NUMERICAL STABILITY OF FINE MESH TORUS MODELS

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ABSTRACT

Numerical stability in Torus Models can only be verified with a full size model. Instabilities may occur when there are a large number of shell type elements around the circumference. Shell elements do not have inplane rotational stiffness, therefore the use of many elements around the circumference may cause some small rotational terms in the Stiffness Matrix. To study the effects and magnitudes of these terms a 30 degree segment of a Torus Model was analysised utilizing Dynamic Reduction in Normal Modes Analysis.

INTRODUCTION

This study was conducted to verify the numerical stability of MSC/NASTRAN. To accomplish this verification, a Torus was modeled using a number of different modeling techniques. Each model was then analyzed with Solution 25, Normal Modes Using Dynamic Reduction. This particular type of analysis was chosen for two reasons. First, Normal Modes Analysis is a standard type of analysis for Torus models. Second, Eigenvalue analysis will require a large number of matrix operations, therefore, it may be subjected to numerical instabilities and round-off errors. The reason numerical instabilities may arise in a Torus model is as follows; when two adjacent shell elements (specifically QUAD4 and TRIA3 elements) are co-planer, a singularity exists. Singularities occur because shell elements do not have in-plane rotational stiffness. In a fine mesh Torus model the elements around the circumference intersect at a relatively small angle, (see figure 1).

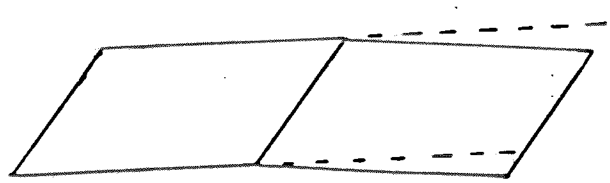


FIGURE 1

Regardless of the displacement coordinate systems defined for the grid points, there will be some rotational stiffness terms which will be relatively small in respect to the other rotational terms. MSC/NASTRAN performs a singularity check on three by three partitions of the KGG (Global Stiffness) Matrix and flags any potential singularities when Ri<EPZERO.

where: $Ri = \frac{Ki}{|Kmax|}$

$EPZERO(default) = 1 \times 10^{-8}$

Ki = each individual term in the 3 x 3 partiti

|Kmax| = the absolute value of the maximum term in the 3 x 3 partition.

(Reference: MSC/NASTRAN APPLICATIONS MANUAL SECTION 2.19)

One may be concerned with the affect of round-off errors on the final results of the analysis when some stiffness terms are very small and pass the singularity test.

The method chosen to investigate this question is as follows:

1) Perform a Normal Modes (Solution 25) analysis of a typical fine mesh Torus model, printing the Eigenvalue summary, printing the diagonal of the Stiffness Matrix and plotting the first seven Eigenvectors.

2) Find the rotational terms in the Stiffness Matrix that are small, increase them by means of CELAS Cards to approximately 10% of the higher terms, and run the analysis a second time.

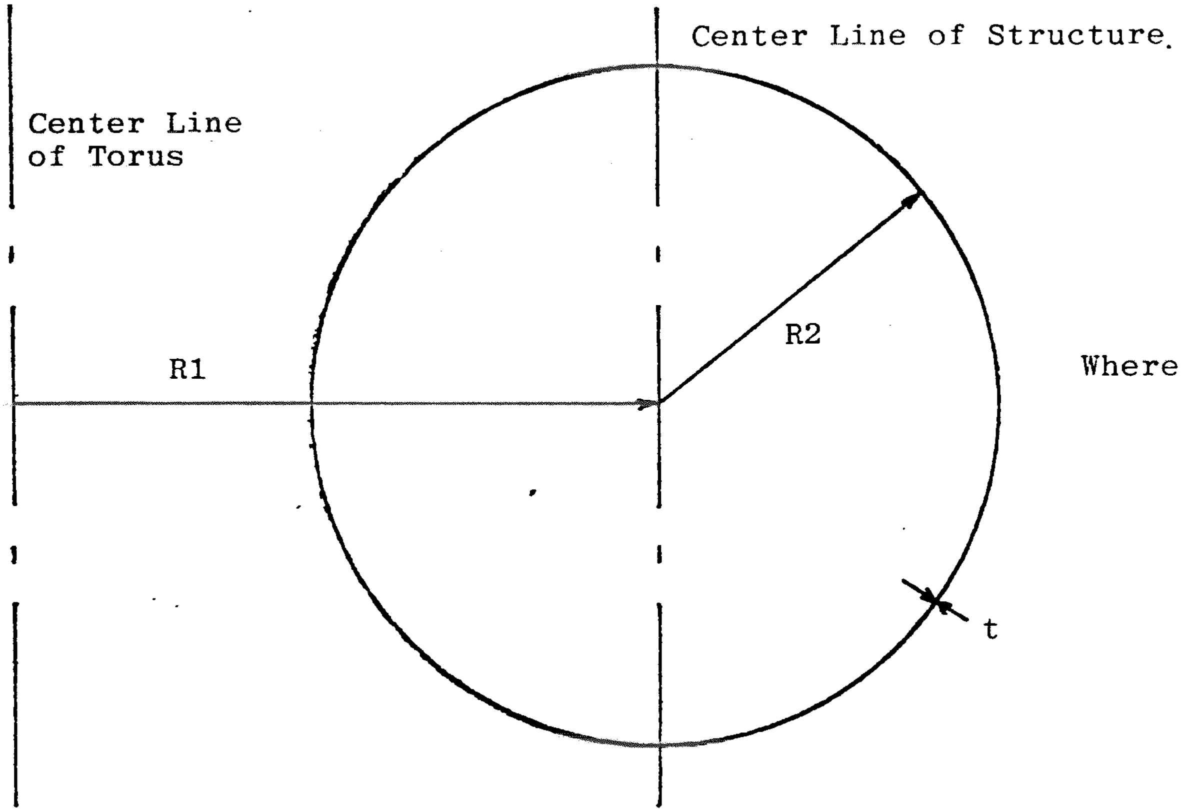
3) Re-run the analysis using three other modeling techniques.

4) Compare the Eigenvalue Summaries and the Eigenvector plots for all five analyses.

Since additional modeling techniques will be used, as mentioned in item 3 above, this study will also check the effects of out of plane nodes in QUAD4 elements and the validity of the RSPLINE element. These topics will be discussed in more detail when the individual models are presented.

STRUCTURE TO BE ANALYZED

The structure chosen is a 30 degree segment of a Torus, see figure 2.



Where: R1=320 inches
R2=170 inches
t =0.6 inches

FIGURE 2

The Material Properties (A36 steel):

Young's Modules	29×10^6 PSI
Poisson's Ratio	.3
Density	7.246×10^{-4} lbm/in^3

ANALYSIS TYPE

All five models where analyzed using Normal Modes (Solution 25) with Dynamic Reduction (RF 25$70). The analyses were run using MSC/NASTRAN Version 48 on a CDC CYBER 175 at University Computing Company. The parameters for the analyses are as follows:

BULK DATA DECK

Input Data Card EIGR Real Eigenvalue Extraction Data

Description: Defines data needed to perform real eigenvalue analysis

Format and Example:

1	2	3	4	5	6	7	8	9	10
EIGR	SID	METHØD	F1	F2	NE	ND	NZ	E	+abc
EIGR	13	INV	1.9	15.6	10	12	0	1.3	ABC
+abc	NØRM	G	C						
+BC	PØINT	32	4						

Field	Contents
SID	Set identification number (Unique integer > 0)
METHØD	Method of eigenvalue extraction, one of the BCD values "INV," "DET," "GIV," "SDET," "UINV," or "UDET" INV - Inverse power method, symmetric matrix operations. SDET - Determinant method, symmetric matrix operations GIV - Givens' method of tridiagonalization MGIV - Modified Givens' Method UINV - Inverse power method, unsymmetric matrix operations UDET - Determinant method, unsymmetric matrix operations DET - Determinant method, unsymmetric matrix operations
F1, F2	Frequency range of interest (Real ≥ 0.0; F1 < F2). (Required for METHØD = "DET," "SDET," "INV," "MGIV," "UDET," or "UINV." Frequency range in which eigenvectors will be computed for METHØD = "GIV," except if ND > 0, in which case the eigenvectors for the first ND positive roots are computed. F2 is used to internally compute a shift parameter for the "MGIV" method.
NE	Estimate of number of roots in range (Required for METHØD = "DET," "INV," "UDET," "SDET," or "UINV") (Integer > 0)
ND	Desired number of roots (eigenvalues and eigenvectors) for METHØD = "DET," "INV," "SDET," "UDET," or "UINV" (Default is 3 NE) (Integer > 0). Desired number of eigenvectors for METHØD = "GIV" and "MGIV" (Default is zero) (Integer ≥ 0)
NZ	Number of free body modes (Optional - used only if METHØD = "DET," "SDET" or "UDET") (Integer ≥ 0)
E	Mass orthogonality test parameter (Default is 0.0, which means no test will be made) (Real > 0.0) This is convergence test for "INV" or "UNIV." Inverse power limits this value to $10^{-4} \leq E \leq 10^{-6}$

NØRM	Method for normalizing eigenvectors, one of the BCD values "MASS," "MAX" or "PØINT"
	MASS - Normalize to unit value of the generalized mass
	MAX - Normalize to unit value of the largest component in the analysis set
	PØINT - Normalize to unit value of the component defined in fields 3 and 4 - defaults to 'MAX" if defined component is zero.
G	Grid or scalar point identification number (Required if and only if NØRM = "PØINT") Integer $\geq$ 0)
C	Component number (One of the integers 16) (Required if and only if NØRM = "PØINT") and G is a geometric grid point)

where METHOD = MGIV
F2 = 1.0
ND = 7
NORM = MAX

BULK DATA DECK

Input Data Card DYNRED

Description: Defines data needed to perform dynamic reduction.

