DYNAMIC STRUCTURAL RESPONSES TO RIGID BASE ACCELERATION

Thomas G. Butler, BUTLER ANALYSES

Introduction

NASTRAN has rigid formats which determine dynamic strctural responses to excitations which are specified in terms of force time histories, but they do not address themselves to those cases for which the excitiation is specified in terms of motion time histories, e.g. accelerations. A method is presented herein which is based on modifiying the existing rigid formats with DMAP ALTER packets and/or using preprocessors or by composing DUMMOD's in new rigid formats. Only particular cases of motion time histories can be handled in the method being presented. The method restricts the excitatins to rigid base accelerations. This implies that all input accelerations be synchronous and all excited points satisfy rigid body relationships. Methods are developed for responses tro steady sinusoidal or transient accelerations to be solved by either direct integration or by the modal method. Specifically, modifications are provided to enable

solutions via R.F.'s 8, 9, 11, and 12. This technique will handle most cases of support motion such as ground motion to building foundations, motion to bases of machinery, and shaker motion input to a test article.

Theory

As a perlude to the theory, a simple 4 mass system shown in figure 1 in one dimension with a base acceleration will be used to expose certain characteristics that might elucidate their operation before being cast in matrix form.

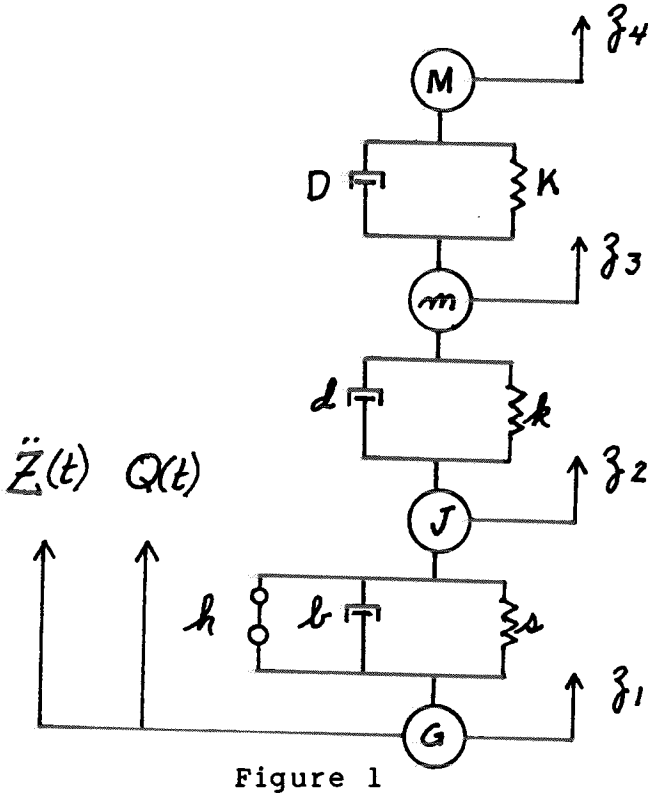


Figure 1

The set of dynamic equations is:

$$\text{Mass 4} \quad -M\ddot{z}_4 - D(\dot{z}_4 - \dot{z}_3) - K(z_4 - z_3) = \emptyset \qquad (1)$$

$$\text{Mass 3} \quad -m\ddot{z}_3 - (D\dot{z}_3 - \dot{z}_4) - d(\dot{z}_3 - \dot{z}_2) - K(z_3 - z_4) - k(z_3 - z_2) = \emptyset$$

$$\text{Mass 2} \quad -J\ddot{z}_2 - d(\dot{z}_2 - \dot{z}_3) - b(\dot{z}_2 - \dot{z}_1) - k(z_2 - z_3) - s(z_2 - z_1) = \emptyset$$

$$\text{Base Mass} \quad -G\ddot{z}_1 - b(\dot{z}_1 - \dot{z}_2) - s(z_2 - z_2) + Q(t) = \emptyset$$

$$\text{and} \quad \ddot{z}_1(t) = \ddot{Z}(t) \qquad \text{Base acceleration.}$$

Since the objective is to transform the acceleration statements so as to be in a form that NASTRAN can accept, the scheme is to look for a conversion to forces. One way is to transform the coordinates from an external to a relative system. Let the variables z_4, z_3, z_2, and z_1 be written in terms of motion relative to z_1. Thus $z_i = r_i + z_1$ for $(i=1,2,3,4)$ can be substituted into equations (1) and a change in sign to give:

$$M(\ddot{r}_4+\ddot{z}_1) + D(\dot{r}_4+\dot{z}_1-\dot{r}_3-\dot{z}_1) + K(r_4+z_1-r_3-z_1) = \emptyset \qquad (2)$$

$$m(\ddot{r}_3+\ddot{z}_1) + D(\dot{r}_3+\dot{z}_1-\dot{r}_4-\dot{z}_1) + d(\dot{r}_3+\dot{z}_1-\dot{r}_2-\dot{z}_1) + K(r_3+z_1-r_4-z_1) + k(r_3+z_1-r_2-z_1) = \emptyset$$

$$J(\ddot{r}_2+\ddot{z}_1) + d(\dot{r}_2+\dot{z}_1-\dot{r}_3-\dot{z}_1) + b(\dot{r}_2+\dot{z}_1-\dot{r}_1-\dot{z}_1) + k(r_2+z_1-r_3-z_1) + s(r_2+z_1-r_1-z_1) = \emptyset$$

$$G(\ddot{r}_1+\ddot{z}_1) + b(\dot{r}_1+\dot{z}_1-\dot{r}_2-\dot{z}_1) + s(r_1+z_1-r_2-z_1) = Q(t).$$

Recognizing that the motion of z_1 with respect to z_1 is zero, it follows that $r_1 = \dot{r}_1 = \ddot{r}_1 = \emptyset$. Apply this with $\ddot{z}_1 = \ddot{Z}$, then the equations can be rearranged with only terms in r on the L.H.S.

$$M\ddot{r}_4 + D(\dot{r}_4 - \dot{r}_3) + K(r_4 - r_3) = -M\ddot{Z} \qquad (3)$$

$$m\ddot{r}_3 + D(\dot{r}_3 - \dot{r}_4) + d(\dot{r}_3 - \dot{r}_2) + K(r_3 - r_4) + k(r_3 - r_2) = -m\ddot{Z}$$

$$J\ddot{r}_2 + d(\dot{r}_2 - \dot{r}_3) + b\dot{r}_2 + k(r_2 - r_3) + sr_2 = -J\ddot{Z}$$

$$- b\dot{r}_2 - sr_2 = Q(t) - G\ddot{Z}$$

$-M\ddot{Z}$, $-m\ddot{Z}$, $-J\ddot{Z}$ and $-G\ddot{Z}$ are all known quantities and they are all accelerations of mass so for brevity they can be called respectively: F_4, F_3, F_2 and F_1. Q(t) is the unknown base force required to maintain the base acceleration according to the prescribed pattern of $\ddot{Z}(t)$. The 3 equations in the 3 unknowns r_4, r_3, and r_2 can be solved as an independent set. Once solved, the values of r_2 and $\dot{r}_2$ can be substituted into the last equation to obtain Q(t). This shows that it is possible to transform a problem, defined as a system with acceleration onto external coordinates, into a problem defined by a system of relative coordinates excited by known forces. The final form of the equations of equilibrium for the model in fiugure 1 in relative coordinates is:

$$M\ddot{r}_4 + D(\dot{r}_4 - \dot{r}_3) + K(r_4 - r_3) = F_4 \qquad (4)$$

$$m\ddot{r}_3 + D(\dot{r}_3 - \dot{r}_4) + d(\dot{r}_3 - \dot{r}_2) + K(r_3 - r_4) + k(r_3 - r_2) = F_3$$

$$J\ddot{r}_2 + d(\dot{r}_2 - \dot{r}_3) + b\dot{r}_2 + k(r_2 - r_3) + sr_2 = F_2$$

$$- b\dot{r}_2 \qquad - sr_2 - Q(t) = F_1$$

When this simple system is described in matrix notation, the extension to problems in a great number of variables will become apparent. Start by putting the set (1) into matrices.

