Dynamic Analysis of Optical Scan Systems Using MSC/NASTRAN

by
William J. Nowak
Project Engineer
Xerox Corporation

ABSTRACT

This paper describes practical dynamic modeling and analysis of optical scan systems using MSC/NASTRAN. Simple spring and mass finite elements are used to represent the dynamic behavior of the system. Frequency and transient response solutions are presented based on representative dynamic loads induced during scan and re-scan operation. System response is characterized by image motion during image scanning and represented in the dynamic model by multi-point constraint equations. Linearization of the dynamic model, determination of representative forcing functions, and calculation of image motion coefficients are described along with listings of the actual NASTRAN data decks.

INTRODUCTION

Today's business market is seeking higher copy volume at lower copying costs. The use of more sophisticated analytical tools to evaluate optical scan system design with respect to reproduction quality is becoming more apparent. Since the function of a scan system is the syncronous transfer of an image from the document platen to the photoreceptor, dynamic analysis of these systems is necessary to determine image motion during processor operation. As the number of dynamic degrees of freedom increases to accommodate the accuracy of the analysis, the use of finite element programs becomes advantageous. MSC/NASTRAN is well suited to evaluate the dynamic response of an optical scan system consisting of any number of degrees of freedom with any general time or frequency dependent loading.

Typical Optical Scan System

While there are many possible mirror/lens combinations to transfer an image from the document platen to the photoreceptor drum, one of the most common is a full-rate/half-rate system shown in Figure 1. The image is transferred from the platen to the full-rate mirror, moving at a constant linear velocity, V_{SCAN}, to the half-rate mirror moving at 1/2 V_{SCAN}, and on to the lens mirror. This portion of the imaging is referred to as the "Front conjugate" and its length is formulated in Equation (1).

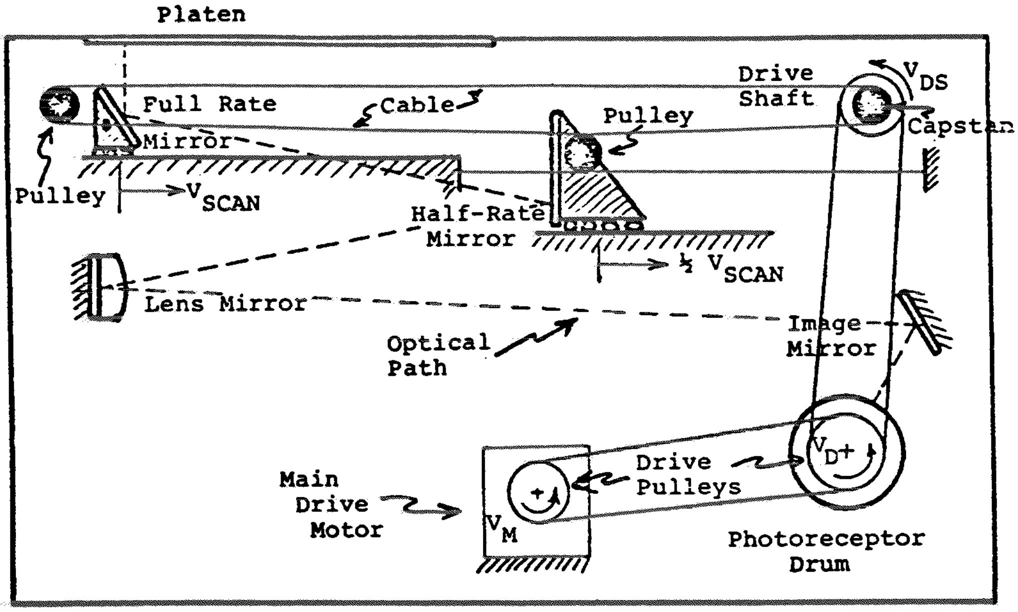


Figure 1. Typical Optical Scan System

The image then reflects off of the lens mirror, to the image mirror, and on to the photoreceptor drum moving at a constant angular velocity, V_D. This portion of the imaging is referred to as the "rear conjugate" and its length is formulated in Equation (2).

$$\text{Front Conjugate} = \frac{\text{Focal Length X (Magnification + 1)}}{\text{Magnification}} \qquad (1)$$

$$\text{Rear Conjugate} = \text{Focal Length X (Magnification + 1)} \qquad (2)$$

The front and rear conjugate must remain constant for the proper transfer of the image. The full-rate/half-rate system is the mechanism which maintains a constant front conjugate during document scanning.