Format and Example:

1	2	3	4	5	6	7	8	9	10
DYNRED	SID	FMAX	NIRV	NIT	IDIR	NCMAX	NCDES	AUTØSEL	
DYNRED	2	20.0						YES	

Field	Contents
SID	Set identification number (Unique Integer > 0)
FMAX	Highest frequency of interest (cycles per unit time)(Real > 0.)
NIRV	Number of initial random vectors (Integer $\geq$ 0 or blank) Default $\geq$ 3
NIT	Number of iteration (100 $\geq$ Integer > 0 or blank) Default = 20
IDIR	Integer used to select starting point to generate initial random vectors (any of the integers 0 thru 9 or blank)

NCMAX — Maximum number of generalized coordinates (Integer > 0). If a q-set exists, the default is size of the q-set, and NCMAX must be blank.

NCDES — Number of generalized coordinates to be used on present computation (Integer > 0 or blank, ≤ NCMAX). Default = NCMAX if AUTØSEL not selected.

AUTØSEL — Request for automatic choice of NCDES if = 'YES' (BCD) Default is a blank field

where: FMAX = 40.0
NIRV = 7
NIT = 8
NCDES = 50
AUTOSEL = YES

BULK DATA DECK

Input Data Card PARAM Parameter

Description: Specifies values for parameters used in DMAP sequences (including rigid formats).

Format and Example :

1	2	3	4	5	6	7	8	9	10
PARAM	N	V1	V2	X	X	X	X	X	
PARAM	IRES	1							

Field — Contents

N — Parameter name (one to eight alphanumeric characters, the first of which is alphabetic)

V1, V2 — Parameter value based on parameter type as follows:

Type	V1	V2
Integer	Integer	Blank
Real, single-precision	Real	Blank
BCD	BCD	Blank
Real, double-precision	Double-precision	Blank
Complex, single-precision	Real	Real
Complex, double-precision	Double-precision	Double-precision

where: N = ASING
V1 = O

This PARAM card is required for AUTOSEL=YES to automatically SPC Scalar points not used in the analysis (Reference: MSC/NASTRAN Applications Manual Sect. 2.4)

THE GENERAL MODEL

The model will be divided into six rings of grid points and five rings of elements, see figure 3.

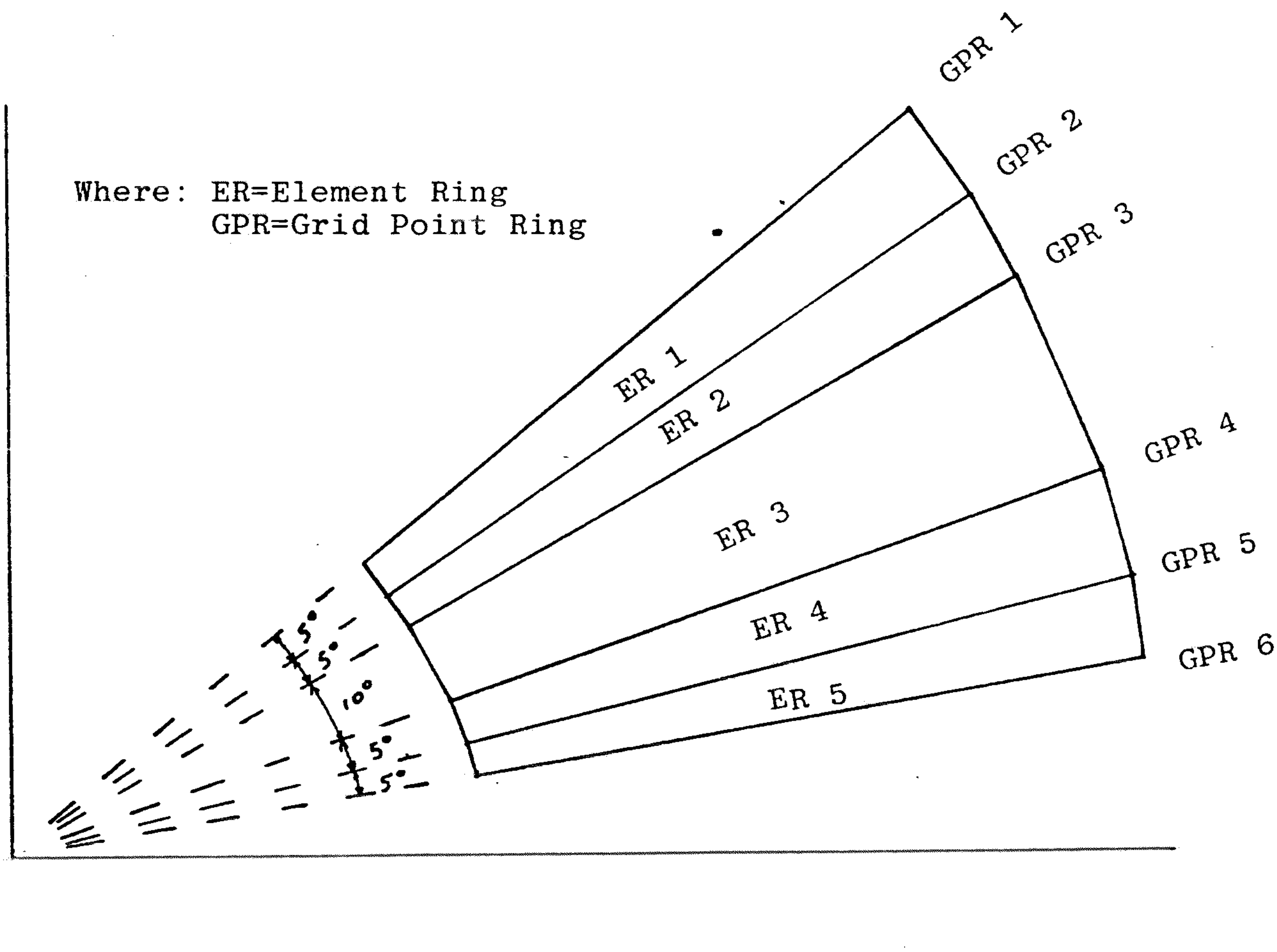


FIGURE 3

Element Rings one and five will consist of 72 QUAD4's each. Element Ring three will consist of 36 QUAD4's. The remaining two rings will vary according to the individual models.

For all models, grid point rings one and six will have cylindric displacement coordinate systems about the center line of the Torus. The use of cylindrical displacement coordinate systems will allow symmetrical boundary conditions by constraining the out-plane translation and the two in-plane rotations.

There will be eight grid points constrained in the radial direction, two at the top and bottom of grid point rings three and four. This prevents the center of the model from translating parallel to the center line of the Torus.

MODEL ONE

The first model uses 108 TRIA3 elements in element ring two and 108 TRIA3 elements in element ring four. TRIA3's are used to connect the fine mesh of element rings one and five to the course mesh of element ring three. All the displacement coordinate systems will be cylindrical.

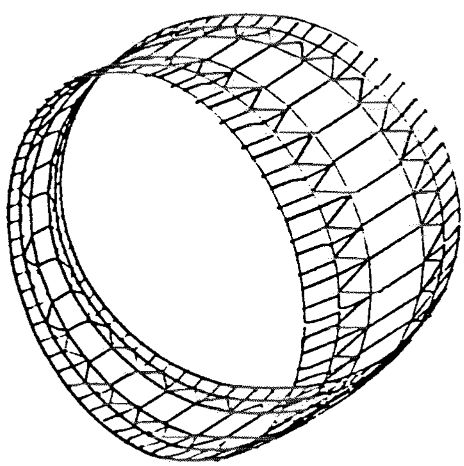


FIGURE 4

MODEL TWO

The second model will be similar to model one except CELAS2 elements will be used to increase the stiffness of all rotational terms below 1 x 10^6. The diagonal of the Stiffness Matrix shows that the majority of rotational terms are in the order of 1 x 10^7. All the rotational terms about the radial axes are below 10^6; therefore, 216 CELAS2 elements will be used.

