INTRODUCTION

At the 1980 Conference, A. O. Smith presented newly developed RF6 elastic-plastic capability in MSC/NASTRAN. This new capability was a thin-wall beam element and a triangular shell element which would yield and deform plastically under bending loads.

In the following will be presented a third elastic-plastic element - a quadrilateral shell element. The addition of this element has greatly enhanced our shell modeling RF6 analysis. The quad element is the principal 'workhorse' element because there are fewer elements required and because the element has much better in-plane or membrane stiffness characteristics than the triangular element.

THEORY AND APPLICATION OF THE QUADRILATERAL SHELL ELEMENT

The quadrilateral shell element used here was essentially that due to Kanok-Nukulchai*. This element was evolved from an eight-node three dimensional solid element. The displacement field associated with this element was used in a degenerate form to obtain a shell element. The transverse shear energy was retained in the element formulation, consequently, the element is applicable to either thick or thin shells.

This element was implemented in MSC/NASTRAN as the dummy element CDUM5. One ADUM5 card is necessary. The connection cards CDUM5 are the same format as CQUAD4 card. The PDUM5 card has the same format as the PQUAD2 card. The Bulk Data cards are shown in Figure 1. These cards will be produced from the A. O. Smith RF6 preprocessor from CQUAD4 and PSHELL cards.

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*W. Kanok-Nukulchai, "A Simple and Efficient Finite Element for General Shell Analysis", International Journal for Numerical Methods in Engineering, Vol. 14, 1979, pp. 179-200.

The elastic-plastic stress-strain law used is the well-known Prandtl-Reuss law which assumes the Hencky-Mises yield condition. Three options are available. Elastic-ideally plastic (zero strain hardening) is one option. Also available is an elastic plastic option with linear strain hardening. The most general option is specifying elastic-plastic behavior from table input stress-strain data. These options are selected on the MAT1 card which requires special consideration as shown in Figure 2.

Capability of the quadrilateral CDUM5 shell element will be shown through correlation with three sample problems for which both elasticity and plastic limit analysis solutions are known. The first problem is a rectangular cantilever beam which will test the in-plane or membrane properties of the element. The NASTRAN model of the beam is pictured in Figure 3. The results from an elastic analysis are shown in Figure 4. It is seen the quadrilateral gives good agreement with beam theory including shear deformation for this relatively coarse mesh. At the beam tip the error in displacement is 4.5%. This is much better than the 16% achieved with an equivalent mesh of triangular CDUM4 elements. This improved in-plane flexibility with isoparametric quadrilateral elements is well-known and it is one of the reasons the quadrilateral should be used whenever practicable. The comparison with stresses is also seen to be good with the largest difference from theory being less than 3%.

A NASTRAN RF6 plastic collapse analysis was made on this same model. The initial yield load was determined from the linear RF24 analysis and then added in increments to failure. Figure 5 shows the load-deflection curve. The triangular and quadrilateral elements show very similar results with collapse about 7% below the limit analysis load.

The second problem is a square, simply supported, laterally loaded plate. This problem will test the bending properties of the quadrilateral CDUM5 element. Convergence studies were made for both uniform and concentrated loads in linear analysis. Symmetry allows analysis with only one-fourth of the plate modeled. The square plate was divided into uniform meshes as shown in Figure 6 except quad elements were used instead of triangles. Linear analyses were made for the meshes for N of 2, 3 and 4 and the results plotted in Figure 6. It is seen the quadrilateral CDUM5 is more flexible than the triangular CDUM4. This is due to the transverse shear deformation. Using as the standard, the results from a CQUAD4 analysis with shear deformation for mesh N=4, it is seen the CDUM5 converges to the correct value with shear deformation included.

A NASTRAN RF6 analysis was made on the square plate under uniform load using mesh number 4. Figure 7 shows the resulting load-deflection curve. No exact solution is known but close upper and lower bounds have been obtained. The CDUM5 model is seen to collapse at the lower limit load.

The third problem is an infinite, circular, cylindrical shell with an axially symmetric ring load. Both large hoop stresses and localized bending around the applied ring load will test the combined bending and membrane capabilities of the CDUM5 quad element. Because the displacements and stresses damp out rapidly the infinite shell can be approximated by a long finite shell. The radial displacements from a linear analysis are shown in Figure 8. The agreement with theory is seen to be good. The CDUM5 quad element gives generally better agreement than the triangular CDUM4 element. For both elements the agreement tends to deteriorate at a long distance from the load point because of the finite shell approximation. However, the stresses and displacements in this region are small.

A NASTRAN RF6 plastic analysis was made on the cylindrical shell model of CDUM5 quad elements. Initial yield was determined from linear analysis and then incremented to failure. The load-deflection curve in Figure 9 shows good agreement with limit analysis theory.

