ABSTRACT

Approach Heat of MSC/NASTRAN can be used to calculate electric current distributions in regions of variable electrical resistivity. This paper presents calculations made of electrochemical currents flowing in cathodic protection systems, the purpose of which is to prevent corrosion. In addition, convective heat boundary (CHBDY) elements are shown to be useful.

INTRODUCTION

Finite element programs such as MSC/NASTRAN were developed for structural and thermal analysis, but may be applied to other areas of engineering. For example, Zienkiewicz has shown [1] that thermal finite elements can be used to calculate electrostatic fields in dielectric media.

This paper discusses applying MSC/NASTRAN to electric current flow in conducting media, with emphasis on electrochemical currents in cathodic protection systems. The corrosion of structures immersed in (and/or containing) fluids can be prevented by a properly designed cathodic protection system, which sets up electric currents that oppose the chemical reaction of corrosion. Until now the current densities produced on the structure could not be accurately calculated, making cathodic protection systems difficult to design.

This paper will first review the basic equations for electric current flow, and then show the analogy with heat conduction. Next the use of MSC/NASTRAN will be discussed, including both conduction and convection elements of Approach Heat. Finally, the cathodic protection system of a water heater will be analyzed.

LAW OF CONSERVATION OF CHARGE

The basic differential equation governing electric current flow in conducting media is the law of conservation of charge, often called the continuity equation. If the current density per unit volume is $\overline{J}$, then

$$\nabla \cdot \overline{J} = -(\partial\rho/\partial t) \quad (1)$$

where ρ is charge density (coulombs per cubic meter), and t is time. For steady (DC) currents the right hand side of Equation (1) is zero, and, (1) becomes the differential form of Kirchoff's current law.

From basic electromagnetic theory, if $\bar{E}$ is the electric field, then

$$\bar{J} = \sigma \bar{E} \tag{2}$$

where σ is electrical conductivity. Also,

$$\bar{E} = -\nabla \phi \tag{3}$$

where ϕ is electric (scalar) potential or voltage.

Substituting (2) and (3) in (1) gives

$$\nabla \cdot \sigma \nabla \phi = (\partial\rho/\partial t) \tag{4}$$

ANALOGY BETWEEN ELECTRIC CURRENT AND HEAT CONDUCTION

The basic differential equation governing the temperature T throughout thermally conductive media is

$$\nabla \cdot k \nabla T + \dot{q} = \rho c(\partial T/\partial t) \tag{5}$$

where k is thermal conductivity, $\dot{q}$ is internal heat generation, and ρc is volume heat capacity. MSC/NASTRAN can solve for the temperature field T that satisfies (5).

Equations (5) and (4) are duals, that is, analogous. Their analogous variables are included in Table I.

TABLE I. Analogous Electrical and Thermal Quantities

Electrical Symbol	Thermal Symbol	MSC/NASTRAN name
ϕ	T	temperature
σ	k	conductivity
$\bar{E}$	$-\nabla T$	-(temperature gradient)
$\bar{J}$	$-k \nabla T$	flux
$(\partial\rho/\partial t)$	$-\dot{q}$	-(internal heat generation)
I	Q	total heat flow
1/R	h	convective film coefficient

MSC/NASTRAN INPUT AND OUTPUT FOR ELECTRIC CURRENT PROBLEMS

If the relations of Table I are kept in mind, interpretation of MSC/NASTRAN input and output for electric current problems is fairly straightforward. Executive, Case Control, and Bulk Data decks are prepared as for Approach Heat.

In the Bulk Data deck, SPC's now constrain voltages rather than temperatures. For steady-state linear (Sol 51) or nonlinear (Sol 53) problems, where the right hand term of Equation (5) is zero, $\dot{q}$ of (5) corresponds to $-(\partial\rho/\partial t)$ of (4). Therefore QVOL bulk data cards (loads) now represent internal current generators. Finite elements available include one-dimensional CONRODs, which are the equivalent of lumped parameter circuit resistors, two-dimensional CQUADs, and three-dimensional CHEXAs and CPENTAs.