Scan system dynamics is defined as the vibratory motion of the optical and mechanical components in the direction of scan or re-scan. To simplify the analysis, assume that all major optical and mechanical components shown in Figure 1 are rigid structures represented by individual lumped mass elements (CONM2). Also assume that the lens and image mirrors are rigidly constrained from all motion and are not contributors to the dynamics of the scan system. Further assume that cables, belts, pulleys, etc. all contribute to the stiffness and damping (CELAS2) between each component mass. The simplified structure is a five degree of freedom spring/mass/dashpot model shown in Figure 2.

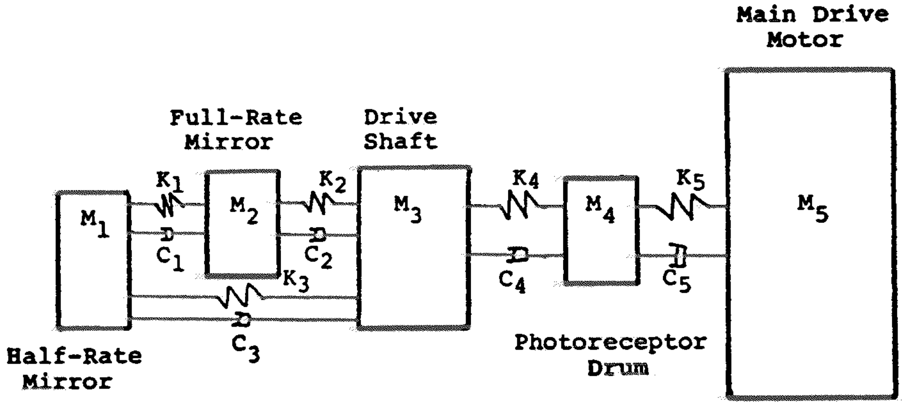


Figure 2. Dynamic Representation of Scan System

Calculation of Linear Spring Rates and Masses at a Reference Point

Because a simplified dynamic modeling approach is being used, it is not required that the actual kinematics of the scan system be defined geometrically. Therefore, all calculated or experimentally determined springs and masses must be referenced to some common point of motion. This point of motion is chosen as the full-rate mirror motion. Thus all other masses and springs, whether they are linear or rotational, must be referenced to the linear motion of the full-rate mirror.

To maintain the correct dynamic behavior of the half-rate mirror connected between the full-rate mirror and the drive shaft capstan, the mass of the half-rate carriage must be divided by the square of the velocity ratio as given in Equation (3).

$$M_1 = \frac{M_{HALF-RATE}}{(V_{SCAN}/\frac{1}{2}V_{SCAN})^2} \qquad (3)$$

or

$$M_1 = \frac{M_{HALF-RATE}}{4}$$

This procedure maintains the correct kinetic energy balance with respect to scan or process velocity, V_{SCAN}. The mass of the full-rate is simply the measured or calculated mass of the component since the full-rate mirror is the designated reference point.

The effective masses of the drive shaft, photoreceptor drum, and main drive motor are calculated based on the relative angular velocities of each component referenced to the linear process velocity, V_{SCAN}. For the drive shaft and the photoreceptor drum, the procedure involves two steps. First the inertia, I, about the center of rotation of each, is calculated using standard mathematical formulas. Since the capstan surface and the photoreceptor drum surface are moving at an effective linear velocity equal to V_{SCAN}, all that is required is to linearize the inertia of each component by dividing them by the surface radius, R, squared as given in Equation (4) and (5).

$$M_3 = \frac{I_{DRIVE\ SHAFT}}{R^2_{CAPSTAN\ SURFACE}} \tag{4}$$

and

$$M_4 = \frac{I_{DRUM}}{R^2_{DRUM\ SURFACE}} \tag{5}$$

The effective mass of the main drive motor requires one more intermediate step. Again the inertia of the rotor is calculated. Since the rotor angular velocity is usually 20 to 30 times larger than the drive shaft angular velocity, the apparent rotor inertia referenced to the rest of the scan system is larger by the square of the velocity ratio between the rotor speed, V_M, and the drive shaft speed, V_{DS}, as given in Equation (6). Since the rotor inertia is referenced to the capstan inertia, all that remains is to linearize the apparent rotor inertia by dividing it by the capstan surface radius squared as given in Equation (7).

$$I_{ROTOR\ APPARENT} = I_{ROTOR} \times \frac{V_M^2}{V_{DS}^2} \tag{6}$$

$$M_5 = \frac{I_{ROTOR\ APPARENT}}{R^2_{CAPSTAN\ SURFACE}} \tag{7}$$

Typically the effective mass of the main drive motor will be 2 or 3 orders of the magnitude larger than the other masses of the system. This means optical scan systems are essentially base excitation dynamic problems.

