ABSTRACT

MSC/NASTRAN's heat transfer capability can be used to calculate electrostatic fields in three-dimensional regions of variable permittivity. This paper presents calculations of the electrostatic fields along a pole-type three-phase high voltage transmission line. Graphical displays of contours of constant voltage are presented at various locations along the line.

INTRODUCTION

Overhead electric transmission lines consist of wire cables suspended from towers or poles. Increasingly high transmission line voltages produce high electric fields that affect the adjacent environment and determine line reliability and losses. To minimize the electric field and associated corona losses, environmental impact, and probability of arcing, it is desirable to be able to calculate the electric field produced by lines having a variety of geometric configurations and material permittivities.

While the finite element method is often used in the structural analysis of transmission lines, it has seldom been used to calculate their electric fields. A recent IEEE subcommittee report [1] compares seven methods of calculating transmission line electric fields, but does not mention finite elements. The only paper found on finite element analysis of transmission line electric fields is restricted to one-dimensional elements [2].

It is known that electrostatic fields are analogous to thermal fields [3], and therefore they can be calculated by MSC/NASTRAN [4] - [6]. This paper will first review the electric field analogy and the analogous MSC/NASTRAN inputs and outputs. Then it will discuss the generation and solution of a three-dimensional finite element model for the electric fields of a steel-pole-type three-phase transmission line.

ELECTROSTATIC FIELD ANALOGY

One of Maxwell's four basic electromagnetic equations is the continuity equation for the electric field $\bar{E}$:

$$\nabla \cdot \varepsilon \bar{E} = \rho \tag{1}$$

where ε is permittivity (dielectric constant) and ρ is volumetric charge density. $\bar{E}$ is related to electrostatic voltage (potential) ϕ by

$$\bar{E} = -\nabla \phi \tag{2}$$

Substituting (2) in (1) gives the differential equation for electrostatic fields:

$$\nabla \cdot \varepsilon \nabla \phi + \rho = 0 \tag{3}$$

This is recognized as the basic diffusion equation. It is analogous to the equation of heat diffusion:

$$\nabla \cdot k \nabla T + \dot{q} = \rho c \, (\partial T/\partial t) \tag{4}$$

where k is thermal conductivity, T is temperature, $\dot{q}$ is internal heat generation per unit volume, and ρc is heat capacity per unit volume.

The heat diffusion equation (4) is solved by MSC/NASTRAN. Table I lists the analogous variables for equations (3) and (4), along with their names in the MSC/NASTRAN input and output. Also, Table I lists additional electrostatic and thermal quantities that are volumetric, having no meaning at a point.

The volumetric thermal parameters include total heat flow Q, defined as

$$Q = \int_v \dot{q}\, dv \tag{5}$$

where v is volume. Related to Q is the convective film coefficient h, according to:

$$Q = h\ s\ (\Delta T) \tag{6}$$

where s is surface area and Δ indicates a difference.

In analogy with Equations (5) and (6), in electrostatic problems charge Q_e is defined as

$$Q_e = \int_v \rho\, dv \tag{7}$$

TABLE I.

ANALOGOUS THERMAL AND ELECTROSTATIC PARAMETERS

MSC/NASTRAN Name	Thermal Symbol Eqs. 4 or 6	Electrostatic Symbol Eqs. 3 or 8
Temperature	T	ϕ
Conductivity	k	ε
Temperature Gradient	∇T	$-\overline{E}$
Heat Flux	$-k\nabla T$	$\overline{D}=\varepsilon\overline{E}$
Internal Heat Generation	$\dot{q}$	ρ
Total Heat Flow	Q	Q_e
Convective Film Coefficient	h	C/S

Also, capacitance C is related to charge and potential difference ($\Delta\phi$) as

$$Q_e = C\ (\Delta\phi) \qquad (8)$$

If the relations of Table I are kept in mind, the use of MSC/NASTRAN for electrostatic problems is straightforward. Executive, Case Control, and Bulk Data decks are prepared as for Approach Heat.

Executive control should specify SOL 24 (linear steady state) with NASTRAN HEAT = 1. In the Bulk Data deck, SPC's constrain voltages rather than temperatures. MAT4 cards contain permittivities, and QVOL cards specify charge densities (if any). All the two- and three-dimensional heat conduction elements are available for modelling complex dielectric and conductor shapes.