MODEL THREE

The third model is similar to model one except that grid point rings two, three, four and five have rectangular displacement coordinate systems. Of course, the eight grid points constrained for translation in the radial direction will need cylindrical displacement coordinate systems. This model does not contain CELAS2 elements.

MODEL FOUR

The fourth model uses 72 QUAD4 elements in element ring number two and 72 QUAD4 elements in element ring number four. RSPLINE elements are included in the model to connect the fine mesh of element rings two and four to the course mesh of element ring three. No CELAS2 elements are used and all displacement coordinate systems are cylindrical.

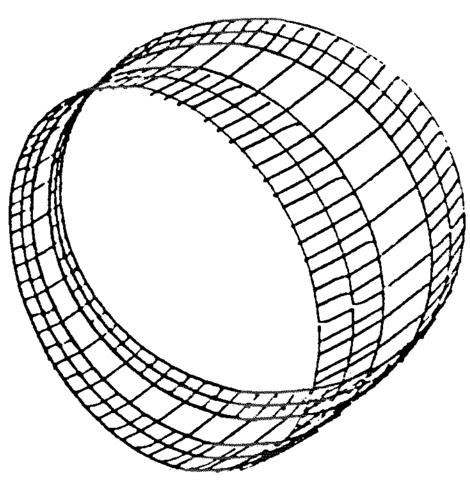


FIGURE 5

MODEL FIVE

The fifth model will similar to model four except, the dependent grid points of the RSPLINE elements will have a radius of 169.353 inches instead of 170 inches. This smaller radius will put each of these grid points in the plane of the elements of element ring three. Consequently, all of the QUAD4 elements in element rings two and four will have out of plane nodes.

RESULTS

The following table shows the maximum percent deviation of all five models for the first twelve Modes.

Mode Number	Eigenvalue	Cycles
1	6.2%	3.0%
2	2.1%	1.0%
3	3.9%	1.9%
4	2.1%	1.0%
5	6.7%	3.3%
6	3.9%	1.9%
7	9.0%	4.4%
8	8.7%	4.2%
9	7.9%	3.9%
10	7.2%	3.5%
11	4.8%	2.4%
12	5.8%	2.9%

Model five has the lowest value for Eigenvalues and Cycles in all twelve modes. The highest values for Eigenvalues and Cycles alternated between Models Two and Four. The Eigenvalue Summaries and Mode Shapes can be seen in the following sheets.

MODEL ONE FEBRUARY 8, 1979 NASTRAN 12/ 4/78

R E A L   E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZ STIFFNES
1	1	3.456012E+02	1.859035E+01	2.958747E+00	1.000158E+00	3.456557E
2	2	2.202586E+03	4.693171E+01	7.469415E+00	1.000194E+00	2.203012E
3	3	1.380761E+04	1.175058E+02	1.870163E+01	1.000238E+00	1.381089E
4	4	1.568338E+04	1.252333E+02	1.993150E+01	1.001122E+00	1.570098E
5	5	2.893684E+04	1.701083E+02	2.707356E+01	1.894565E+00	5.482273E
6	6	3.117313E+04	1.765591E+02	2.810026E+01	2.557163E+00	7.971476E
7	7	3.591476E+04	1.895119E+02	3.016176E+01	1.986736E+00	7.135316E
8	8	3.871795E+04	1.957688E+02	3.115672E+01	0.0	0.0
9	9	5.654556E+04	2.377931E+02	3.784595E+01	0.0	0.0
10	10	5.859520E+04	2.420644E+02	3.852575E+01	0.0	0.0
11	11	5.950122E+04	2.439287E+02	3.882246E+01	0.0	0.0
12	12	6.916190E+04	2.629865E+02	4.185560E+01	0.0	0.0
13	13	7.072116E+04	2.659345E+02	4.232479E+01	0.0	0.0
14	14	7.604986E+04	2.757714E+02	4.389038E+01	0.0	0.0
15	15	7.969898E+04	2.823101E+02	4.493105E+01	0.0	0.0
16	21	8.598148E+04	2.932260E+02	4.666837E+01	0.0	0.0
17	18	9.949627E+04	3.154303E+02	5.020229E+01	0.0	0.0
18	20	1.052448E+05	3.244146E+02	5.163219E+01	0.0	0.0
19	19	1.090009E+05	3.301528E+02	5.254546E+01	0.0	0.0
20	17	1.134251E+05	3.367865E+02	5.360124E+01	0.0	0.0
21	16	1.278259E+05	3.575275E+02	5.690227E+01	0.0	0.0

MODEL TWO FEBRUARY 8, 1979 NASTRAN 12/ 4/78

R E A L   E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZ STIFFNES
1	1	3.462355E+02	1.860741E+01	2.961461E+00	1.000157E+00	3.462899E
2	2	2.206010E+03	4.696819E+01	7.475219E+00	1.000191E+00	2.206431E
3	3	1.388304E+04	1.178263E+02	1.875264E+01	1.000268E+00	1.388676E
4	4	1.580696E+04	1.257258E+02	2.000988E+01	1.001109E+00	1.582449E
5	5	2.904305E+04	1.704202E+02	2.712322E+01	2.091086E+00	6.073152E
6	6	3.136741E+04	1.771085E+02	2.818769E+01	2.345158E+00	7.356155E
7	7	3.600138E+04	1.897403E+02	3.019811E+01	2.121564E+00	7.637926E
8	8	3.879582E+04	1.969665E+02	3.134820E+01	0.0	0.0
9	9	5.666123E+04	2.380362E+02	3.788464E+01	0.0	0.0
10	10	5.898180E+04	2.428617E+02	3.865264E+01	0.0	0.0
11	11	5.969141E+04	2.443183E+02	3.888446E+01	0.0	0.0
12	13	7.008167E+04	2.647294E+02	4.213300E+01	0.0	0.0
13	12	7.085783E+04	2.661913E+02	4.236567E+01	0.0	0.0
14	14	7.684504E+04	2.772094E+02	4.411924E+01	0.0	0.0
15	15	8.160203E+04	2.856607E+02	4.546431E+01	0.0	0.0
16	17	8.681908E+04	2.946508E+02	4.689513E+01	0.0	0.0
17	21	1.001282E+05	3.164304E+02	5.036147E+01	0.0	0.0
18	20	1.058069E+05	3.252797E+02	5.176987E+01	0.0	0.0
19	19	1.117288E+05	3.342585E+02	5.319890E+01	0.0	0.0
20	18	1.204038E+05	3.469925E+02	5.522557E+01	0.0	0.0
21	16	1.329682E+05	3.646480E+02	5.803554E+01	0.0	0.0

MODEL THREE FEBRUARY 8, 1979 NASTRAN 12/ 4/78

R E A L E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERA STIFF
1	1	3.456012E+02	1.859035E+01	2.958747E+00	1.236215E+00	4.2723
2	2	2.202586E+03	4.693171E+01	7.469415E+00	1.236215E+00	2.7228
3	3	1.380761E+04	1.175058E+02	1.870163E+01	1.034033E+00	1.4277
4	4	1.568338E+04	1.252333E+02	1.993150E+01	1.035170E+00	1.6234
5	5	2.893684E+04	1.701083E+02	2.707358E+01	1.882810E+00	5.4482
6	6	3.117312E+04	1.765591E+02	2.810026E+01	1.349351E+00	4.2063
7	7	3.591465E+04	1.895116E+02	3.016171E+01	2.928922E+00	1.0519
8	8	3.871809E+04	1.967691E+02	3.131678E+01	0.0	0.0
9	9	5.652308E+04	2.377458E+02	3.783843E+01	0.0	0.0
10	11	5.884062E+04	2.425708E+02	3.860635E+01	0.0	0.0
11	10	5.997274E+04	2.448933E+02	3.897598E+01	0.0	0.0
12	14	6.792116E+04	2.606169E+02	4.147847E+01	0.0	0.0
13	15	7.105311E+04	2.665579E+02	4.242401E+01	0.0	0.0
14	18	7.492993E+04	2.737333E+02	4.356601E+01	0.0	0.0
15	20	8.416832E+04	2.901178E+02	4.617368E+01	0.0	0.0
16	21	8.614532E+04	2.935052E+02	4.671281E+01	0.0	0.0
17	19	9.075936E+04	3.012629E+02	4.794749E+01	0.0	0.0
18	17	1.011820E+05	3.180912E+02	5.062579E+01	0.0	0.0
19	16	1.062047E+05	3.258906E+02	5.186710E+01	0.0	0.0
20	13	1.243872E+05	3.526857E+02	5.613168E+01	0.0	0.0
21	12	1.335264E+05	3.654126E+02	5.815722E+01	0.0	0.0