As these are new, user defined elements, a preprocessor has been developed for converting existing NASTRAN data. This has been added to our MSC/NASTRAN catalogued procedure and it is invoked by setting one parameter and specifying set cards (ALL is permitted) that define the elements which will be allowed to go plastic. In addition to the set cards the user must specify the load increments and the stress-strain table if that option is desired. A sample deck is illustrated in Figure 10.

CONCLUSIONS

A quadrilateral shell element which will yield and deform plastically under combined states of bending, twisting and axial load has been implemented in Rigid Format 6 of MSC/Nastran as user element CDUM5. Together with the CDUM4 triangular shell element, the user has a full complement of shell modeling capability in RF6.

ADUM5 - only one card permitted

Enter exactly as shown

ADUM5	NG	NC	NP	ND	ELNM				
ADUM5	4	2	1						

CDUM5 - Connection Card

CDUM5	EID	PID	G1	G2	G3	G4			
CDUM5	16	4	2314	2316	2416	2414			

EID - element identification number

PID - identification number of corresponding PDUM5 card

G1, G2, G3, G4 - Grid point numbers of connection points

PDUM5 - Property Card

PDUM5	PID	MID	T						
PDUM5	4	6	1.2						

PID - property identification number

MID - material identification number of MAT1 card

T - thickness (Real>0.0)

FIGURE 1. ELASTIC-PLASTIC QUADRILATERAL SHELL ELEMENT BULK DATA

Elastic-Ideally Plastic Option (No Strain Hardening)

MAT1	MID	E	G	NU	RHO	A	TREF	GE	+M1
MAT1	5	3.+7		.25					+M1

+M1	ST	SC	SS	MCSID					
+M1	33000.								

Elastic-Plastic Option with Linear Strain Hardening

MAT1	MID	E	G	NU	RHO	A	TREF	GE	+M1
MAT1	5	3.+7		.25					+M1

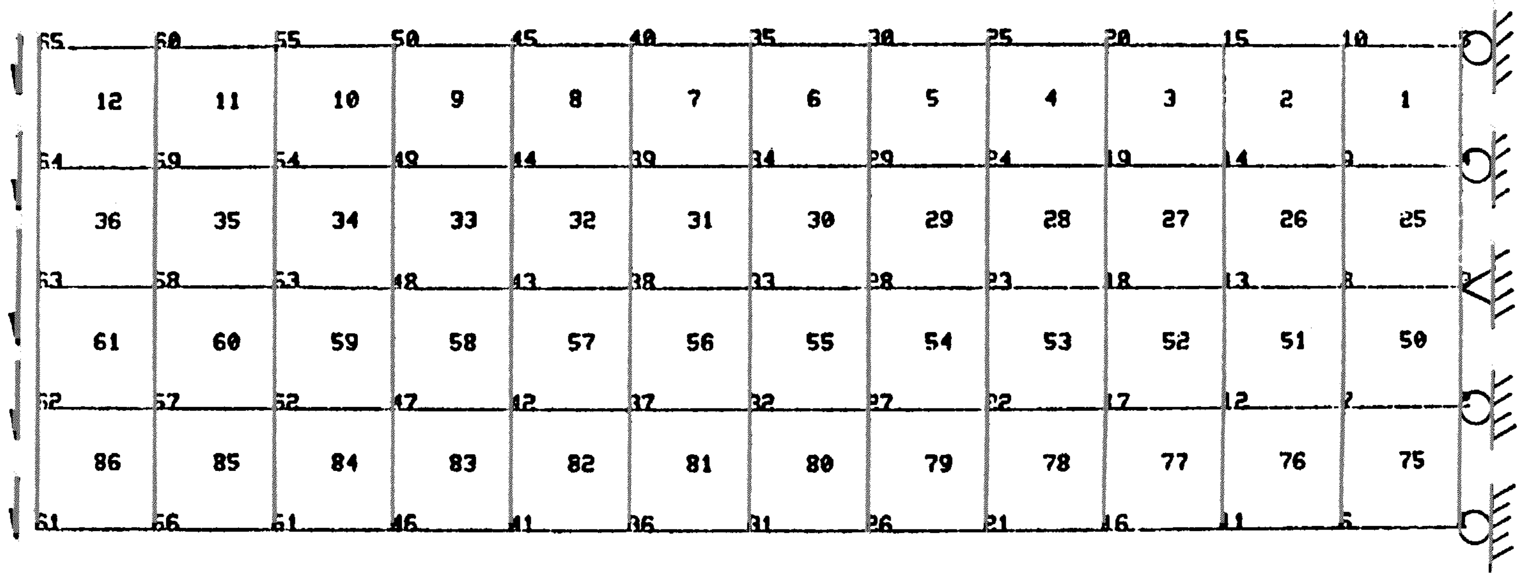
+M1	ST	SC	SS	MCSID					
+M1	33000.		1000.						