TRANSMISSION LINE MODEL

Figure 1 shows a typical three-phase pole-type transmission line. Each steel pole has three steel arms with dielectric insulators from which the three energized phase wires are suspended. An additional wire is attached without insulators to the top of all the poles to provide protection against lightning.

A proper finite element model for the electric fields of the transmission line will be very different from a structural model of the line. Unlike structural models, electrostatic models must include finite elements that cover all of the adjacent environment. Therefore the finite element model of Figure 1 must include many air elements.

FIGURE 1.

Typical three-phase transmission line suspended from steel poles.

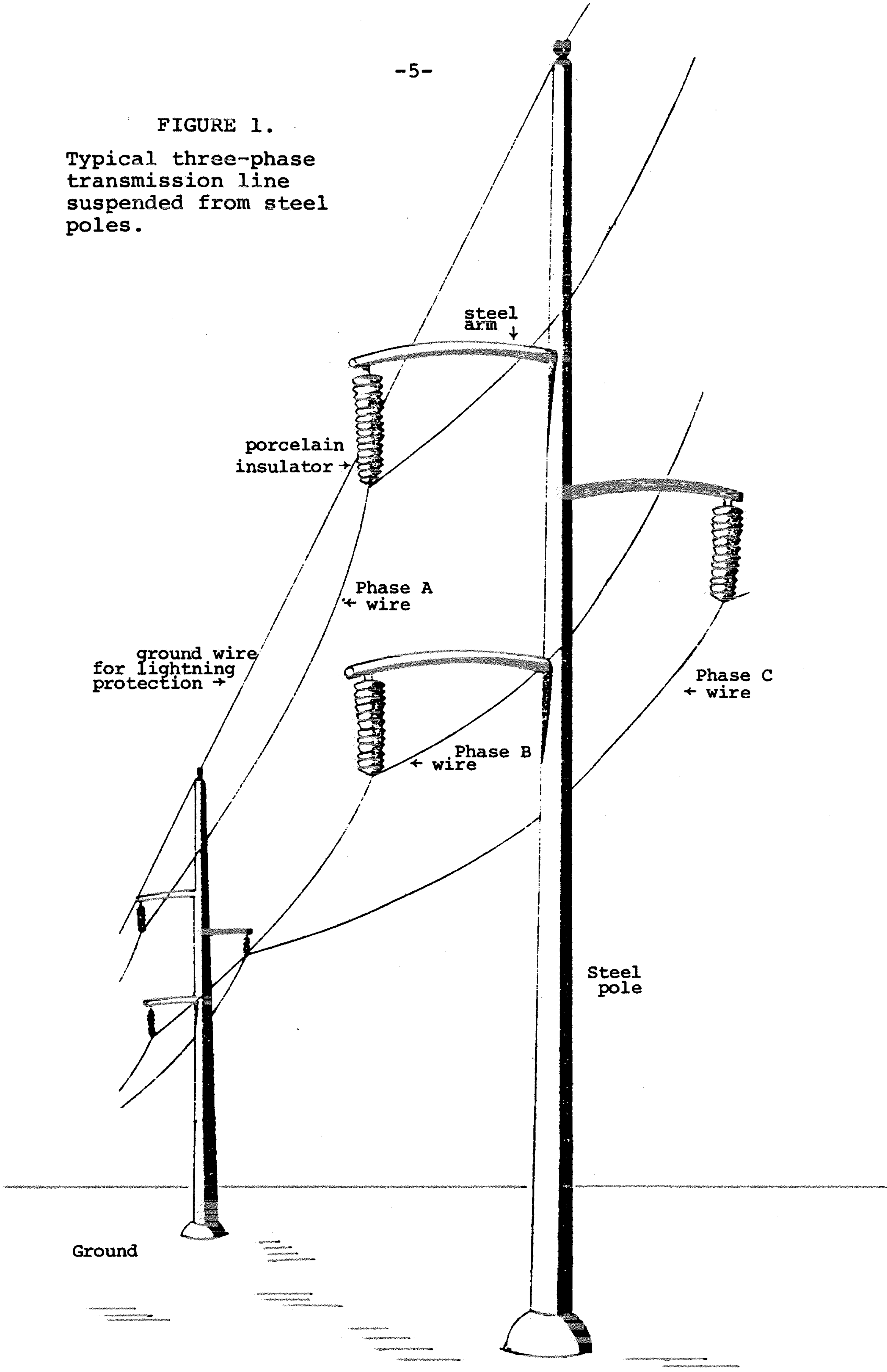

Figure 2 shows the finite element model developed for one-half span of the line of Figure 1. The span between poles is 300 meters, and thus Figure 2 shows a model extending 150 meters along the line's z direction. The ground is assumed (for simplicity only) to be flat and to be of high electrical conductivity. Thus the ground plane at y=0 is at ground potential of zero volts, and the earth below the surface need not be modelled. The steel pole and arms have such high conductivity that they are also at ground potential, and they are thin enough so that they can be considered as lines. Thus Figure 2 shows that the pole and arms are straight lines connecting certain grid points in the z=0 plane which are constrained to zero potential via SPC1 cards.

The four suspended wires are also shown in Figure 2 as straight lines extending in the -z direction. The dielectric insulators above all of the three energized phase wires are each modelled using solid three-dimensional CHEXA elements. There are 6 such CHEXA elements having the permittivity (specified on their MAT4 card) of porcelain, which is 5.0 times the permittivity of air.

As Figure 2 shows, there are very many finite elements of air, in fact, 2514 of them. They are smaller in the regions near the high voltage wires, where the electric field is expected to be highest and most rapidly changing with position. They extend 60 meters in the x and y directions, and thereby include all of the "adjacent" environment. The 2520 finite elements and 3150 GRID point input cards were all generated with the aid of the Generate New Data command of AOS/GRAFAX, a proprietary finite element processor program [7].

