INTERLAMINAR STRESSES IN A LAMINATED ANGLE BRACKET

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1. ABSTRACT

The interlaminar stress problem in an advanced composite laminated tension joint angle fitting has been investigated with the help of finite element, theoretical, and experimental methods. The finite element solution, using the MSC/NASTRAN program, was obtained by analyzing two models; first model represented a solid laminate construction and provided the displacement/force distribution to be applied on the refined laminated model of the critical strip. Two preprocessors were developed to automatically generate the input data required for the NASTRAN analysis on the basis of the given fitting parameters. The results obtained from the finite element and the theoretical method were found to be in good agreement.

2. INTRODUCTION

For the past two decades, a considerable research and development effort has been sponsored, particularly in the aerospace industry, to exploit the very attractive structural efficiency achievable through the use of advanced composite structural materials. Advanced composites offer promise of substantial weight savings and cost reduction relative to current metallic structures.

The field of theoretical analysis of structural composites has recently made significant advances (References 1 thru 6), yet, no comprehensive analytical method has been established to determine the stress field within a three-dimensional multilayered anisotropic composite structure. However, the finite element technique has made such gains in the realm of general structural analysis that even the commonly available programs such as MSC/NASTRAN may be utilized for the analysis of irregularly shaped, non-linearly influenced structures composed of anisotropic materials.

The scope of this paper is focused towards the development of analytical methods for determining the interlaminar stresses in the vicinity of "turning the corner" in advanced composite, laminated tension joint fittings. An angle bracket was chosen, for its simplicity, to investigate the problem, which consists of several complexities (discontinuities): such as the tension joint, the turn around the corner, and the anistotropic material. A theoretical method for determining the interlaminar stresses has been introduced based on the thick/thin laminated plate/shell theory. A numerical (finite element) method has been employed wherein two special preprocessors were developed to automatically generate respective models of the bracket composed of isoparametric solid elements. The first model of a solid laminate was to determine the displacement/force distribution, and the second consisted of a critical strip extracted from the first model but laminated so as to determine the interlaminar stresses. This paper demonstrates the manner in which the finite element technique can be conveniently used to accurately solve the above defined complex fitting problem. The corresponding theoretical solution involved long and tedious effort.

This investigation was conducted at Hughes Helicopters under the sponsorship of the U.S. Army. The developed methodology is intended to be used to support the conceptual design and analysis of advanced composites generic joints and fittings. The computer approach is very flexible and can lead to the development of a sophisticated super element, the "Lambend" (laminated bend) element.

3. THE INVESTIGATIONS

3.1 GENERAL

The object of this research was to examine the behaviour of laminated tension joint fittings subjected to interlaminar stresses, and to develop efficient methodology which could be employed by practicing engineers to safely design composite fittings against interlaminar stress failures.

It was judged that a bolted corner bracket would function as the simplest structural model to investigate the significant interlaminar stress problems characteristically associated with the composite laminated fittings. Such a bracket is presented in slide 1[*] along with a statement of the typical discontinuities which make the problem complex.

The following assumptions were made:

- i. Environmental effects are not considered at this stage.
- ii. Material is homogeneous and orthotropic in load axis, with matrix elastic properties in the lateral direction.
- iii. Applied loads are uniformly distributed.
- iv. Bolted washer is imposing a rigid constraint.

Three approaches were adopted to investigate the problem:

- a. Theoretical - The finite difference scheme
- b. Finite Element Method - MSC/NASTRAN
- c. Experimental

3.2 THEORETICAL ANALYSIS

The equations outlining the theoretical approach are presented in slides 2 through 6. First the relations defining the interlaminar shearing stresses between laminae j and k are defined for laminated anistropic plates and cylindrical shells (slides 3-5). Subsequently, interlaminar tension equations derived on the basis of the thick plate theory are given (slide 6).

3.3 FINITE ELEMENT ANALYSIS

The finite element approach, using the MSC/NASTRAN program, was designed to encompass most of the foreseeable aspects of the problem. Primarily, answers were sought for the following three questions:

- a. Which region (strip) of the bracket develops maximum stress/strain gradients?
- b. What is the magnitude of the interlaminar stresses in the critical

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[*] Slides are included in the Appendix in a viewgraphical form showing the figures and the highlighted points.

region?

c. How does the geometry and composition of the bracket influence its structural efficiency?

IDEALIZATION

The considerations used for the finite element solution are stated in slide 7. Because of symmetry only half the bracket needed to be analyzed (slide 8). In order that the desired results may be accomplished economically, it was decided to build two types of finite element models for the bracket (slide 9). The first (C-1) model idealized the bracket as a solid laminate (single layer) having several strips across the width (slides 14 and 15). The second model (C-2) represented the critical strip with as many layers across the thickness as demanded by the actual laminate construction (slide 18). Three - dimensional solid (HEXA and PENTA) isoparametric elements were used for both the models. Analysis of the first model helped in identifying the region or strip under maximum distress, and also defined the displacement boundary conditions to be imposed on the subsequent multilayer model. The first model was further used for studying the influence of various geometric and material properties of the bracket. The second model was meant to provide the magnitude of the interlaminar stresses.

The boundary conditions used for the C-1 and C-2 models are shown in slide 10. Anisotropic material properties of the bracket were computed on the basis of the individual lamina composition and specified in a matrix form (MAT9/MATT9 cards) for the NASTRAN program (slide 11). The fitting was subjected at the free edge to two types of loads: uniform tension and a couple (slide 12).

PREPROCESSORS

In order that design engineers may use the developed finite element procedure conveniently and efficiently, it was decided to develop two preprocessors which would generate the NASTRAN Bulk data, for the C-1 and C-2 models, simply on the basis of the input bracket geometry and design parameters (slides 9 and 13). Special care was taken to ensure proper combination of well shaped PENTA and HEXA elements around the washer circumference. Five different coordinate systems had to be employed to accomplish proper mesh generation, consistent material alignment, and desired orientation of displacement and stress output (slide 14).

SAMPLE RESULTS

Sample results for a specific bracket are presented in slides 15 through 21. The bracket was composed of GR/EP - T300/5208, $(0_{\#}/45_{\#}/0_{\#})$ * laminate construction having nine layers each equal to 0.014 inches. Preprocessor produced discretization, as well as the NASTRAN plots and results are shown. In this case the middle-region strip was found to be critical, leading to the second stage interlaminar analysis of a single-strip nine-layer C-2 model shown in slide 18. Values of interlaminar shear and normal stresses thus obtained are plotted on slides 19, 20 and 21.

3.4 EXPERIMENTAL ANALYSIS

The experimental work was conducted on laminated brackets composed of four types of materials to observe the actual response and delamination of the fittings under eccentric tensile loading. Typical behaviour is expressed by the annotated load deflection curve in slide 22. Failure loads for different material compositions and different laminate thicknesses are also shown on the same slide.

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* The symbol '#' signifies woven fabric materials.

3.5 SUMMARY OF ANALYSIS METHODS

The values of interlaminar stresses recovered through the theoretical procedure were in excellent agreement with the results obtained from the finite element (C-2 model) analysis. However, the computations involved were quite laborious and the presented relations for evaluating the interlaminar stresses require that the stress gradients for the laminate solid be predetermined by some other means.

The experimental approach proved useful by revealing the overall performance and delamination process of the fitting, but the actual interlaminar stresses at any stage could not conceivably be measured.

The finite element method, using the two types (C-1 and C-2) of models was found to be most satisfactory. Judicious application of the method permits accurate evaluation of the overall behaviour as well as the local interlaminar state of stress in any region of the fitting.

Some of the conclusions drawn from the application of the three methods to the bracket problem, along with a mutual comparison, are presented in slide 23.

4. PARAMETRIC STUDY

Adopting the established finite element methodology, it was decided to further examine the influence of geometry and material properites on the structural efficiency of the fitting. Using as coarse, and thus the most economical, a finite element model as would not cause any significant loss in accuracy, a number of brackets with different parameters (shown in slide 9) were analyzed. Variations studied by using the coarse C-1 model, shown in slide 24, are tabulated in slide 25, and the corresponding results are graphically shown in slides 26 and 27. The multilayer C-2 model was employed for examining the effect of different laminae stacking sequences whose results are plotted on slide 28.

5. CONCLUSIONS

Investigations made to recover interlaminar stresses developed in an angle bracket, having composite laminated construction, under tensile and bending environment have been presented. The maximum interlaminar shear stresses obtained from the NASTRAN solution, by analyzing first a single layer multistrip and then a multilayer single strip model, were well corroborated by the theoretical analysis. NASTRAN indicated the interlaminar normal stress field to be in compression, which is apparently owing to the economical two model approach adopted for this investigation. In this regard a better insight was gained through the experimental work which revealed that composite fittings turning a corner tend to develop transverse tensile field (as inferred from the observed laminae seperation) near the free edge of the bracket. It is envisioned, that should a finite element solution of a bracket model having a multistrip multilayer discretization be made, the existence of the interlaminar tensile field would be confirmed.

It is recommended that amongst the presently available methods of analysis, judiciously planned application of the finite element technique offers the most practical approach for relatively conveniently and accurately analyzing/designing composite fittings.

The parametric study conducted with the aid of the finite element method indicated that the efficiency of composite fittings can be significantly improved by carefully adjusting the geometry, the laminae material properties and the stacking sequence.

6. REFERENCES

1. Calcote, Lee R.: "The Analysis of Laminated Composite Structures", Van Nostrand Reinhold Company, 1969.

2. Jones, Robert M.: "Mechanics of Composite Materials", Scripta Book Company, Washington, 1975.

3. Mansfield, E.H.: "Stress Analysis of Fibre Reinforced Plates with Polar Orthotropy", Aeronautical Research Council Reports and Memoranda No. 3796, May 1975, London.

4. Pipes, R. B. and Pagano, N. J.: "Interlaminar Stresses in Composite Laminates - An approximate Elasticity Solution", Transactions of the ASME, September 1974.

5. Schaefer, W. H., Bender, R.E., Dunbar, D. R., et al: "Advanced Composite Wing and Empennage to Fuselage Attachement Fittings", Technical Report AFML-TR-74-5, General Dynamics Convair Aerospace Divison, San Diego, California, January 1974.

6. "Stress Fields in Composite Laminates", Technical Report AFML-TR-77-114, Mechanics and Surface Interactions Branch, Wright-Patterson Air Force Base, Ohio 45433, August 1977.

7. MSC/NASTRAN Finite Element Program, User's and Application Manuals.

APPENDIX

(SLIDES)

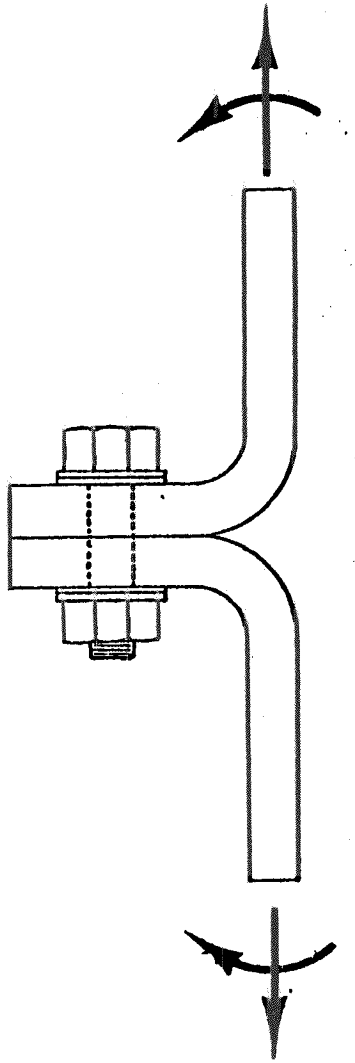
One of the problems that warranted the attention is the laminated corner of tension joints.

Hence, the scope of this presentation is mainly focused towards the development of the analytical methods for determining the interlaminate stresses in the vicinity of turning the corner in an advanced composite, laminated tension joint.

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

PROBLEM DEFINITION



- TYPICAL CORNER BRACKET TENSION JOINT
- NATURE OF THE PROBLEM — EXISTING DISCONTINUITIES
  (a) TENSION JOINT
  (b) THE CORNER - PREMATURE DELAMINATIONS
  (c) MATERIAL - ANISOTROPIC
  - HETEROGENEOUS
  - LAMINATED

One of the problems that warranted the attention is the laminated corner of tension joints.

Hence, the scope of this presentation is mainly focused towards the development of the analytical methods for determining the interlaminate stresses in the vicinity of turning the corner in an advanced composite, laminated tension joint.

The angle bracket was chosen for its simplicity to be the best means for defining this problem.

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

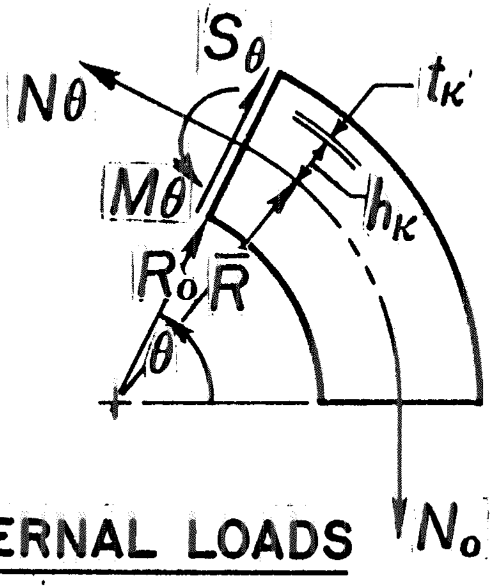
THEORETICAL

WITH AN APPLIED LOAD N_o (#/IN)

$$N\theta = N_o \cos\theta$$

$$S\theta = N_o \sin\theta$$

$$M\theta = N_o \bar{R}\left[1-\cos\theta\right]$$



INTERNAL LOADS

LET $\sigma_o = N_o / t$

HENCE THE KTH LAMINA HOOP STRESS IS

$$\frac{\sigma_\theta^{(\kappa)}}{\sigma_o} = \left\{\frac{t\left(R_o + t/2\right)\left[1-\cos\theta\right]\left(h_\kappa + t_\kappa/2\right)}{D_{\theta\theta}} + \frac{\cos\theta}{Q_{\theta\theta}}\right\} Q_{\theta\theta}^{(\kappa)}$$

DUE TO THE SHEAR S_θ

$$\frac{\tau_{r\theta}^{(\kappa)}}{\sigma_o} = \frac{t\,\sin\theta}{2\,G_{r\theta}}\left[\left(\frac{t}{2}\right)^2 - \left(h_\kappa + \frac{t_\kappa}{2}\right)^2\right] G_{r\theta}^{(\kappa)}$$

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

THEORETICAL (CONTINUED)

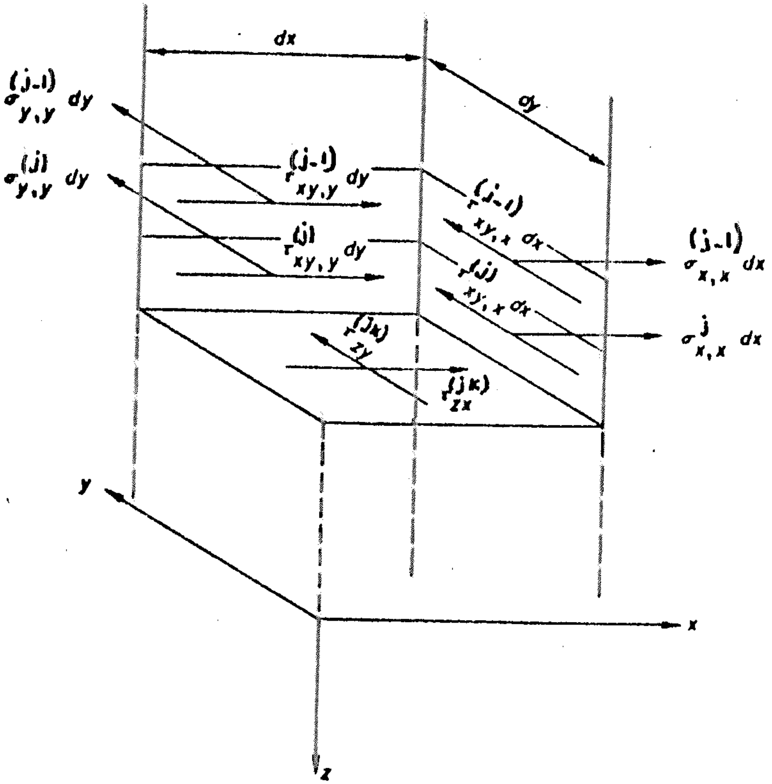
INTERLAMINAR SHEARING STRESSES BETWEEN
LAMINAS J AND K

$$\tau_{zx}^{(jk)}\,dx\,dy = -\int_{h_0}^{h_j}\left(\sigma_{x,x}\,dx + \tau_{xy,y}\,dy\right)dz$$

$$\tau_{zy}^{(jk)}\,dx\,dy = -\int_{h_0}^{h_j}\left(\sigma_{y,y}\,dy + \tau_{xy,x}\,dx\right)dz$$

WHERE THE INTERLAMINAR SHEARING STRESSES
ARE A FUNCTION OF THE INPLANE STRESS GRADIENTS
OVER THE LAMINATE THICKNESS.

