ADVANCED VISCOPLASTIC FINITE-ELEMENT STRUCTURAL ANALYSES USING MARC PROGRAM

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Abstract

This paper presents finite element solution methodologies developed at the National Aeronautics and Space Administration (NASA) Lewis Research Center for use with the general purpose finite element program MARC. The solution technologies are illustrated for two viscoplastic models, one developed by Robinson and the other by Freed and Walker. The developed MARC-based finite element solution technologies for these viscoplastic models are then applied to some mutiaxial structural engineering problems (involving thrust chambers and cowl lip). The results are discussed and applicability of the MARC program to the solution of complex, nonlinear structural engineering problems is demonstrated. It is believed that the results from the present work will encourage engineers/researchers for using MARC program for solving complex structural engineering problems encountered in aerospace and other industries.

Introduction

Classical creep-plasticity constitutive models treat creep and plastic strains as independent noninteracting entities. These models are, therefore, incapable of accounting for the observed interactions between creep and plastic strains at high temperatures. Viscoplastic models, however, consider all the inelastic strain (including plasticity, creep, relaxation, etc.) as a single, unified, time-dependent quantity, and thus, automatically include interactions that occur among them. Viscoplastic models, therefore, provide more realistic descriptions of time-dependent inelastic behavior of materials at high temperatures. Viscoplastic models become more realistic when as much material physics as possible is included in the models. This, however, results in complex mathematical frameworks for viscoplastic models. The constitutive differential equations of viscoplastic models that govern the flow and evolution laws are generally highly nonlinear and mathematically stiff. The closed-form solutions for structural engineering problems are virtually intractable when viscoplastic models are used to define the stress-strain relationship. To assess the advantages offered by more realistic viscoplastic models one must, therefore, employ numerical solution methodology involving, for example, the finite element method or the boundary element method.

Toward this aim, this paper presents a review of the finite element solution methodologies developed at the National Aeronautics and Space Administration (NASA) Lewis Research Center for use with the general purpose finite element program MARC [1]. The methodologies, designed for use with viscoplastic models, are demonstrated in this paper for viscoplastic models put forth by Robinson [2] and Freed and Walker [3]. However, because the methodologies are general in nature, these can easily be adapted for use with other viscoplastic models. For completeness, the paper includes brief descriptions of the two viscoplastic models. The methodologies are illustrated by applying them to multiaxial problems.

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Viscoplastic Models

Robinson's Viscoplastic Model

Robinson's model [2] employs a dissipation potential to derive the flow and evolutionary laws for the inelastic strain and internal state variables. The model incorporates a single internal state variable representing kinematic hardening. The material behavior is elastic for all the stress states within the dissipation potential and is viscoplastic for the stress states outside. A small displacement and a small strain formulation are employed.

The total strain rate $\dot{\epsilon}_{ij}$ is decomposed into elastic, $\dot{\epsilon}_{ij}^{el}$, inelastic $\dot{\epsilon}_{ij}^{in}$ (including plastic, creep, relaxation, etc.), and thermal $\dot{\epsilon}_{ij}^{th}$ strain-rate components. Thus

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{el} + \dot{\epsilon}_{ij}^{in} + \dot{\epsilon}_{ij}^{th} \qquad (i,j = 1,2,3) \tag{1}$$

The elastic strain rate for an isotropic material is governed by Hooke's law:

$$\dot{\epsilon}_{ij}^{el} = \frac{1+\nu}{E}\,\dot{\sigma}_{ij} - \frac{\nu}{E}\,\dot{\sigma}_{kk}\,\delta_{ij} \tag{2}$$

where E is the Young's modulus, ν is the Poisson 's ratio and σ_{ij} is the stress. The repeated subscripts in equation (2) and elsewhere imply summation over their range, and δ_{ij} is the Kronecker delta function. A dot over a symbol denotes its derivative with respect to time t.