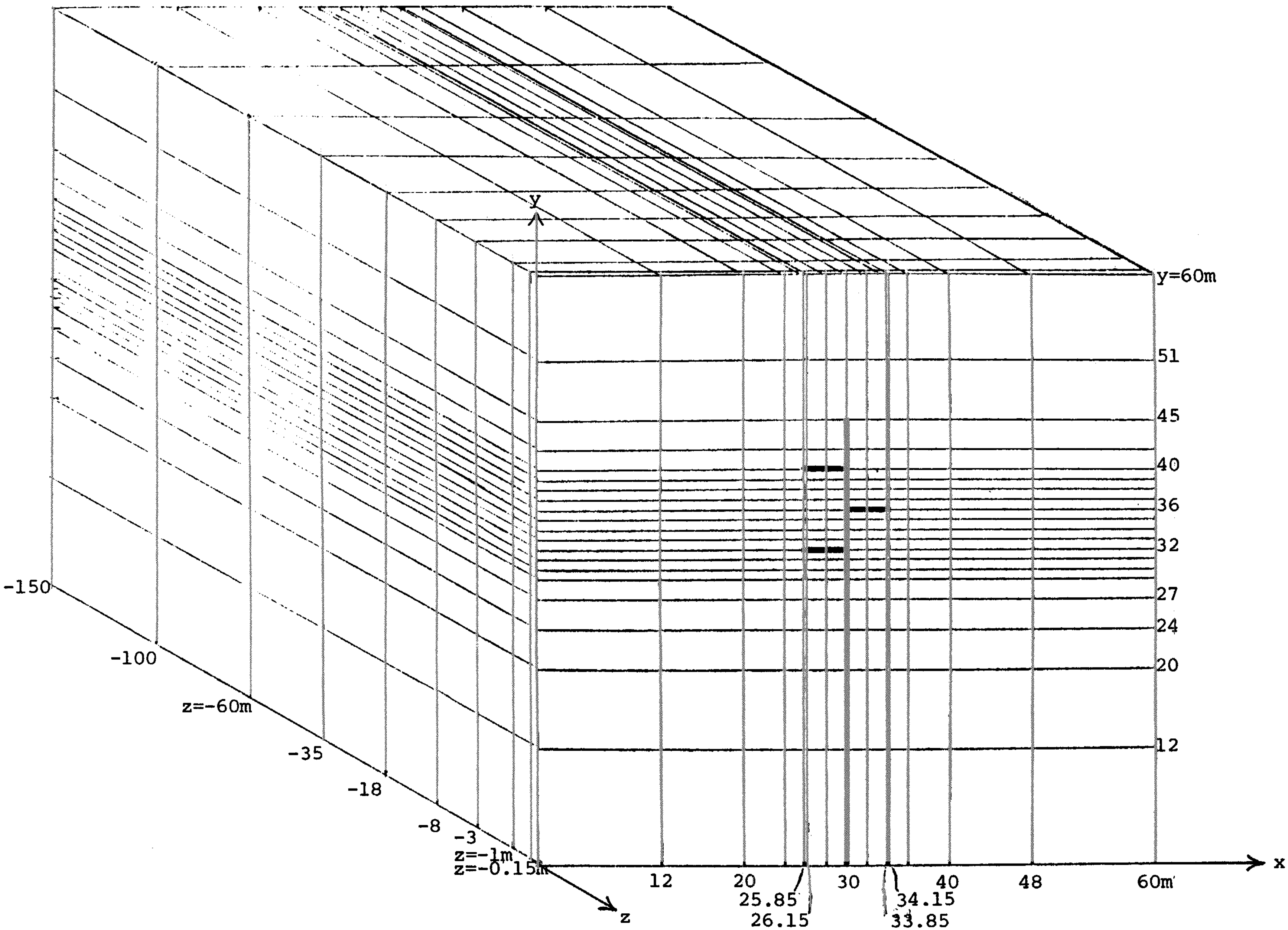

FIGURE 2.
Finite Element Model of one-half span of Figure 1.

CALCULATED FIELDS AND VOLTAGES

The voltages applied to the three phase wires vary sinusoidally with time at three different phase angles. Thus the electric fields and voltages throughout the region will also vary with time.

The voltages of phase wires A, B, and C for a balanced system are:

$$V_A = \sqrt{2}\ V_{rms} \cos(\omega t) \quad (9)$$

$$V_B = \sqrt{2}\ V_{rms} \cos(\omega t+120^\circ) \quad (10)$$

$$V_C = \sqrt{2}\ V_{rms} \cos(\omega t-120^\circ) \quad (11)$$

where V_{rms} is the rms transmission voltage, which ranges from 34.5 KV to 1200 KV today, and will probably increase in future years.