Elastic-Plastic Option Obtaining Material Data from TABLES1 Card

MAT1	MID	E	G	NU	RHO	A	TREF	GE	+M1
MAT1	6	1.+7		.3					+M1

+M1	ST	SC	SS	MCSID					
+M1		-1.							

FIGURE 2. MAT1 CARD EXAMPLES

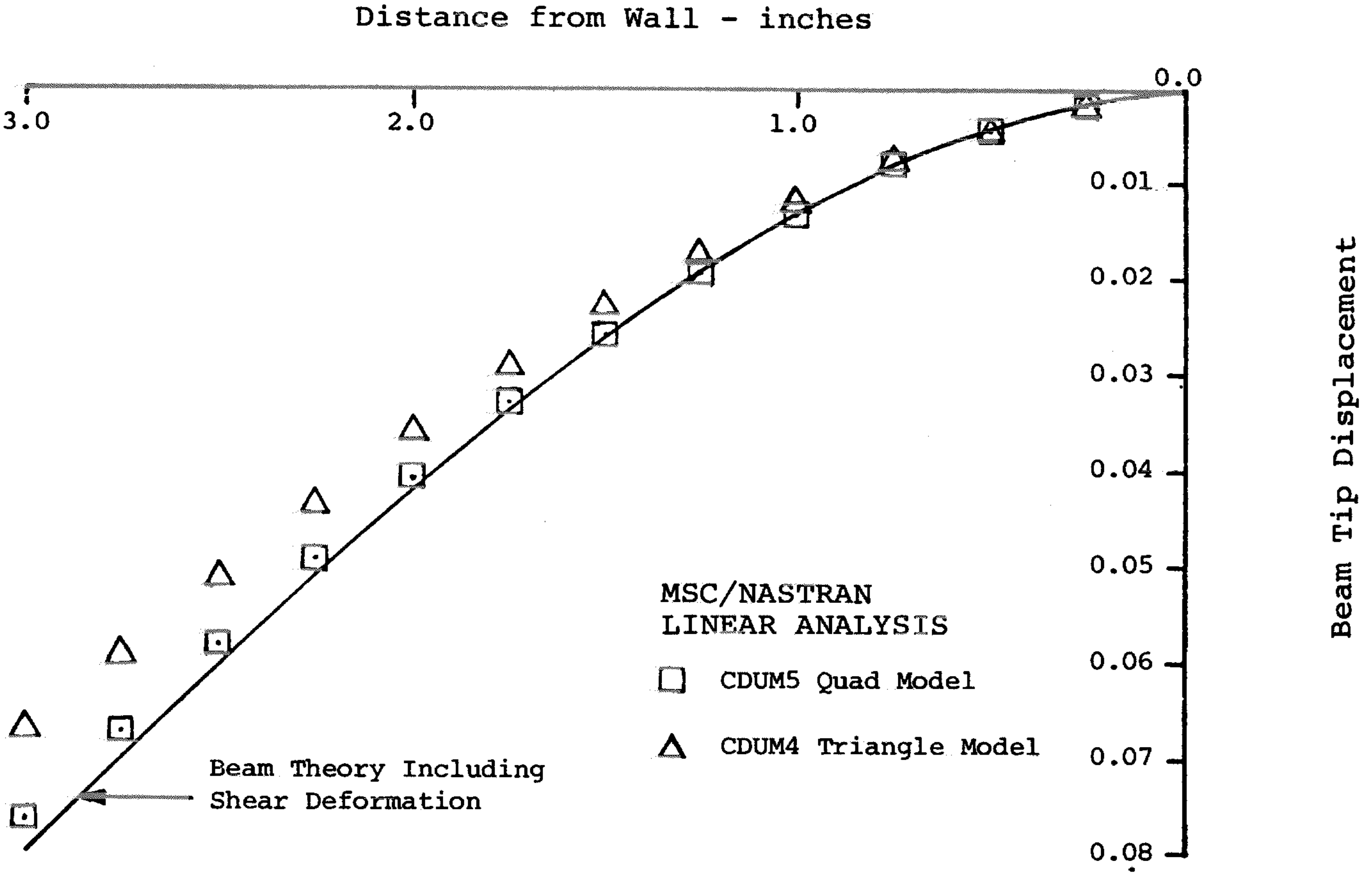


L = 3.0 inches E = 3.E7 psi

d = 1.0 inches ν = 0.3

t = 0.1 inches P = 2000 lbs.