MULTI-DIRECTIONAL PLANE
ANISOTROPIC PLATES



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SLIDE 3

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

LAMINATED PLATE THEORY

EXPANSIONS FOR THE INTEGRALS:

$$\tau_{zx}^{(jk)} = -\sum_{i=1}^{j}\Bigg[\lfloor g_{11}^{(i)} \quad g_{12}^{(i)} \quad g_{13}^{(i)}\rfloor \begin{Bmatrix} U_{,yyx} \\ U_{,xxx} \\ -U_{,xyx} \end{Bmatrix}$$

$$+\lfloor h_{11}^{(i)} - p_{11}^{(i)} \quad h_{12}^{(i)} - p_{12}^{(i)} \quad h_{13}^{(i)} - p_{13}^{(i)}\rfloor \begin{Bmatrix} w_{,xxx} \\ w_{,yyx} \\ 2w_{,xyx} \end{Bmatrix}$$

$$+\lfloor g_{31}^{(i)} \quad g_{32}^{(i)} \quad g_{33}^{(i)}\rfloor \begin{Bmatrix} U_{,yyy} \\ U_{,xxy} \\ -U_{,xyy} \end{Bmatrix}$$

$$+\lfloor h_{31}^{(i)} - p_{13}^{(i)} \quad h_{32}^{(i)} - p_{23}^{(i)} \quad h_{33}^{(i)} - p_{33}^{(i)}\rfloor \begin{Bmatrix} w_{,xxy} \\ w_{,yyy} \\ 2w_{,xyy} \end{Bmatrix}\Bigg]$$

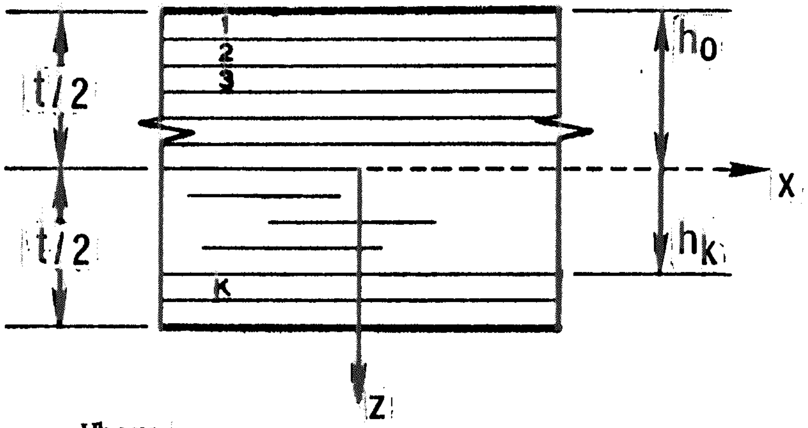
In a similar manner

$$\tau_{zy}^{(jk)} = -\sum_{i=1}^{j}\Bigg[\lfloor g_{21}^{(i)} \quad g_{22}^{(i)} \quad g_{23}^{(i)}\rfloor \begin{Bmatrix} U_{,yyy} \\ U_{,xxy} \\ -U_{,xyy} \end{Bmatrix}$$

$$+\lfloor h_{21}^{(i)} - p_{12}^{(i)} \quad h_{22}^{(i)} - p_{22}^{(i)} \quad h_{23}^{(i)} - p_{23}^{(i)}\rfloor \begin{Bmatrix} w_{,xxy} \\ w_{,yyy} \\ 2w_{,xyy} \end{Bmatrix}$$

$$+\lfloor g_{31}^{(i)} \quad g_{32}^{(i)} \quad g_{33}^{(i)}\rfloor \begin{Bmatrix} U_{,yyx} \\ U_{,xxx} \\ -U_{,xyx} \end{Bmatrix}$$

$$+\lfloor h_{31}^{(i)} - p_{13}^{(i)} \quad h_{32}^{(i)} - p_{23}^{(i)} \quad h_{33}^{(i)} - p_{33}^{(i)}\rfloor \begin{Bmatrix} w_{,xxx} \\ w_{,yyx} \\ 2w_{,xyx} \end{Bmatrix}\Bigg]$$



Where

$$[e^{(k)}] = \begin{bmatrix} e_{11}^{(k)} & e_{12}^{(k)} & e_{13}^{(k)} \\ e_{21}^{(k)} & e_{22}^{(k)} & e_{23}^{(k)} \\ e_{31}^{(k)} & e_{32}^{(k)} & e_{33}^{(k)} \end{bmatrix} = [\bar{C}^{(k)}][a \quad]$$

$$[f^{(k)}] = \begin{bmatrix} f_{11}^{(k)} & f_{12}^{(k)} & f_{13}^{(k)} \\ f_{21}^{(k)} & f_{22}^{(k)} & f_{23}^{(k)} \\ f_{31}^{(k)} & f_{32}^{(k)} & f_{33}^{(k)} \end{bmatrix} = [\bar{C}^{(k)}][b \quad]$$

The unsymmetric matrices

$$[g^{(k)}] = \int_{h_{k-1}}^{h_k} [e^{(k)}]\,dz = (h_k - h_{k-1})[e^{(k)}]$$

$$[h^{(k)}] = \int_{h_{k-1}}^{h_k} [f^{(k)}]\,dz = (h_k - h_{k-1})[f^{(k)}]$$

And the symmetric matrix

$$[p^{(k)}] = \int_{h_{k-1}}^{h_k} z(\bar{C}^{(k)})\,dz = \tfrac{1}{2}(h_k^2 - h_{k-1}^2)[\bar{C}^{(k)}]$$

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COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

LAMINATED CYLINDRICAL SHELL THEORY

EXPANSIONS FOR THE INTEGRALS:

$$\tau_{zx}^{(Jk)} = -\sum_{l=1}^{J}\Bigg[\lfloor g_{11}^{(l)} \quad g_{12}^{(l)} \quad g_{13}^{(l)} \rfloor \begin{Bmatrix} \dfrac{U_{,\theta\theta x}}{r^2} \\ U_{,xxx} \\ -\dfrac{U_{,x\theta x}}{r} \end{Bmatrix}$$

$$+ \lfloor h_{11}^{(l)} - p_{11}^{(l)} \quad h_{12}^{(l)} - p_{12}^{(l)} \quad h_{13}^{(l)} - p_{13}^{(l)} \rfloor \begin{Bmatrix} w_{0,xxx} \\ \dfrac{w_{0,\theta\theta x}}{r^2} \\ \dfrac{2w_{0,x\theta x}}{r} \end{Bmatrix} + \lfloor g_{31}^{(l)} \quad g_{32}^{(l)} \quad g_{33}^{(l)} \rfloor \begin{Bmatrix} \dfrac{U_{,\theta\theta\theta}}{r^3} \\ \dfrac{U_{,xx\theta}}{r} \\ -\dfrac{U_{,x\theta\theta}}{r^2} \end{Bmatrix}$$

$$+ \lfloor h_{31}^{(l)} - p_{13}^{(l)} \quad h_{32}^{(l)} - p_{23}^{(l)} \quad h_{33}^{(l)} - p_{33}^{(l)} \rfloor \begin{Bmatrix} \dfrac{w_{0,xx\theta}}{r} \\ \dfrac{w_{0,\theta\theta\theta}}{r^3} \\ \dfrac{2w_{0,x\theta\theta}}{r^2} \end{Bmatrix} \Bigg]$$

and

$$\tau_{x\theta}^{(Jk)} = -\sum_{l=1}^{J}\Bigg[\lfloor g_{21}^{(l)} \quad g_{22}^{(l)} \quad g_{23}^{(l)} \rfloor \begin{Bmatrix} \dfrac{U_{,\theta\theta\theta}}{r^3} \\ \dfrac{U_{,xx\theta}}{r} \\ -\dfrac{U_{,x\theta\theta}}{r^2} \end{Bmatrix}$$

$$+ \lfloor h_{21}^{(l)} - p_{12}^{(l)} \quad h_{22}^{(l)} - p_{22}^{(l)} \quad h_{23}^{(l)} - p_{23}^{(l)} \rfloor \begin{Bmatrix} \dfrac{w_{0,xx\theta}}{r} \\ \dfrac{w_{0,\theta\theta\theta}}{r^3} \\ \dfrac{2w_{0,x\theta\theta}}{r^2} \end{Bmatrix} + \lfloor g_{31}^{(l)} \quad g_{32}^{(l)} \quad g_{33}^{(l)} \rfloor \begin{Bmatrix} \dfrac{U_{,\theta\theta x}}{r^2} \\ U_{,xxx} \\ -\dfrac{U_{,x\theta x}}{r} \end{Bmatrix}$$

$$+ \lfloor h_{31}^{(l)} - p_{13}^{(l)} \quad h_{32}^{(l)} - p_{23}^{(l)} \quad h_{33}^{(l)} - p_{33}^{(l)} \rfloor \begin{Bmatrix} w_{0,xxx} \\ \dfrac{w_{0,\theta\theta x}}{r^2} \\ \dfrac{2w_{0,x\theta x}}{r} \end{Bmatrix} \Bigg]$$

COEFFICIENT ROW MATRICES ARE SAME AS FOR PLATES.

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COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

INTERLAMINAR TENSION

BASED ON THE THICK PLATE THEORY

3-DIMENSIONAL ANISOTROPY ELASTIC PROPERTIES

$$\{\sigma\} = [\bar{C}_{ij}]\ \{\epsilon\}$$

$$\{\sigma\}^{(K)} = [\bar{C}_{ij}]^{(K)}\{\epsilon\}^{(K)} \quad K^{th} \text{ LAMINA}$$

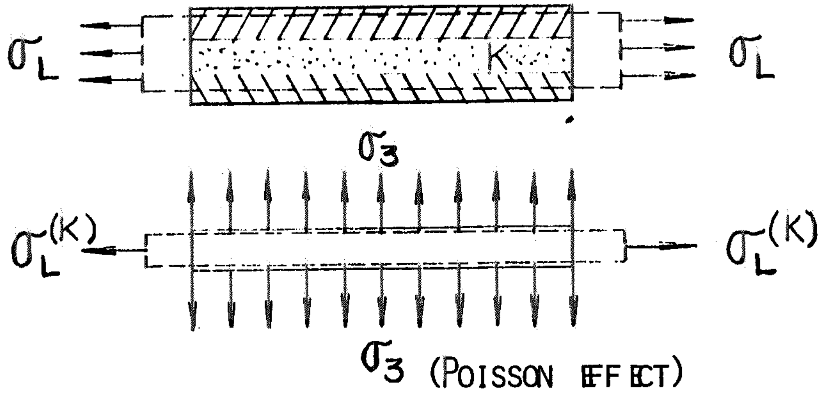
$$\text{BUT} \quad \{\epsilon\}^{(K)} = \{\epsilon\} = [\bar{S}_{ij}]\{\sigma\}$$

$$\text{WHERE : } [\bar{S}_{ij}] = [\bar{C}_{ij}]^{-1}$$

$$\therefore \{\sigma\}^{(K)} = [C_{ij}]^{(K)}[S_{ij}]\ \{\sigma\}$$

$[C_{ij}]$ = STIFFNESS MATRIX

$[S_{ij}]$ = COMPLIANCE MATRIX



$$\{\sigma\} = \begin{Bmatrix} \sigma_L \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

FINITE ELEMENT

- PREPROCESSOR - MESH GENERATOR "GENGRID"
    - IDEALIZATION
    - PARAMETERS
    - ELEMENTS
- BOUNDARY CONDITIONS
- MATERIAL PROPERTIES
- APPLIED LOADS
- NASTRAN EXAMPLE

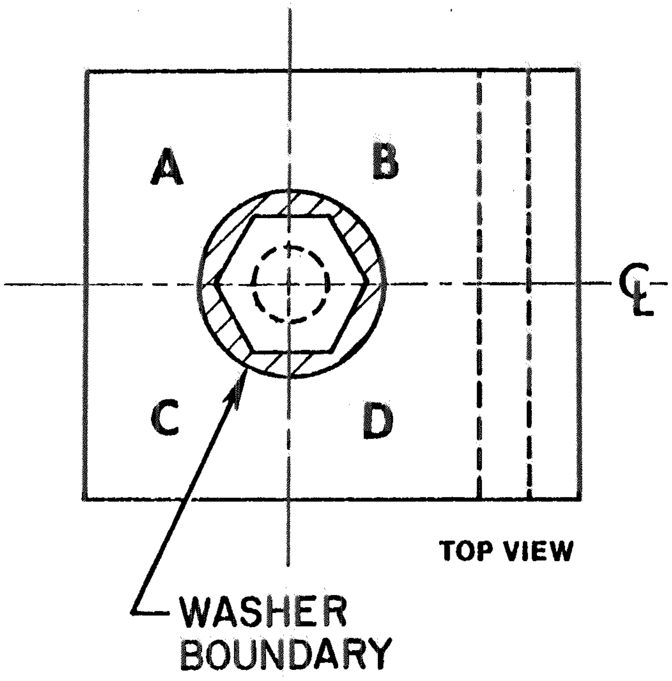
COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