The spring ratio between the half-rate and full-rate mirrors, drive shaft, photoreceptor drum, and main drive motor can be calculated by using standard mathematical formulas. Since cable and belt stiffnesses are usually very dependent on pre-tension, an experimental approach using dial indicators and force gauges is recommended. By locking certain components of an actual machine or of a fixture, and loading adjoining components, the spring rates can be determined between components. As was the case for calculation of masses and inertias, a completely linear system referenced to the full-rate mirror is required. Thus, torsional spring rates must be referenced to the capstan outer radius as formulated in Equation (8).

$$K_{LINEAR} = \frac{K_{TORSIONAL}}{R^2_{CAPSTAN}} \tag{8}$$

Typical values for linear spring rates and effective masses are given in Table 1. These values are used for dynamic calculations in the following sections.

Table 1. Mass and Spring Rate Values for the Scan System Dynamic Model Shown in Figure 2.

M_1	=	0.0030	lbs sec^2/in	K_1	= 375.0	lbs/in
M_2	=	0.0061	lbs sec^2/in	K_2	= 1450.0	lbs/in
M_3	=	0.0182	lbs sec^2/in	K_3	= 325.0	lbs/in
M_4	=	0.0068	lbs sec^2/in	K_4	= 4500.0	lbs/in
M_5	=	1.3258	lbs sec^2/in	K_5	= 2075.0	lbs/in

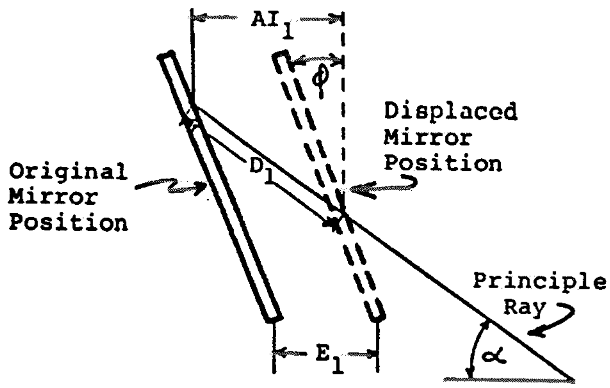
Derivation of Image Motion Equation

Image motion in the scan direction is defined as the difference in displacement between the aerial image, δ_{AI}, and the drum, δ_{DR}. For the scan model shown in Figure 2, the aerial image displacement is the difference between the full-rate mirror displacement, δ_{FR}, and the half-rate mirror displacement, δ_{HR}, multiplied by C_1 and C_2, respectively. The complete image motion equation is given by (9).

$$\delta_{IM} = C_1\delta_{FR} - C_2\delta_{HR} - \delta_{DR} \tag{9}$$

To obtain the constants, C_1 and C_2, a geometric derivation is shown in Figure 3 based on the projected shift of a principle ray referenced to the lens mirror. Assuming a unit displacement error of E_1 and E_2, an aerial image motion AI_1 and AI_2 can be calculated. The ratio of the aerial image motion to the induced displacement error are the coefficients C_1 and C_2 of Equation (9). Typical values for C_1 amd C_2 are 1.21 and 0.42, respectively. The coefficient for drum displacement is unity, since the drum surface is linearized and the incoming principle ray is normal to the drum surface.