For illustrative purposes, consider t=0, the instant when the voltage in phase A is peaking and phases B and C both have voltages that are -0.5 times that of phase A. These voltage constraints were input to MSC/NASTRAN and the resulting instantaneous electrostatic fields and voltages were calculated.

The resulting contours of constant voltage are shown in Figures 3 through 10. Each figure is for a plane of constant z, where z decreases from 0 to -150 m. Notice that at z=0 (Figures 3 and 4) the zero volt contour clearly conforms with the pole and its arms. At z=-0.15 m (Figure 5) the pole contour disappears, but the other voltage contours are not changed much at all. At z=-1.0 m the contours change, and continue to change perceptibly through z=-35 meters (Figure 10).

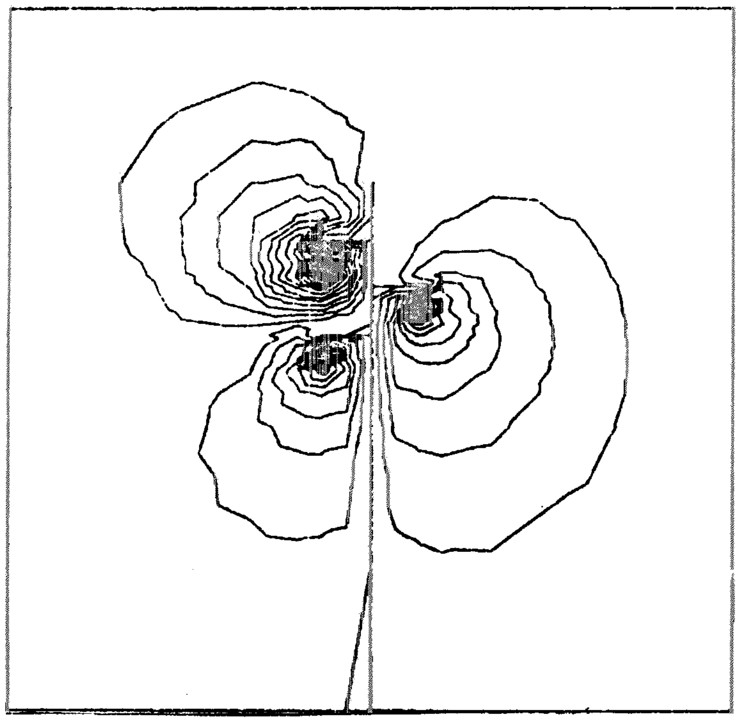

FIGURE 3.