IDEALIZATION

SYMMETRY

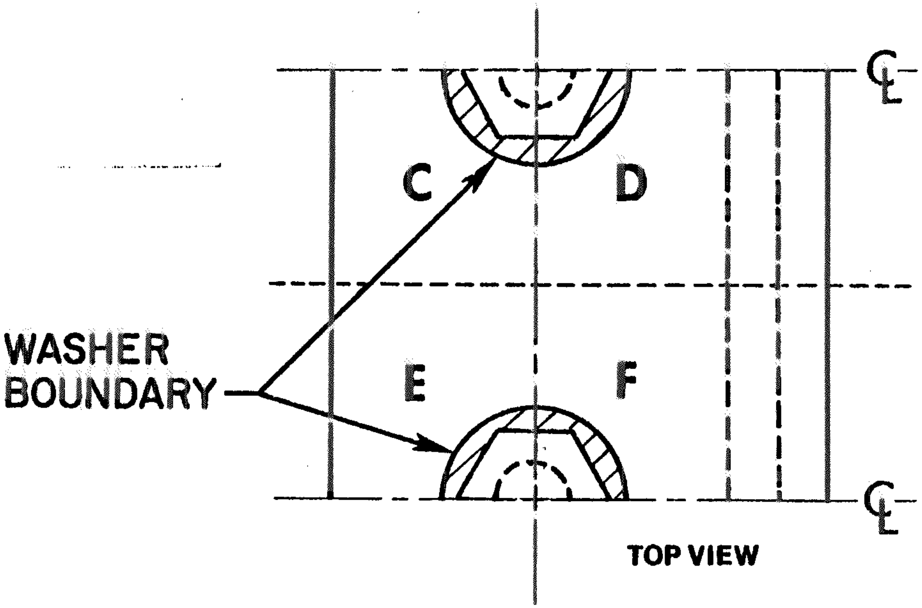
C & D ARE A MIRROR
IMAGE OF B & A



(OPTIONAL)

SYMMETRY

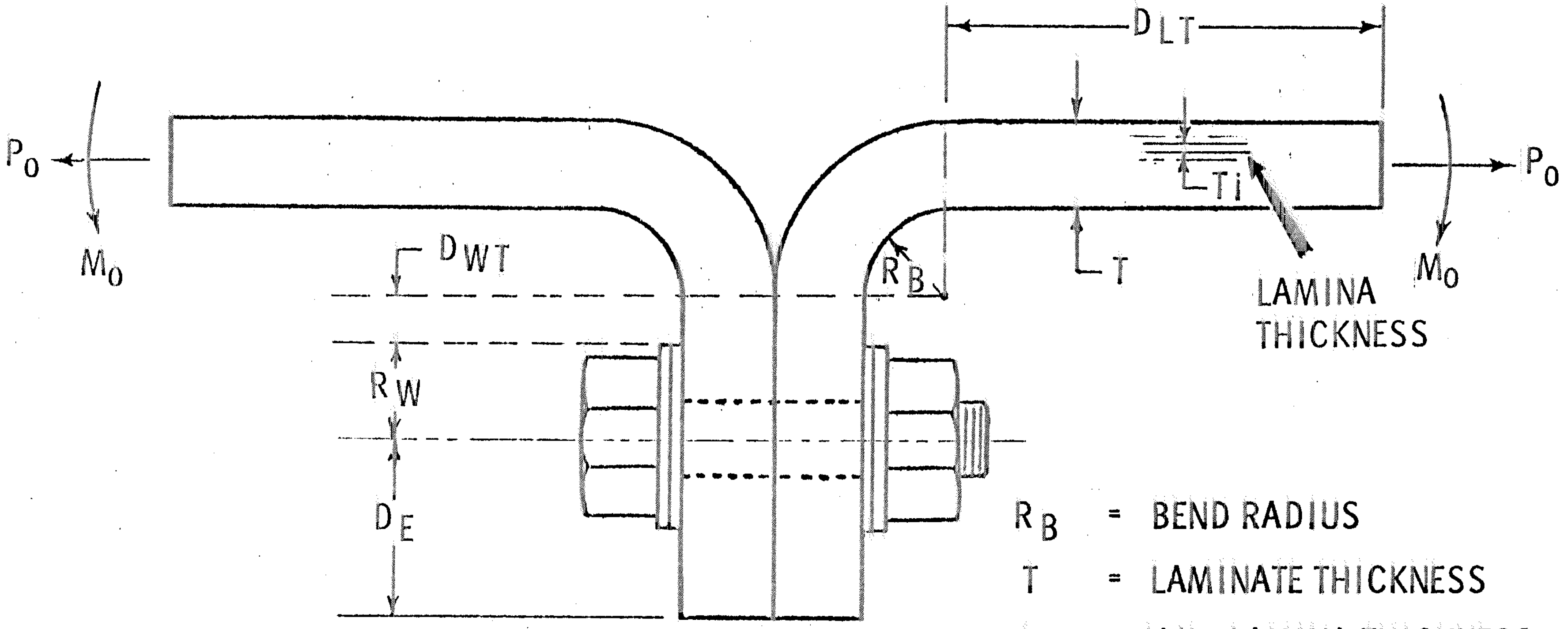
C & D ARE A MIRROR
IMAGE OF E & F



COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

STURUCTURAL MODEL AND DESIGN PARAMETERS



R_B = BEND RADIUS

T = LAMINATE THICKNESS

T_i = I'th LAMINA THICKNESS

R_W = WASHER RADIUS

D_{WT} = WASHER TO TANGENT LINE DISTANCE

D_{LT} = LOAD TO TANGENT LINE DISTANCE

D_E = EDGE DISTANCE

P_0 = APPLIED NORMAL LOAD

M_0 = APPLIED BENDING MOMENT

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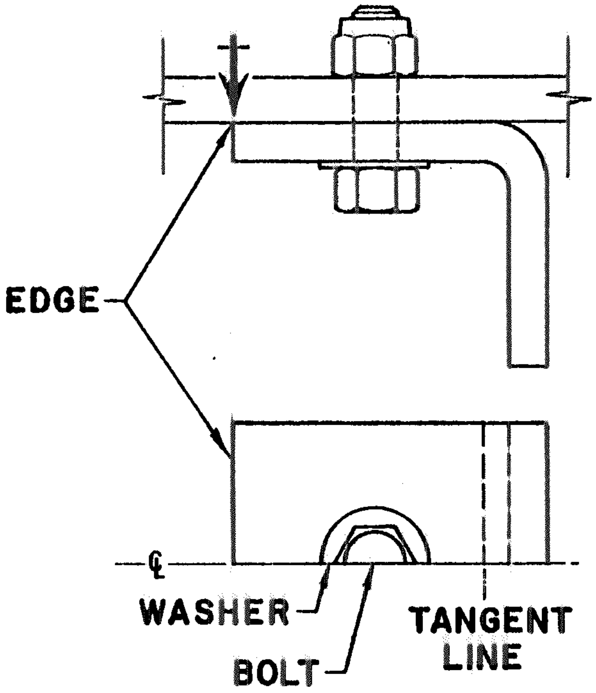
COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

BOUNDARY CONDITIONS

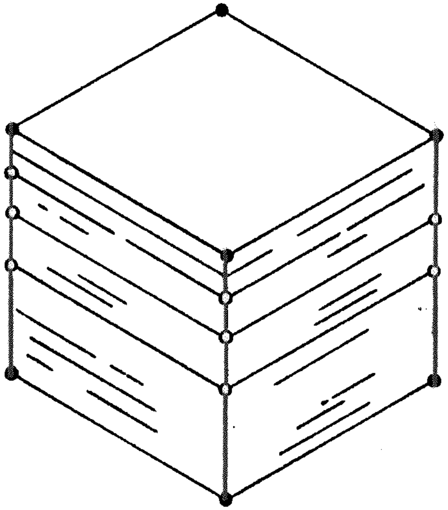
MODEL C-1

- CONSTRAINED @ ℄ FOR SYMETRY
- WASHER BOUNDARY IS FIXED
- EDGE CONSTRAINED NORMALLY



MODEL C - 2

- ● SINGLE POINT CONSTRAINTS & ENFORCED DISPLACEMENTS — FOR GRIDS @ THE SURFACES AS IS IN THE C-1 OUTPUT
- ○ MULTIPOINT CONSTRAINTS (RSPLINE) - FOR THE INNER GRIDS (NEW GRIDS CREATED FOR C-2)



MATERIAL PROPERTIES

MAT9 CARD

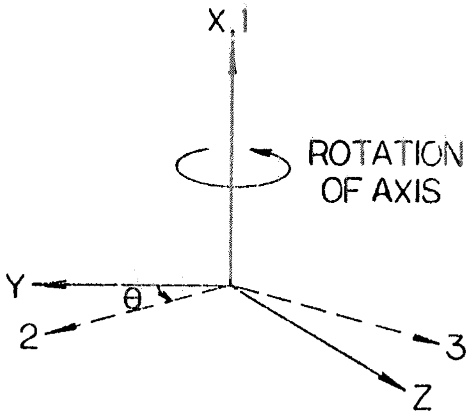
$$\left[G_{ij}\right] = \begin{bmatrix} \frac{(1-V_{23}V_{32})E_{11}}{V} & \frac{(V_{21}+V_{23}V_{31})E_{11}}{V} & \frac{(V_{31}+V_{12}V_{32})E_{11}}{V} & 0 & 0 & 0 \\ & \frac{(1-V_{31}V_{13})E_{22}}{V} & \frac{(V_{32}+V_{12}V_{31})E_{22}}{V} & 0 & 0 & 0 \\ & & \frac{(1-V_{12}V_{21})E_{33}}{V} & 0 & 0 & 0 \\ & & & G_{12} & 0 & 0 \\ & \text{SYM.} & & & G_{23} & 0 \\ & & & & & G_{31} \end{bmatrix}$$

WHERE : $V = 1 - V_{12}\ V_{21} - V_{23}\ V_{32} - V_{31}\ V_{13} - 2V_{12}\ V_{23}\ V_{31}$

$$\left[\bar{G}^{(K)}\right] = \left[T^{(K)}\right]^{-1}\left[G^{(K)}\right]\left[T^{(K)}\right]$$

$$\left[T^{(K)}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & M^2 & N^2 & 0 & 2MN & 0 \\ 0 & N^2 & M^2 & 0 & -2MN & 0 \\ 0 & 0 & 0 & M & 0 & -N \\ 0 & -MN & MN & 0 & M^2-N^2 & 0 \\ 0 & 0 & 0 & N & 0 & M \end{bmatrix}$$

WHERE:
$M = \cos\theta$
$N = \sin\theta$



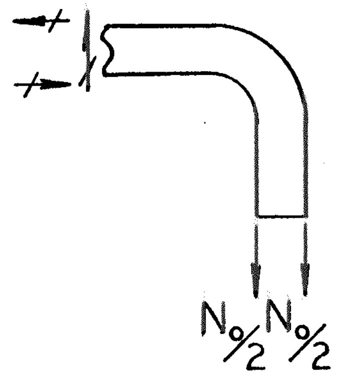
COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

APPLIED LOADS

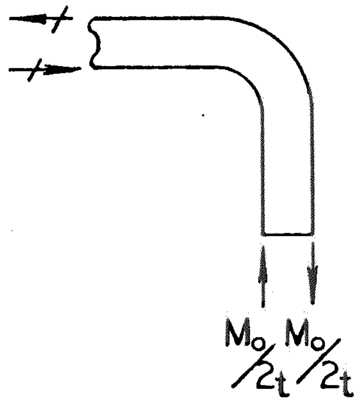
CASE 1

UNIFORMLY DISTRIBUTED TENSION



CASE 2

IN CASE OF AN INITIAL ECCENTRICITY
A COUPLE IS APPLIED UNIFORMLY
ACROSS THE EDGES

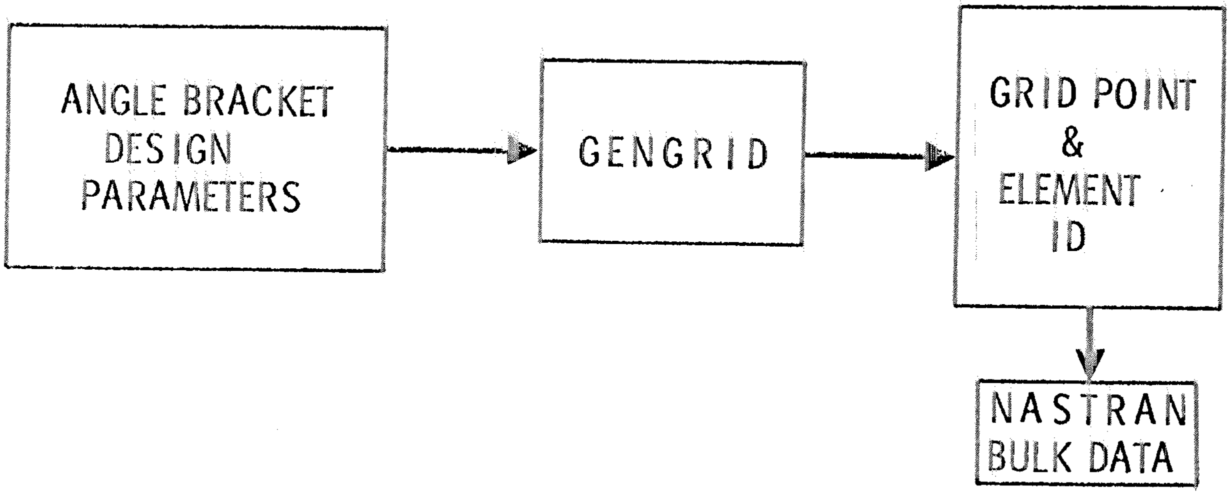


Hughes Helicopters

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

<u>GENGRID</u> - IS A CUSTOM TAILORED PREPROCESSOR, DESIGNED TO AUTOMATICALLY GENERATE TWO FINITE ELEMENT MODELS, C1 & C2