The nonisothermal multiaxial inelastic constitutive equations for the model are given below:

Flow Law

$$\dot{\epsilon}_{ij}^{in} = \begin{cases} \dfrac{AF^{n}\,\Sigma_{ij}}{\sqrt{J_2}} & F > 0 \text{ and } S_{ij}\,\Sigma_{ij} > 0 \\ 0 & F \le 0 \text{ or } F < 0 \text{ and } S_{ij}\,\Sigma_{ij} \le 0 \end{cases} \tag{3}$$

Evolutionary Law

$$\dot{a}_{ij} = \begin{cases} \dfrac{H}{G^{\beta}}\,\dot{\epsilon}_{ij}^{in} - \dfrac{RG^{m-\beta}}{\sqrt{I_2}}\,a_{ij} & G < G_o \text{ and } S_{ij}\,a_{ij} > 0 \\ \dfrac{H}{G_o^{\beta}}\,\dot{\epsilon}_{ij}^{in} - \dfrac{RG_o^{m-\beta}}{\sqrt{I_2}}\,a_{ij} & G \le G_o \text{ or } G > G_o \text{ and } S_{ij}\,a_{ij} \le 0 \end{cases} \tag{4}$$

where

$$F = \frac{J_2}{K^2} - 1 \tag{5}$$

$$G = \frac{I_2}{K_o^2} \tag{6}$$

$$J_2 = \frac{1}{2}\,\Sigma_{ij}\,\Sigma_{ij} \tag{7}$$

$$I_2 = \frac{1}{2}\,a_{ij}\,a_{ij} \tag{8}$$

$$\Sigma_{ij} = S_{ij} - a_{ij} \tag{9}$$

$$\left.\begin{aligned} S_{ij} &= \sigma_{ij} - \frac{1}{3}\,\sigma_{kk}\,\delta_{ij} \\ a_{ij} &= \alpha_{ij} - \frac{1}{3}\,\sigma_{kk}\,\delta_{ij} \end{aligned}\right\} \tag{10}$$

In the preceding equation, S_{ij} is the deviatoric stress, and a_{ij} and K are the internal state variables. The variable, α_{ij} called the back stress, accounts for the kinematic hardening, whereas K, taken here to be a constant, is called the drag stress and represents the isotropic hardening of the material. The value of K at the reference temperature is denoted by K_o and the minimum value attainable by G is denoted by G_o. The inequalities in equations (3) and (4) define boundaries across which the flow and evolutionary laws change form discontinuously. To facilitate the numerical computations, these boundaries are smoothed by using a spline function given in the next section. Seven numerical parameters are required to characterize a given material. These parameters are A, n, m, H, R, G_o and $K(K_o)$. The values of these parameters for the copper alloy NARloy-Z, taken from reference 4, are listed in table I.

Spline Function

The discontinuous boundaries in Robinson's viscoplastic model should be smoothed to facilitate numerical computations. This smoothing is achieved by defining a "spline function" P(x) (refs. 2 and 5) on the interval (-1,1) as

$$P(x) = \begin{cases} \dfrac{(1+x)^2}{2} & -1 \le x < 0 \\ 1 - \dfrac{(1-x)^2}{2} & 0 \le x \le 1 \\ 1 & x > 1 \\ 0 & x < -1 \end{cases} \qquad (11)$$

Freed and Walker Model

The viscoplastic model used herein was recently put forth by Freed and Walker [3]. The model contains one scalar internal state variable D, called the drag strength, and an internal variable B_{ij}, called the back stress. The back stress B_{ij} is assumed to be composed of two back stresses that are denoted by B^s_{ij} ; and B^ℓ_{ij} . (A small displacement and a small strain formulation is employed in the model.)

Again as in equation (1), the total strain rate is written as the sum of elastic, inelastic (including plasticity, creep, relaxation, etc.) , and thermal strain rate components. The stress σ_{ij} is taken to be related to elastic strain by Hooke's law , equation (2).

The deviatoric total strain rate $\dot{E}_{ij}$ has the following expression

$$\dot{E}_{ij} = \dot{\varepsilon}_{ij} - \frac{1}{3}\dot{\varepsilon}_{kk}\,\delta_{ij} \qquad (12)$$

The back stress B_{ij} is the sum of two back stress components B^s_{ij} and B^ℓ_{ij},

$$B_{ij} = B^s_{ij} + B^\ell_{ij} \qquad (13)$$

The effective stress Σ_{ij} is defined as

$$\Sigma_{ij} = S_{ij} - B_{ij} \qquad (14)$$