FIGURE 3. CANTILEVER BEAM MODEL OF CDUM5 ELEMENTS



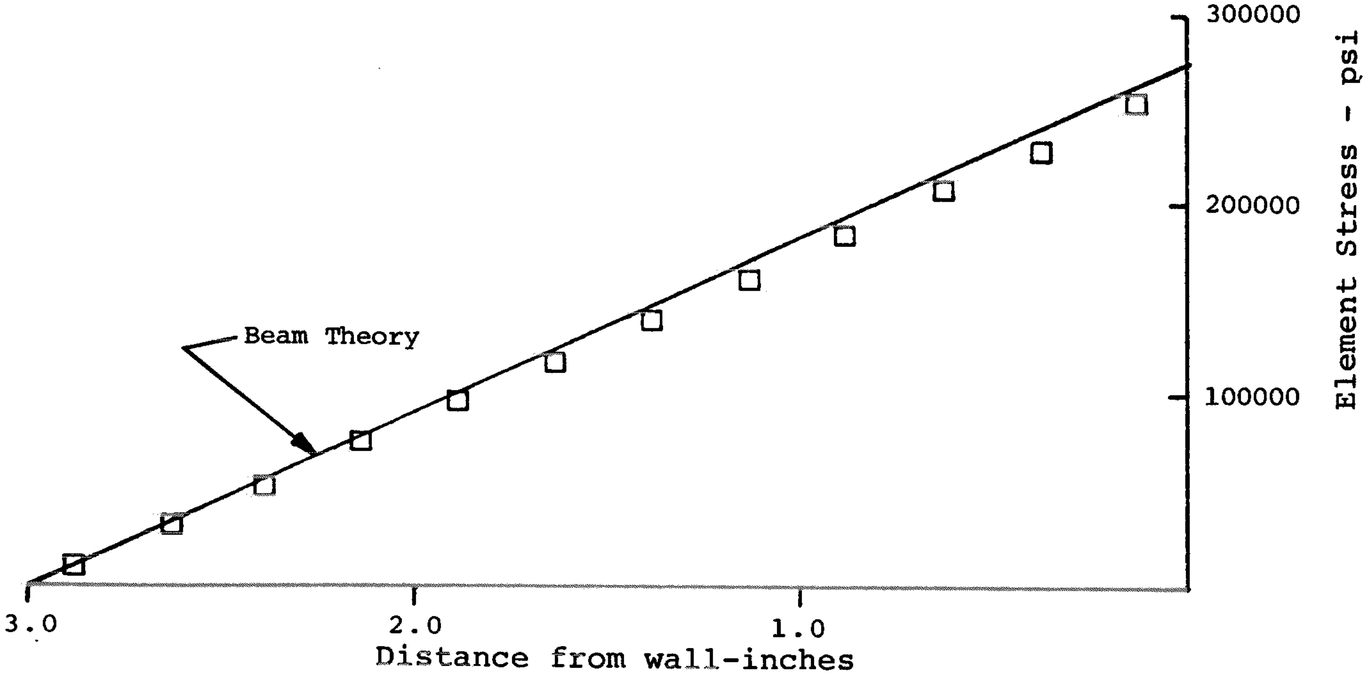