MODEL FOUR FEBRUARY 8, 1979 NASTRAN 12/ 4/78

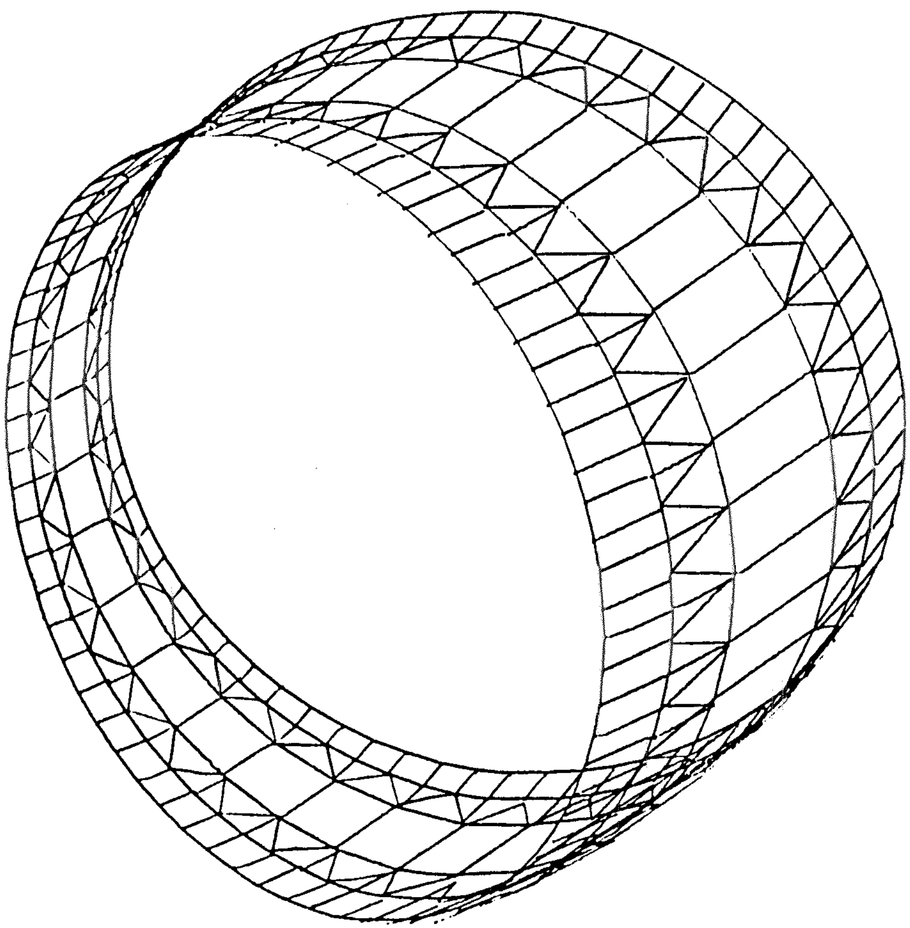
R E A L E I G E N V A L U E S

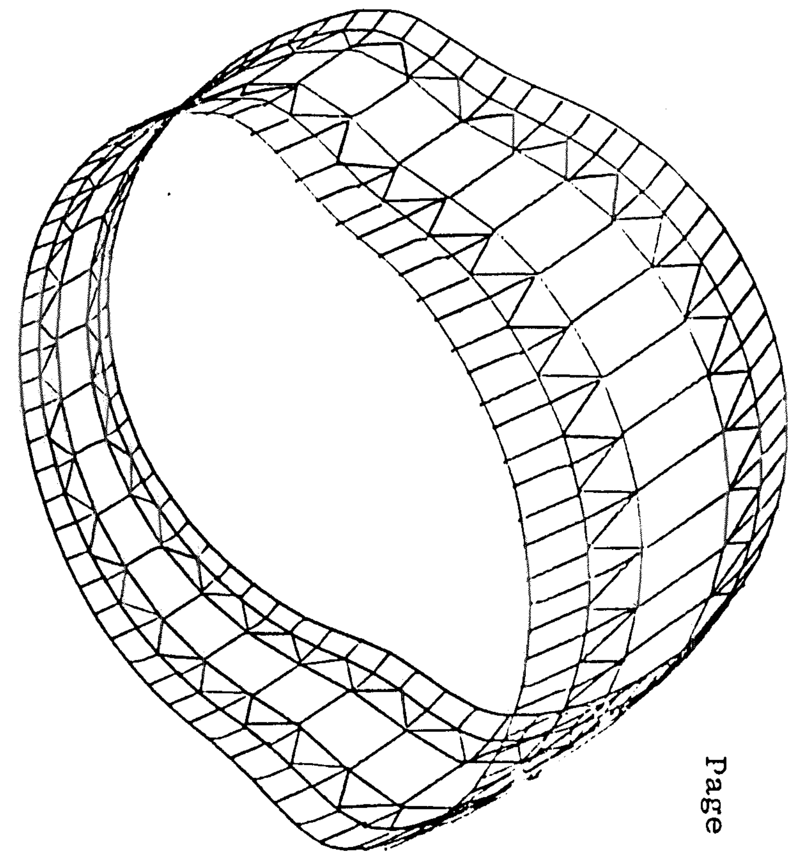
MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERA STIFF
1	1	3.369623E+02	1.835653E+01	2.921533E+00	1.000272E+00	3.3705
2	2	2.231579E+03	4.723959E+01	7.518415E+00	1.000291E+00	2.2322
3	3	1.353524E+04	1.163410E+02	1.851625E+01	1.065086E+00	1.4416
4	4	1.587316E+04	1.259887E+02	2.005173E+01	1.077174E+00	1.7098
5	5	2.748189E+04	1.657766E+02	2.638417E+01	1.281955E+00	3.5230
6	6	3.142314E+04	1.772657E+02	2.821272E+01	1.255785E+00	3.9460
7	7	3.347242E+04	1.829547E+02	2.911814E+01	1.819431E+00	6.0900
8	8	3.658421E+04	1.912700E+02	3.044157E+01	0.0	0.0
9	9	5.272779E+04	2.296253E+02	3.654601E+01	0.0	0.0
10	10	5.552828E+04	2.356444E+02	3.750397E+01	0.0	0.0
11	11	5.771472E+04	2.402389E+02	3.823521E+01	0.0	0.0
12	16	6.683647E+04	2.585275E+02	4.114593E+01	0.0	0.0
13	17	6.875451E+04	2.622108E+02	4.173215E+01	0.0	0.0
14	19	7.572792E+04	2.751871E+02	4.379738E+01	0.0	0.0
15	20	7.614124E+04	2.759370E+02	4.391674E+01	0.0	0.0
16	21	8.300459E+04	2.881052E+02	4.585336E+01	0.0	0.0
17	18	8.703103E+04	2.950102E+02	4.695234E+01	0.0	0.0
18	15	1.001422E+05	3.164526E+02	5.036499E+01	0.0	0.0
19	14	1.045460E+05	3.233358E+02	5.146049E+01	0.0	0.0
20	13	1.117459E+05	3.342842E+02	5.320298E+01	0.0	0.0
21	12	1.345557E+05	3.668183E+02	5.838095E+01	0.0	0.0