$$-\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \ddot{z}_4 \\ \ddot{z}_3 \\ \ddot{z}_2 \\ \ddot{z}_1 \end{Bmatrix} - \begin{bmatrix} D & -D & 0 & 0 \\ -D & (D+d) & -d & 0 \\ 0 & -d & (d+b) & -b \\ 0 & 0 & -b & b \end{bmatrix} \begin{Bmatrix} \dot{z}_4 \\ \dot{z}_3 \\ \dot{z}_2 \\ \dot{z}_1 \end{Bmatrix} - \begin{bmatrix} K & -K & 0 & 0 \\ -K & (K+k) & -k & 0 \\ 0 & -k & (k+s) & -s \\ 0 & 0 & -s & s \end{bmatrix} \begin{Bmatrix} z_4 \\ z_3 \\ z_2 \\ z_1 \end{Bmatrix}$$

$$+ \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -Q(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (5)$$

and $\ddot{z}_1(t) = \ddot{Z}(t)$.

Let $z_i = r_i + z_1$ (6)

$$\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \ddot{r}_4+\ddot{z}_1 \\ \ddot{r}_3+\ddot{z}_1 \\ \ddot{r}_2+\ddot{z}_1 \\ \ddot{r}_1+\ddot{z}_1 \end{Bmatrix} + \begin{bmatrix} D & -D & 0 & 0 \\ -D & (D+d) & -d & 0 \\ 0 & -d & (d+b) & -b \\ 0 & 0 & -b & b \end{bmatrix} \begin{Bmatrix} \dot{r}_4+\dot{z}_1 \\ \dot{r}_3+\dot{z}_1 \\ \dot{r}_2+\dot{z}_1 \\ \dot{r}_1+\dot{z}_1 \end{Bmatrix} \tag{7}$$

$$+ \begin{bmatrix} K & -K & 0 & 0 \\ -K & (K+k) & -k & 0 \\ 0 & -k & (k+s) & -s \\ 0 & 0 & -s & s \end{bmatrix} \begin{Bmatrix} r_4+z_1 \\ r_3+z_1 \\ r_2+z_1 \\ r_1+z_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ Q(t) \end{Bmatrix}$$

Retain the terms in r on the L.H.S. and remove all others to the R.H.S.

$$\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \ddot{r}_4 \\ \ddot{r}_3 \\ \ddot{r}_2 \\ \ddot{r}_1 \end{Bmatrix} + \begin{bmatrix} D & -D & 0 & 0 \\ -D & (D+d) & -d & 0 \\ 0 & -d & (d+b) & -b \\ 0 & 0 & -b & b \end{bmatrix} \begin{Bmatrix} \dot{r}_4 \\ \dot{r}_3 \\ \dot{r}_2 \\ \dot{r}_1 \end{Bmatrix} + \begin{bmatrix} K & -K & 0 & 0 \\ -K & (K+k) & -k & 0 \\ 0 & -k & (k+s) & -s \\ 0 & 0 & -s & s \end{bmatrix} \begin{Bmatrix} r_4 \\ r_3 \\ r_2 \\ r_1 \end{Bmatrix} =$$

$$- \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_1 \\ \ddot{z}_1 \\ \ddot{z}_1 \end{Bmatrix} - \begin{bmatrix} D & -D & 0 & 0 \\ -D & (D+d) & -d & 0 \\ 0 & -d & (d+b) & -b \\ 0 & 0 & -b & b \end{bmatrix} \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_1 \\ \dot{z}_1 \\ \dot{z}_1 \end{Bmatrix} - \begin{bmatrix} K & -K & 0 & 0 \\ -K & (K+k) & -k & 0 \\ 0 & -k & (k+s) & -s \\ 0 & 0 & -s & s \end{bmatrix} \begin{Bmatrix} z_1 \\ z_1 \\ z_1 \\ z_1 \end{Bmatrix}$$

$$+ \begin{Bmatrix} 0 \\ 0 \\ 0 \\ Q(t) \end{Bmatrix} \tag{8}$$

Inspection of the 2nd and 3rd terms on the R.H.S. shows that they are identically zero. Set $r_1=\dot{r}_1=\ddot{r}_1=\emptyset$ and impose the prescribed excitation $\ddot{z}_1 = \ddot{Z}(t)$.

$$\begin{bmatrix} M & \emptyset & \emptyset & \emptyset \\ \emptyset & m & \emptyset & \emptyset \\ \emptyset & \emptyset & J & h \\ \emptyset & \emptyset & h & G \end{bmatrix} \begin{Bmatrix} \ddot{r}_4 \\ \ddot{r}_3 \\ \ddot{r}_2 \\ \emptyset \end{Bmatrix} + \begin{bmatrix} D & -D & \emptyset & \emptyset \\ -D & (D+d) & -d & \emptyset \\ \emptyset & -d & (d+b) & -b \\ \emptyset & \emptyset & -b & b \end{bmatrix} \begin{Bmatrix} \dot{r}_4 \\ \dot{r}_3 \\ \dot{r}_2 \\ \emptyset \end{Bmatrix} + \begin{bmatrix} K & -K & \emptyset & \emptyset \\ -K & (K+k) & -k & \emptyset \\ \emptyset & -k & (k+s) & -s \\ \emptyset & \emptyset & -s & s \end{bmatrix} \begin{Bmatrix} r_4 \\ r_3 \\ r_2 \\ \emptyset \end{Bmatrix}$$

$$= (-) \begin{bmatrix} M & \emptyset & \emptyset & \emptyset \\ \emptyset & m & \emptyset & \emptyset \\ \emptyset & \emptyset & J & h \\ \emptyset & \emptyset & h & G \end{bmatrix} \begin{Bmatrix} \ddot{Z} \\ \ddot{Z} \\ \ddot{Z} \\ \ddot{Z} \end{Bmatrix} + \begin{Bmatrix} \emptyset \\ \emptyset \\ \emptyset \\ Q(t) \end{Bmatrix} \qquad (9)$$

If the mass between stations 1 and 2 were distributed and a coupled mass modeling of this distributed mass were used then an off-diagonal term would appear in the 3,4 and 4,3 positions of the mass matrix, say "h" as shown above. In the interest of maintaining symmetry in all matrices, the temptation to transfer the unknown base force Q(t) to the L.H.S. with the other unknowns was resisted and the zeroes in the motion vectors are retained in order to generate the coupling between base and adjoining d.o.f.'s. The equation in base force with complete coupling becomes

$$h\ddot{r}_2 - b\dot{r}_2 - sr_2 = (-)(h + G)\ddot{z} + Q(t) \qquad (10).$$

Almost all features of the simplified analogue have been touched on in anticipation of extending these concepts into a general nth order matrix system. A remaining item is the nature of the excitation. So long as the rigid body base motion is confined to translations, the preparation thus far is complete. However, if rigid body rotations are included, then there is a non-uniform distribution of translations such that the value of Z would vary from row to row, and the damping and stiffness forcings on the R.H.S. of equation (8) would not be identically zero. This complication is left for another time. Suffice it to say that for the present scheme, the acceleration excitations will be confined to translations. The seven permutations of the three translational accelerations of the base are the only admissable ones. Let any of them be represented by the expression $\{\ddot{U}\}$. The task at hand is to generalize equation (9) to n degrees of freedom for any kind of translational excitation. In general, the dynamic equation for a structure is:

$$[M_{GG}]\{\ddot{u}_G\} + [B_{GG}]\{\dot{u}G\} + [KG_G]\{u_G\} = \{P_G(t)\}. \qquad (12)$$