FIGURE 4. CANTILEVER BEAM IN-PLANE LOADING

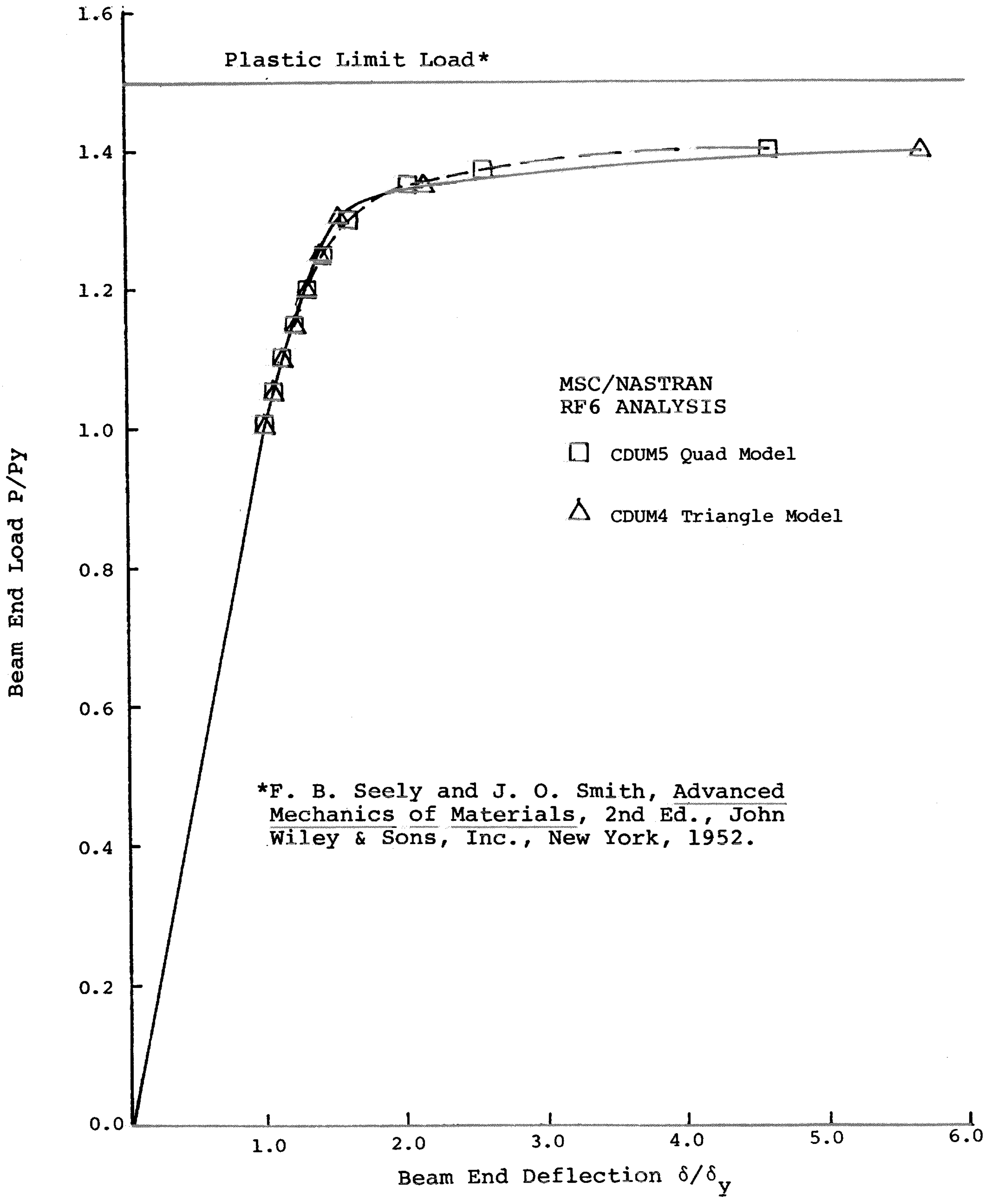