Calculated voltage contours in z=0 plane.

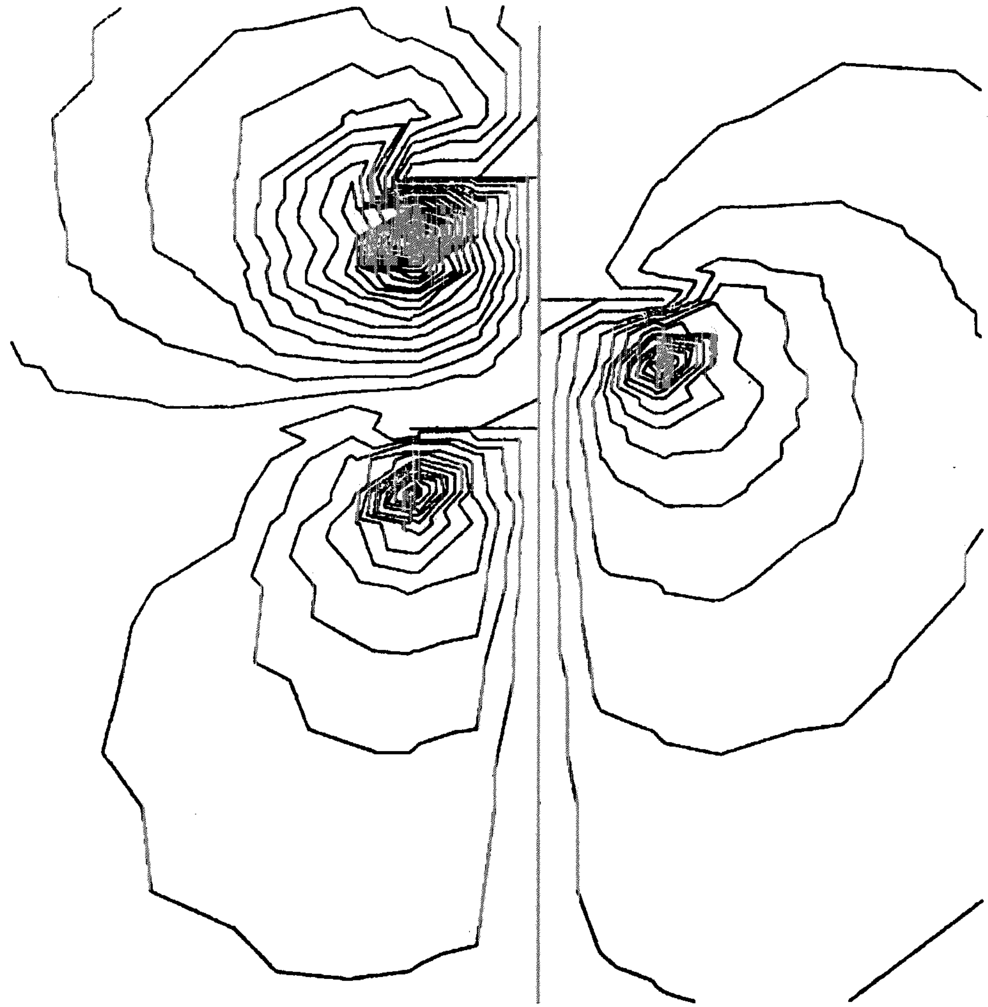

FIGURE 4.

Detail of Figure 3.