MODEL C-1

AN IDEALIZATION OF THE ANGLE BRACKET TO STUDY THE INTERNAL DISPLACEMENT/LOAD DISTRIBUTION

MODEL C-2

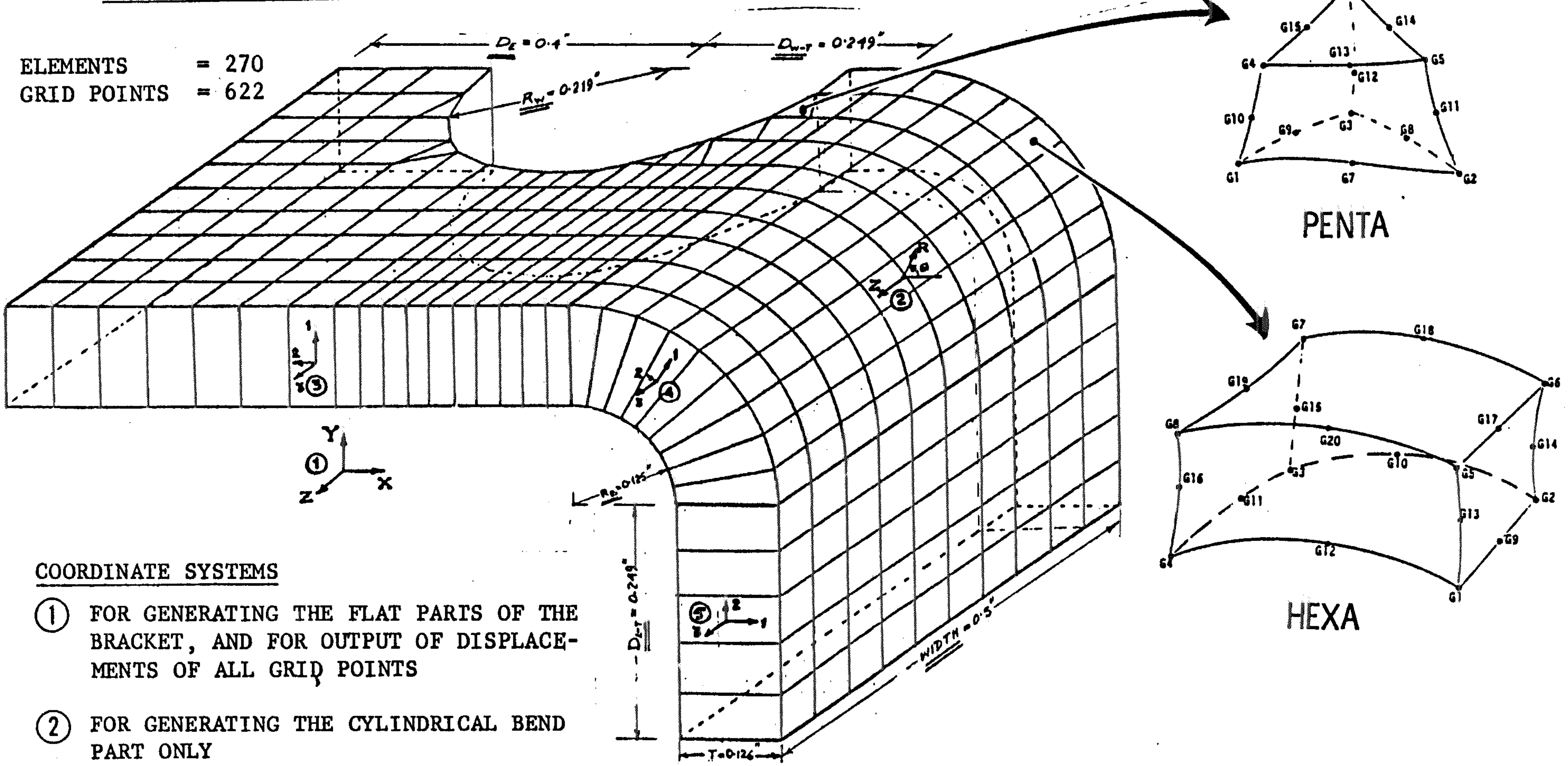
A "LAMINATED STRIP" TAKEN FROM C-1 TO DETERMINE THE INTERLAMINAR STRESSES

SLIDE 13

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

PREPROCESSOR GENERATED NASTRAN MODEL (C1-0), AND THE COORDINATE SYSTEMS



ELEMENTS = 270
GRID POINTS = 622

COORDINATE SYSTEMS

① FOR GENERATING THE FLAT PARTS OF THE BRACKET, AND FOR OUTPUT OF DISPLACEMENTS OF ALL GRID POINTS

② FOR GENERATING THE CYLINDRICAL BEND PART ONLY

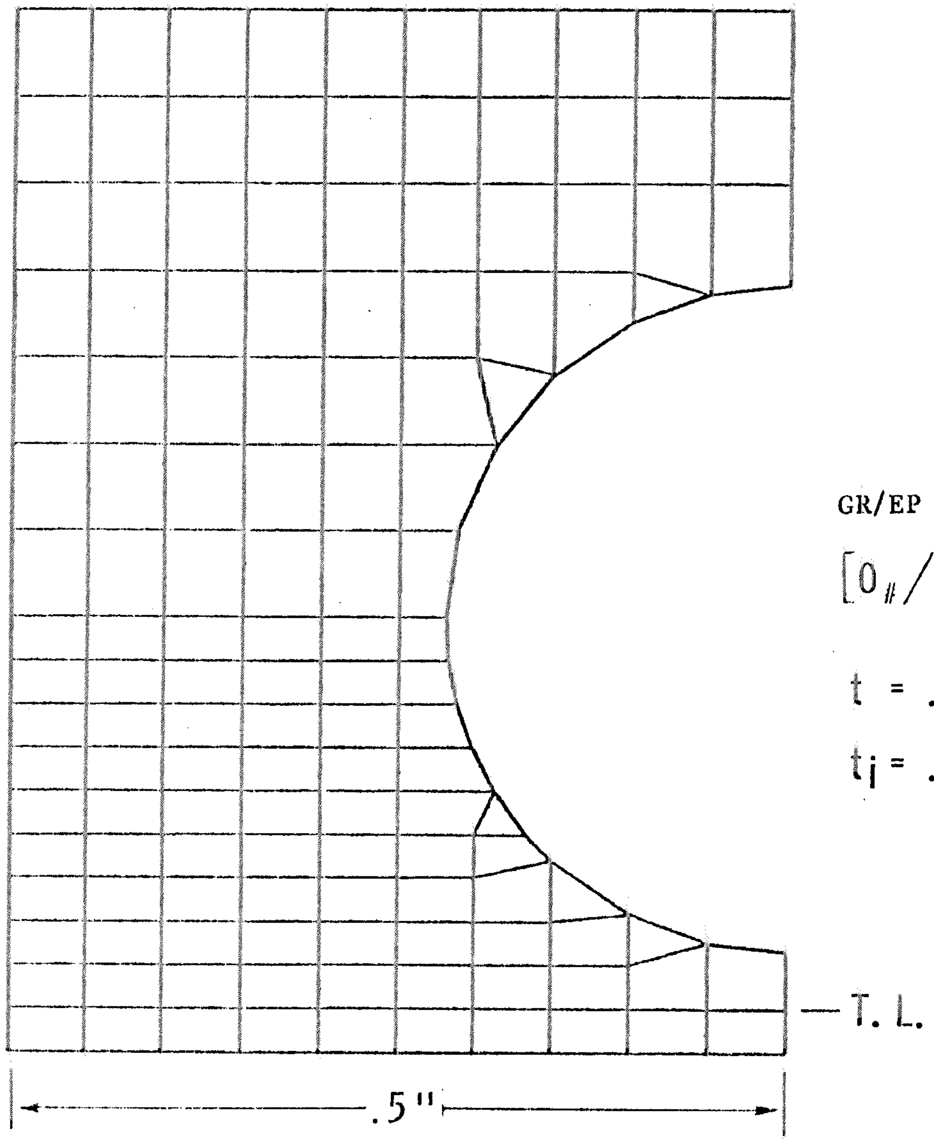
③-⑤ FOR MATERIAL ORIENTATION, AND FOR OUTPUT OF STRESSES FOR THE CORRESPONDING PARTS

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

SAMPLE BRACKET:

MODEL C 1

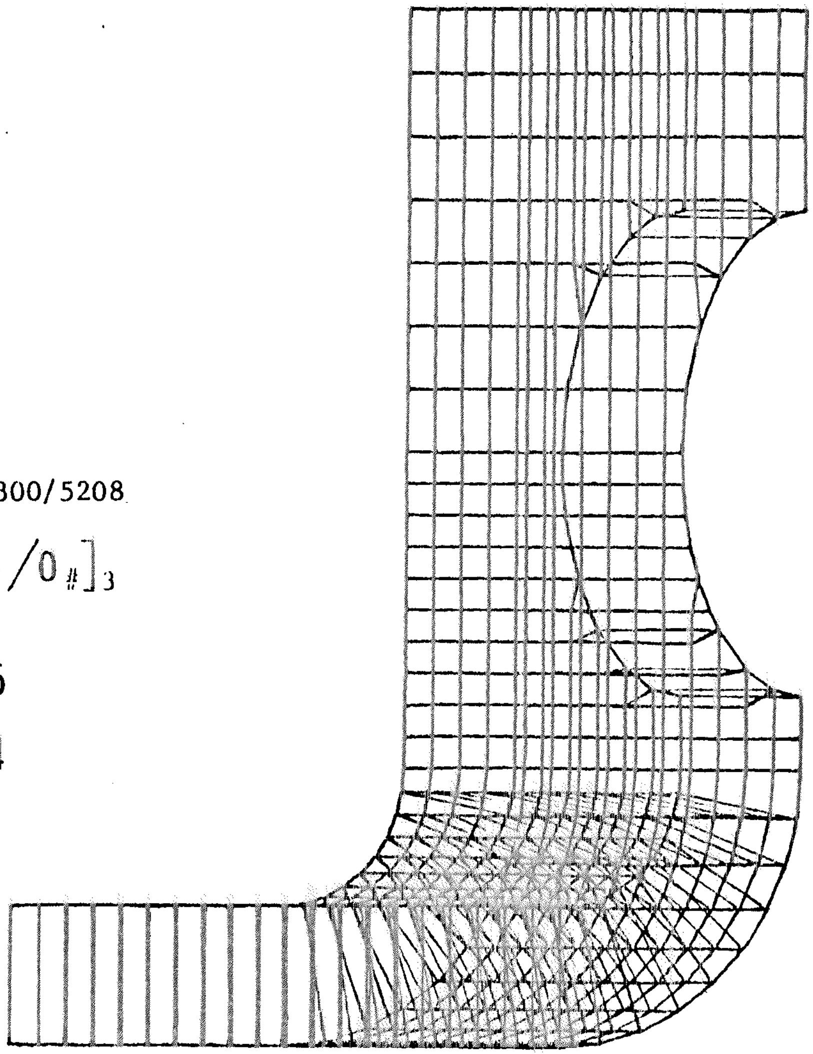


GR/EP - T300/5208

$[0_{\#}/45_{\#}/0_{\#}]_3$

$t = .126$

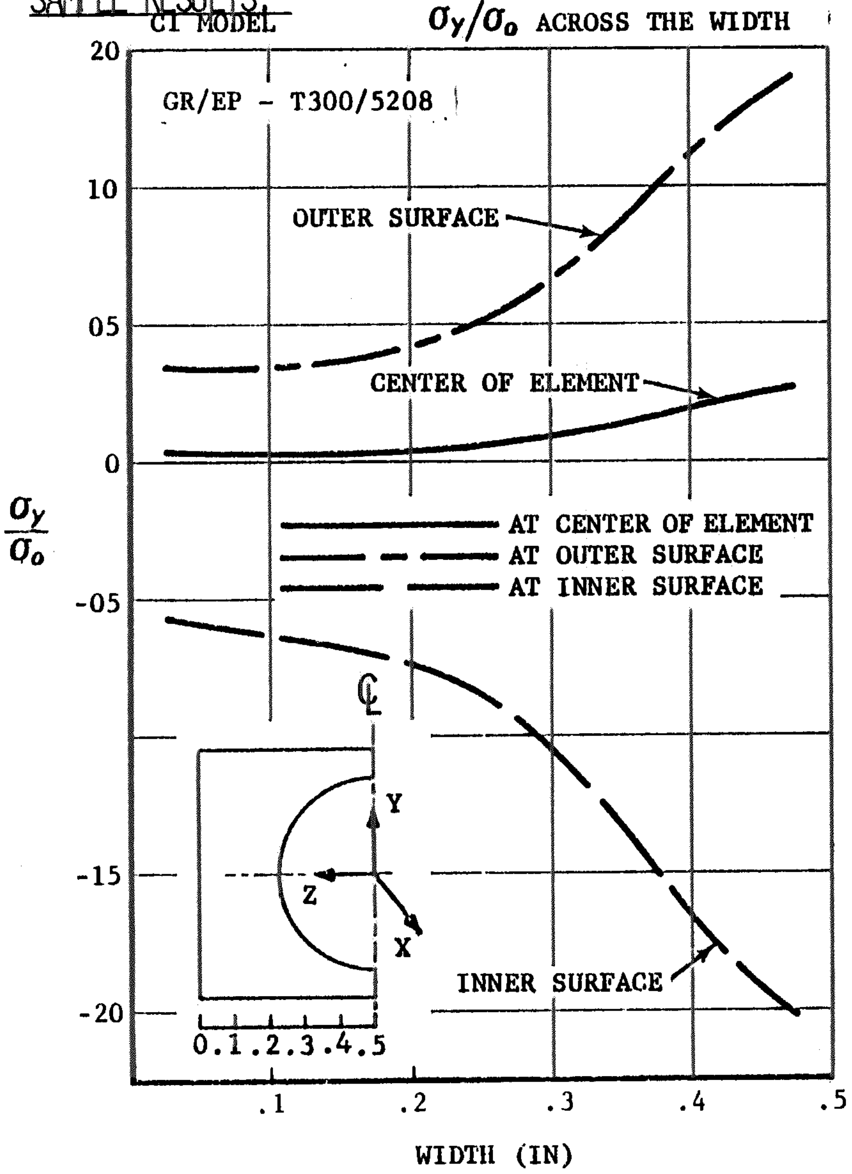
$t_i = .014$

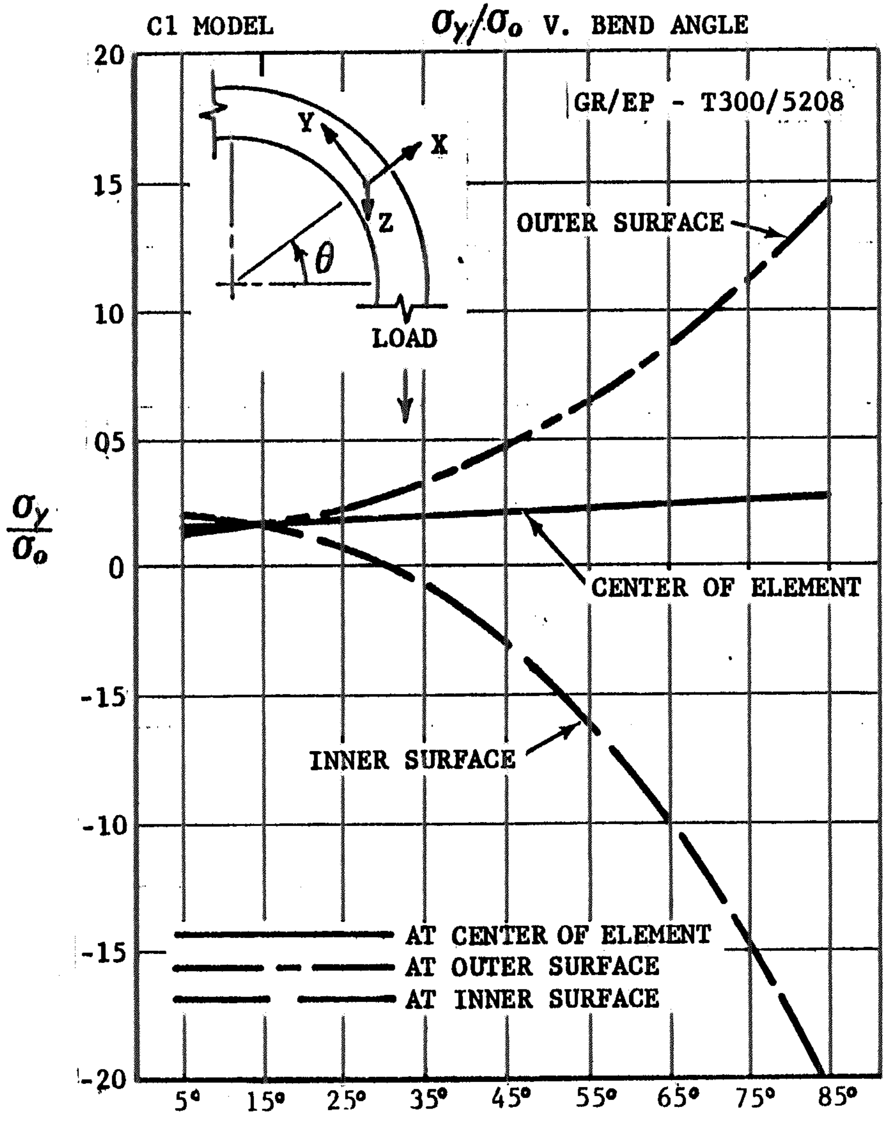


COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

SAMPLE RESULTS:





