STATIC ANALYSIS WITH MODIFIED ELEMENTS

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1. INTRODUCTION

A new efficiency improvement is described for recalculating the static deflections after a change of a few structural elements. MSC/NASTRAN previously had modules (MØDTA and MØDEMG) which calculate the change in the stiffness matrix due to a change of element input data. The new improvement is an alternate solution procedure which avoids the decomposition of the modified matrix. The new procedure should be efficient only if the number of degrees of freedom associated with the changed elements is small, say less than one-fourth of the bandwidth of the stiffness matrix.

2. USER INTERFACE

The new method is designed for RESTARTS in SØL 24 (static analysis using EMG). No special ALTER's are necessary in the initial CHKPNT run. The user must observe the following rules and limitations:

1. Elements may be changed by modifying element connections or property bulk data cards. This is done on RESTART by deleting and/or adding the necessary bulk data cards.

2. Geometry (GRID bulk data cards) and constraint (SPC, MPC, ØMIT) changes are not allowed.

3. The changed elements may not be connected to degrees of freedom which are dependent in multipoint constraints (U_m) or dependent in matrix partitioning (U_o).

4. GENEL elements and DMIG input may not be changed.

5. Loads may be changed. This includes element loads (gravity, pressure, or thermal) associated with the element changes.

6. SPC and MPC changes between subcases, which would result in looping, are forbidden. Use SØL 24,1 which removes loops from the solution sequence.

7. An ALTER, given in section 3 of this paper, must be added to the RESTART run.

The major use of this alter is to allow changes to a few elements in an efficient manner. The superelement technology offers an alternate method to achieve this goal.

3. THE ALTER PACKAGE

The RF ALTER is shown below. This alter will be added to the MSC/NASTRAN ALTER library to be accessible to the user community.

```
$ BEGIN RF ALTER 24$XX
$ EFFICIENCY FOR RESTART WITH ELEMENT CHANGES
$ RIGID FORMAT 24      OCT 1978      RLH
$  1. THIS ALTER IS INTENDED TO ALLOW AN EFFICIENT SOLUTION WHEN ONLY A
$     FEW ELEMENTS ARE CHANGED ON RESTART.
$  2. THIS ALTER IS USED ONLY FOR THE RESTART, NO ALTER IS NEEDED FOR
$     THE CHKPNT RUN.  USE THE BULK DATA EDITING FEATURE TO CHANGE DATA.
$  3. YOU MAY CHANGE ELEMENT (CONNECTION AND PROPERTY), PELAS, AND LOAD
$     BULK DATA CARDS.
$  4. YOU MAY NOT CHANGE CELAS, GRID, MATERIAL, CONSTRAINT, GENEL, OR
$     OMIG BULK DATA CARDS.
$  5. USE SOL 24,1.  INERTIA RELIEF (SUPORT) AND LOOPING (SUBCASES WITH
$     CONSTRAINT CHANGES) ARE NOT ALLOWED.
$  6. CHANGED ELEMENTS MAY NOT BE CONNECTED TO DEGREES OF FREEDOM IN THE
$     M-SET OR O-SETS.
$
ALTER 6 $
COND     RFERR,NOMOD $  METHOD REQUIRES NOMOD = +1
ALTER 112,144 $
ALTER 150,159 $
SSG3     LLL,,KLL,PL,LOO,,KOO,PO/ULW,UOOV,,,/V,N,OMIT/-1 $
VEC      USET/VECGL/C,N,G/C,N,COMP/C,N,L $
PARTN    KGGO,VECGL,/,,,KLLO/ $
DIAGONAL KLLO/VECLX/ $
PARTN    KLLO,VECLX,/,,KLX,/C,N,1 $
FBS      LLL,,KLX/GLX/ $
PARTN    GLX,,VECLX/,GXX,,/C,N,1 $
DIAGONAL GXX/IXX/C,N,SQUARE/C,N,0. $
ADD      IXX,GXX/IIXX/ $
PARTN    ULW,,VECLX/,UXW,,/C,N,1 $
SOLVE    IIXX,UXW/UXV/ $
MPYAD    GLX,UXV,ULW/ULV//C,N,-1 $
$ END RF ALTER 24$XX
```

4. THEORETICAL DEVELOPMENT

The equilibrium equation for static analysis is

$$\left[K_{aa}\right] \left\{U_a\right\} = \left\{P_a\right\} \quad . \tag{1}$$

In (1), K_{aa} is the stiffness matrix, U_a is the solution displacement vector (or matrix of vectors), and P_a is the load vector (or matrix of vectors). The NASTRAN subscript "a" is used instead of "ℓ", since SUPØRT degrees of freedom are excluded. Assume that (1) has been solved, and it is required to solve again with a change of K_{aa}, and possibly P_a. We assume that the decomposition factors of K_{aa} are available from the previous solution.