In the output of MSC/NASTRAN Approach Heat, an SPCFORCE is the total heat flow Q required to constrain the temperature at a grid point. Total heat flow is related to $\dot{q}$ by

$$Q = \int_V \dot{q} \, dv \qquad (6)$$

where v is the volume over which internal generation takes place. By analogy using $\dot{q}$ from Table I, in electrical problems the SPCFORCES are

$$-I = \int_V \left(-\frac{\partial\rho}{\partial t}\right) dv \qquad (7)$$

That is, each SPCFORCE output by MSC/NASTRAN is interpreted as a current I at a grid point that is required to constrain its voltage.

CHBDY ELEMENTS FOR ELECTRIC CURRENTS

In many electric current flow problems, metal electrodes are present which are more easily modelled using convection elements (CHBDYs) instead of conduction finite elements. Typical electrodes include plates ("grids") of batteries, complex underwater structures, and scratches on coated surfaces. Modelling such electrodes with CHEXAs, CQUADs, or other conduction elements would require very intricate finite element meshes.

Like convection boundary problems in heat transfer, the determining factors for electrode performance are surface area, film resistance, and location. Therefore an electrode can be represented by CHBDY elements.

To prepare the CHBDY, PHBDY, and MAT4 cards, the following relations are helpful. Recall that for thermal problems,

$$Q = h\ A\ (\Delta T) \qquad (8)$$

where h is the convection film coefficient and A is surface area. Using the analogies discussed above, (8) becomes for electric current flow,

$$I = h\ A\ (\Delta V) \qquad (9)$$

where (ΔV) is the voltage drop across the film. Defining current density as

$$J = I/A \qquad (10)$$

then (9) becomes

$$J = h\ (\Delta V) \qquad (11)$$

An alternative expression for (9) is

$$I = (\Delta V)/R \qquad (12)$$

which is the familiar Ohm's law, where the film resistance R obeys

$$R = (1/(hA) \qquad (13)$$

In electrochemical problems, film resistance R is useful in specifying the effects of coatings deposited on electrodes, where R is independent of J but can vary with time as the chemical deposits build up. Also, R can be a function of J in problems where J lies in the nonlinear portion of a polarization (ΔV vs J) curve, in which case MSC/NASTRAN can analyze the problem by specification of a nonlinear h in Sol 53.

ANALYSIS OF CATHODIC PROTECTION OF WATER HEATER

A rather simple example of cathodic protection is a system used in a water heater. To prevent corrosion, a magnesium anode is hung inside the water-filled tank. The tank is glass coated, but defects in the coating are possible, leaving bare steel areas which must be protected electro-chemically.

Figure 1 shows a plane lying in one-half of a typical cylindrical water heater [2]. Due to symmetry, the other 180° section of the heater is the mirror image of the 180° shown in Figure 1. The finite element mesh shown was developed using MSGMESH to generate isoparametric CQUAD4 elements for the water, except for CTRIA3 elements near the one-half magnesium anode. Figure 1 also shows the four bare "spots" arbitrarily assumed to be 60° apart.

Table II summarizes results for three conditions of bare surface area and film coefficient. The areas of the four bare spots are all 1.E-3 square meters in Cases A and B. In Case C bare spots 2 and 3 have their areas reduced to 1.E-4m^2. Table II shows how the currents at the four bare spots vary for the three cases. In all cases, the four bare spot currents add up to equal the calculated anode current.

MSC/NASTRAN contour plots of Cases A, B, and C are shown in Figures 2, 3, and 4 respectively. These plots were obtained using a special combination of alters RF51$31 and RF51$33. The contours shown are lines of constant voltage. From (2) and (3), the current flow lines are perpendicular to the contours of constant voltage.

Figures 1 through 4 illustrate a simple two-dimensional problem. Much larger three-dimensional cathodic protection problems have been solved, containing over 5000 CHEXA elements. One such solution recently showed the value of resequencing with the new SEQP method (RF24$74). Resequencing time was one tenth that taken by WAVEFRONT, and SEQP gave significantly smaller bandwidths.

