate transformations that operate on an element level rather than on a node-by-node level. Process multiple solution vectors for coordinate transformations as well.
- Reduce the number of matrix-vector products by almost 50% for typical models.

The details of the above enhancements are proprietary in nature, and cannot be discussed here. The improvements in performance due to the above enhancements are demonstrated in section 9 for an Automotive NVH optimization analysis.

4. NEC SX-4 and SX-5 Series Supercomputers

NEC has provided state-of-the-art supercomputing products since 1983. The SX-4 Series was first delivered in the 4^{th} quarter 1995. The newest model range is the SX-5 Series. The SX-4 and SX-5 Series, which are parallel vector supercomputers, provide solutions for a broad range of application requirements involving intensive computation, very large main memory, very high performance main memory and very high input-output rates. Both, SX-4 and SX-5 are constructed using air-cooled CMOS VLSI technology. CMOS enables low costs for system acquisition, low operational power consumption, and high system reliability through stable chip technology. Each processor board contains a vector unit and a scalar unit.

The SX-4 Series include a wide range of models ranging from the Compact models to the Single-node (a maximum of 32 CPUs and 8 Gigabytes main memory) and Multi-node configurations (a maximum of 512 CPUs and 128 Gigabyte main memory). The System Peak performance of each processor is 2 GigaFLOPS, resulting in a System Peak performance of 64 GigaFLOPS for a Single-node model and 1 TeraFLOPS for a Multi-node system.

The SX-5 Series is the newest member of the family of parallel vector supercomputers from NEC and includes a wide range of models ranging from cabinet models to Single-node models (a maximum of 16 CPUs and 128 Gigabytes main memory) and Multi-node configurations (a maximum of 512 CPUs and 4 Terabytes main memory). The System Peak performance of each processor in SX-5 is 8GigaFLOPS, resulting in a System peak performance of 128 GigaFLOPS for a Single-node SX-5 and 4 TeraFLOPS for the top-of-the-line SX-5 Multi-node system.

5. Performance Studies

Two customer datasets are presented to demonstrate the performance of MSC.Nastran V70.7 for Brake Squeal analysis as well as for NVH Optimization. The first example is from GM and it performs brake squeal simulations, while the second one is a typical NVH Optimization dataset provided by MSC. Performance comparisons are made between MSC.Nastran V70, V70.5 and V70.7.

5.1 Brake Squeal Simulation Example

This model was provided by General Motors. It performs brake squeal simulations up to 12,000 Hz. It is built of solid elements containing approximately 120,000 degrees of freedom. The model consisted of rotor, pad, caliper and the mounting bracket. These were modeled with solid elements. A rigid piston and load spreader were used to beam the loads to the piston. The fluid was modeled as a linearized spring. It was necessary to model a pin connection between the liner and brake shoe accurately. The solution is done in two steps: a measured contact simulation with friction is followed by a complex modes solution. The physical phenomenon occurring at braking assumes that the stationary brake pads are in full contact with the rotating disk at all times and that the entire interface is set at slip (section 2.2). Effects of rotational inertia are assumed to be negligible.