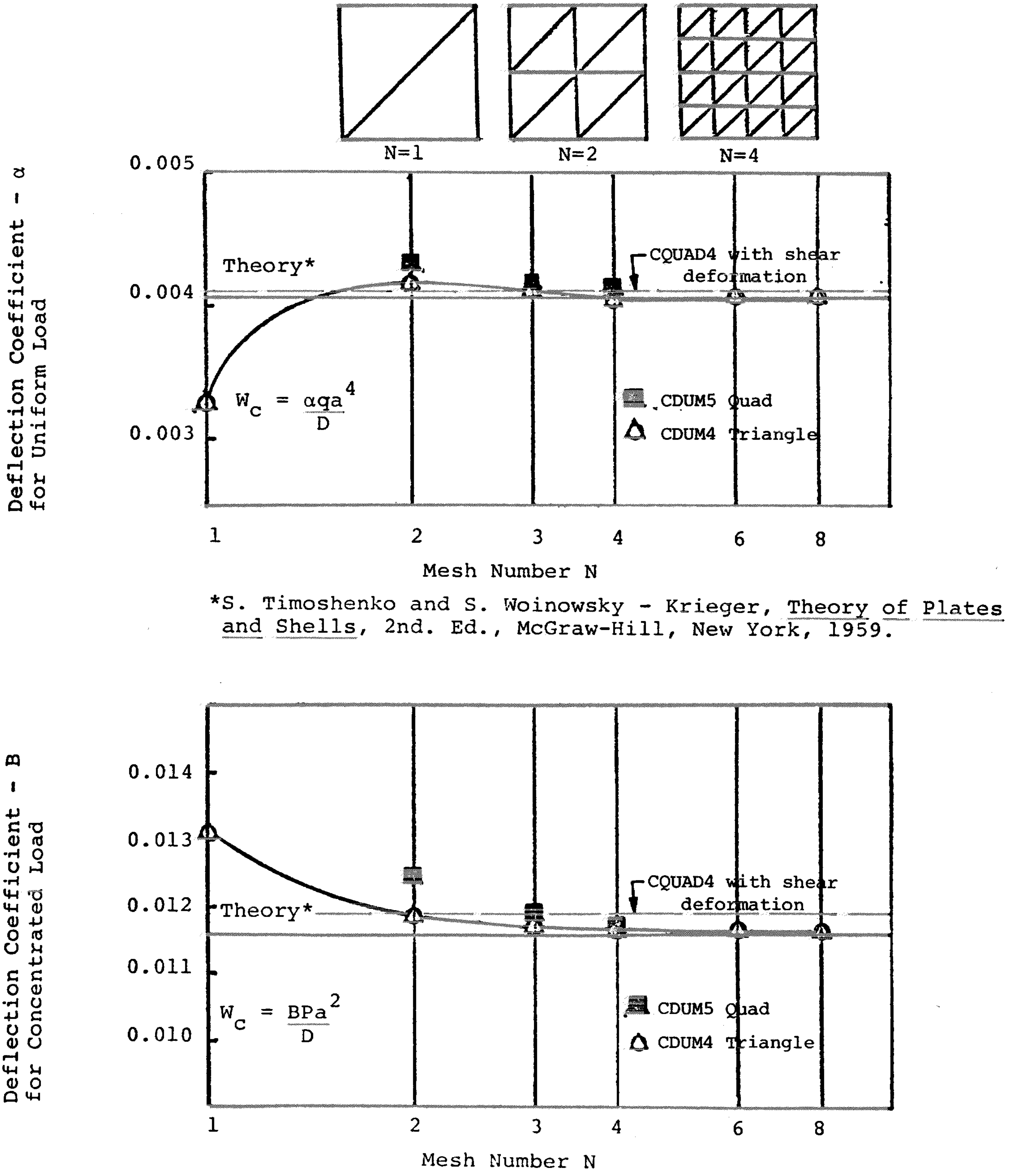
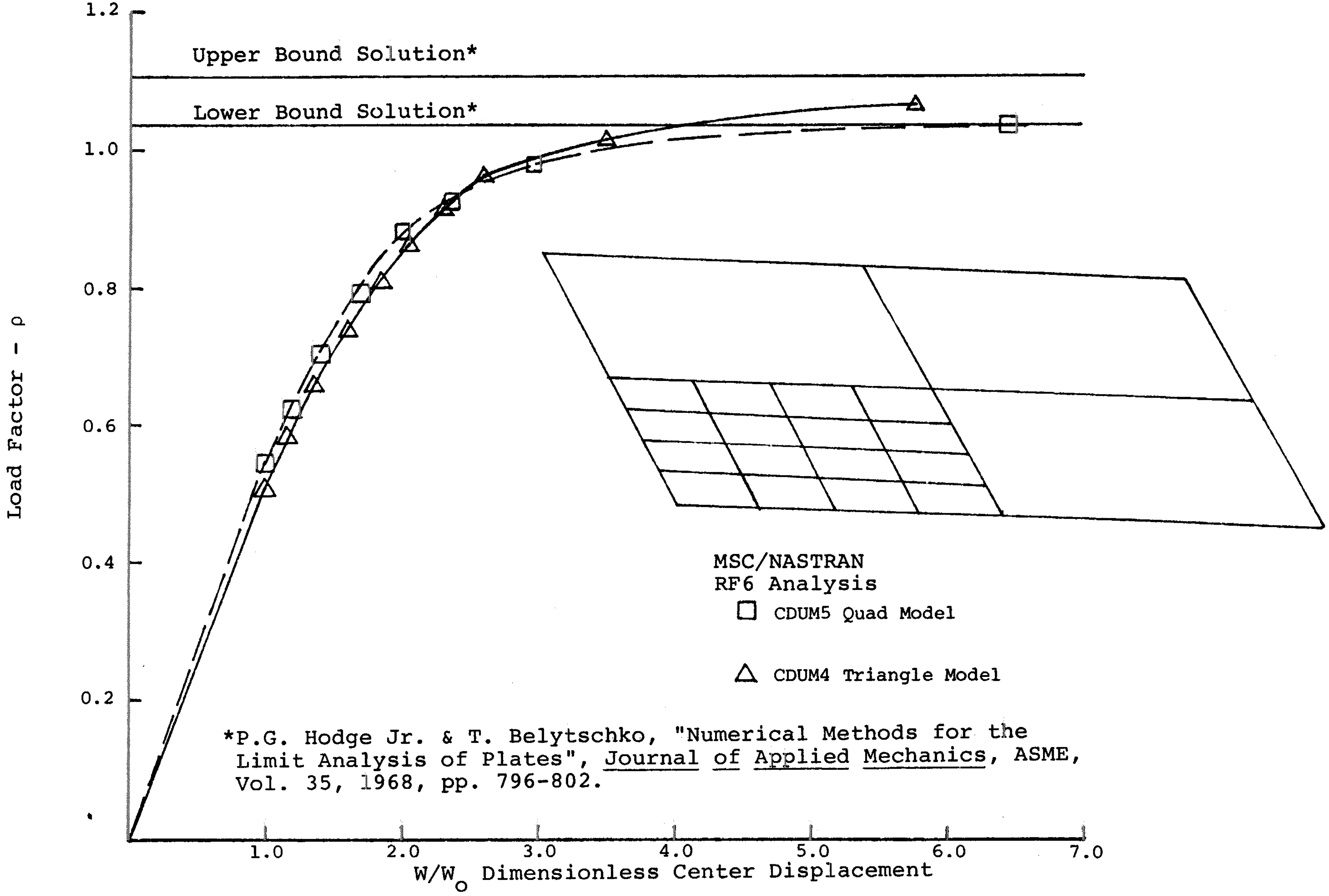