When the excitation is specified in terms of base motion $\{\ddot{U}\}$ only, then $\{P(t)\}$ reduces to the unknown support force $\{Q_B(t)\}$ needed to produce that acceleration. The corollary to equation (6) for transforming the general system to relative coordinates is

$$\{u\} = \{r\} + \{R\}. \tag{13}$$

Eventually $\{R\}$ sill be defined, but first consider an example in which $\{\ddot{U}\} = \lfloor 5 \ 0 \ 0 \ 0 \ 0 \ 0 \rfloor^T$, implying that the base excitation will consist of translations in x only. The transformation to relative coordinates for this example would entail:

$$\{\ddot{u}\} = \begin{Bmatrix} \ddot{u}_x^1 \\ \ddot{u}_y^1 \\ \ddot{u}_z^1 \\ \ddot{y}_X^1 \\ \ddot{u}_Y^1 \\ \ddot{u}_Z^1 \\ \text{---} \\ \ddot{u}_x^2 \\ \ddot{u}_y^2 \\ \ddot{u}_z^2 \\ \ddot{u}_X^2 \\ \ddot{u}_Y^2 \\ \ddot{u}_Z^2 \\ \text{---} \\ \ddot{u}_x^3 \\ \cdot \\ \cdot \\ \text{---} \\ \cdot \\ \cdot \\ \cdot \\ \text{---} \\ \ddot{u}_x^n \\ \ddot{u}_y^n \\ \cdot \\ \cdot \\ \ddot{u}_z^n \end{Bmatrix} = \begin{Bmatrix} \ddot{r}_x^1 + 5 \\ \ddot{r}_y^1 \\ \ddot{r}_z^1 \\ \ddot{r}_X^1 \\ \ddot{r}_Y^1 \\ \ddot{r}_Z^1 \\ \text{---} \\ \ddot{r}_x^2 + 5 \\ \ddot{r}_y^2 \\ \ddot{r}_z^2 \\ \ddot{r}_X^2 \\ \ddot{r}_Y^2 \\ \ddot{r}_Z^2 \\ \text{---} \\ \ddot{r}_x^3 + 5 \\ \cdot \\ \cdot \\ \text{---} \\ \cdot \\ \cdot \\ \cdot \\ \text{---} \\ \ddot{r}_x^n + 5 \\ \ddot{r}_y^n \\ \cdot \\ \cdot \\ \ddot{r}_z^n \end{Bmatrix} \quad \text{or briefly} \quad \begin{Bmatrix} \ddot{u}^1 \\ \text{---} \\ \ddot{u}^2 \\ \text{---} \\ \ddot{u}^3 \\ \text{---} \\ \cdot \\ \cdot \\ \cdot \\ \text{---} \\ \ddot{u}^n \end{Bmatrix} = \begin{Bmatrix} \ddot{r}^1 \\ \text{---} \\ \ddot{r}^2 \\ \text{---} \\ \ddot{r}^3 \\ \text{---} \\ \cdot \\ \cdot \\ \cdot \\ \text{---} \\ \ddot{r}^n \end{Bmatrix} + \begin{Bmatrix} \ddot{U} \\ \text{---} \\ \ddot{U} \\ \text{---} \\ \ddot{U} \\ \text{---} \\ \cdot \\ \cdot \\ \cdot \\ \text{---} \\ \ddot{U} \end{Bmatrix}$$

so the acceleration reference vector $\{\ddot{R}\}$ can be defined as:

$$\{\ddot{R}\} = \begin{Bmatrix} \ddot{U} \\ --- \\ \ddot{U} \\ --- \\ . \\ . \\ . \\ --- \\ \ddot{U} \end{Bmatrix} \qquad (14)$$

which says that the transformation from external to relative coordinates consists of subtracting the reference vector $\{\ddot{R}\}$ from the external vector $\{\ddot{u}\}$ wherein the reference vector is an array of identical translatinal displacements which are repeated for every grid point outside of the array of support points.

In order to get a picture of how the matrices in relative coordinates is going to fit into a NASTRAN analysis, the partitioning of the stiffness matrix in two different ways will be used as an example. Since subscript "r" already has been pre-empted by NASTRAN to mean the d.o.f.'s that are given D'Alembert reactions, subscript "c" for cousin will be used as a "near" relative coordinate.

NORMAL

$$\left[K_{GG}\right]$$

G--> M, N

$$\left[\begin{array}{c|c} K_{MM} & K_{MN} \\ \hline K_{NM} & K_{NN} \end{array}\right]$$

N--> S, F

$$\left[\begin{array}{cc|c} K_{MM} & K_{MS} & K_{MF} \\ K_{SM} & K_{SS} & K_{SF} \\ \hline K_{FM} & K_{FS} & K_{FF} \end{array}\right]$$

S--> SG, SB

$$\left[\begin{array}{cc|cc} K_{MM} & K_{M,SG} & K_{M,SB} & K_{M,F} \\ K_{SG,M} & K_{SG,SG} & K_{SG,SB} & K_{SG,F} \\ \hline K_{SB,M} & K_{SB,SG} & K_{SB,SB} & K_{SB,F} \\ K_{FM} & K_{F,SG} & K_{F,SB} & K_{FF} \end{array}\right]$$

RELATIVE

$$\left[K_{GG}\right]$$

G--> SB, C

$$\left[\begin{array}{c|c} K_{SB,SB} & K_{SB,C} \\ \hline K_{C,SB} & K_{CC} \end{array}\right]$$

K_{CC} is now the stiffness matrix partition for relative coordinates, i.e. the complement of the base contraint partition with respect to the G-set

C--> M, N

$$\left[\begin{array}{cc|c} K_{SB,SB} & K_{SB,M} & K_{SB,N} \\ K_{M,SB} & K_{M,M} & K_{M,N} \\ \hline K_{N,SB} & K_{N,M} & K_{N,N} \end{array}\right]$$

N--> SG, F

$$\left[\begin{array}{ccc|c} K_{SB,SB} & K_{SB,M} & K_{SB,SG} & K_{SB,F} \\ K_{M,SB} & K_{MM} & K_{M,SG} & K_{MF} \\ K_{SG,SB} & K_{SG,M} & K_{SG,SG} & K_{SG,F} \\ \hline K_{F,SB} & K_{F,M} & K_{F,SG} & K_{FF} \end{array}\right]$$

At this point the matrices have all the same component partitions, but arrived at in different sequences. From here on the sequence remains the same; i.e. F--> O/A. Next the E-Point d.o.f.'s are added to the A matrices to inflate them to D-size; i.e. A/E--> D.