FULL-RATE AERIAL IMAGE



$$AI_1 = D_1 \cos \alpha$$

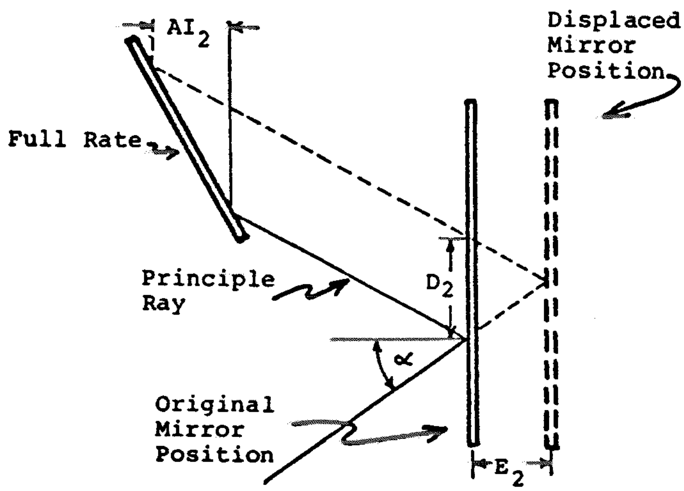
$$D_1 = \frac{E_1 \sin(90+\phi)}{\sin(90-\phi-\alpha)}$$

$$AI_1 = E_1 C_1$$

where

$$C_1 = \frac{\sin(90+\phi)\cos\alpha}{\sin(90-\phi-\alpha)}$$

HALF-RATE AERIAL IMAGE



$$AI_2 = D_2 \cdot \cos\alpha$$

$$D_2 = 2 \cdot E_2 \tan\alpha$$

$$AI_2 = E_2 C_2$$

where

$$C_2 = 2 \sin\alpha$$

Figure 3. Derivation of Image Motion Coefficients, C_1 and C_2.

A multipoint constraint equation (MPC) can be used to represent Equation (9). However, it is suggested that the relative displacement of the full-rate, half-rate and drum, with respect to the main drive motor, be calculated first using MPC's. This will omit rigid body displacement of the scan dynamic model, since it is a semi-definite system.

Dynamic Loading of Scan Systems

There are many sources of dynamic excitation affecting an optical scan system. While gear noise, fan vibration and xerographic developing hardware should not be totally ignored, the dominant sources of excitation are (1) "re-scan" of the carriages to initial scan position, and (2) velocity error at the main drive motor. The first is the transient condition of re-scan which shocks the scan system and causes a diminishing cyclic motion during the next scan cycle as shown in Figure 4. If the zero-to-peak amplitude of the first overshoot during the scan cycle is too large, correct transfer of image to the drum will not occur and the resulting copy quality will suffer.