CONCLUSION

MSC/NASTRAN Approach Heat can solve for the electric current distributions in regions of variable conductivity. Both conduction and convection finite elements are useful for analysis of cathodic protection systems.

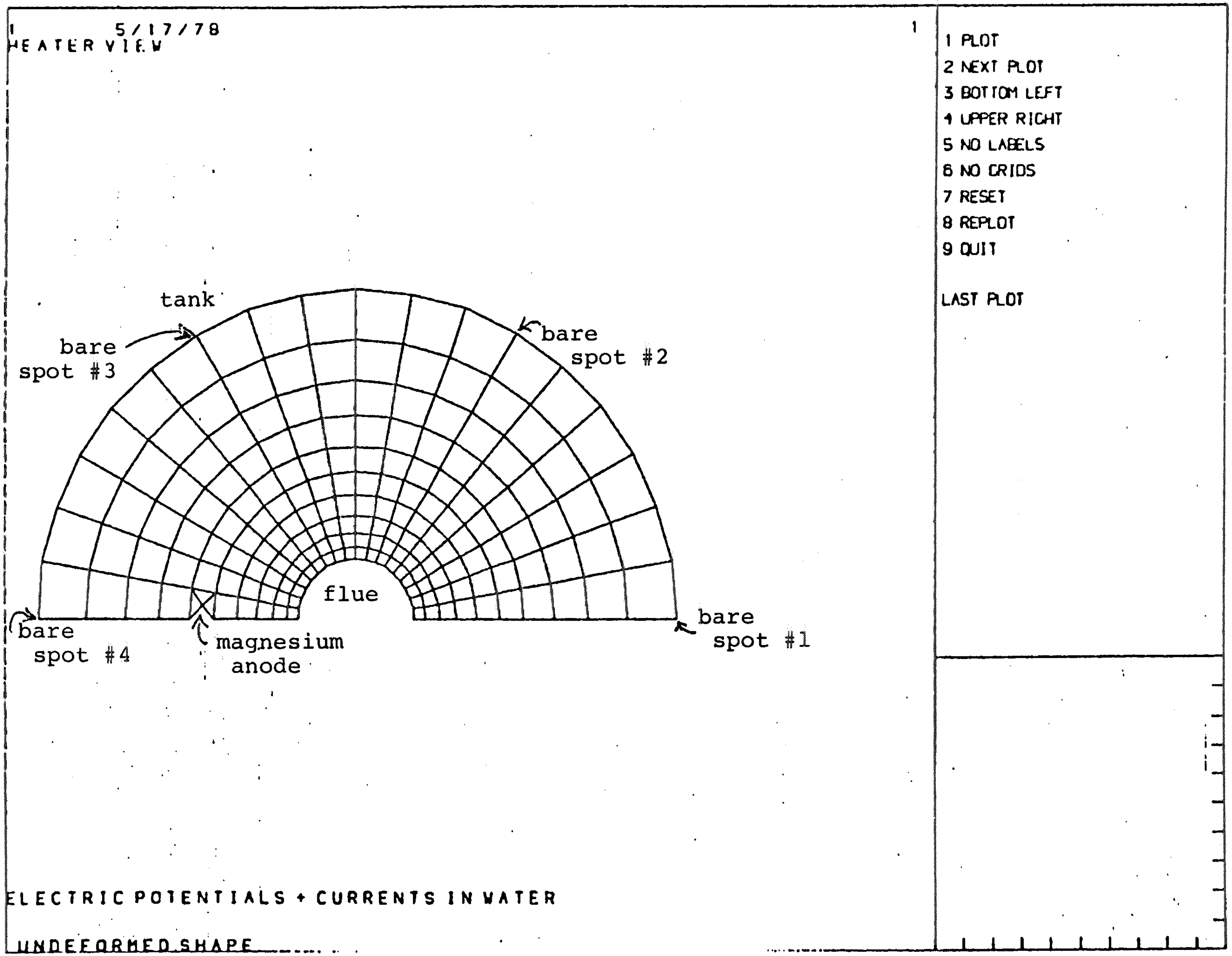


FIGURE 1 FINITE ELEMENT MESH FOR ONE-HALF OF

TABLE II

Currents (SPCFORCEs) calculated for bare steel areas of water heater for anode at 1.60 volts.