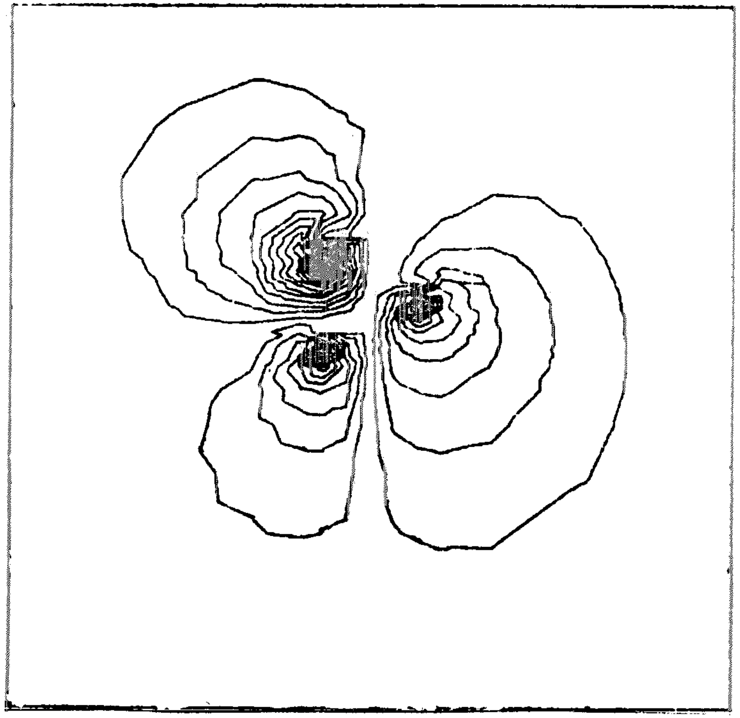

FIGURE 5.

Calculated voltage contours in z=-0.15m plane.

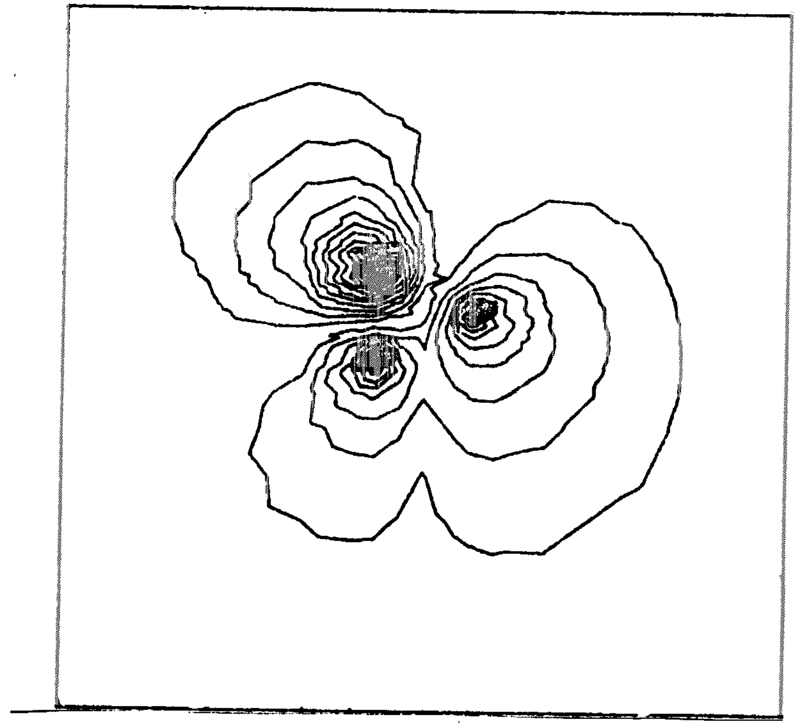

FIGURE 6.

Calculated voltage contours in z=-1.0m plane.

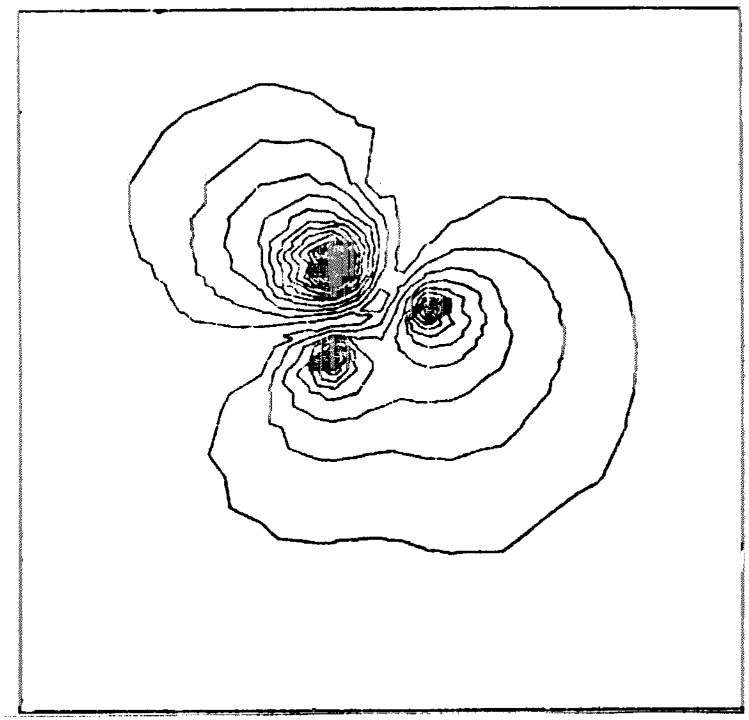

FIGURE 7.