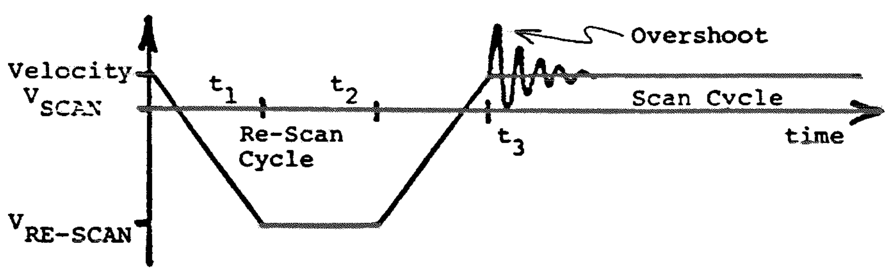


Figure 4. Idealized Re-Scan Cycle and Scan Overshoot

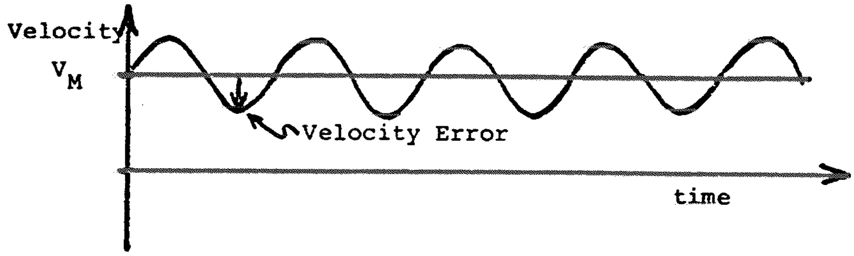
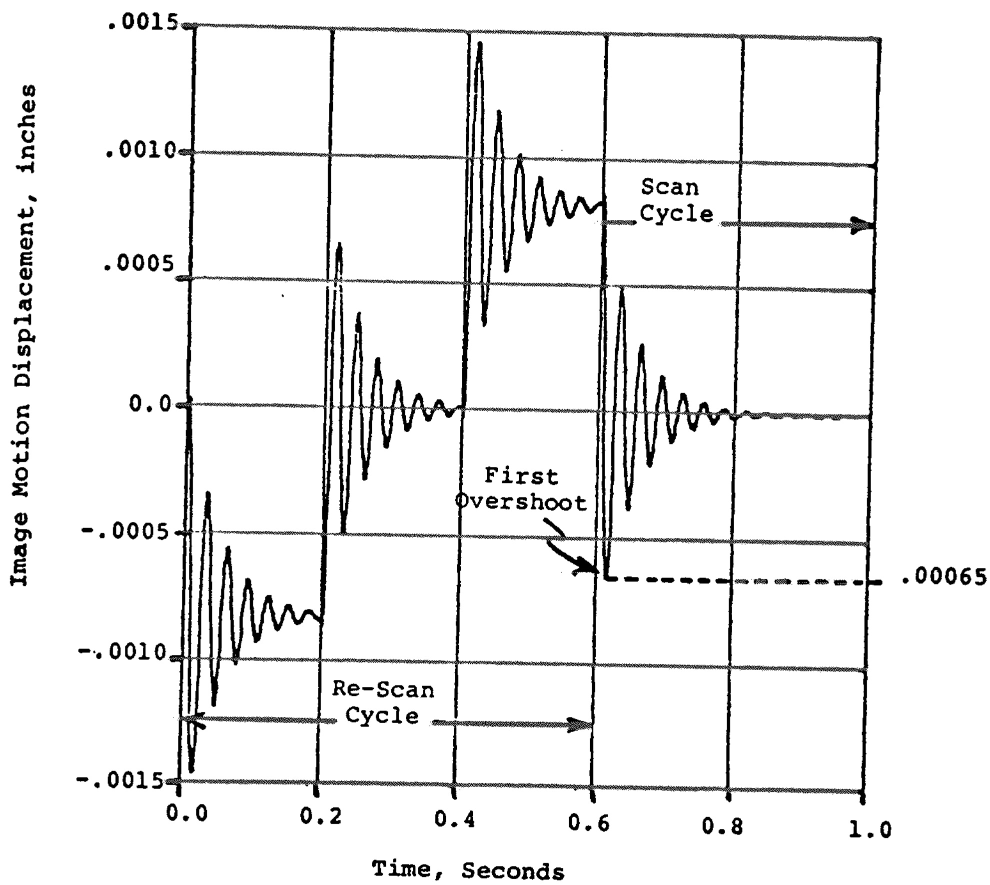


Figure 5. Main Drive Motor Velocity Error

The second source is a constant velocity error inherent in AC motors. The main drive motor is moving at a constant angular velocity, V_M. Superimposed on this is a sinisoidal velocity error as shown in Figure 5. The error acts continuously during scan and if it is large enough, it could cause poor copy quality over the entire scan range.

<u>Dynamic Analysis and Results</u>

A modal transient and frequency response calculation was completed using MSC/NASTRAN solutions 31 and 30, respectively. For the transient calculation, values of V_{SCAN} and $V_{RE-SCAN}$ in Figure 4 were totaled. The system was considered initially at rest. The main drive motor was accelerated to a constant total velocity of 40.0 in/sec, and then decelerated back to zero. Values of t_1, t_2 and t_3 are typically 0.2, 0.4 and 0.6 seconds, respectively, and a value of damping between all components of the scan system was assumed to be 10% of critical. The transient response of image motion is shown in Figure 6, and the first overshoot at the beginning of the scan cycle is calculated to be 0.00065 inches. This transient diminshes quickly as the scan cycle continues.



<u>Figure 6. Transient Response of Image Motion</u>

For the frequency response calculation, a velocity error at the main drive motor was assumed to be 5% of V_M. This value was assumed to be constant over a frequency range from 0 to 200 hz, even though the actual error exists only at an excitation frequency of 120 hz (60 cycle AC motor). The image motion response due to this error is shown in Figure 7. The velocity error amplitude, A, is calculated by applying Equation (10), where F(t) is the frequency dependent forcing function, M is the effective mass of the main drive motor (M_5) and ω is the circular frequency. The image motion response at 120 hz is 2.7 X 10^{-5} inches.