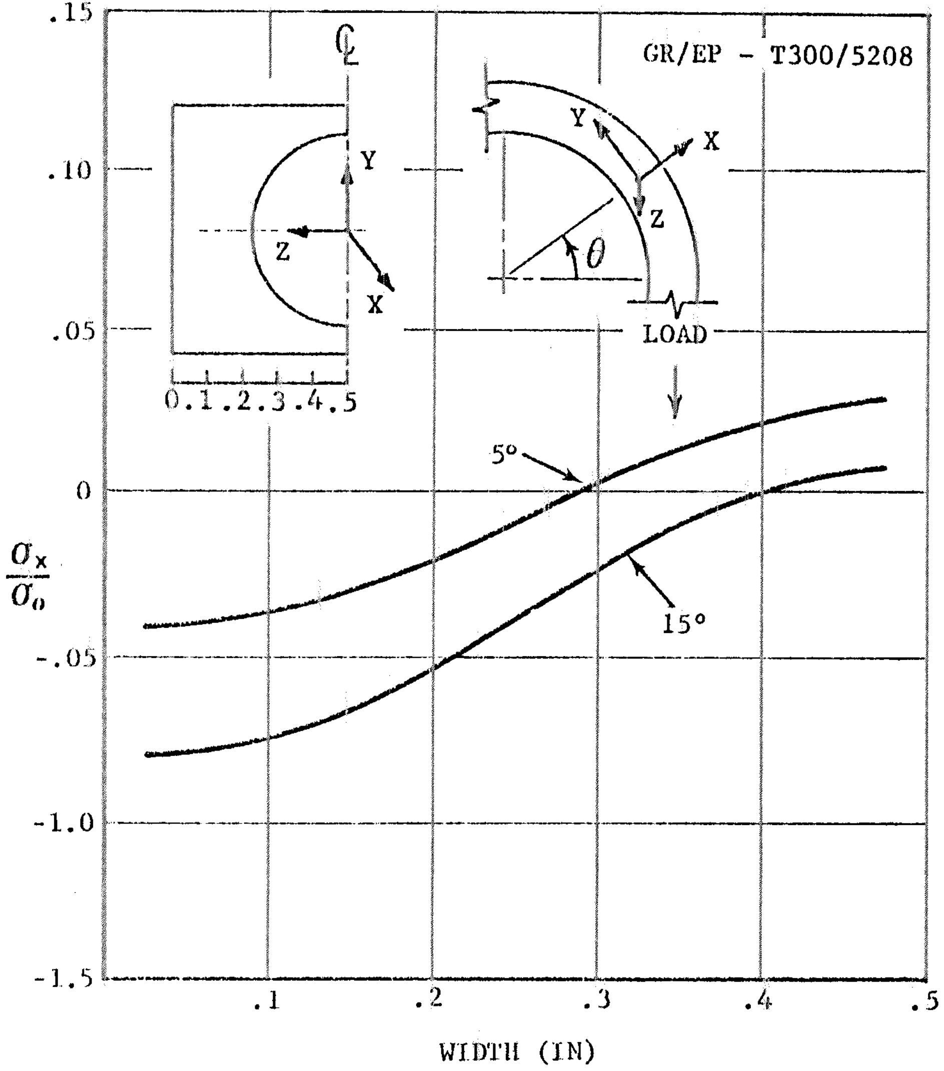
℄ STRIP

COMPOSITE JOINTS AND FITTINGS

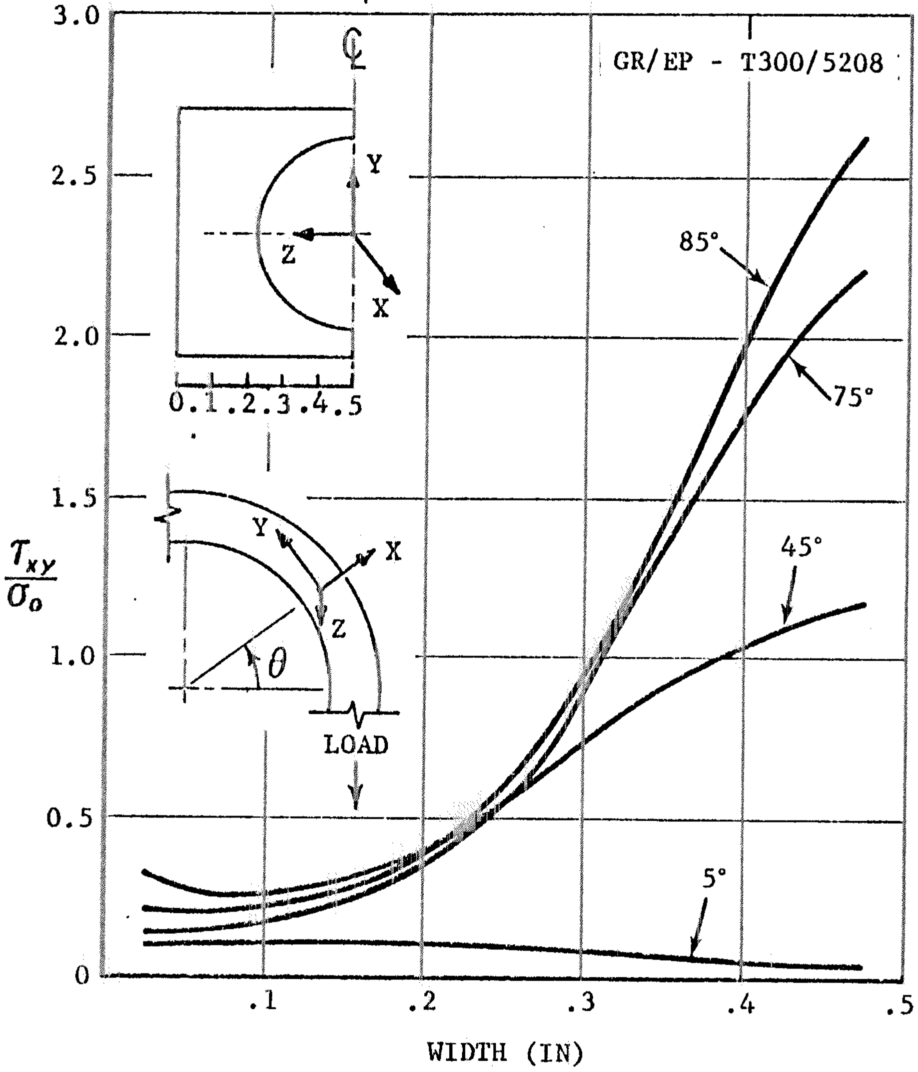
ANGLE BRACKET

MODEL C1: UNIFORM TENSION

σ_x/σ_o ACROSS THE WIDTH



τ_{xy}/σ_o ACROSS THE WIDTH

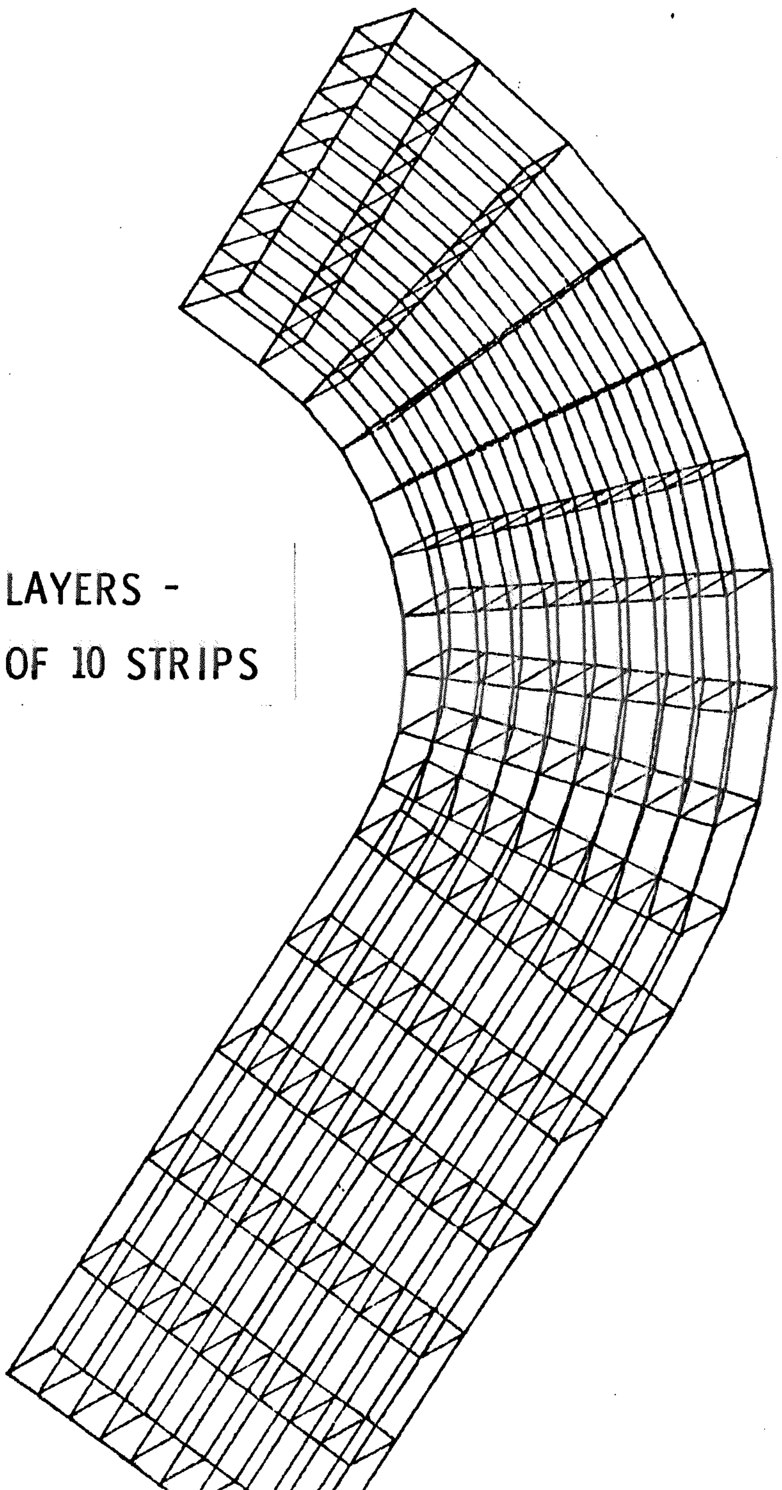


Hughes Helicopters

SLIDE 17

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

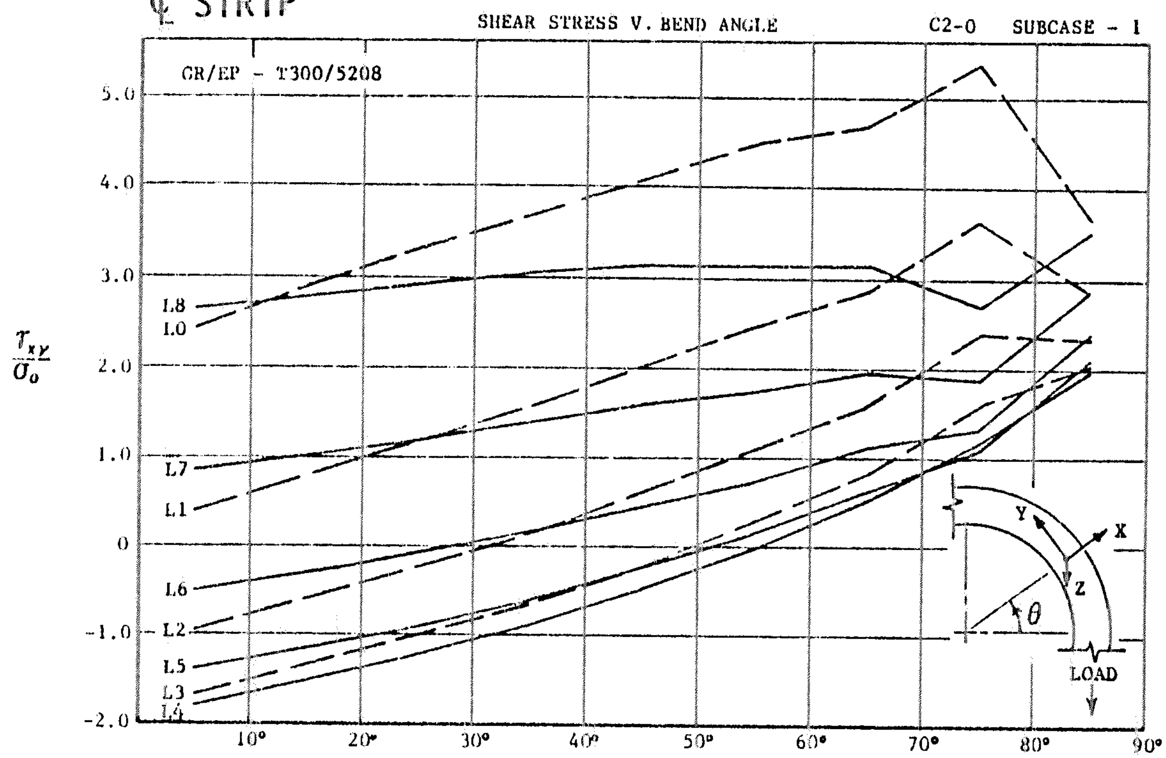


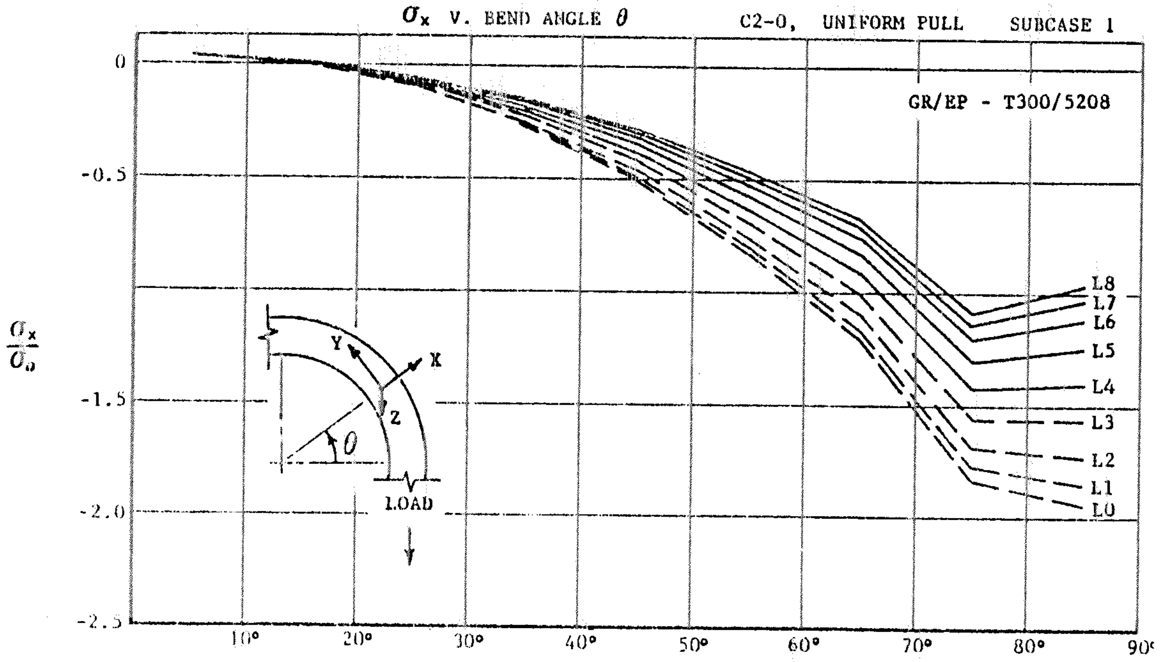
MODEL C 2 OF 9 LAYERS -
FROM MODEL C 1 OF 10 STRIPS