FIGURE 5. ELASTIC-PLASTIC COLLAPSE OF CANTILEVER BEAM



*S. Timoshenko and S. Woinowsky - Krieger, Theory of Plates and Shells, 2nd. Ed., McGraw-Hill, New York, 1959.

FIGURE 6. DEFLECTION CONVERGENCE FOR LATERALLY LOADED, SIMPLY SUPPORTED SQUARE PLATES



*P.G. Hodge Jr. & T. Belytschko, "Numerical Methods for the Limit Analysis of Plates", <u>Journal of Applied Mechanics</u>, ASME, Vol. 35, 1968, pp. 796-802.

FIGURE 7. ELASTIC-PLASTIC COLLAPSE OF LATERALLY LOADED PLATE

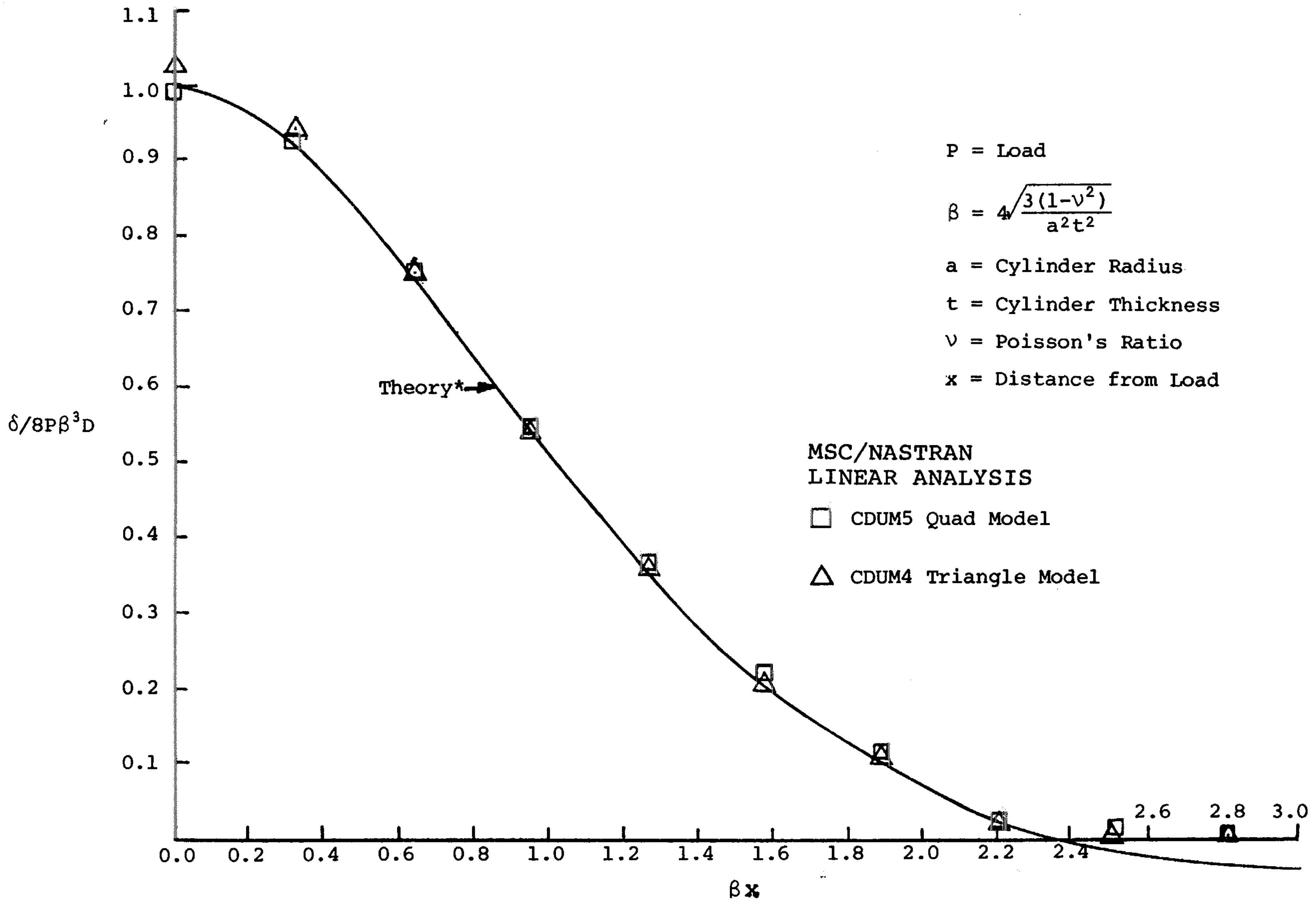
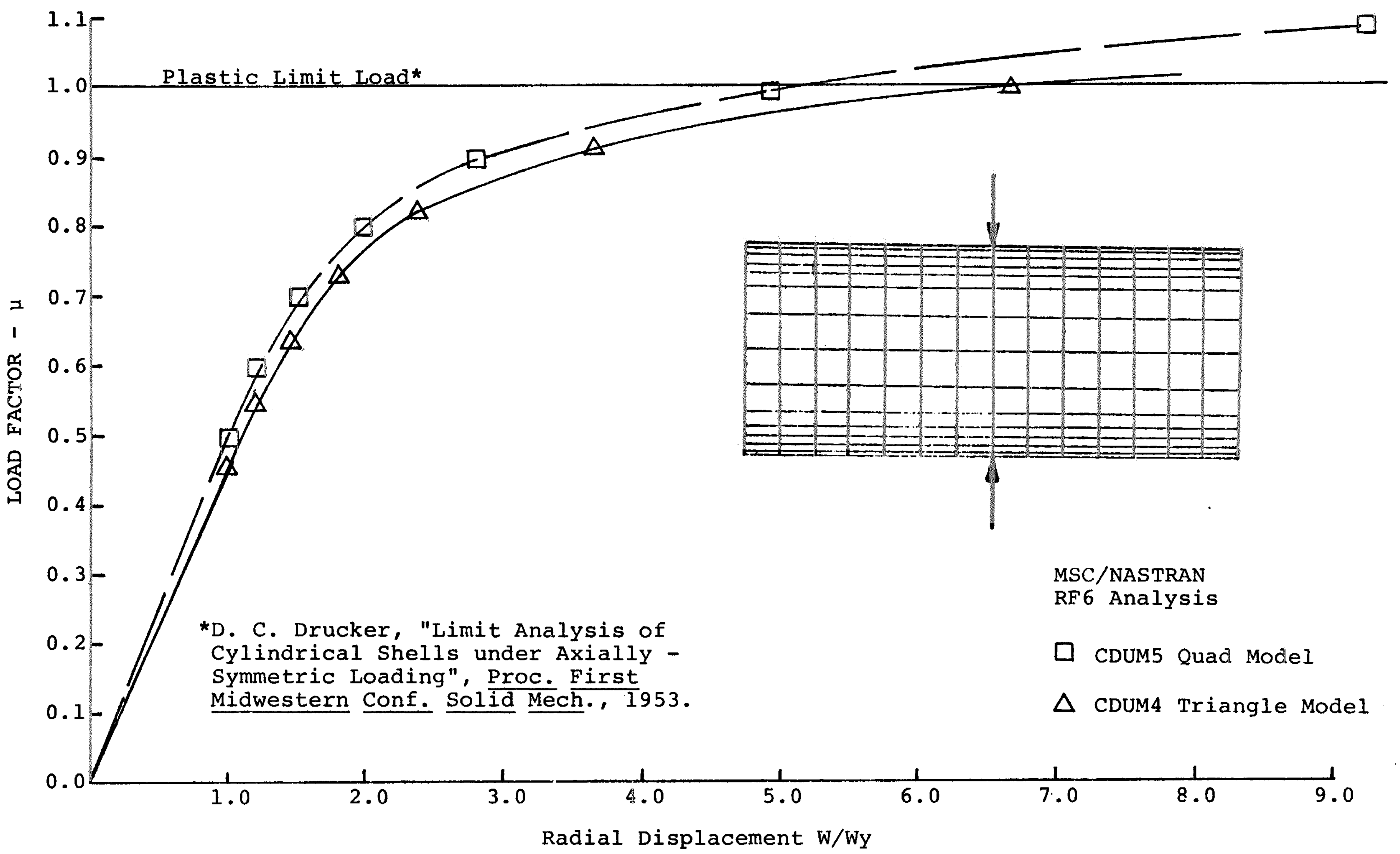