R E A L E I G E N V A L U E S

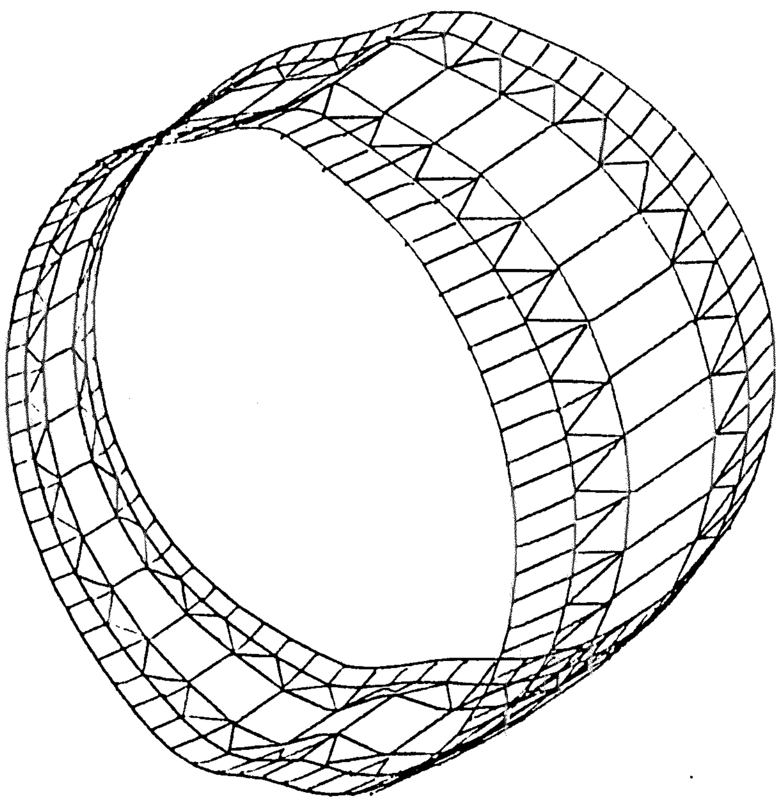
MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZ STIFFNES
1	1	3.261508E+02	1.805965E+01	2.874282E+00	1.000259E+00	3.262353E
2	2	2.186548E+03	4.676054E+01	7.442171E+00	1.000277E+00	2.187153E
3	3	1.335766E+04	1.155754E+02	1.839439E+01	1.126060E+00	1.504153E
4	4	1.554841E+04	1.246932E+02	1.984555E+01	1.131319E+00	1.759021E
5	5	2.720715E+04	1.649459E+02	2.625196E+01	1.294219E+00	3.521201E
6	6	3.024580E+04	1.739132E+02	2.767915E+01	1.110080E+00	3.357526E
7	7	3.302474E+04	1.817271E+02	2.892277E+01	1.627507E+00	5.374799E
8	8	3.570525E+04	1.889583E+02	3.007355E+01	0.0	0.0
9	9	5.250746E+04	2.291451E+02	3.646957E+01	0.0	0.0
10	10	5.501305E+04	2.345485E+02	3.732957E+01	0.0	0.0
11	11	5.694717E+04	2.386361E+02	3.798011E+01	0.0	0.0
12	13	6.621440E+04	2.573216E+02	4.095400E+01	0.0	0.0
13	14	6.908743E+04	2.628449E+02	4.183306E+01	0.0	0.0
14	18	7.422628E+04	2.724450E+02	4.336097E+01	0.0	0.0
15	19	7.698915E+04	2.774692E+02	4.416059E+01	0.0	0.0
16	21	8.504014E+04	2.916164E+02	4.641220E+01	0.0	0.0
17	20	8.586212E+04	2.930224E+02	4.663596E+01	0.0	0.0
18	17	1.000522E+05	3.163103E+02	5.034235E+01	0.0	0.0
19	16	1.069785E+05	3.270757E+02	5.205572E+01	0.0	0.0
20	15	1.114568E+05	3.338514E+02	5.313411E+01	0.0	0.0
21	12	1.269864E+05	3.563515E+02	5.671511E+01	0.0	0.0

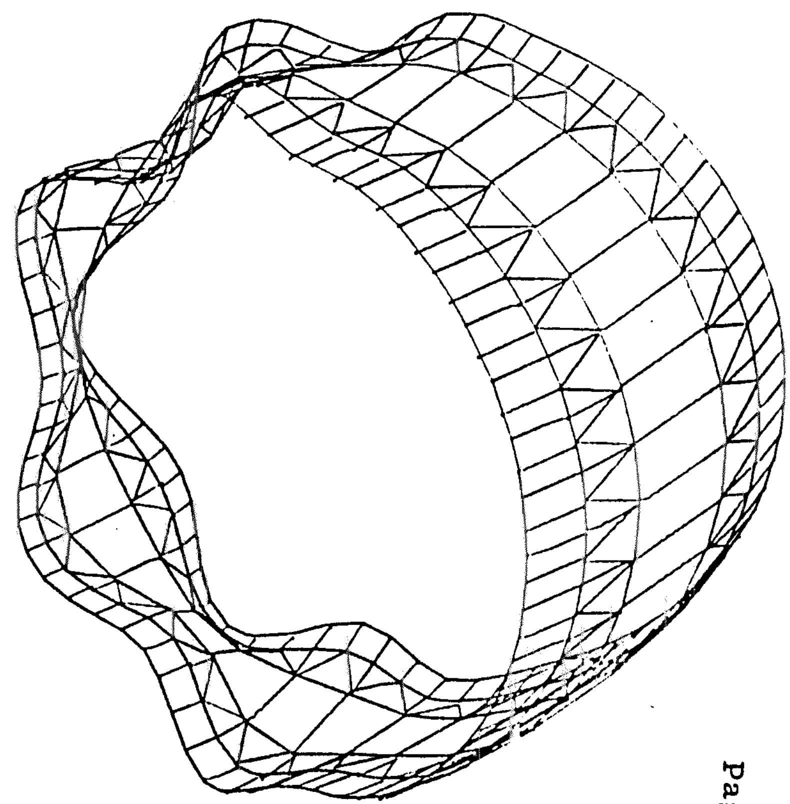
MODEL ONE





MODEL ONE





MODEL ONE

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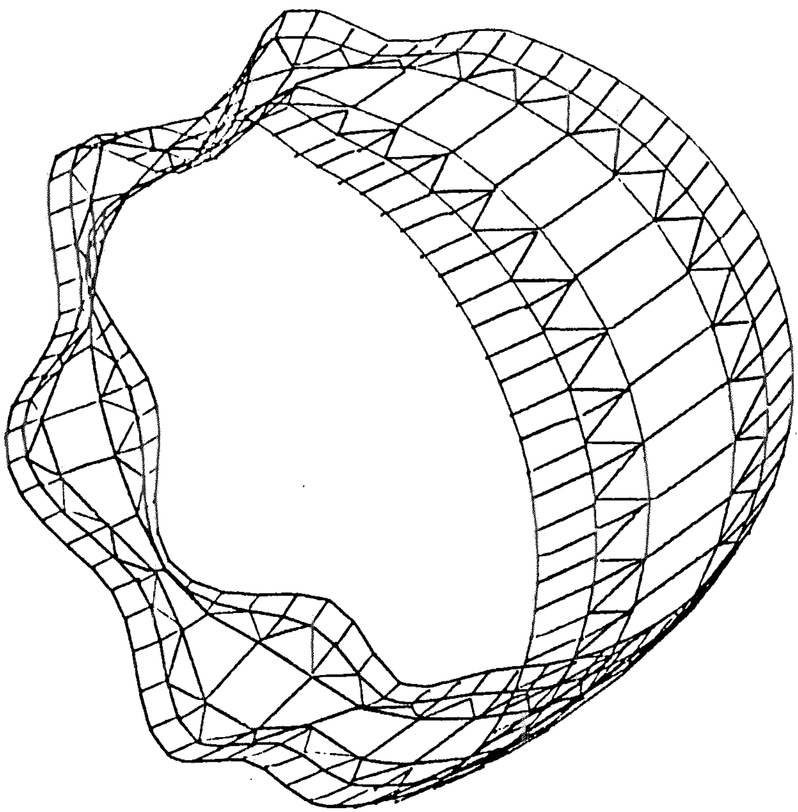
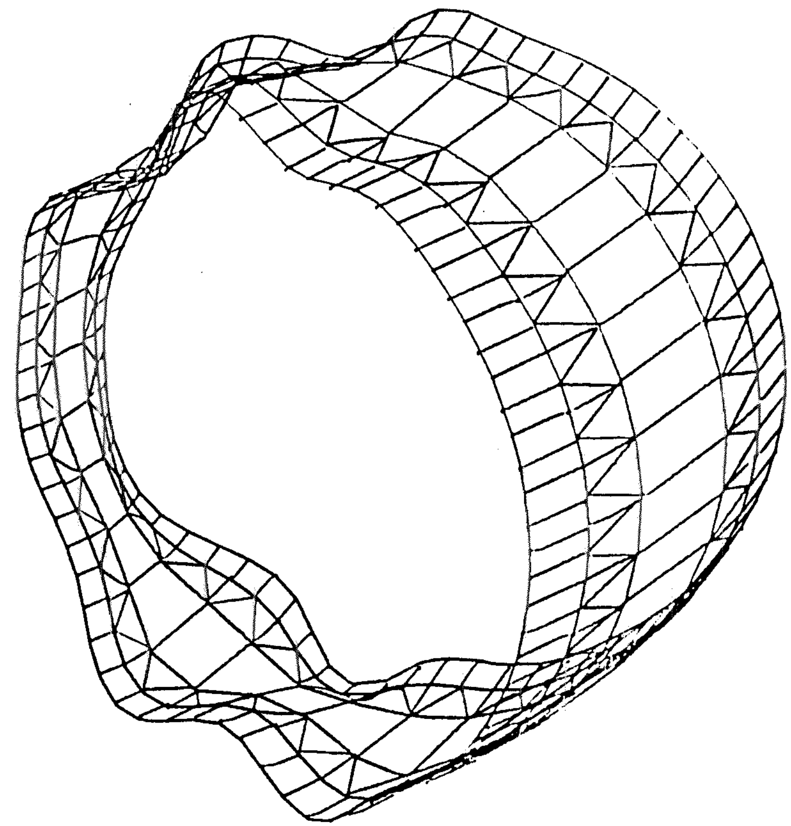