The modified equation is

$$\left[K_{aa} + K^*_{aa}\right] \left\{U^*_a\right\} = \left\{P^*_a\right\} \quad . \tag{2}$$

K^*_{aa} represents the change in stiffness, and should be very sparse. P^*_a and U^*_a are the new loads and displacements. Define the x-set degrees of freedom to be the subset of a, for which there are stiffness changes. Partition K^*_{aa} to give

$$\left[K_{aa}\right] \left\{U^*_a\right\} + \left[K^*_{ax}\right] \left\{U^*_x\right\} = \left\{P^*_a\right\} \quad . \tag{3}$$

Premultiply (3) by K^{-1}_{aa} (i.e. do FBS operations on K^*_{ax} and P^*_a) to get

$$\left\{U_a{}^*\right\} = - \left[G_{ax}\right] \left\{U^*_x\right\} + \left\{U_a{}^1\right\} \tag{4}$$

Where

$$\left[K_{aa}\right] \left[G_{ax}\right] = \left[K^*_{ax}\right] \quad , \text{ and} \tag{5}$$

$$\left[K_{aa}\right] \left\{U_a{}^1\right\} = \left\{P^*_a\right\} \quad . \tag{6}$$

The x-set rows of equation (4) yields

$$\left[I + G_{xx}\right] \left\{U_x^*\right\} = \left\{U_x^1\right\} \tag{7}$$

Equation (7) can be solved for the displacements in the x-set. The complete a-set displacements are found from equation (4).

The operations required to solve equation (2) have been shown. They involve FBS operations on a matrix with x-size columns, a solution of x-size set of unsymmetric simultaneous equations, plus a few other operations.

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PURPOSE: REANALYZE A STRUCTURE AFTER A STIFFNESS CHANGE. THE CHANGE IS ASSUMED TO AFFECT ONLY A FEW STRUCTURAL ELEMENTS.

METHOD: THE CHECKPOINT-RESTART PROCEDURE OF MSC/NASTRAN IS USED. ELEMENTS ARE CHANGED IN THE STANDARD MANNER (DELETE, REENTER).

ALTERS: NO SPECIAL PROCEDURE IS NEEDED FOR CHECKPOINT RUN. AN ALTER IS REQUIRED FOR RESTART. NO SPECIAL DATA CARDS ARE USED.

RULES AND LIMITATIONS

- THINGS THAT MAY BE CHANGED:

  1. ELEMENT CONNECTION CARDS (EXCEPT CELAS)
  2. ELEMENT PROPERTY CARDS (INCLUDING PELAS)
  3. LOAD DATA (INCLUDING GRAVITY, PRESSURE, AND THERMAL LOADS OF CHANGED ELEMENTS)
  4. THE NUMBER OF LOADING CONDITIONS (SUBCASES)

- THINGS THAT MAY NOT BE CHANGED:

  1. COORDINATE SYSTEM AND GRID DATA
  2. MATERIAL DATA
  3. GENEL AND DMIG
  4. CONSTRAINTS (SPC, MPC, ØMIT)
  5. ELEMENTS WHICH ARE CONNECTED TO DEGREES OF FREEDOM IN THE m-SET OR o-SET

- SPECIAL RESTRICTIONS:

  1. SUBCASES FOR CONSTRAINT CHANGES (SPC, MPC) ARE FORBIDDEN.
  2. INERTIA RELIEF IS NOT ALLOWED.

THEORY MATRIX FACTORING

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{33} & 0 & K_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{53} & 0 & K_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ K_{33} & K_{35} \\ 0 & 0 \\ K_{53} & K_{55} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ K_{33} & K_{35} \\ 0 & 0 \\ K_{53} & K_{55} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} U_3 \\ U_5 \end{Bmatrix}$$