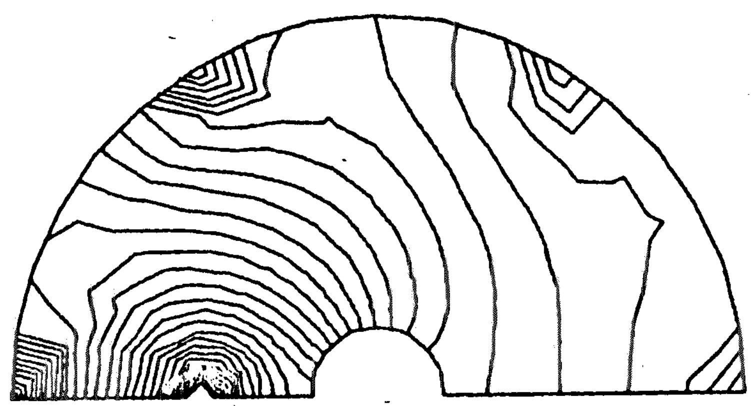
	Case A	Case B	Case C
Film coeff. h	1.E20	1.0	1.0
$A_1, A_4 (m^2)$	1.E-3	1.E-3	1.E-3
$A_2, A_3 (m^2)$	1.E-3	1.E-3	1.E-4
I_1	1.05E-3	8.731E-4	1.041E-3
I_2	2.65E-3	1.004E-3	1.308E-4
I_3	4.75E-3	1.112E-3	1.382E-4
I_4	3.93E-3	1.116E-3	1.191E-3
I_{anode}	12.39E-3	4.106E-3	2.501E-3
Figure	2	3	4

1 2/13/79 MAX-DEF. = 1.59999940
HEATER VIEW

1

1 PLOT
2 NEXT PLOT
3 BOTTOM LEFT
4 UPPER RIGHT
5 NO LABELS
6 NO GRIDS
7 RESET
8 REPLOT
9 QUIT

LAST PLOT



CATHODIC PROTECTION USING CHBDYS

STATIC DEFOR. SUBCASE 1 LOAD 0

FIGURE 2. CONTOURS OF CONSTANT VOLTAGE

1 2/20/79 MAX-DEF. = 1.59999940
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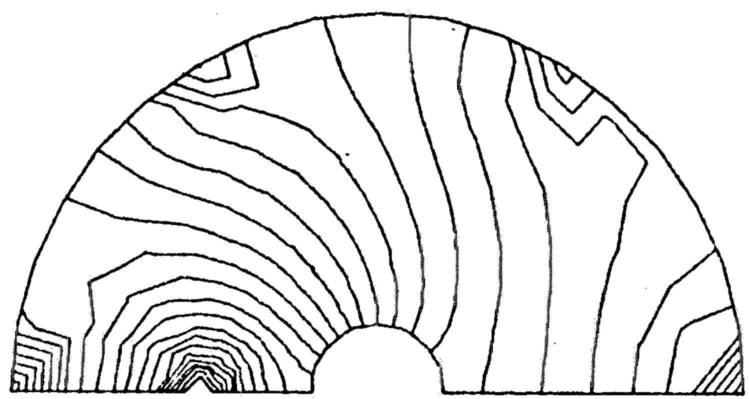