MODEL ONE

MODAL DEFOR. SUBCASE 4 MODE 4 FREQ. 13.9316D

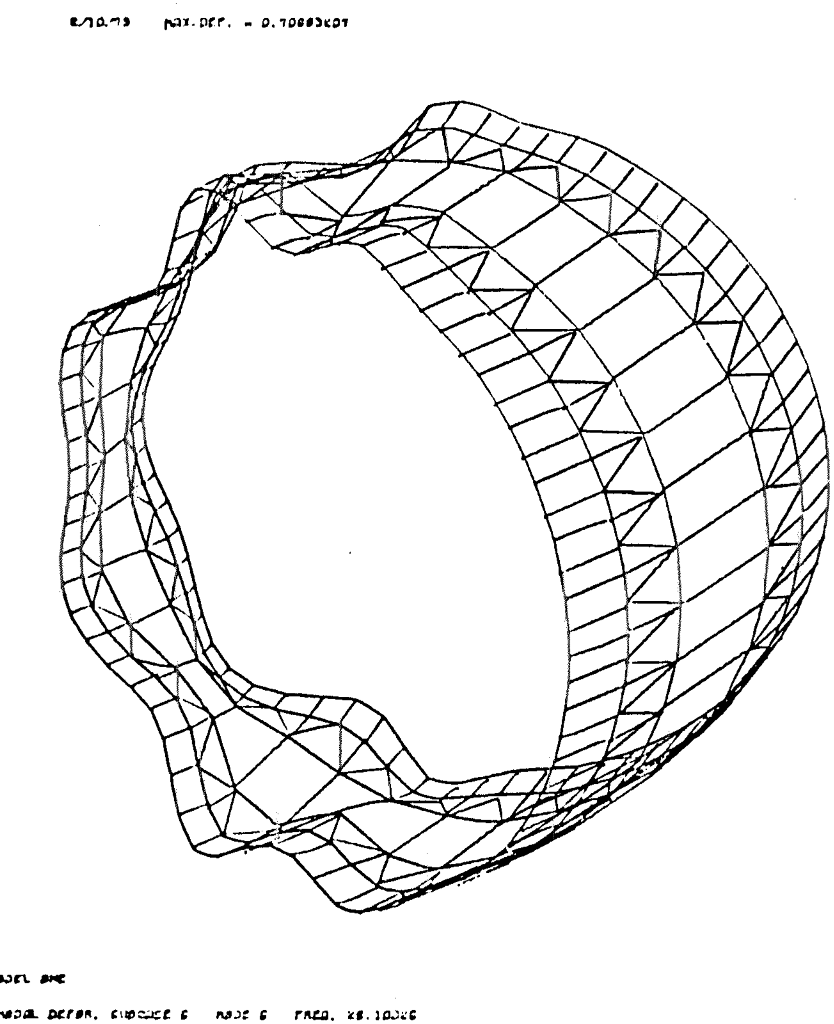
5 6 8/10/79 MAX-DEF. = 0.56493113



MODEL ONE

MODAL DEFOR. SUBCASE 5 MODE 5 FREQ. 27.0736E

MODEL ONE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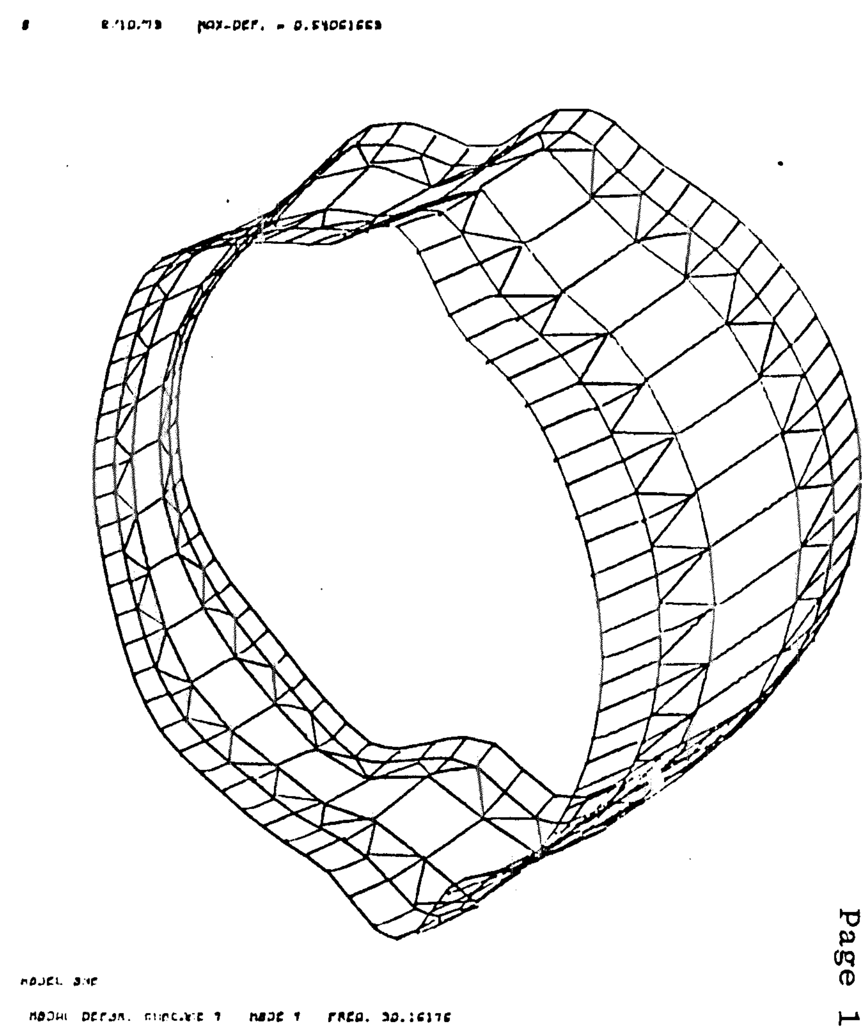
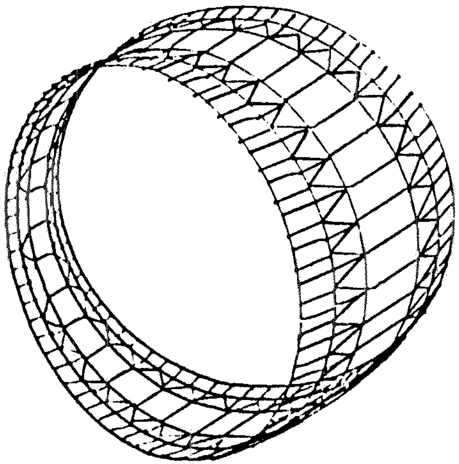
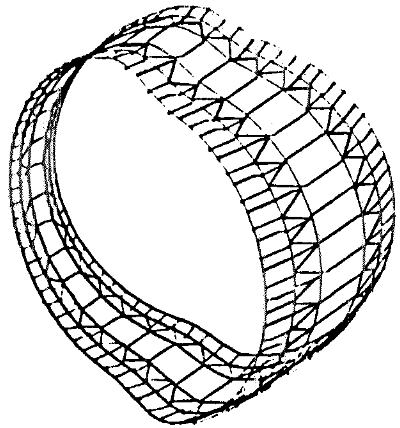
MAX-DEF.
MODAL DEFOR.
MODE 6
FREQ.