THEORY FOR MODIFIED STIFFNESS

$$[K_{aa}]\{U_a\} = \{P_a\} \tag{1}$$

$$[K_{aa} + K^*_{aa}]\{U^*_a\} = \{P^*_a\} \tag{2}$$

$$[K_{aa}]\{U^*_a\} + [K^*_{ax}]\{U^*_x\} = \{P^*_a\} \tag{3}$$

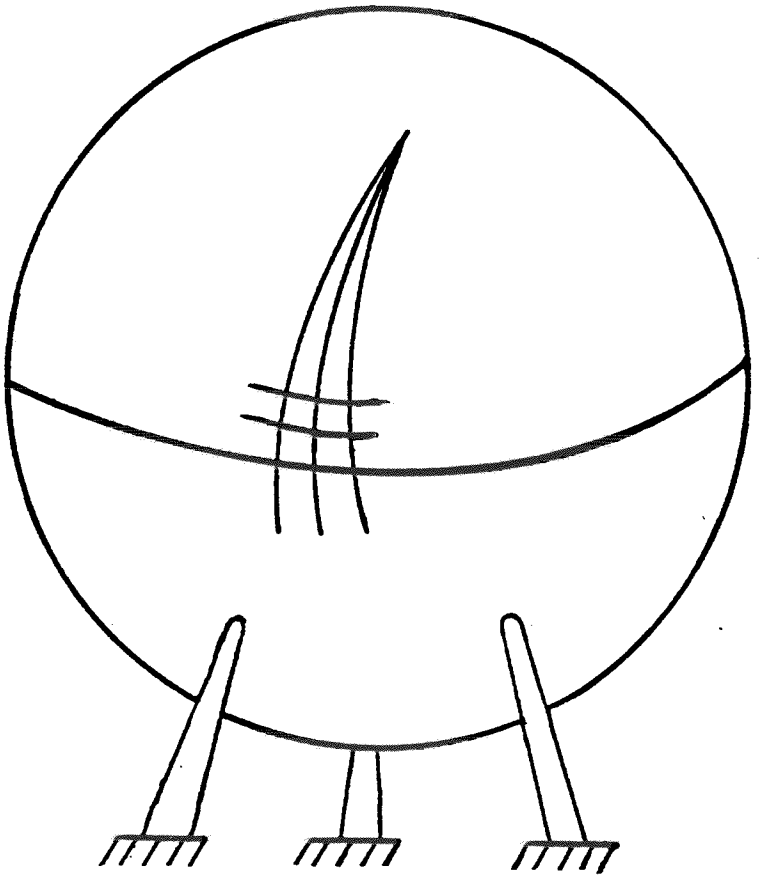
$$\{U^*_a\} = -[G_{ax}]\{U^*_x\} + \{U^1_a\} \tag{4}$$

$$[K_{aa}][G_{ax}] = [K^*_{ax}] \tag{5}$$

$$[K_{aa}]\{U^1_a\} = \{P^*_a\} \tag{6}$$

$$[I + G_{xx}]\{U^*_x\} = \{U^1_x\} \tag{7}$$

SAMPLE PROBLEM: SPHERE

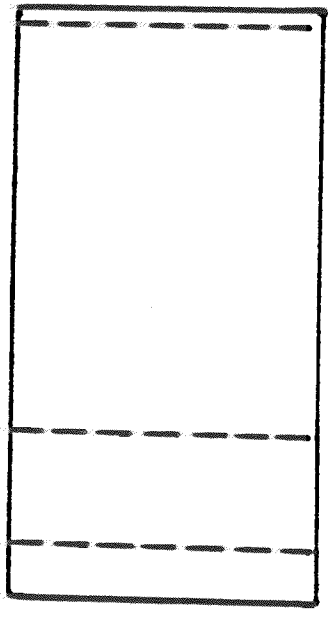


GRID POINTS:	617
PLATE ELEMENTS:	648
BEAM ELEMENTS:	3
LOADING CONDITIONS:	3
INDEPENDENT D.O.F.:	3684
BANDWIDTH:	226 R.M.S.
MODIFIED ELEMENTS:	3 BEAMS
STIFFNESS CHANGE D.O.F.:	18

COST ANALYSIS OF SIMPLE RUN
BASED UPON C.P.U. TIME

NOTE: C. P. U. TIME MAY NOT BE REPRESENTATIVE OF YOUR CHARGE ALGORITHM.