The point to be made is that by the time the equations are partitioned to D-size they are exactly the same for both routes, so it is immaterial which sequence of partitioning is used. The TRD module solves the D-sized problem; then VDR & SDR recover the data by merging back up the ladder. In this case the end of the ladder is C-sized for relative behavior, but NASTRAN recovers all the way back to G-size. The G-sized results would ordinarily provide all that might be needed except that the single point constraint forces are not computed according to equation (27). Stresses can be recovered in relative coordinates, because strain is a differencing process. A constant added to two quantities that are later to be subtracted produce the same value with or without the added quantity.

Apply the relative coordinate sequence for theoretical purposes and go back to see how much of the normal sequence can be preserved. Start with equation (12).

$$[M_{GG}]\{\ddot{u}_G\} + [B_{GG}]\{\dot{u}_G\} + [K_{GG}]\{u_G\} = \emptyset \qquad (16).$$

Partition off the base (support) corrdinates.

$$\begin{bmatrix} M_{CC} & M_{C,SB} \\ M_{SB,C} & M_{SB,SB} \end{bmatrix} \begin{Bmatrix} \ddot{u}_C \\ \ddot{u}_{SB} \end{Bmatrix} + \begin{bmatrix} B_{CC} & B_{C,SB} \\ B_{SB,C} & B_{SB,SB} \end{bmatrix} \begin{Bmatrix} \dot{u}_C \\ \dot{u}_{SB} \end{Bmatrix} + \begin{bmatrix} K_{CC} & K_{C,SB} \\ K_{SB,C} & K_{SB,SB} \end{bmatrix} \begin{Bmatrix} u_C \\ u_{SB} \end{Bmatrix} = \begin{Bmatrix} \emptyset \\ \emptyset \end{Bmatrix} \qquad (17)$$

Separate the u_C displacements into relative coordinates using the definitions:

$$\{u_C\} = \{R_C + r_C\} \quad \text{and} \quad \{R_C\} = \lfloor u_{SB}, u_{SB}, u_{SB}, u_{SB}, u_{SB}, \ldots\ldots u_{SB} \rfloor_C^T \qquad (18)$$

$$\begin{bmatrix} M_{CC} & M_{C,SB} \\ M_{SB,C} & M_{SB,SB} \end{bmatrix} \begin{Bmatrix} \ddot{r}_C + \ddot{R}_C \\ \emptyset + \ddot{R}_{SB} \end{Bmatrix} + \begin{bmatrix} B_{CC} & B_{C,SB} \\ B_{SB,C} & B_{SB,SB} \end{bmatrix} \begin{Bmatrix} \dot{r}_C + \dot{R}_C \\ \emptyset + \dot{R}_{SB} \end{Bmatrix} + \begin{bmatrix} K_{CC} & K_{C,SB} \\ K_{SB,C} & K_{SB,SB} \end{bmatrix} \begin{Bmatrix} r_C + R_C \\ \emptyset + R_{SB} \end{Bmatrix} = \begin{Bmatrix} \emptyset \\ \emptyset \end{Bmatrix} \qquad (19)$$

Isolate the terms in r_C on the L.H.S.

$$\begin{bmatrix} M_{CC} & M_{C,SB} \\ M_{SB,C} & M_{SB,SB} \end{bmatrix} \begin{Bmatrix} \ddot{r}_C \\ \emptyset \end{Bmatrix} + \begin{bmatrix} B_{CC} & B_{C,SB} \\ B_{SB,C} & B_{SB,SB} \end{bmatrix} \begin{Bmatrix} \dot{r}_C \\ \emptyset \end{Bmatrix} + \begin{bmatrix} K_{CC} & K_{C,SB} \\ K_{SB,C} & K_{SB,SB} \end{bmatrix} \begin{Bmatrix} r_C \\ \emptyset \end{Bmatrix} =$$
$$- \begin{bmatrix} M_{CC} & M_{C,SB} \\ M_{SB,C} & M_{SB,SB} \end{bmatrix} \begin{Bmatrix} \ddot{R}_C \\ \ddot{R}_{SB} \end{Bmatrix} \begin{bmatrix} B_{CC} & B_{C,SB} \\ B_{SB,C} & B_{SB,SB} \end{bmatrix} \begin{Bmatrix} \dot{R}_C \\ \dot{R}_{SB} \end{Bmatrix} \begin{bmatrix} K_{CC} & K_{C,SB} \\ K_{SB,C} & K_{SB,SB} \end{bmatrix} \begin{Bmatrix} R_C \\ R_{SB} \end{Bmatrix} \qquad (20)$$

In equation (8) it was shown that the damping and stiffness terms on the R.H.S. are identically zero for translational excitations. Now constrain all of the $\ddot{u}_{SB}$ in $\ddot{R}$ to the prescribed base excitation value

$$\ddot{u}_{SB} = S(x,y,z)T(t). \tag{21}$$

The amount of time varying force needing to be aaplied to the base to maintain the prescribed base acceleration is the unknown $Q_{SB}(t)$.

$$\begin{bmatrix} M_{CC} & M_{C,SB} \\ M_{SB,C} & M_{SB,SB} \end{bmatrix} \begin{Bmatrix} \ddot{r}_C \\ \emptyset \end{Bmatrix} + \begin{bmatrix} B_{CC} & B_{C,SB} \\ B_{SB,C} & B_{SB,SB} \end{bmatrix} \begin{Bmatrix} \dot{r}_C \\ \emptyset \end{Bmatrix} + \begin{bmatrix} K_{CC} & K_{C,SB} \\ K_{SB,C} & K_{SB,SB} \end{bmatrix} \begin{Bmatrix} r_C \\ \emptyset \end{Bmatrix} =$$
$$- \begin{bmatrix} M_{CC} & M_{C,SB} \\ M_{SB,C} & M_{SB,SB} \end{bmatrix} \begin{Bmatrix} \ddot{R}_C \\ \ddot{R}_{SB} \end{Bmatrix} + \begin{Bmatrix} \emptyset \\ Q_{SB}(t) \end{Bmatrix} \tag{22}$$

Expand the upper partition of equation (22) to extract the equation for solution of r_C.

$$[M_{CC}]\{\ddot{r}_C\} + [B_{CC}]\{\dot{r}_C\} + [K_{CC}]\{rC\} = (-)[M_{CC}\ M_{SB,SB}] \left\{ \begin{array}{c} \ddot{R}_C \\ \hline \ddot{R}_{SB} \end{array} \right\} \tag{23}$$

Expand the lower partition of equation (22) to extract the equation for solution of $Q_{SB}(t)$.

$$[M_{SB,C}]\{\ddot{R}_C\} + [B_{SB,C}]\{\dot{R}_C\} + [K_{SB,C}]\{R_C\} = (-)[M_{SB,C}\ M_{SB,SB}] \left\{ \begin{array}{c} \ddot{R}_C \\ \hline \ddot{R}_{SB} \end{array} \right\} +$$
$$\{QS_B(t)\}. \tag{24}$$

Designate the distributed loading due to base acceleration by $P_C(t)$, then,

$$\{P_C(t)\} = (-)[M_{CC}\ M_{C,SB}]\begin{Bmatrix}\ddot{R}_C \\ \hline \ddot{R}_{SB}\end{Bmatrix} = (-)[M_{CG}]\{\ddot{R}_G\}. \tag{25}$$

Substitute (25) into equation (23) to arrange the solution equation for r_C in final form.

$$[M_{CC}]\{\ddot{r}_C\} + [B_{CC}]\{\dot{r}_C\} + [K_{CC}]\{r_C\} = \{P_C(t)\} \tag{26}$$

The equation of constraints can be rearranged according to separate physical actions.

$$\{Q_{SB}(t)\} = \underset{\substack{\text{Coupled}\\ \text{Inertia}}}{[M_{SB,C}]\{\ddot{r}_C+\ddot{R}_C\}} + \underset{\substack{\text{Coupled}\\ \text{Damping}}}{[B_{SB,C}]\{\dot{r}_C\}} + \underset{\substack{\text{Coupled}\\ \text{Elastic}}}{[K_{SB,C}]\{r_C\}} + \underset{\substack{\text{Direct}\\ \text{Base Inertia}}}{[M_{SB,SB}]\{\ddot{R}_{SB}\}} \tag{27}$$

Equations (26) and (27) are now ready for implementation.