Hughes Helicopters

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET



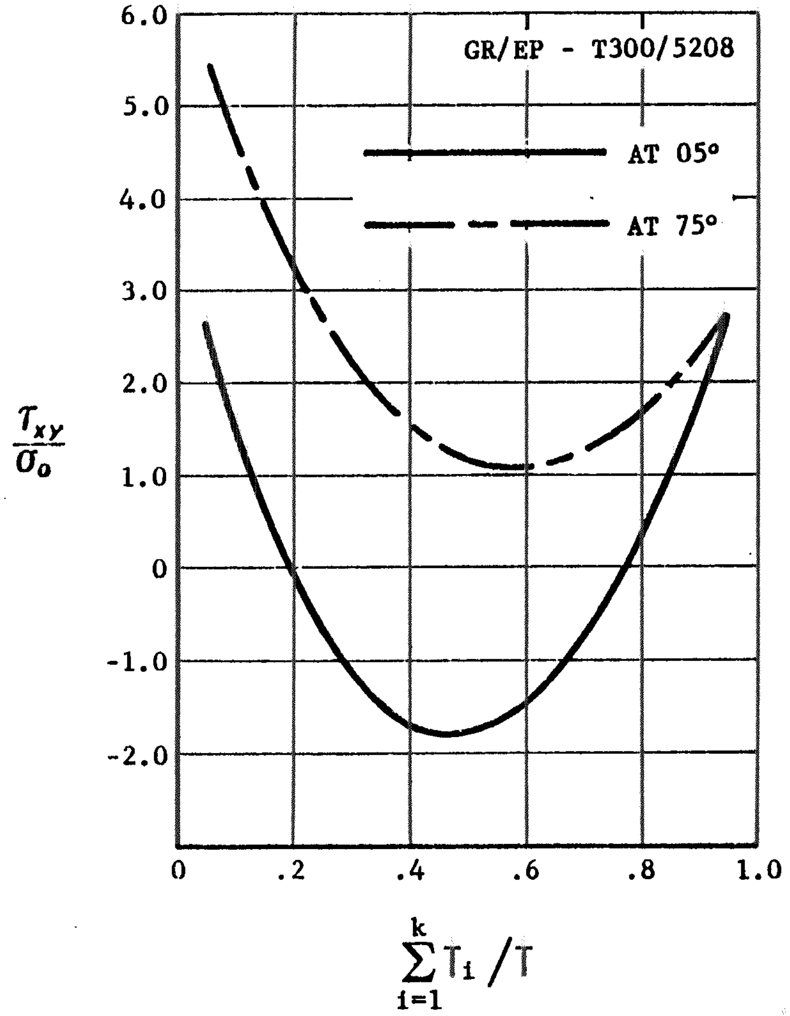


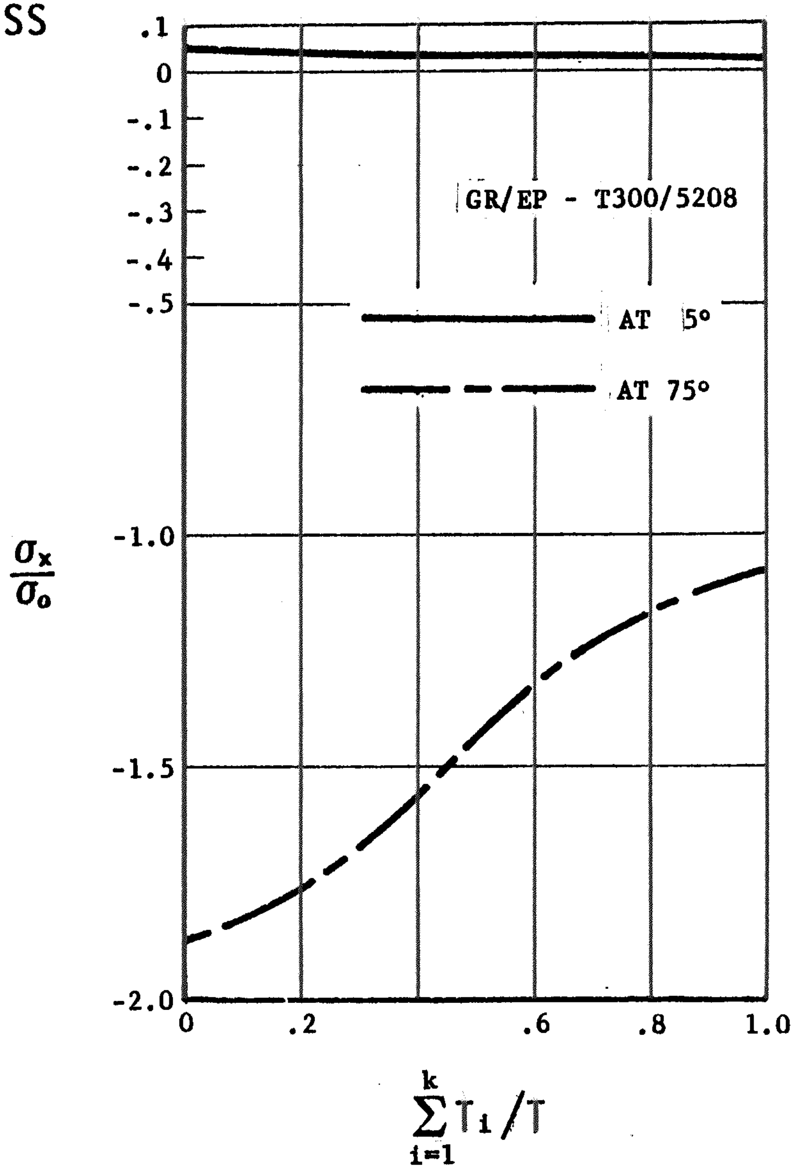
SLIDE 19

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

INTERLAMINAR STRESSES ACROSS THE THICKNESS ARE MOST CRITICAL AT ℄ STRIP



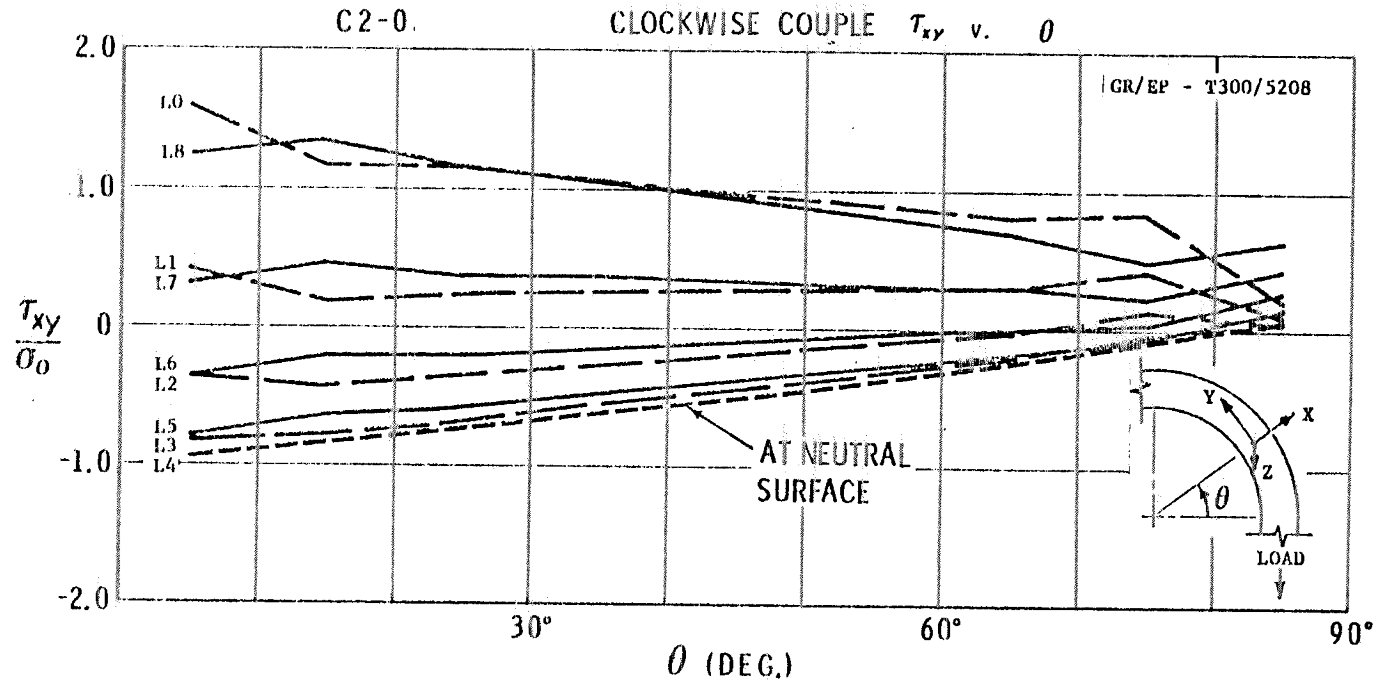


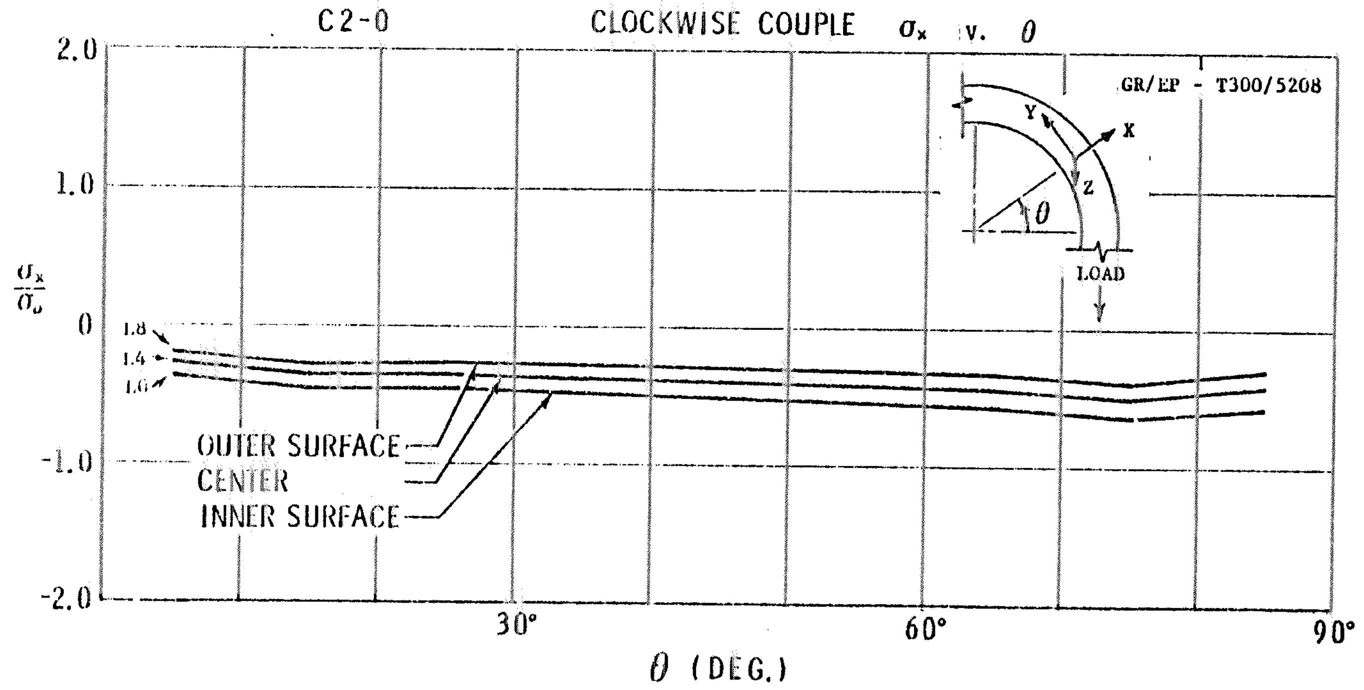
Hughes Helicopters

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

INTERLAMINAR STRESSES
MOST CRITICAL AT ℄ STRIP

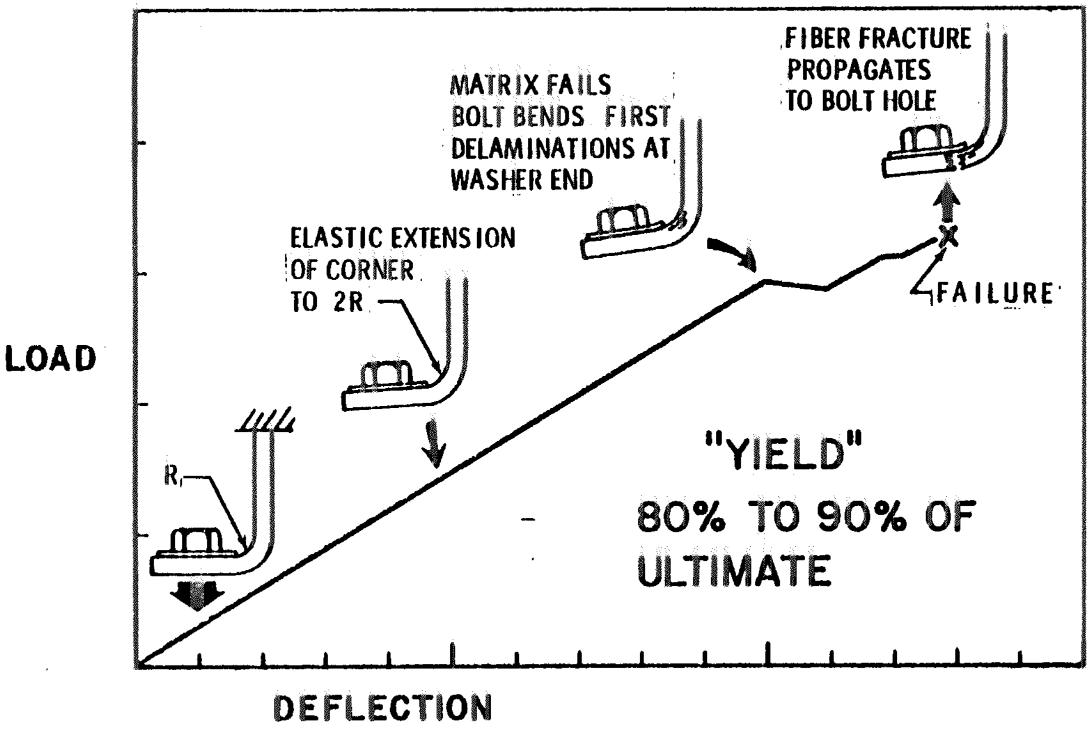


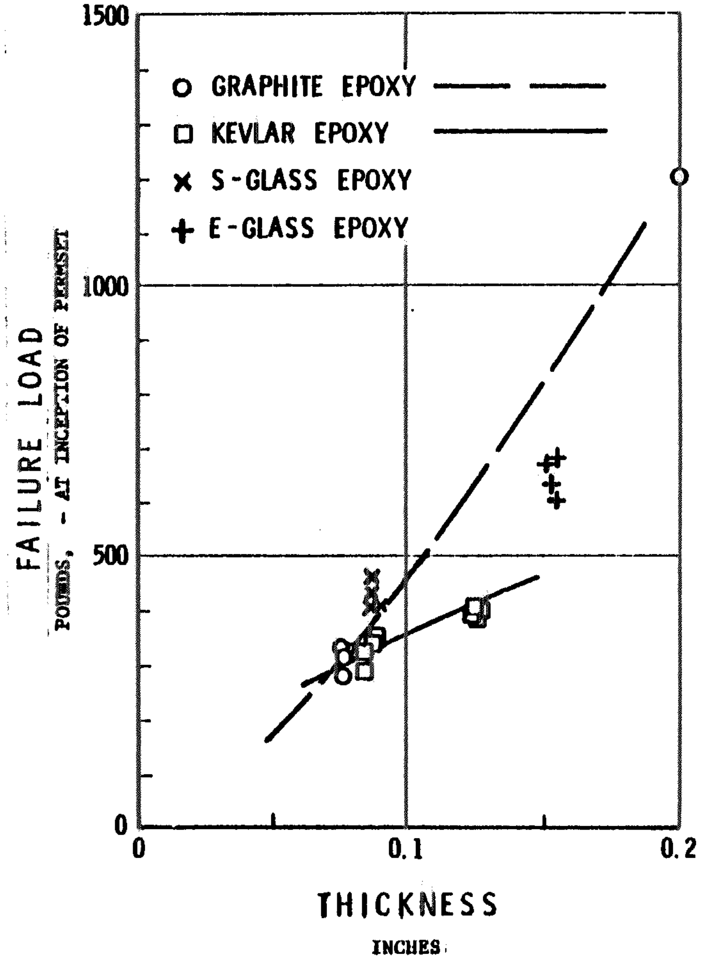


COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

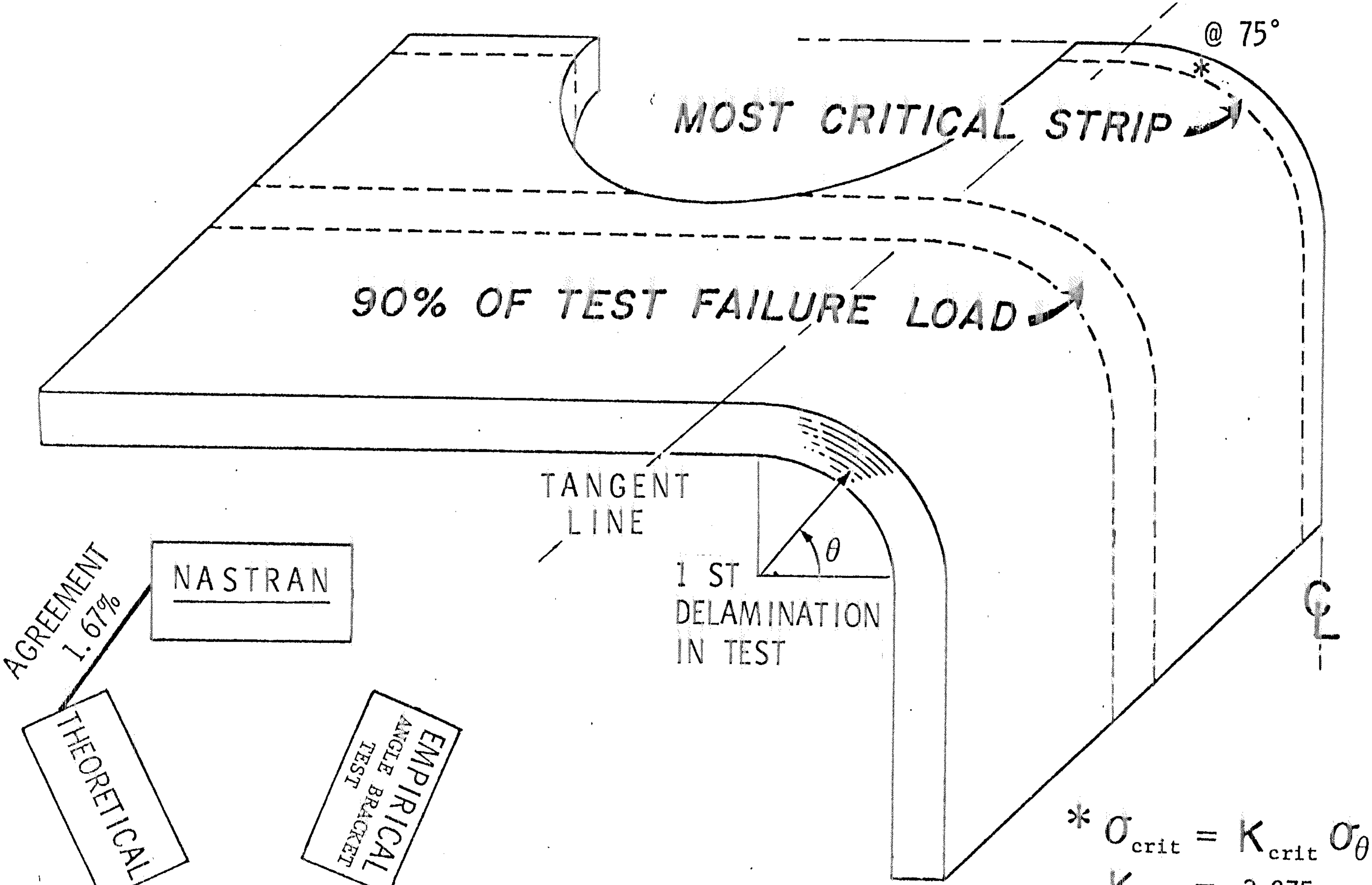