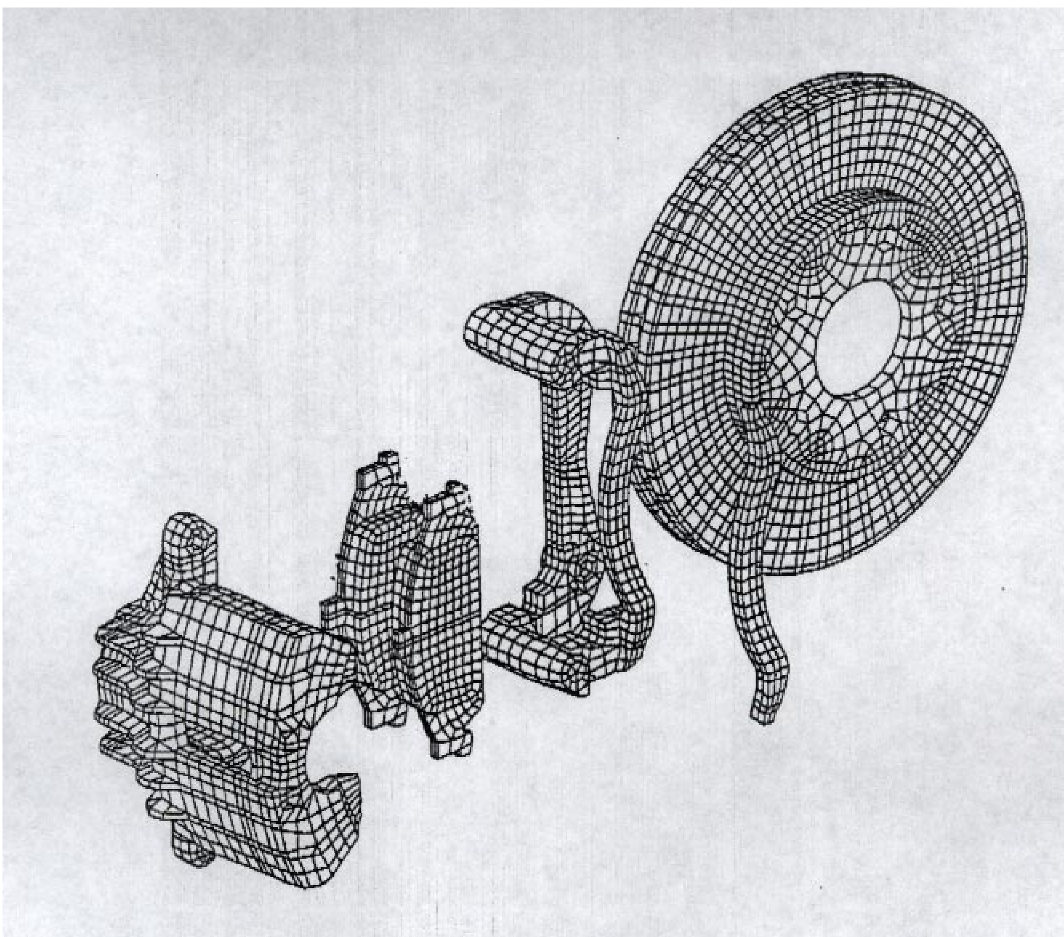


Figure 2: Geometry of Brake Simulation Example

Since in high frequency problems, the modes are localized compared to low frequency problems where the modes are global, therefore a localized model of the brake system constrained to ground at the mounting attachment is used. As brake squeal occurs in the high frequency range 2 Khz < f < 20 Khz, it is appropriate to model only the brake system with the mounting bracket constrained to ground.