8
MAX-DEF.
MODAL DEFOR.
MODE 7
FREQ. 30.16176

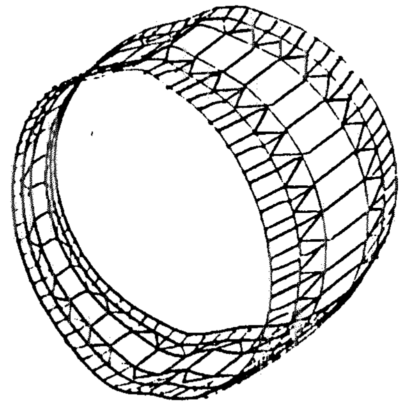
MODEL TWO



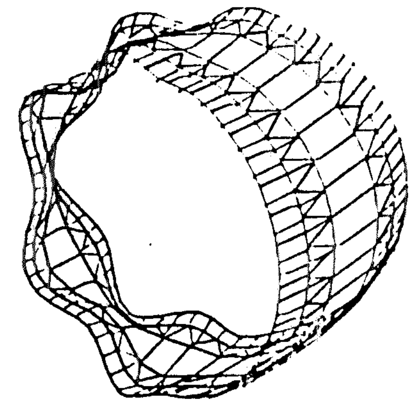
UNDEFORMED



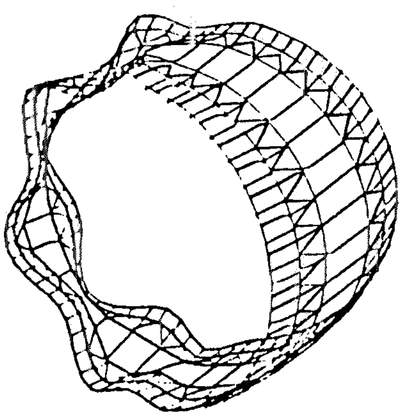
MODE 1



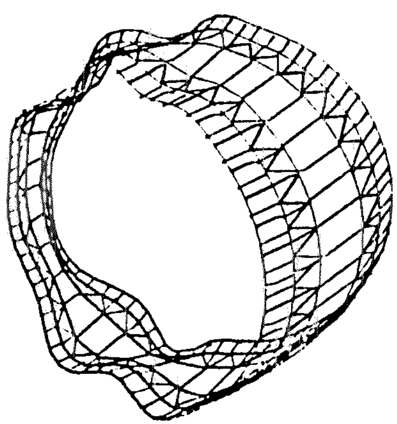
MODE 2



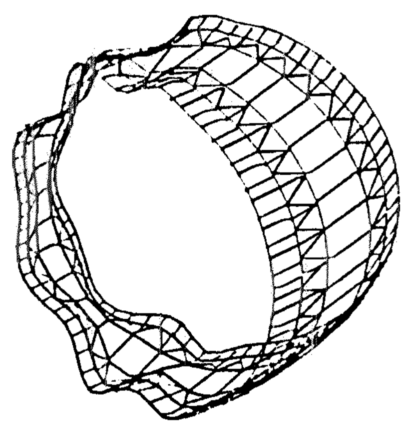
MODE 3



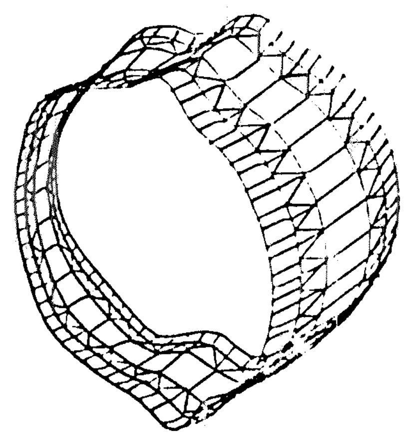
MODE 4



MODE 5

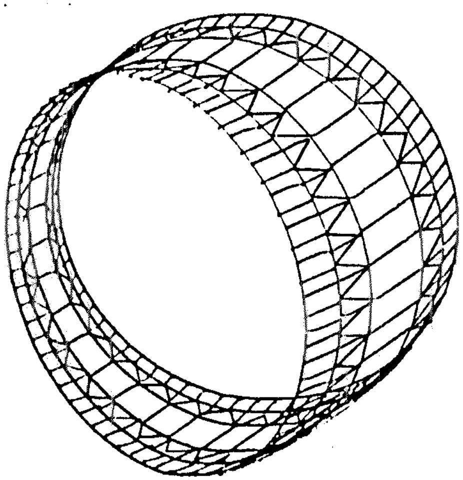


MODE 6



MODE 7

MODEL THREE



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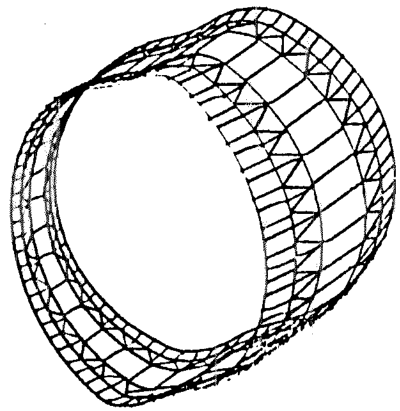
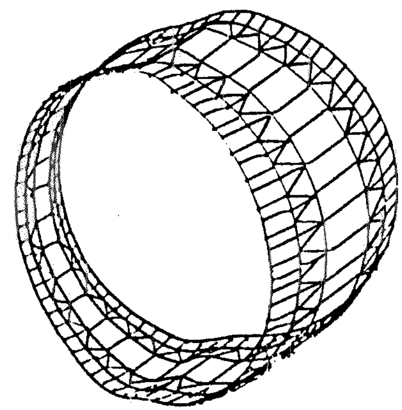


MODE 1



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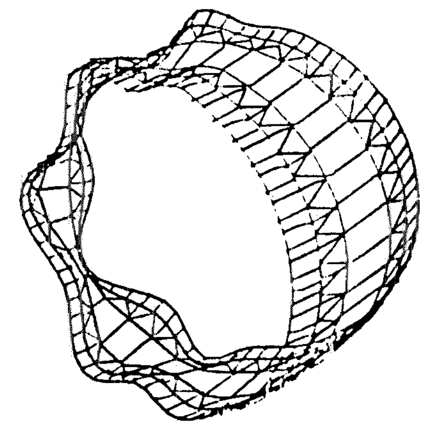

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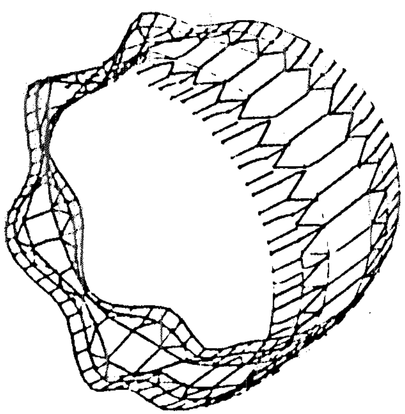

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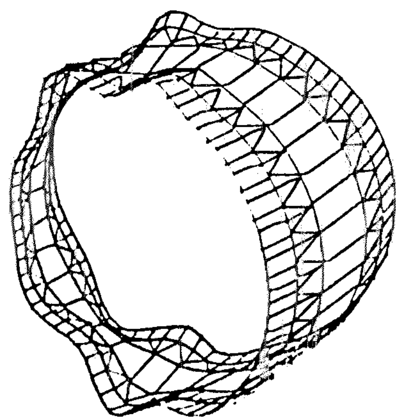
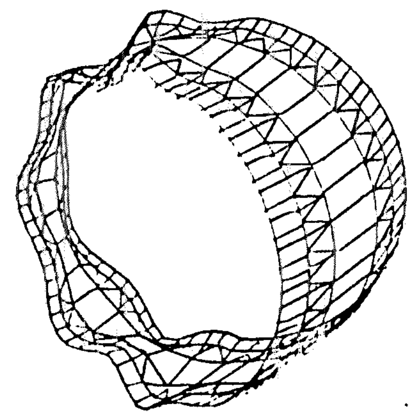
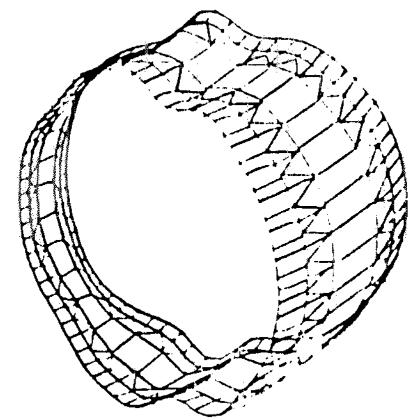


MODE 5

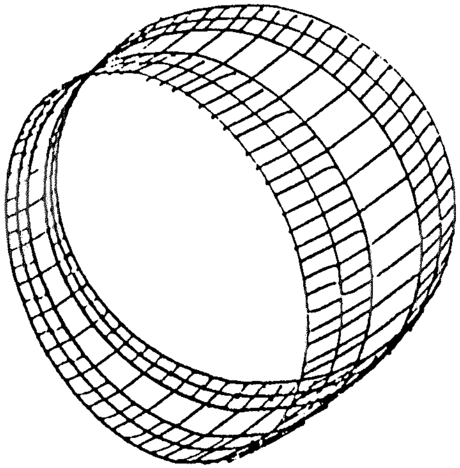


MODE 6



MODE 7

<u>MODEL FOUR</u>



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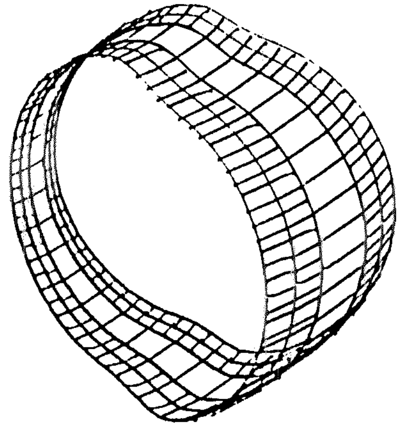
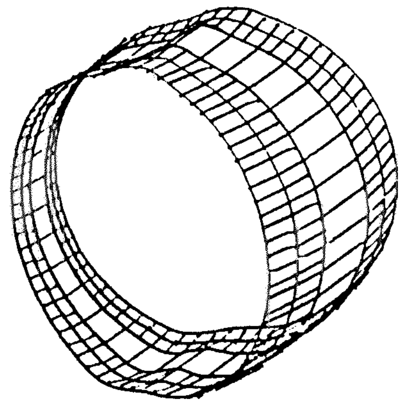