CATHODIC PROTECTION USING CHBDYS

STATIC DEFOR. SUBCASE 1 LOAD 0

FIGURE 3. CONTOURS OF CONSTANT VOLTAGE
CALCULATED FOR CASE B OF TABLE II

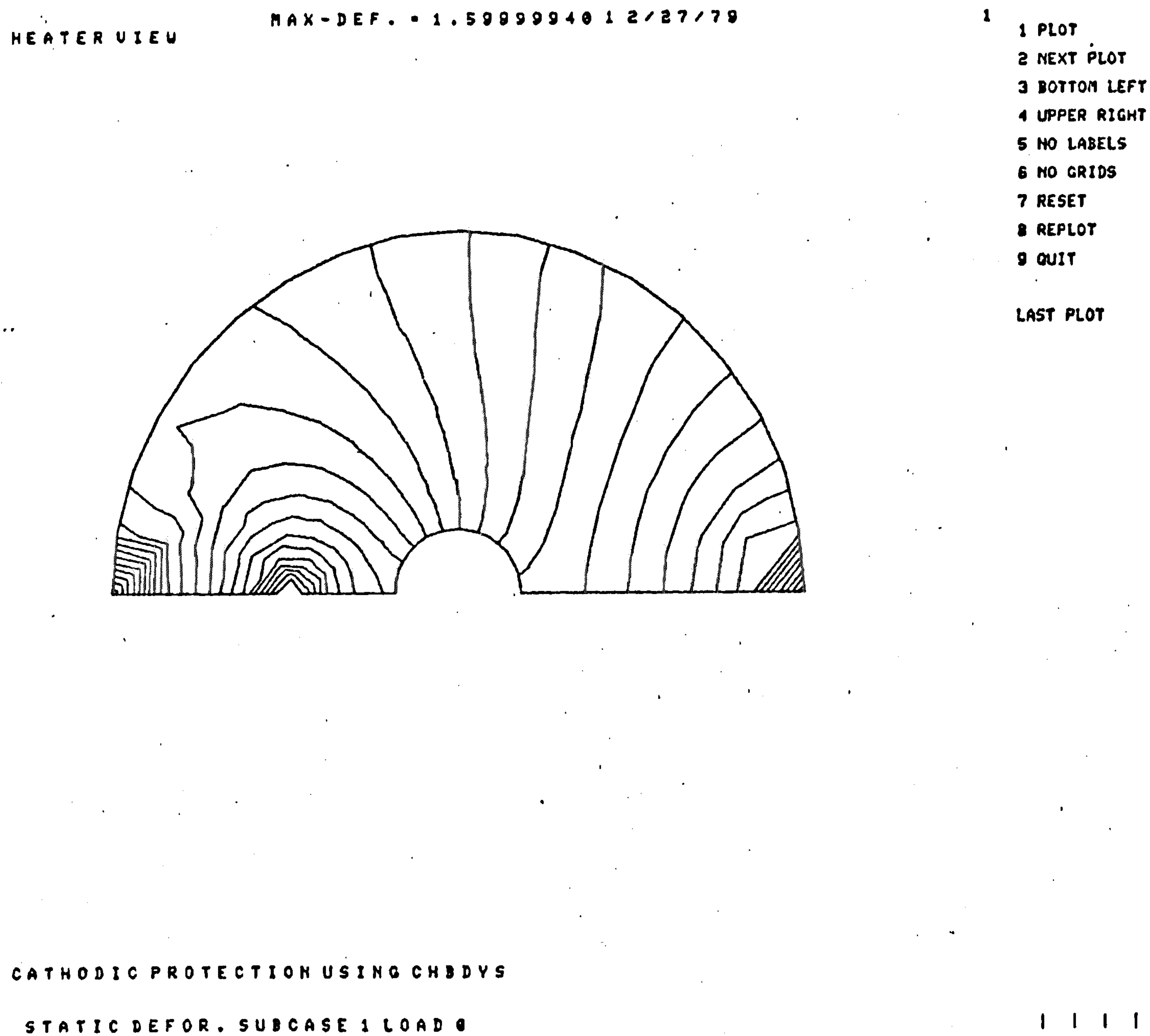

FIGURE 4. CONTOURS OF CONSTANT VOLTAGE

ACKNOWLEDGEMENT

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REFERENCES

[1] O. C. Zienkiewicz, P. L. Arlett, and A. K. Bahrani, "Solution of Three-Dimensional Field Problems by the Finite Element Method", The Engineer, 27 October 1967.

[2] J. R. Brauer, "Finite Element Analysis of Electric Fields Using MSC/NASTRAN", Proceedings of Conference on Computer Techniques for Electrostatic Fields, University of California at Santa Barbara, July 1978.