ANGLE BRACKET TEST





COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET



NASTRAN

AGREEMENT 1.67%

THEORETICAL

EMPIRICAL ANGLE BRACKET TEST

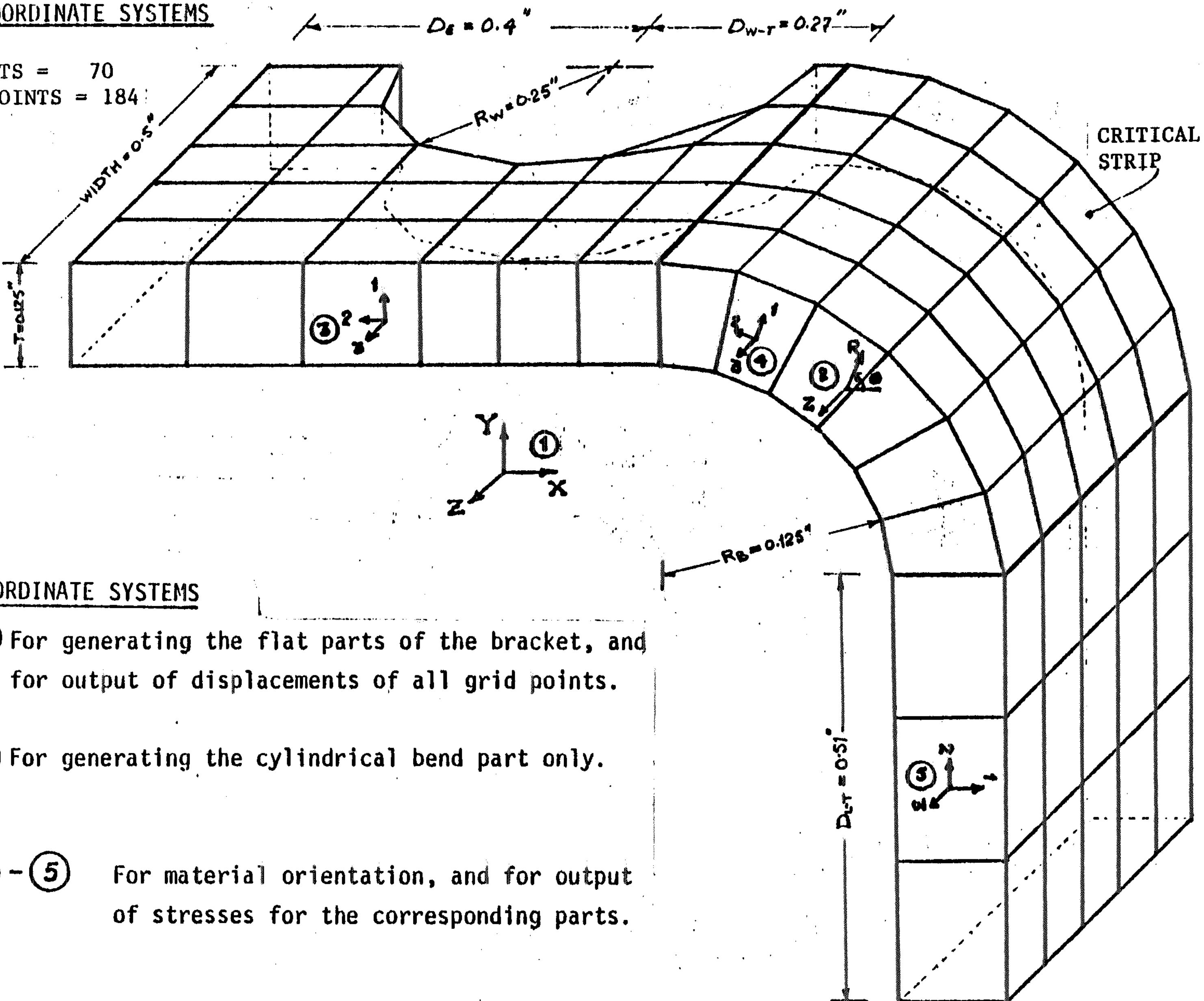
$$* \sigma_{crit} = K_{crit}\,\sigma_{\theta}$$
$$K_{crit} = 3.975$$

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

PREPROCESSOR GENERATED NASTRAN MODEL (C1-8), AND THE COORDINATE SYSTEMS

ELEMENTS = 70
GRID POINTS = 184



COORDINATE SYSTEMS

① For generating the flat parts of the bracket, and for output of displacements of all grid points.

② For generating the cylindrical bend part only.

③ - ⑤ For material orientation, and for output of stresses for the corresponding parts.