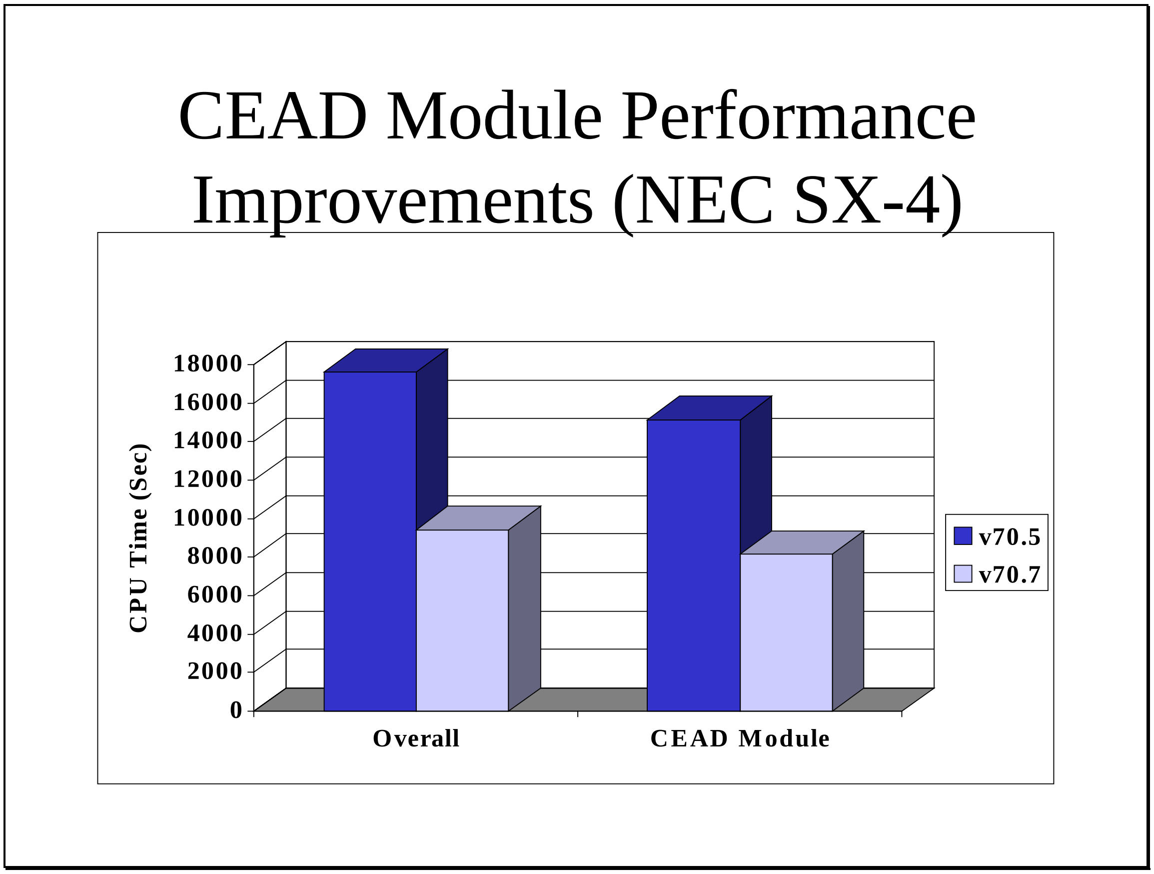
At the system level, the finite element results correlated with experimental. The frequencies correlated as explained below. For unstable modes, laser vibrometer operating deformed shapes visually correlated with the MSC.Nastran complex mode shapes. Real modes were compared as well as frequency response data with a stationary rotor. There were 4 roots with negative damping: 6.07 KHz, 6.36 KHz, 6.46 KHz. and 10.4 KKz. For this dataset, 110 modes were found in the range up to 12 KHz.

The roots picked up the major histograms that were generated by a brake dyno. The test varied pressure and velocity according to a schedule. This is due to the conservative nature of the approximation. It over estimates the roots with negative damping. Eventually, all unstable roots are activated due to the schedule variations. Many design changes were tried to stabilize the system. When two motions are decoupled it was easy for two other neighboring motions to couple. Many analyses were required to fully stabilize the system due to this effect. This is due to the many closely spaced modes in the system. Moreover, the shape of the pad liner (chamfer) was changed until the system stabilized itself. This change stabilized the brake system on a test vehicle and brake squeal was eliminated.

Table 1: CEAD Performance Improvements for GM Brake Simulation Example

NASTRAN Version	**NEC Platform**	**Real Cputime**	**User Cputime**	**System Cputime**	**CEAD Module Cputime**	**% Improv. in CEAD**
MSC/N V70.5	SX-4	26,578	16,107	1,501	15,149	-
MSC/N V70.7	SX-4	12,530	8,860	571	8,173	46%

The results tabulated in Table 1, are represented in the following barchart. It is seen that the performance of CEAD module is approximately twice as fast in V70.7 as compared with V70.5. The overall run time has also reduced by almost 50% in V70.7. The amount of I/O to disk has reduced by almost 70% in V70.7. These improvements are primarily due to the latest enhancements (see section 2.3) to the CEAD module and the adaptive block Lanczos algorithm. These runs were made using NEC's High Performance I/O library.



5.2 Van Body NVH Optimization Example

This second example is a very large finite element model that is considered representative of a state of the art design task. The geometry of the model is shown in Figure 3.