$$F(t) = A \cdot \sin\omega t \qquad (10)$$

where

$$A = \omega \cdot (0.05) \cdot M$$

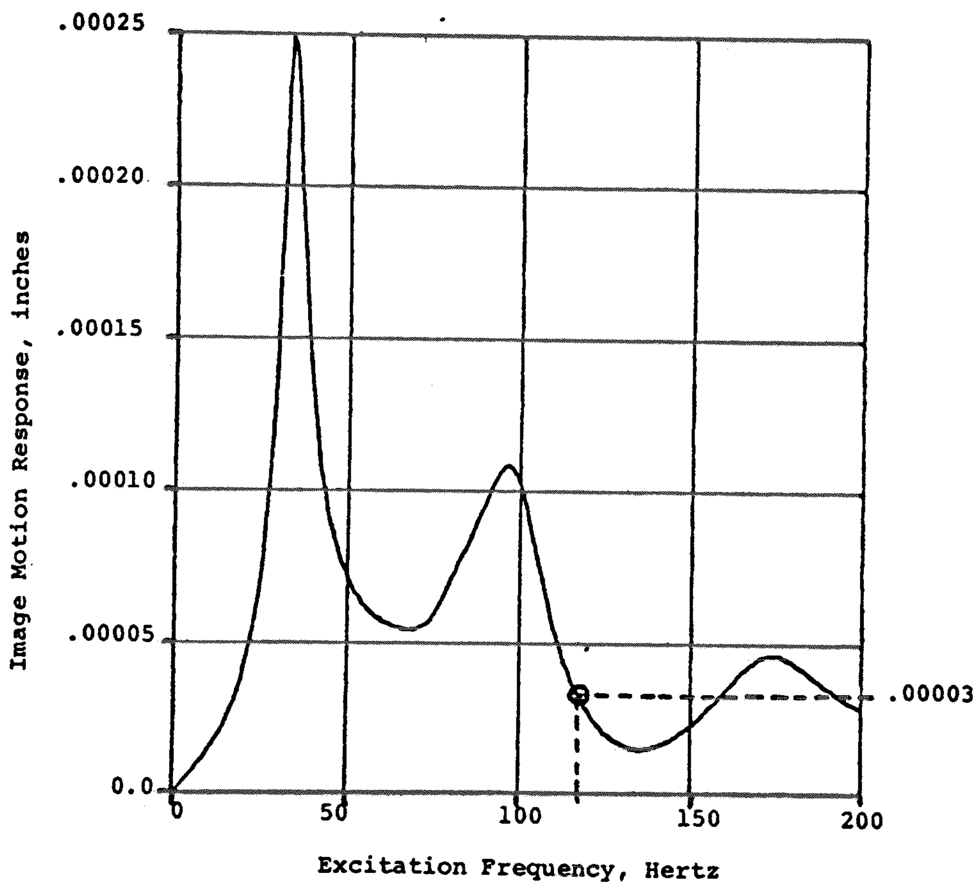


Figure 7. Frequency Response of Image Motion

Listings of the NASTRAN decks used to generate the transient and frequency response results are given in the Appendix.

CONCLUSIONS AND RECOMMENDATIONS

Calculation of image motion does not complete the evaluation of image transfer. The image motion error must be combined with the optical modulation transfer function to determine the overall image quality. This procedure is beyond the scope of the paper.

This analytical procedure assumes idealized forcing functions applied to a simplified dynamic model. For instance, the idealized velocity profile in Figure 4 will not maintain constant velocity and acceleration values during re-scan. Also, the actual full-rate and half-rate mirrors, and photoreceptor drum do not function entirely as a rigid body in the scan direction. These components could respond in other translational and torsional directions, and could contain modes of their own which couple with other components. The same applies to the lens and image mirrors, which were previously considered rigid. Even though they are not affected by scan motion, other sources of excitation could cause these components to add to image motion response if they were considered in the finite element model.

It is easy to understand how fast the number of degrees-of-freedom could grow as a more detailed analysis is required. However, with superelement and component mode synthesis capabilities available in MSC/NASTRAN, the detailed representation of an entire optical scan system can be accommodated.