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

PARAMETRIC ANALYSIS

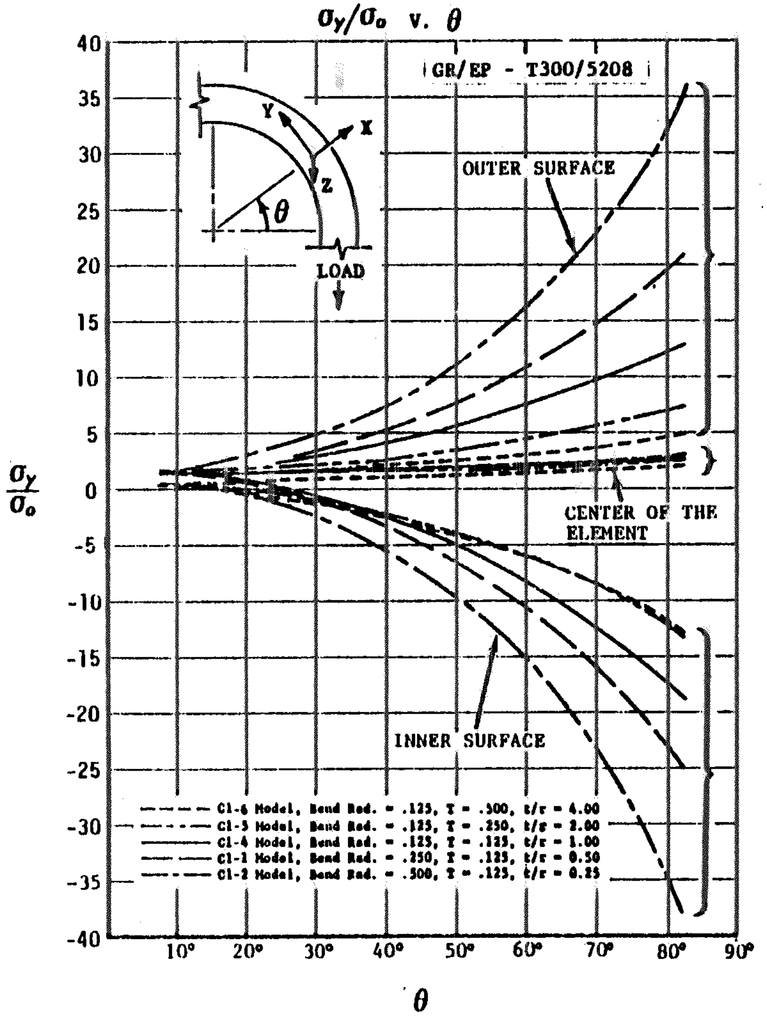
MODEL C - 1

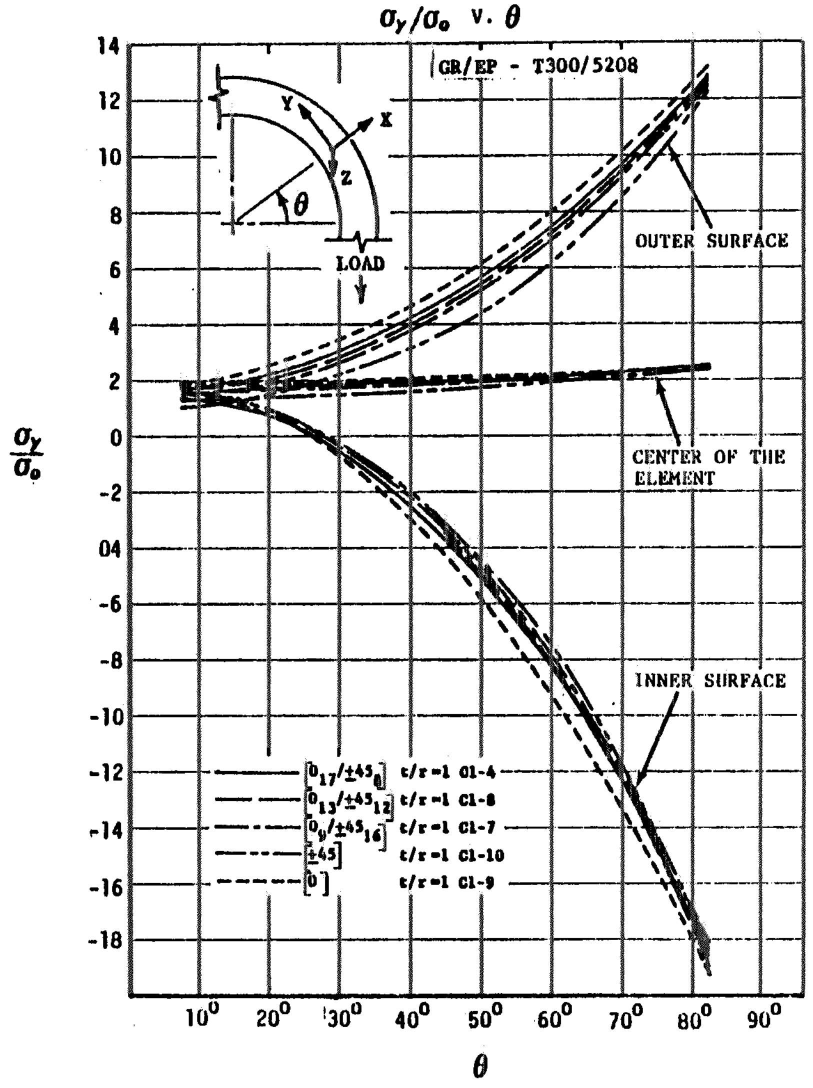
RUN	BEND RADIUS	t	WASH RAD	MATERIAL GR/EP - T300/5208	E.D.	D_{wt}	ΔT	D_{LTV}
1	.25	.125	.25	$[0_{17}/\pm 45_8]$	.4	.27	15°	.51
2	.50	↓	↓	↓	↓	↓	↓	↓
3	.125	↓	.219	$[0_{\#}/45_{\#}/0_{\#}]_3$	↓	.249	↓	.249
4	↓	↓	.25	$[0_{17}/\pm 45_8]$	↓	.27	↓	.51
5	↓	.25	↓	↓	↓	↓	↓	↓
6	↓	.50	↓	↓	↓	↓	↓	↓
7	↓	.125	↓	$[0_9/\pm 45_{16}]$	↓	↓	↓	↓
8	↓	↓	↓	$[0_{13}/\pm 45_{12}]$	↓	↓	↓	↓
9	↓	↓	↓	$[0]$	↓	↓	↓	↓
10	↓	↓	↓	$[\pm 45]$	↓	↓	↓	↓

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET

UNIFORM TENSION LOAD

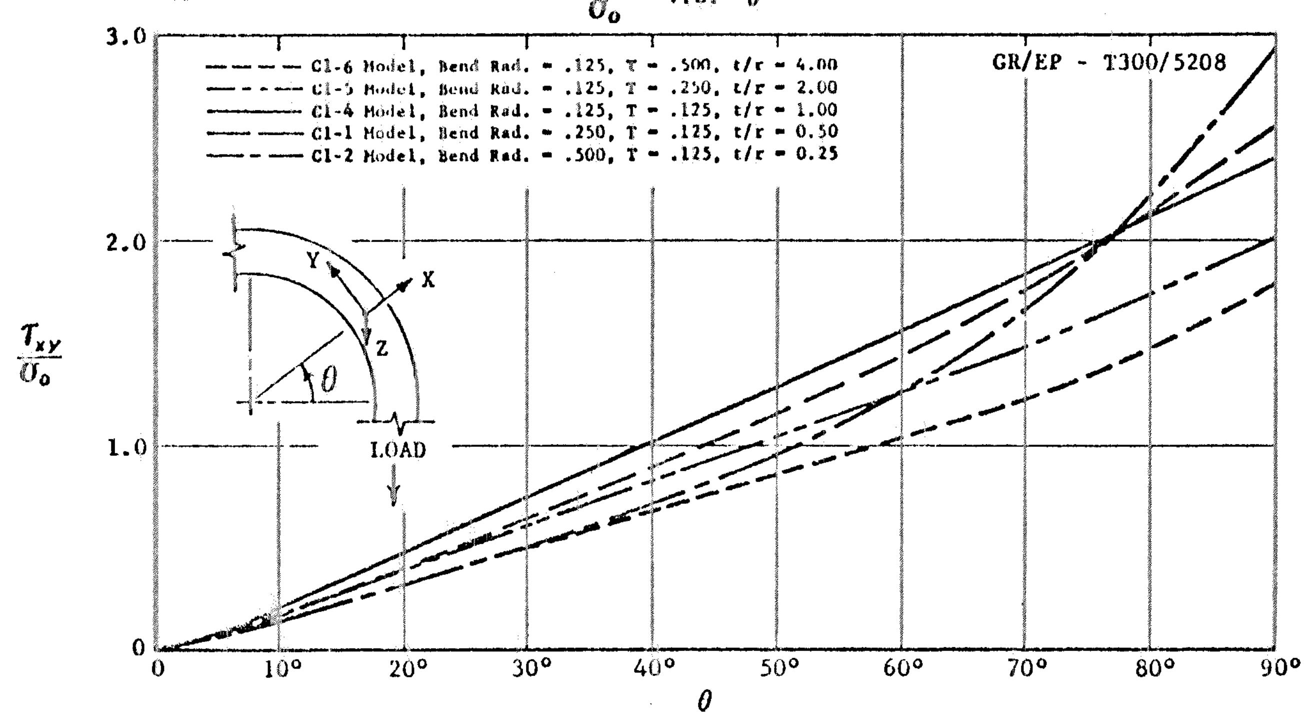




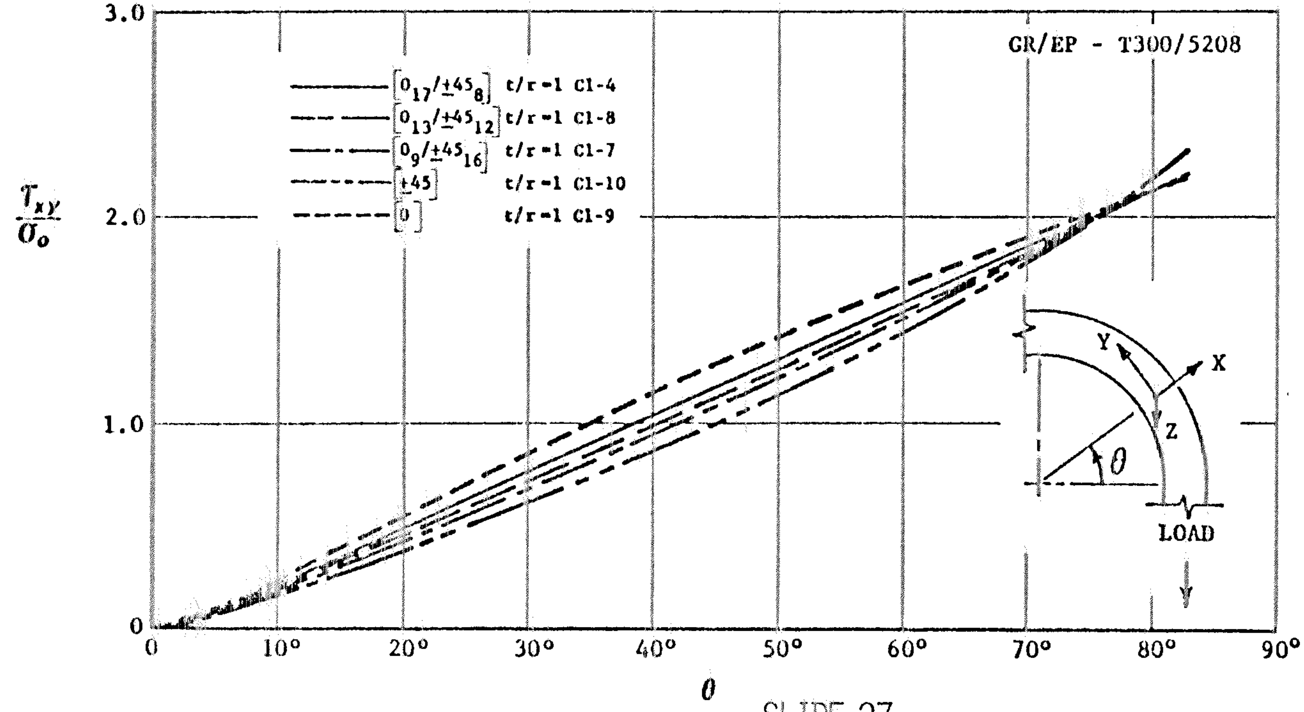
Hughes Helicopters

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET



$\frac{\tau_{xy}}{\sigma_o}$ v.s. θ

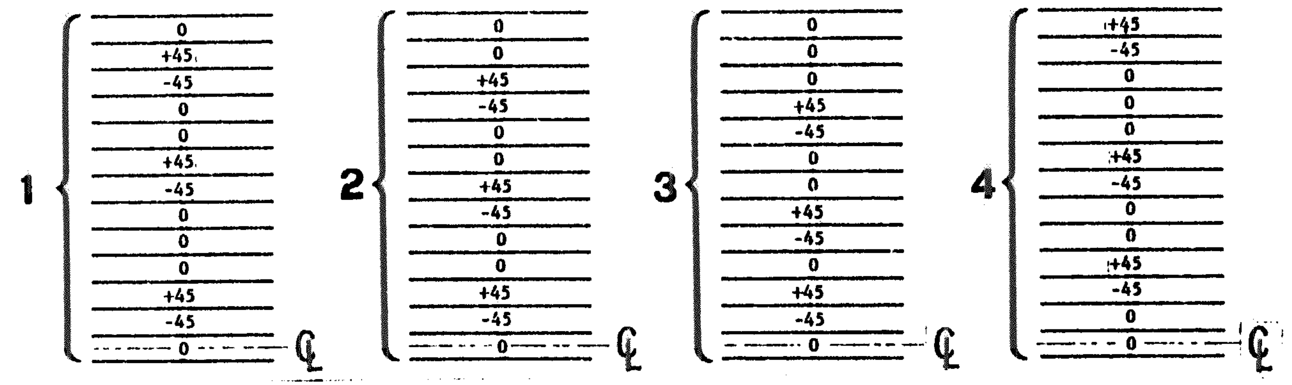


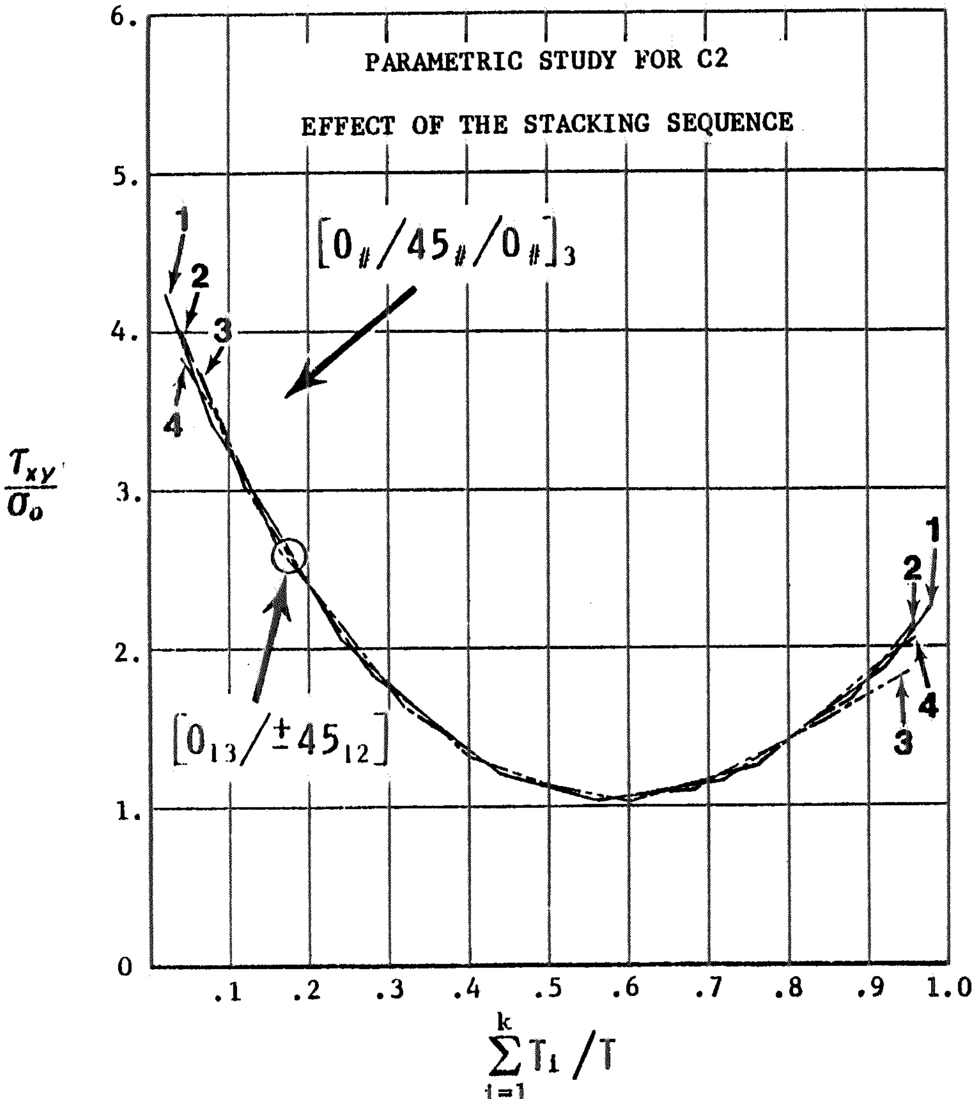
SLIDE 27

Hughes Helicopters

COMPOSITE JOINTS AND FITTINGS

ANGLE BRACKET




PARAMETRIC STUDY FOR C2

EFFECT OF THE STACKING SEQUENCE