Calculated voltage contours
in z=-3.0m plane.

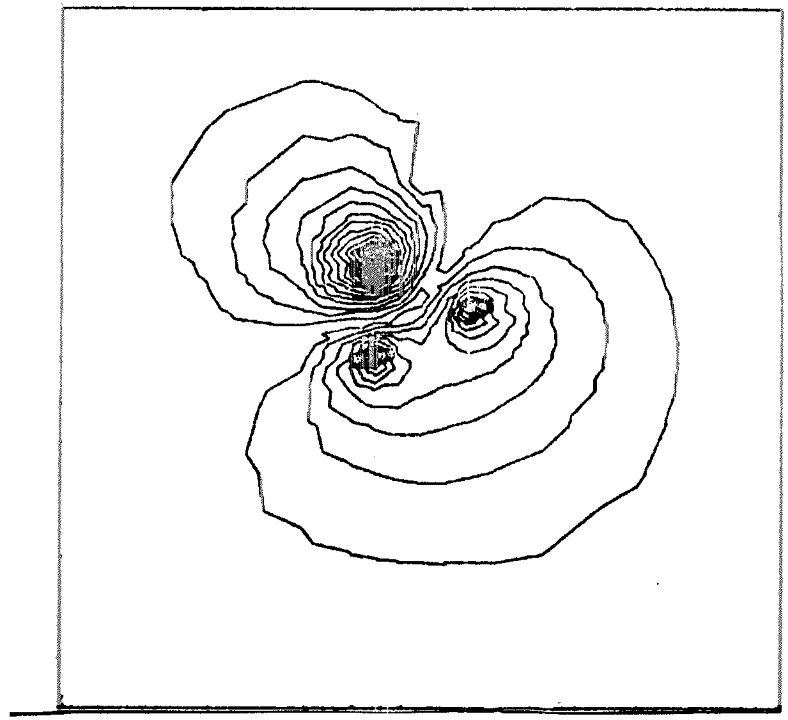

FIGURE 8.

Calculated voltage contours
in z=-8.0m plane.

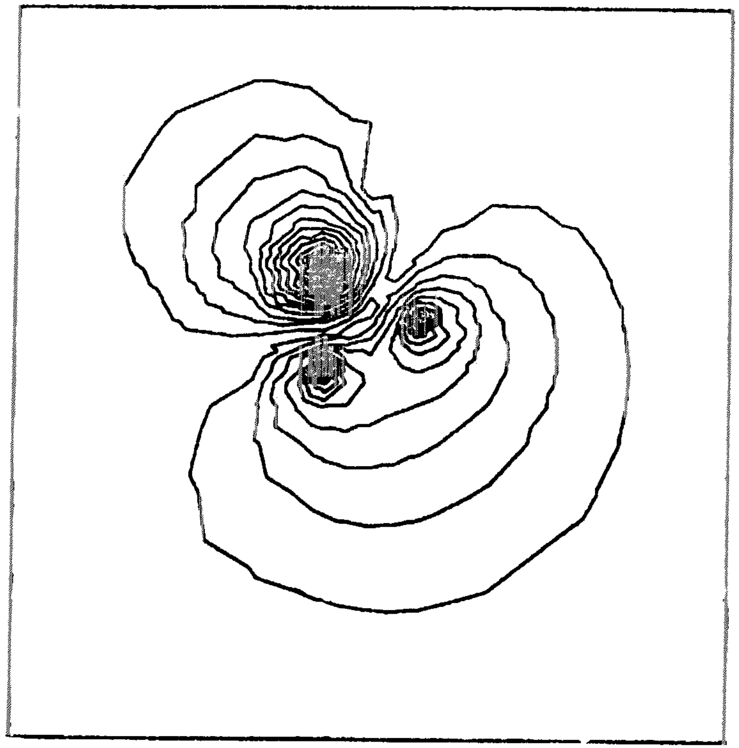

FIGURE 9.