FIGURE 8. LINEAR ANALYSIS OF RING LOADED CYLINDRICAL SHELL DISPLACEMENTS



*D. C. Drucker, "Limit Analysis of Cylindrical Shells under Axially - Symmetric Loading", Proc. First Midwestern Conf. Solid Mech., 1953.

FIGURE 9. PLASTIC COLLAPSE OF RING LOADED CYLINDRICAL SHELL

A. O. SMITH NASTRAN RF6 PREPROCESSOR

PURPOSE

CONVERT MSC/NASTRAN ELEMENTS TO A. O. SMITH RF6 ELASTIC-PLASTIC ELEMENTS

EXAMPLE

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ID A, B
APP DISP
SOL 6, 0
- - -
CEND
TITLE=AOSRF6 PLASTIC COLLAPSE
PLCO=2
- - - -
BEGIN BULK
$RF6 1, 2, 100, 103, 1001, 1010
$RF6 50 THRU 95
$RF6 END
CQUAD4  1  5  201   206  205  202
CQUAD4  4  5  252   257  256  253
CTRIA3  2  7   42    47   46
CTRIA3  3  7   50    53   51
PSHELL  5  6  0.25   6         6
PSHELL  7  6  0.35   6         6
MAT1    6  1. + 7   .3                 +M
+M     3. + 4
PLFACT  2  1.0  1.2  1.3  1.4
- - -
ENDDATA
```

FIGURE 10. A. O. SMITH ELASTIC-PLASTIC ELEMENT PREPROCESSOR