Since the support motion acts like it were engaging a rigid slab, the reference vector of accelerations can be written as a product of a time function and a spatial function of ones. All seven permutations can be obtained by representing the individual component excitations then combining them to obtain their combined action.

The pattern of notation to be adapted for accelerations in each translational coordinate direction is:

First basic coord $\{\ddot{u}\} = \lfloor f(t),0,0,0,0,0 \rfloor^T$

Second basic coord $\{\ddot{v}\} = \lfloor 0,g(t),0,0,0,0 \rfloor^T$

Third basic coord $\{\ddot{w}\} = \lfloor 0,0,h(t),0,0,0 \rfloor^T$.

Let $\ddot{U}_i$ stand for any of the three directions $\ddot{u},\ddot{v}$, or $\ddot{w}$, and let $f_i(t)$ stand for any of the three acceleration functions $f(t)$, $g(t)$, or $h(t)$, and let $\ddot{R}_{Gi}$ stand for any of the relative acceleration vectors $\ddot{R}_1$, $\ddot{R}_2$, or $\ddot{R}_3$, where, for example

$$\{\ddot{R}_{G1}\} = \lfloor f_1(t)\ 0\ 0\ 0\ 0\ 0, f_1(t)\ 0\ 0\ 0\ 0\ 0 \ldots\ldots, f_1(t)\ 0\ 0\ 0\ 0\ 0 \rfloor^T$$

and by factoring out the time function a vector of ones is left:

$$\{\ddot{R}_{G1}\} = f_1(t) \lfloor 1\ 0\ 0\ 0\ 0\ 0, 1\ 0\ 0\ 0\ 0\ 0, 1\ 0\ 0\ 0\ 0\ 0 \ldots., 1\ 0\ 0\ 0\ 0 \rfloor^T$$

Define $\{H_G\}_i$ as the vector arrangement of ones for the i^{th} translational component in basic coordinates, and construct $\{\ddot{R}_i\}$ to be

$$\{\ddot{R}_{Gi}\} = f_i(t)\{H_{Gi}\}. \qquad (28)$$

If the output coordinate systems of some points in the interior of the structure are not parallel to the basic system in which the acceleration excitation is defined, then

the vector $\{H_G\}_i$ would be composed of non-unity direction cosines in those locations corresponding to such points. If the output coordinate system of the base points is basic, then the necessary transformation data can be obtained from the CSTM data block.

Use equation (28) in equation (25) to put the forcing function into NASTRAN format.

$$\{P_C(t)\} = (-)\sum_{1}^{3} f_i(t)\,[M_{CG}]\{H_{Gi}\} \qquad (29)$$

At this point the theoretical development is complete for solution in NASTRAN by the Direct Method of integration. The pertinent equations are 25, 26 and 29.

Development of the theory for solution by the modal method will continue before changing the topic to applications.

Modal Theory

The results of an eigenvalue run on the structure after being partitioned down to A-size equips the analyst with a matrix of eigenvectors $[\phi_{GA}]$. The subscripts indicate that

the eigenvalue solution has recovered the mode shape definition from A-size back to G-size, but the number of frequencies was limited by the number of degrees of freedom retained after partitioning down to A-size. Dynamicists differ in their technique at this point as to how they proceed into the modal route. Some say the number of component modes that are proper to use in a modal expansion are only those that ca safely fall within an acceptable accuracy range. Others say that all modes can be used, regardless of their accuracy in the frequency range out of interest, because the modes can be viewed mathematically as a finite series of orthogonal components that are acceptabale as a basis for an expansion set. In either case the general form of an expansion of a physical displacement vector $\{u_G\}$ into modal coordinates will follow the genral form

$$\{r_G(t)\} = [\emptyset_{Gn}]\{\xi_n(t)\} \qquad (30)$$

where the summation on n can range from 1 to N or from 1 to A depending on the dynamicist's choice. The vector $\{\xi_n(t)\}$ is a time dependent modal coordinate which is a variable that controls to what degree the n^{th} mode participates in the solution for $\{u_G(t)\}$ at any instant of time t.

Theoretically, equation (3Ø) would be partitioned to C-size before substituting into equation (26). In practice, as will be demonstrated in the Applications section, this will not be done, but for theoretical consistency the starting point will be a C-sized problem. Equation (26) becomes

$$[M_{CC}][\Ø_{Cn}]\{\ddot{\xi}_n(t)\} + [B_{CC}][\Ø_{Cn}]\{\dot{\xi}_n(t)\} + [K_{CC}][\Ø_{Cn}]\{\xi_n(t)\} = \{P_C(t)\} \quad (31)$$

Diagonalize the coefficient matrices by pre-multiplying by $[\Ø_{Cn}]^T$ and recognize by virtue of orthogonality of the modes that

$$[\Ø_{Cn}]^T[M_{CC}][\Ø_{Cn}] = [\backslash M_n \backslash] \quad (32)$$

$$[\Ø_{Cn}]^T[K_{CC}][\Ø_{Cn}] = [\backslash K_n \backslash] \quad (33)$$

and replace the second term by a non-diagonal matrix

$$[\Ø_{Cn}]^T[B_{CC}][\Ø_{Cn}] \ (=) \ [B_n] \quad (34)$$

where $[B_n]$ will be defined later. The matrix on the right

hand side becomes the generalized force vector

$$[\phi_{Cn}]^T\{P_C(t)\} = \{P_n\}. \tag{35}$$

Substitution of expressions 32,33,34,35 into equation (31) simplifies it to

$$[\backslash M_n \backslash]\{\ddot{\xi}_n\} + [B_n]\{\dot{\xi}_n\} + [\backslash K_n \backslash]\{\xi_n\} = \{P_n\}. \tag{36}$$

Pre-multiplying equation by $[\backslash M \backslash]^{-1}$ has the effect of using $[\phi_{Cn}]$ in a form whose vectors are normalized to mass, which simplifies by using the following definitions:

$$[\backslash M_n \backslash]^{-1}[B_n] \equiv [\backslash \beta_n \backslash] \tag{37}$$

where $\beta_n \equiv \zeta_n \omega_n$ and $\zeta_n \equiv 2b_n/b_{n\,cr}$, for modal frequency ω_n and for modal critical damping $b_{n\,cr}$. Use the notation for modal frequency and modal loading for compactness

$$[\backslash M_n \backslash]^{-1}[\backslash K_n \backslash] \equiv [\backslash \Omega_n^2 \backslash] \tag{38}$$

$$[\backslash M_n \backslash]^{-1}\{P_n\} \equiv \{\mathcal{P}_n\}. \tag{39}$$

Now equation (36) streamlines to a standard modal formulation

$$\{\ddot{\xi}_n\} + [\gamma_n]\{\dot{\xi}_n\} + [\Omega_n^2]\{\xi_n\} = \{\rho_n\} \quad (40)$$

The theory is now complete for both the modal and the direct methods.