MODE 1



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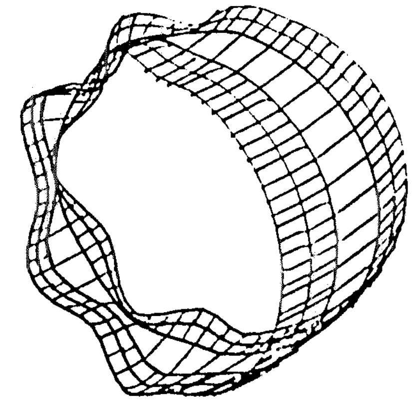

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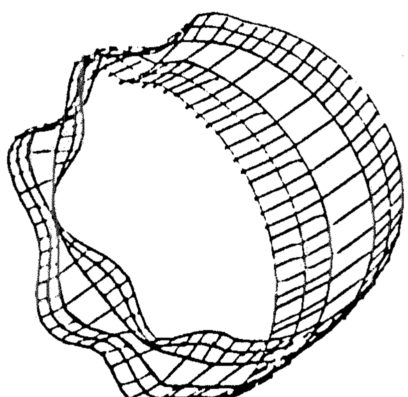

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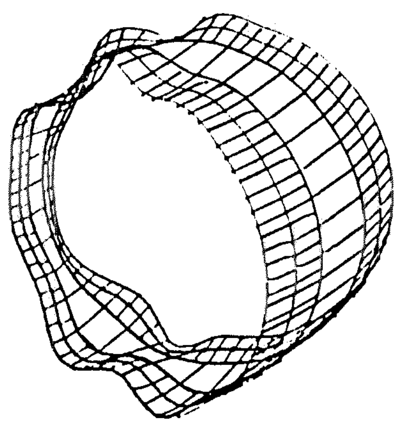

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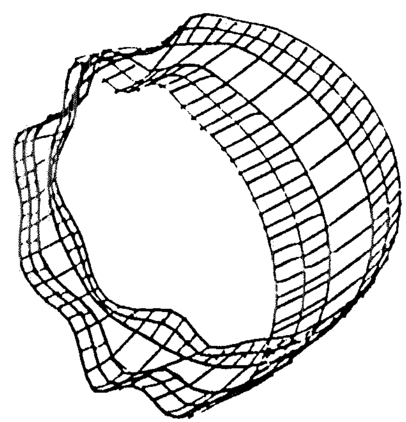

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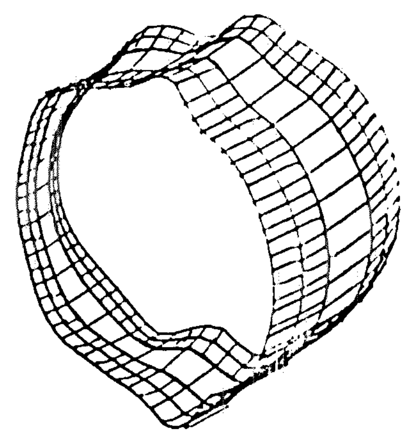
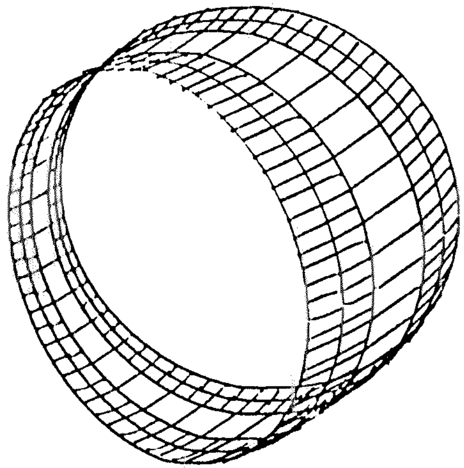


MODE 7

MODEL FIVE



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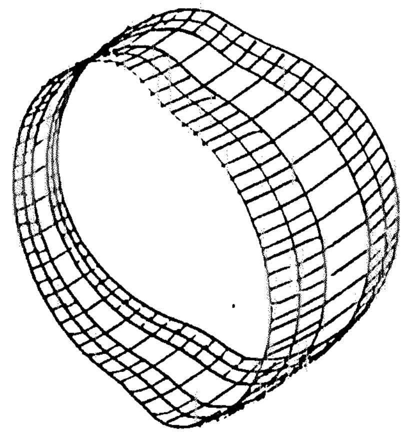
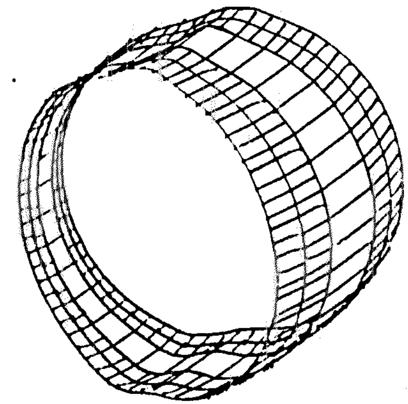


MODE 1



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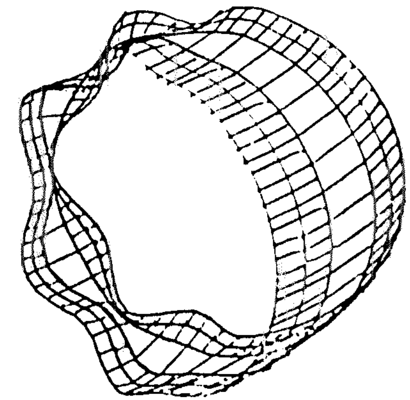

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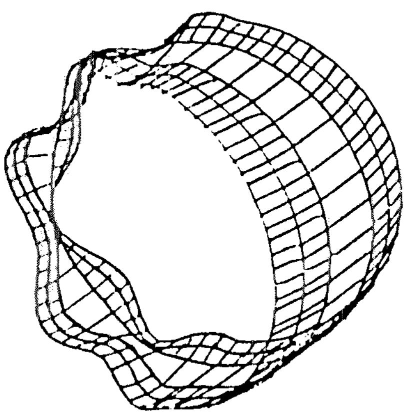

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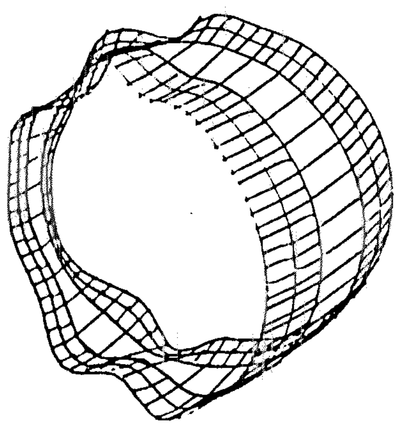

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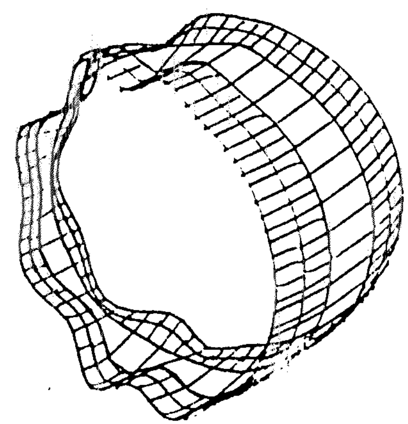
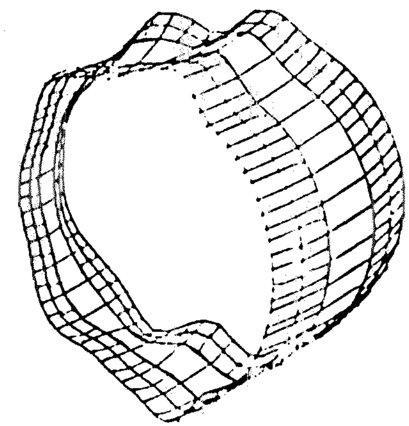


MODE 6



MODE 7

CONCLUSION

It can be seen that MSC/NASTRAN yields highly consistant results regardless of the modeling technique chosen. Also the use of RSPLINE elements and slightly out of plane nodes in QUAD4 elements have relatively little effect on the final results.