APPENDIX

TRANSIENT RESPONSE

```
ID NOWAK,BILL
SOL 31
TIME 5
CEND

TITLE=DYNAMIC ANALYSIS OF OPTICAL SCAN SYSTEMS
SUBTITLE=TRANSIENT RESPONSE / IMAGE MOTION ANALYSIS  ZETA = 0.10
LABEL= TRAPEZOIDAL VELOCITY PROFILE APPLIED TO MOTOR   IDEALIZED RESCAN
MPC=1
METHOD=10
DLOAD=20
TSTEP=30
SDAMP=40
SVECT=ALL
DISP(PHASE,PLOT)=ALL
OUTPUT(XYPLOT)
PLOTTER NASTRAN
XGRID LINES = YES
YGRID LINES = YES
XYPRINT,XYPLOT DISP RESPONSE / 1004(T1RM)
BEGIN BULK
CELAS2  1       375.0   101     1       102     1
CELAS2  2       1450.0  102     1       103     1
CELAS2  3       325.0   101     1       103     1
CELAS2  4       4500.0  103     1       104     1
CELAS2  5       2075.0  104     1       105     1
CONM2   11      101             .0030
CONM2   12      102             .0061
CONM2   13      103             .0182
CONM2   14      104             .0068
CONM2   15      105             1.3258
DAREA   21      105     1       1.3258
EIGR    10      GIV                             5                       +EIGR
+EIGR   MAX
GRID    101             .0                              23456
GRID    102             1.0                             23456
GRID    103             2.0                             23456
GRID    104             3.0                             23456
GRID    105             4.0                             23456
MPC     1       1001    0       1.0     102     1       -1.0            +MPC1
+MPC1           105     1       1.0
MPC     1       1002    0       1.0     101     1       -1.0            +MPC2
+MPC2           105     1       1.0
MPC     1       1003    0       1.0     104     1       -1.0            +MPC3
+MPC3           105     1       1.0
MPC     1       1004    0       1.0     1001    0       -1.21           +MPC4
+MPC4           1002    0       .42     1003    0       1.0
PARAM   LMODES  5
SPOINT  1001    THRU    1004                                            +TAB1
TABDMP1 40
+TAB1   .0      .20     10000.0 .20     ENDT                            +TAB2
TABLED1 22                                                              +TAB3
+TAB2   .0      .0      .2      40.0    .4      40.0    .6      .0
+TAB3   100.0   .0      ENDT
TLOAD1  20      21              2       22
TSTEP   30      1000    .001    1
ENDDATA
```

FREQUENCY RESPONSE

```
ID NOWAK,BILL
SOL 30
TIME 5
CEND
TITLE=DYNAMIC ANALYSIS OF OPTICAL SCAN SYSTEMS
SUBTITLE=FREQUENCY RESPONSE / IMAGE MOTION ANALYSIS  ZETA = 0.10
LABEL=A 5 PERCENT VELOCITY ERROR IS APPLIED AT THE MAIN DRIVE MOTOR
MPC=1
METHOD=10
DLOAD=20
FREQ=30
SDAMP=40
SVEC=ALL
DISP(PHASE,PLOT)=ALL
OUTPUT(XYPLOT)
PLOTTER NASTRAN
XGRID LINES = YES
YGRID LINES = YES
XYPRINT,XYPLOT DISP RESPONSE / 1004(T1RM)
BEGIN BULK
CELAS2  1       375.0   101     1       102     1
CELAS2  2       1450.0  102     1       103     1
CELAS2  3       325.0   101     1       103     1
CELAS2  4       4500.0  103     1       104     1
CELAS2  5       2075.0  104     1       105     1
CONM2   11      101             .0030
CONM2   12      102             .0061
CONM2   13      103             .0182
CONM2   14      104             .0068
CONM2   15      105             1.3258
DAREA   21      105     1       .066
EIGR    10      GIV                             5                       +EIGR
+EIGR   MAX
FREQ1   30      .0      1.0     200
GRID    101             .0                              23456
GRID    102             1.0                             23456
GRID    103             2.0                             23456
GRID    104             3.0                             23456
GRID    105             4.0                             23456
MPC     1       1001    0       1.0     102     1       -1.0            +MPC1
+MPC1           105     1       1.0
MPC     1       1002    0       1.0     101     1       -1.0            +MPC2
+MPC2           105     1       1.0
MPC     1       1003    0       1.0     104     1       -1.0            +MPC3
+MPC3           105     1       1.0
MPC     1       1004    0       1.0     1001    0       -1.21           +MPC4
+MPC4           1002    0       .42     1003    0       1.0
PARAM   LMODES  5
RLOAD2  20      21                      22
SPOINT  1001    THRU    1004
TABDMP1 40                                                              +TAB1
+TAB1   .0      .20     10000.0 .20     ENDT
TABLED1 22                                                              +TAB2
+TAB2   .0      .0      100.0   628.319 1000.0  6283.19 ENDT
ENDDATA
```