Application

As was mentioned in the Introduction there are a number of ways to apply this technique. It will simplify the discussion if the applications are viewed as two broad categories, external and internal. External was used to certify the method and to establish its feasibility. The external route is useful, but somewhat laborious. The internal route is managed internal to NASTRAN which reduces the burden on the analyst to a few input data entries.

In the purely external mode the application of base acceleration forcing involves up to 4 partial executions of the dynamic rigid format each having DMAP ALTER's to prepare matrices for sending to the processor program BASACCC. BASACC uses the data from the interrupted NASTRAN executins to prepare DAREA cards for the distributed inertia load and to prepare matrices for solution recovery. Fewer than these 7 steps are required if some of the solutin refinements are eschewed.

Dynamic loads are formed by TLOAD1 input. Equation (29) is in a form amenable to TLOAD1 format. The vector product $[M_{CG}]\{H_G\}_i$ describes spatial distribution of inertia loads and must be supplied in a DAREA format. $\{H_G\}_i$, henceforth called HAICHi, is to be supplied manually by preparing DMI data input. It is advisable to execute NASTRAN thru GP4 with a DIAG 21 specification in EXECC to get a table of internal sequencing to act as a guide in preparing DMI data for HAICHi. A restart run including the DMI's for HAICHi in the input is needed next to carry it at least thru the generation of the mass matrix. A DMAP ALTER contains commands to multiply the mass matrix into the HAICHi vectors for the load in every mass point beyond the base. This second run can go up to the DPD module then exit. These matrix products are output under the name of DARINi. The function of the first BASACC run is to transform the DARINi matrices into DAREA card images by using the table of DIAG 21 information as a guide. Now the dynamics problem is ready to be solved. Nothing more would have to be done if the user were content with relative coordinate solutins and no refinements. In many applications, the recovery of response accelerations at interior points is needed for specifications to equipment suppliers. Consequently, the third NASTRAN execution does

not run to completion, but only thru module TRLG. It sends data blocks RUOO, HAICHi, TOL, and DIT to BASACC for it to generate matrices needed in the data recovery. There are three options available for running the RECOVERY phase of BASACC: (a) refinement of constraint forces QP, (b) refinement of relative displacements UPV, and (c) the recovery of absolute accelerations. Comparing the vector recovery operations for statics with dynamics one can see that the contribution of displacements due to loads on omitted points is missing from the dynaics rigid formats. In the early days there was a very good reason for this. NASTRAN had to operate on the 7094 direct coupled computer whose secondary storage was tape and operate as well on the 360, 6600 and 1108 machines that were equipped with discs. If a dynamic analyst did a proper job of modeling by condensing points with only insignifigant mass, then the contribution to displacement due to inertia loads on these points would be insignifigant enough to ignore. In an application such as this wherein all loading is inertia loading, the cumulative effect of condensation would not necessarily be insignifigant. For this reason and also because substructuring tends to produce cumulative effects the design of this operation deliverately offers the analyst a chance to recoup the contributions from omitted points.

Equation (27) is described as the equation to recover the base input forces. NASTRAN calls these single point constraint forces. The manual shows that the SPC forces that are recovered come from the equation

$$q_s = -P_s + K^T_{fs} u_f .$$

It consists of the external forces P_s plus the elastic forces that develop from coupling between unconstrained and constrained points. Now examination of equation (27) shows that this design will also recover the P_s forces $[M_{SB,SB}]\{\ddot{R}_{SB}\}$. Coupled elastic forces are also present in the term $[K_{SB,C}]\{r_C\}$. In addition this design provides for recovery of constraint forces due to inertia coupling $[M_{SB,C}]\{\ddot{r}_C+\ddot{R}_C\}$ which inevitably develop whenever PARAM COUPMASS is invoked and due to damping coupling $[B_{SB,C}]\{\dot{r}_C\}$. Implementation of these equations entailed a fair amount of DMAP gymnastics because output data blocks were not in the form needed by this design. Some of these tasks are performed with assistance from BASACC. Specifically, BASACC generates relative accelerations $\ddot{r}_C$ and the relative displacements from omitted points r_o^o. Finally NASTRAN completes its transient solution in relative coordinates and produces plots in relative coordinates. However another DAMP

string has to be appended to repeat the data recovery and plots for absolute accelerations. A summary of these seven steps is given below.

NASTRAN runs thru GP4 to get DIAG 21 output for setting up the HAICHi vectors.

System edit run of DIAG 21 data to prepare a file for BASACC use.

NASTRAN forms $[M_{GG}]\{H_G\}_i$ and passes this to BASACC as data blocks DARINi.

First BASACC run creates a file of DAREA card images.

Third NASTRAN run uses the DAREA data from BASACC and sets up data blocks RUOO, HAICHi, TOL, and DIT for BASACC.

Second BASACC run prepares matrices RDBLDOT and UOOT to refine the recovery of SPCF and UPV.

Final NASTRAN run solves the problem, recovers refined data in relative and absolute coords.

A flow diagram of these operations is shown in figures 2 thru 5 for the case of R.F. 9 on a CDC 6500.

This paper is reporting on work that is currently in progress. So far this design has been adapted to direct transients on a CDC 6500 computer. Adaptations to direct

frequency response and to modal transients and frequency response are being interrupted by preparing this paper. As it stands this technique is awkward and tedious and fraught with room for human error. What has been accomplished is to show that the method is feasible. Now it is time to make it convenient. Plans have been drafted for writing the code for DUMMOD's to automate this whole procedure in the form of a set of new rigid formats. It is expected that it will have been automated and tested on the CDC before the year is out.

Tests on this base acceleration technique have been conducted on a simplified model of a nuclear reactor containment building subjected to seismic accelerations into its base. The model has a variety of elements, constraints, condensations, transfer functions, and nonlinearities so that its interface with a sizeable range of conditions has been satisfactorily explored.