This model is a B-I-W car body consisting of mostly 2-D shell elements. There are 91,378 elements and 569,839 dynamic degrees of freedom. 269 modes were computed and 705 excitation frequencies were applied. This model involves 111 design variables and 618 responses. The analysis involves a typical NVH Optimization, wherein the Modal Frequency response is performed followed by a minimization of the RMS value of responses. The starting value of the Objective function was 9,000 units and the analysis was performed for 1 design cycle only. NEC's High Performance I/O library was used for efficient asynchronous I/O.

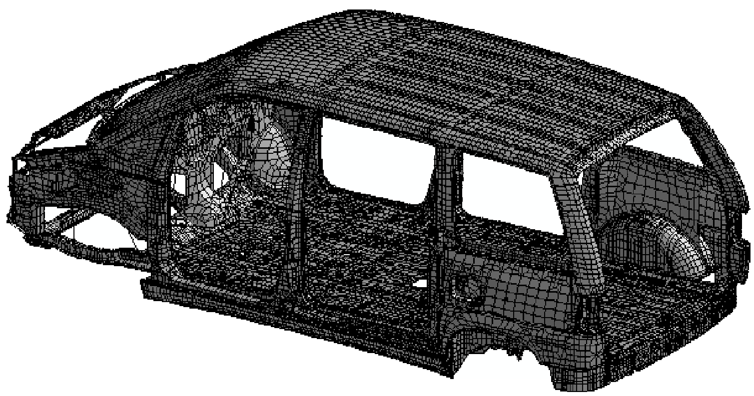


Figure 3. Van Body Model (courtesy of PSA Peugot).

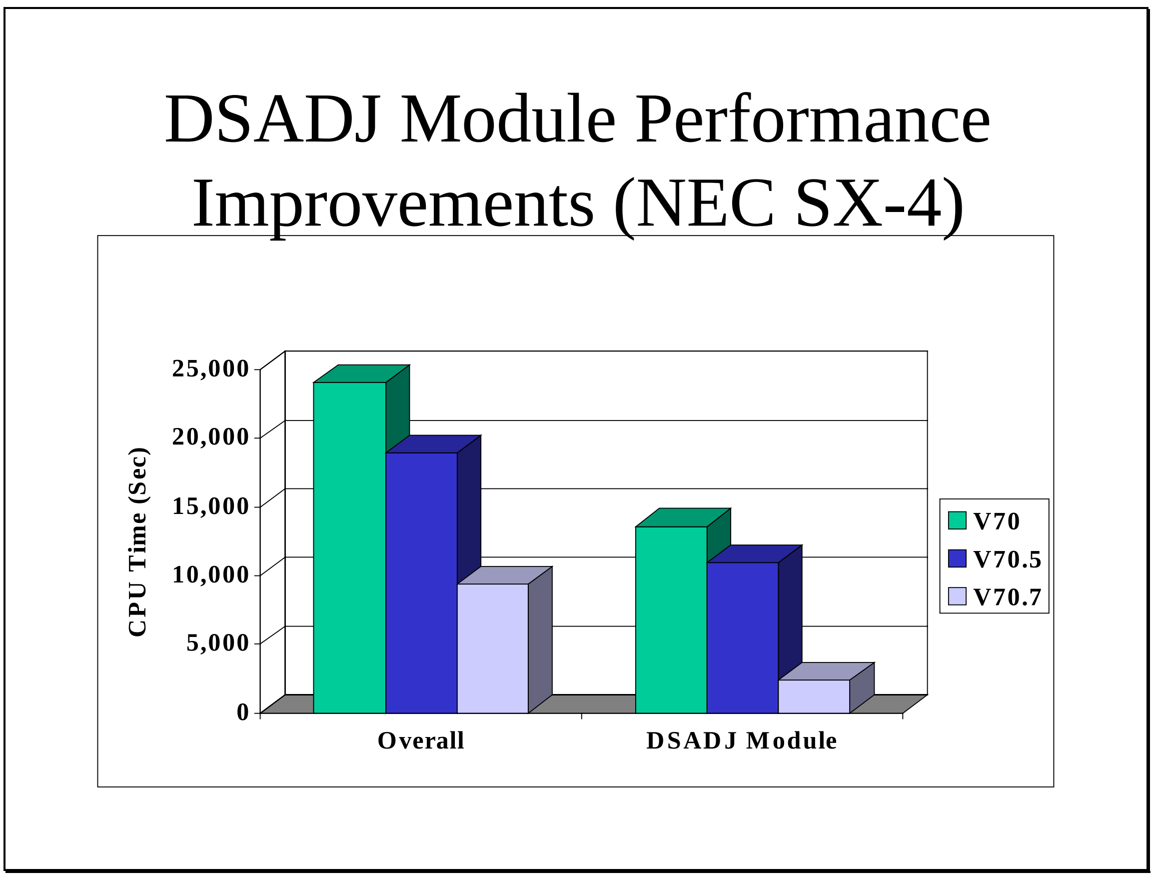
From Table 2, it is evident that the performance of MSC.Nastran V70.7 on SX-4 is twice as fast as MSC.Nastran V70.5. The cputime spent in the DSADJ module has reduced by almost 85% in MSC.Nastran V70.7 as compared with MSC.Nastran V70. This is primarily because of better vectorization and longer vector lengths for operations performed inside the DSADJ module. The net effect is that the overall User cputime has reduced from 6.7 hours using MSC.Nastran V70 to 5.2 hrs with MSC.Nastran V70.5 and only 2.6 hrs using MSC.Nastran V70.7. The turnaround time (elapsed or real) time has also been reduced by more than a factor of two.