SPHERE SAMPLE PROBLEM



.019 DATA RECOVERY

.717 SOLUTION, MAINLY DECOMPOSITION

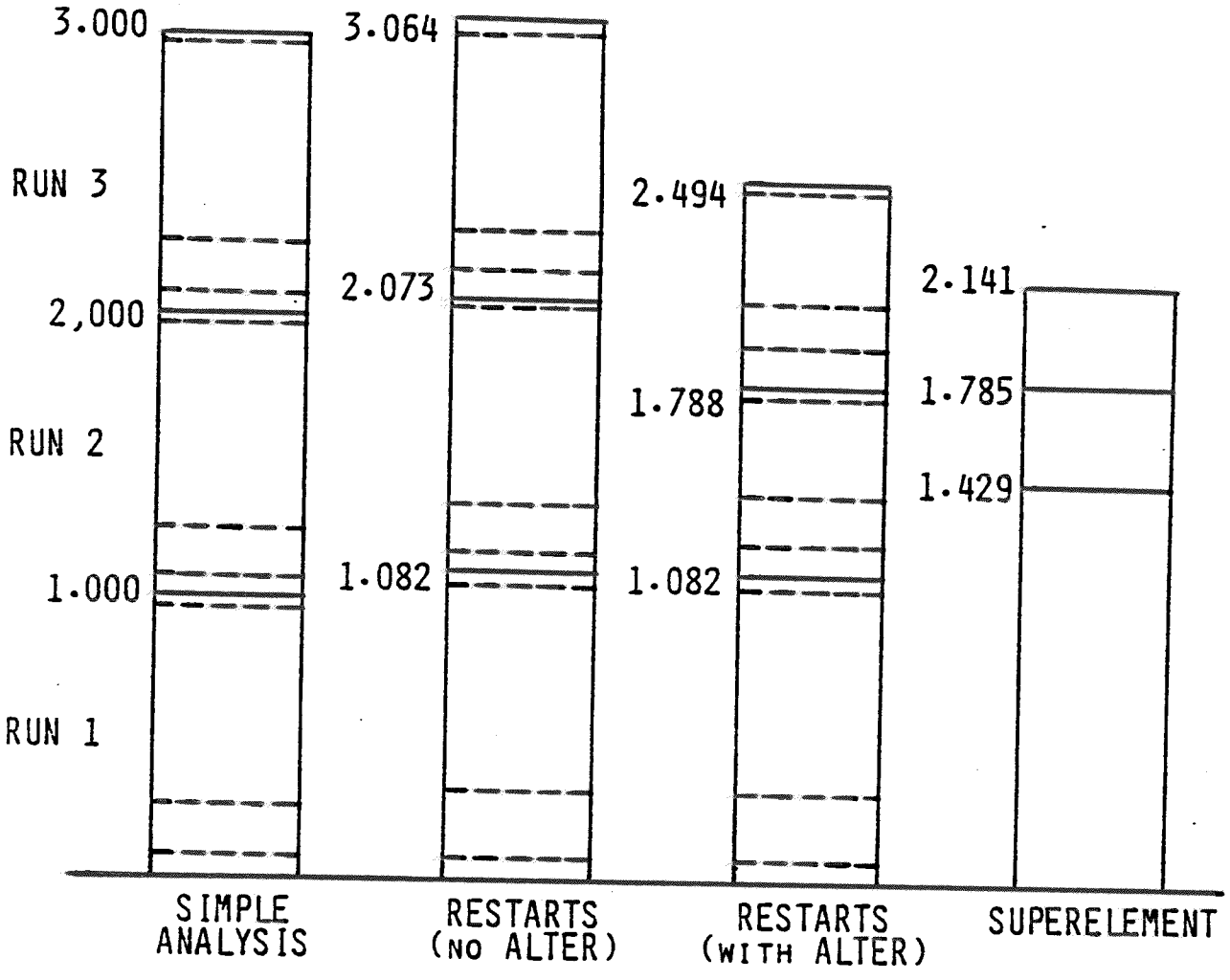
.193 MATRIX ASSEMBLY

.071 IFP + GEOMETRY

CPU TIME FOR REANALYSIS

(NORMALIZED TO SIMPLE ANALYSIS)

SPHERE SAMPLE PROBLEM



CONCLUSIONS

- A PROCEDURE HAS BEEN SHOWN TO REPEAT A STATIC ANALYSIS AFTER A STIFFNESS CHANGE.

- THE NUMBER OF DEGREES OF FREEDOM WITH STIFFNESS CHANGE SHOULD BE LESS THAN ONE-FOURTH OF THE BANDWIDTH.

- THE ONLY SPECIAL PROCEDURE IS AN ALTER TO BE USED ON RESTART.

- IF THE ELEMENTS TO BE CHANGED CAN BE IDENTIFIED IN ADVANCE AND SEVERAL CHANGES WILL BE MADE, USE SUPER-ELEMENT APPROACH.