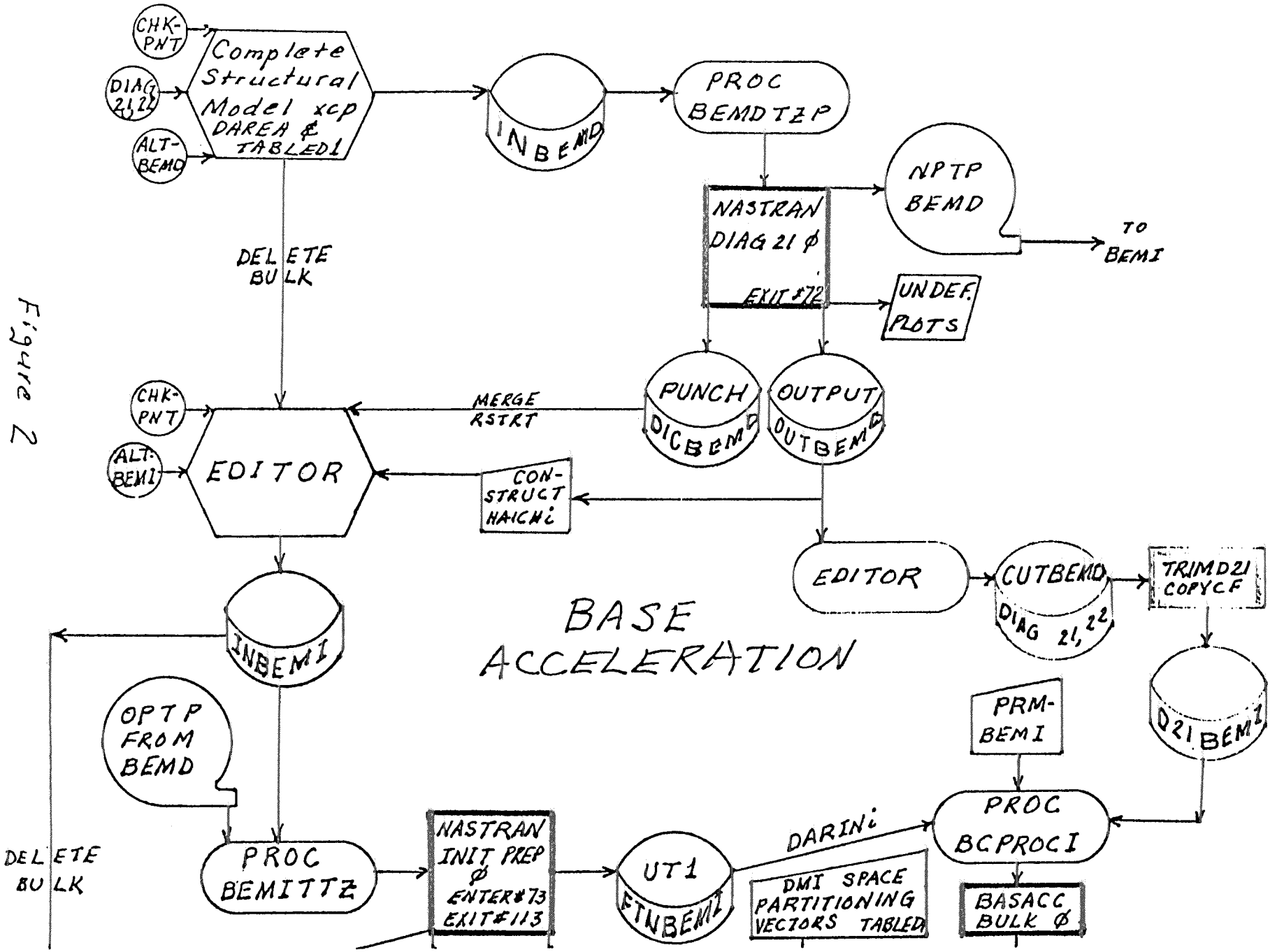
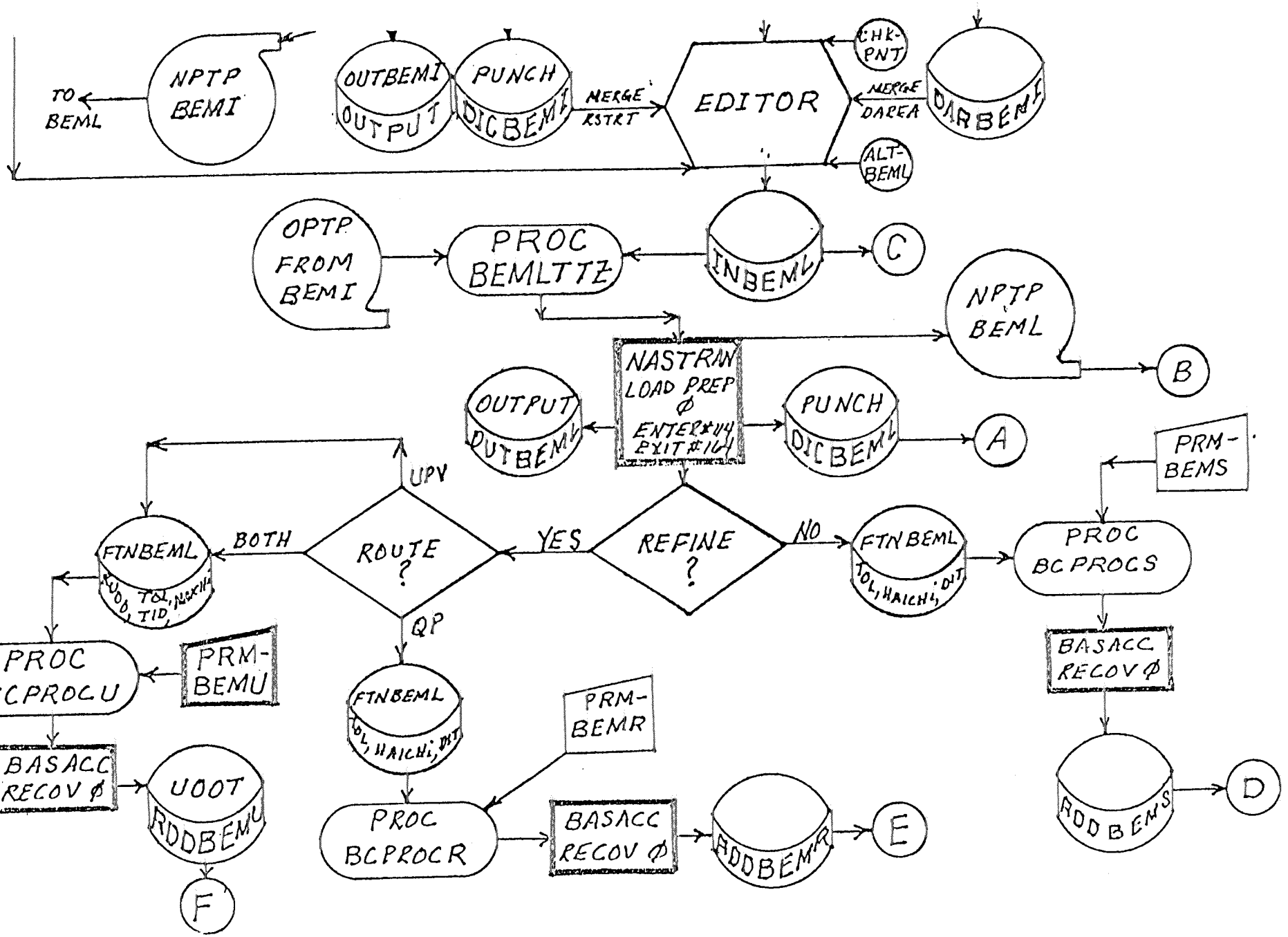

CHK-PNT
DIAG 21,22
ALT-BEMD
Complete Structural Model xcp DAREA & TABLED1
INBEMD
PROC BEMDTZP
NASTRAN DIAG 21 Ø
EXIT #72
NPTP BEMD
TO BEMI
UNDEF. PLOTS
DELETE BULK
PUNCH DICBEMD
OUTPUT OUTBEMD
MERGE RSTRT
CHK-PNT
ALT-BEMI
EDITOR
CON-STRUCT HAICMi
EDITOR
CUTBEMD DIAG 21,22
TRIMD21 COPYCF
D21BEM1
INBEMI
BASE ACCELERATION
OPTP FROM BEMD
PRM-BEMI
PROC BCPROCI
DELETE BULK
PROC BEMITTZ
NASTRAN INIT PREP Ø ENTER#73 EXIT#113
UT1 FTNBEMI
DARINi
DMI SPACE PARTITIONING VECTORS TABLED
BASACC BULK Ø