This analysis required 18 Gbytes of Hiwater Disk, 433 Gbytes of I/O, 13.8 Gbyte of SCRATCH Dbset and 5 Gbyte of SCR300. Clearly, this problem is intractable without the availability of the adjoint method and is still only accessible to powerful computers with extensive available disk space and fast I/O and an efficient implementation of MSC.Nastran. The availability of High Performance Input Output (HPIO) library on NEC SX-4 was instrumental in achieving extremely fast I/O and in reducing the elapsed cputime.

Table 2: Performance Improvements for Van Body

NASTRAN Version	NEC Platform	Real Cputime	User Cputime	System Cputime	DSADJ Module Cputime	% Improv. in DSADJ
MSC/N V70	SX-4 (non-dedicated)	45,798	24,038	1,857	13,562	-
MSC/N V70.5	SX-4 (non-dedicated)	46,377	18,922	1,806	10,910	19.6%
MSC/N V70.7	SX-4 (dedicated)	13,810	9,390	473	2,378	82.5%

The results tabulated in Table 2, are represented in the following barchart.



6. Concluding Remarks

Results from brake simulation indicate that the CEAD module in MSC.Nastran V70.7 is almost twice as fast as that in V70.5 for the GM dataset. Experiences at GM indicate that the complex modes methodology is very effective in eliminating brake squeal with the aid of MSC.Nastran V70.7. Correlation with experimental results indicates the effectiveness of this approach and the robustness of MSC.Nastran V70.7.

NEC's enhancements to the DSADJ module have resulted in a dramatic improvement in performance of MSC.Nastran V70.7 on NEC SX Series Supercomputers for design sensitivity and optimization. Performance improvements ranging from 80-90% were observed in the DSADJ module for NVH Optimization problems. The overall cputime for the complete solution has been reduced by approximately 50%. The performance of MSC.Nastran V70.7 on NEC Series Supercomputers is quite possibly the 'best in class' for NVH Optimization.

7. Acknowledgments

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The brake simulation problem was provided by General Motors, while the Van body model was provided by Peugeot, SA, via Alain Jacq of MSC's Paris office.

8. Disclaimer

MSC supported NEC in the theoretical aspects and integration of DSADJ enhancements in MSC.Nastran. However, nothing in this paper should be interpreted as an endorsement by MSC of NEC systems relative to other computer vendors that MSC supports.

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