Calculated voltage contours in z=-18m plane.

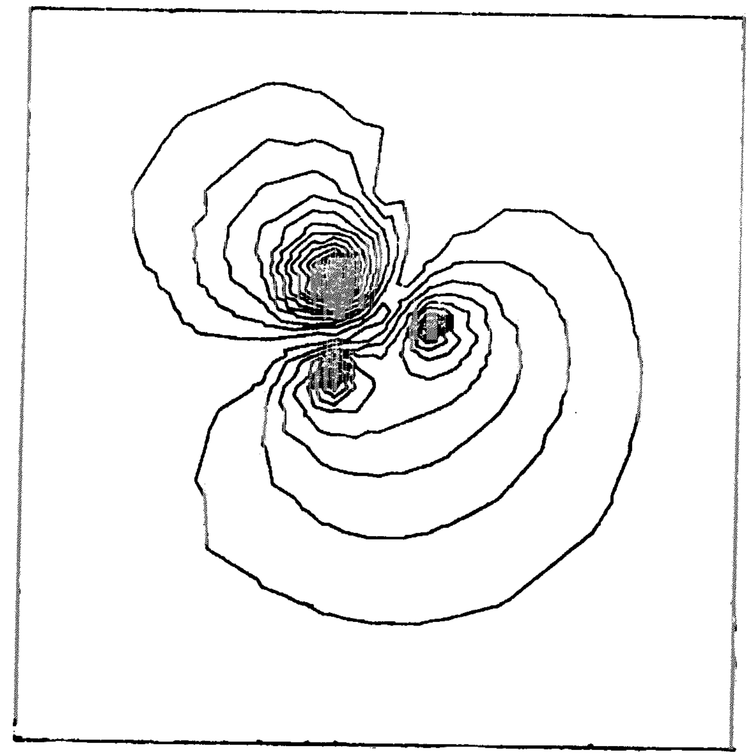

FIGURE 10.

Calculated voltage contours in z=-35m plane. The contours calculated for z=-35m through z=-105m are identical.

From z=-35 through z=-150 meters there is little perceptible change in the voltage contours. The contours are identical in this region because it is far from any steel pole, and because the model of Figure 2 assumes that all four wires have zero sag. The model could be easily altered to include any wire deflections due to gravity and/or wind as calculated by Approach Displacement of MSC/NASTRAN. In any case, it is expected that the highest concentration of voltage contours, and therefore the highest E field, will remain near the three energized phase wires as shown in Figures 3 through 8.

Solution time for the above electrostatic fields was approximately 5 minutes on an IBM 370/165 computer. Only one matrix decomposition is necessary for as many solutions at as many instants of time as desired.

LINE IMPEDANCES

A transmission line has an equivalent circuit made up of impedances related to its electric and magnetic fields. The circuit has a series impedance consisting of a resistance plus an inductive reactance, and a parallel impedance consisting of a capacitive reactance.

Capacitive reactances can be calculated using the output of MSC/NASTRAN. Program output includes the "strain energy", which is the energy stored in the region analyzed. In a region with an electrostatic field set up by constraining (energizing) only two electrodes, one at $\phi = V$ and the other at $\phi = 0$, the stored energy is

$$W = \frac{1}{2} C V^2 \qquad (12)$$

where C is the capacitance between the electrodes. Thus C is easily calculable from the W (strain energy) output by MSC/NASTRAN:

$$C = 2\ W/V^2 \tag{13}$$

While the parallel capacitance of a transmission line is determined by the electrostatic fields, the line's series impedance is determined by its alternating magnetic fields. The series impedance (as a function of frequency) and magnetic field distribution can be calculated using a finite element program called AOS/MAGNETIC. [6]

CONCLUSIONS

MSC/NASTRAN's heat transfer capability can calculate the three-dimensional electrostatic fields of transmission lines. Calculated voltage contour plots show that typical steel poles have a significant effect on the electrostatic field, causing it to vary along the line.

Not only is the electric field calculable, but so is the related line capacitance. Finite element calculation of line capacitance and other impedances should aid in the development of more efficient and reliable electric power transmission systems.

ACKNOWLEDGEMENT

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