Figure 2


NPTP BEMI
TO BEML
OUTBEMI OUTPUT
PUNCH DICBEML
MERGE RSTRT
EDITOR
CHK-PNT
MERGE DAREA
DARBEMF
ALT-BEML
OPTP FROM BEMI
PROC BEMLTTZ
INBEML
C
NPTP BEML
B
NASTRAN LOAD PREP Ø ENTER#114 EXIT#164
OUTPUT OUTBEML
PUNCH DICBEML
A
PRM-BEMS
UPV
FTNBEML
BOTH
ROUTE ?
YES
REFINE ?
NO
FTNBEML
PROC BCPROCS
BASACC RECOVØ
RDDBEMS
D
PROC BCPROCU
PRM-BEMU
BASACC RECOVØ
UOOT RDDBEMU
F
QP
FTNBEML
PRM-BEMR
PROC BCPROCR
BASACC RECOVØ
RDDBEMR
E

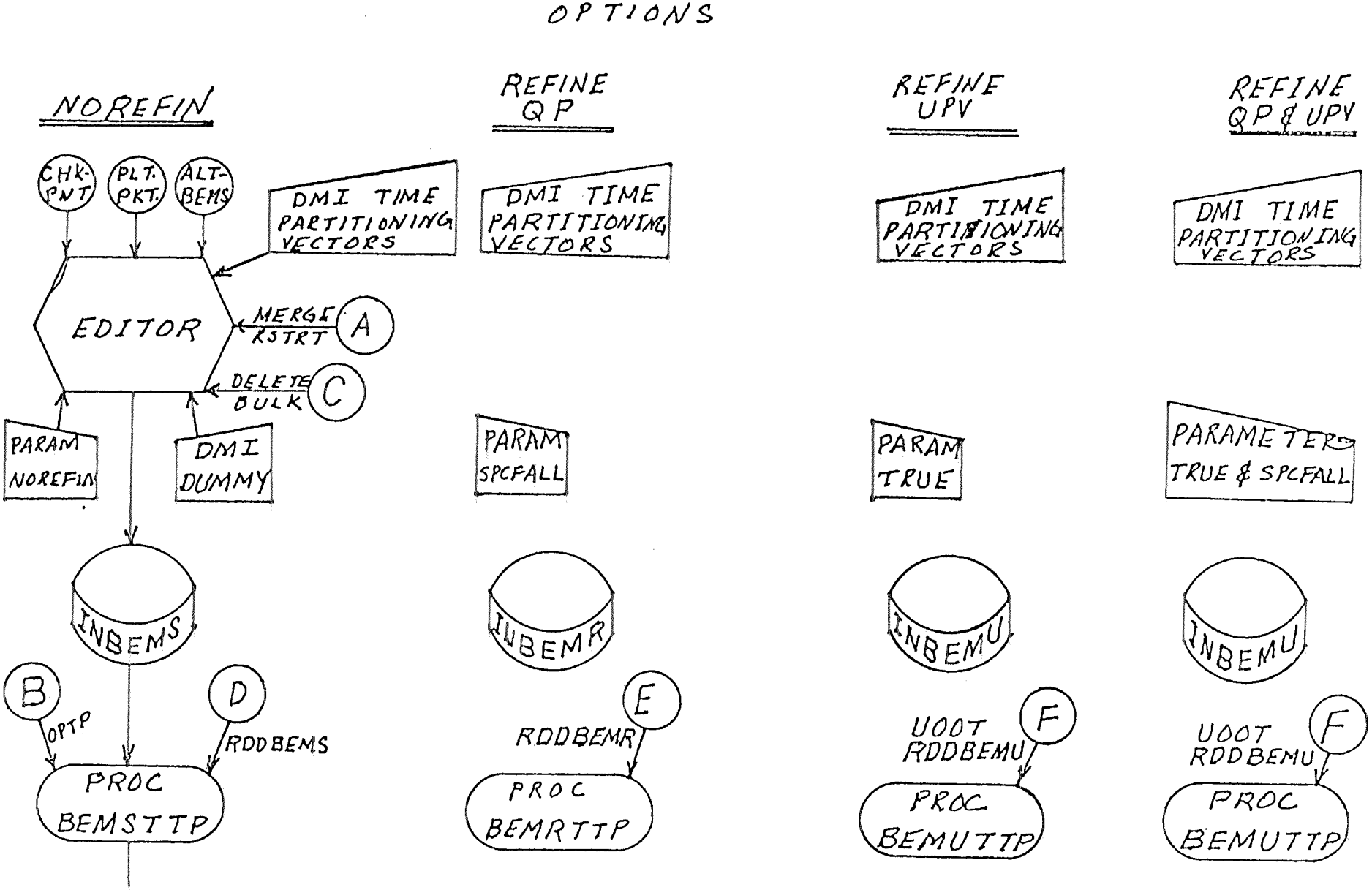
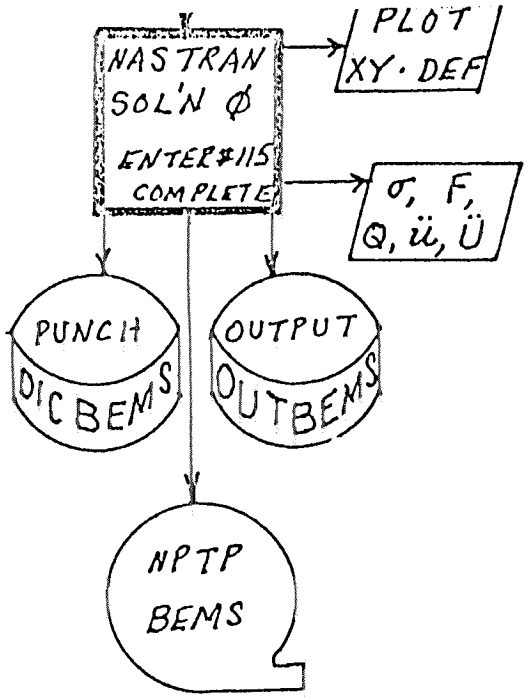
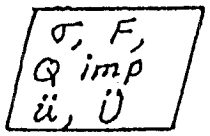
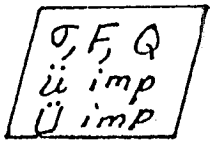

OPTIONS
NOREFIN
REFINE QP
REFINE UPV
REFINE QP & UPV
CHK-PNT
PLT-PKT
ALT-BEMS
DMI TIME PARTITIONING VECTORS
DMI TIME PARTITIONING VECTORS
DMI TIME PARTITIONING VECTORS
DMI TIME PARTITIONING VECTORS
EDITOR
MERGE RSTRT
A
DELETE BULK
C
PARAM NOREFIN
DMI DUMMY
PARAM SPCFALL
PARAM TRUE
PARAMETER TRUE & SPCFALL
INBEMS
INBEMR
INBEMU
INBEMU
B
OPTP
D
RDDBEMS
E
RDDBEMR
UOOT RDDBEMU
F
UOOT RDDBEMU
F
PROC BEMSTTP
PROC BEMRTTP
PROC BEMUTTP
PROC BEMUTTP

Figure 4


NASTRAN
SOL'N Ø
ENTER#1115
COMPLETE
PLOT
XY · DEF
σ, F,
Q, ü, Ü
PUNCH
DICBEMS
OUTPUT
OUTBEMS
NPTP
BEMS


σ, F,
Q imp
ü, Ü


σ, F, Q
ü imp
Ü imp

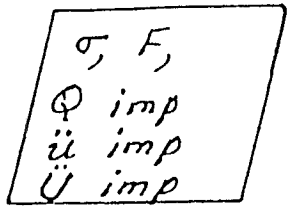

σ, F,
Q imp
ü imp
Ü imp

Figure 5