Modal Analysis of Coupled Fluid-Structure Response

Masaaki Watanabe and Hiroshi Saito
Mitsubishi Research Institute, Inc., Tokyo Japan

SUMMARY

To predict the dynamic characteristics of structures with containment fluid, finite element representation of the pressure field within the fluid is employed. A moderate DMAP modification with MSC/NASTRAN makes a convenient analysis procedure for this purpose. Numerical examples as well as discussions on lumped/consistent formulation of fluid-structure coupling are presented.

INTRODUCTION

To estimate more accurately the dynamic characteristics of large scale oil tanks, off-shore structures snd so on, it is inevitable to include the influence of fluid motions upon the structure in the course of the analysis. Up to the present, various methods of dealing with the coupled fluid-structure dynamic behaviours have been proposed. There are two different approaches adopting different coordinates system. One is the

Lagrangian approach which expresses the fluid motion by the displacement function in the same manner as the structure motion. The fluid is treated as an elastic solid with a finite bulk modulus and a negligibly small shear modulus. When the fluid is imcompressible, this approach has a shortpoint that it requires the special technique such as a hybrid variational principle or a penalty method to supress many rotatry modes to be produced in the fluid.

Another is the Eulerian approach. In this approach the velocity field is expressed by the gradient of a scalor function which represents the velocity potential or the pressure field. As there is only one unknown variable per nodal point, the number of total degrees of freedom is one-third of that of the Lagrangian approach. Since our main purpose is the seismic analysis of the coupled imcompressible fluid-structure, the Lagrangian approach is less preferable to the Eulerian approach.

MSC/NASTRAN provides the virtual mass method based on the boundary integrals of the velocity potential in the Eulerian approach, however, its application to a complex shaped fluid such as contained in a nuclear reactor vessel seems to be inappropriate. Hence the finite element representation of the fluid has been chosen.

BASIC THEORY

Zienkiewicz et al.[1),4)] show the details of the theory used in this paper and so only basic points are described in brief as below.

On the assumptions that deformations of the structure and the fluid are infinitesmal and fluid motion is of the potential flow, the linear theory can be adopted. Tow variational principles are expressed as follows,[4)]

for the structure

$$\iiint_{V_e}(\sigma_{ij}\delta\varepsilon_{ij}+\rho_s\ddot{u}_i\delta u_i)\,dV - \iint_{S_s} p n_i \delta u_i\, dS - \iint_{S_F}\bar{T}_i\delta u_i\, dS = 0 \qquad (1)$$

for the fluid

$$\iiint_{V_f}\frac{1}{\rho_f}p_{,i}\,\delta p_{,i}\,dV + \iint_{S_s}\ddot{u}_i n_i \delta p\, dS = 0 \qquad (2)$$

where

V_e; volume of the structure,

S_s; contacting surface of fluid and structure,

S_F; boundary surface on which external load acts,

V_f; volume occupied by the fluid,

σ_{ij} ; components of stress tensor,

δ ; means variation,

ε_{ij}; components of strain tensor,

ρ_s ; density of the structure,

u_i ; components of displacement,

$\ddot{u}_i$; components of accelration,

n_i ; outward normal direction cosines on the contact boundary,

p ; pressure of the fluid,

$\overline{T}_i$; prescribed boundary force for the structure,

ρ_f ; density of the fluid,

$(\ )_{,i}$; partial differentiation with respect to the i-th, coordinate,

$(\dot{\ })$; partial differentiation with respect to time.

Eq.(2) neglects the free surface waves.

FINITE ELEMENT FORMULATION

According to the finite element displacement formulation, Eqs. (1) and (2) expressed as the following equations of matrices,

$$K_s u + M_s \ddot{u} - A^t p = F \tag{3}$$

$$K_f p + A \ddot{u} = 0 \tag{4}$$

where

K_s; stiffness matrix of the structure,

M_s; mass matrix of the structure,

A ; fluid-structure interaction matrix,

K_f; stiffness matrix of pressure,

F ; nodal point load vector acting on the structure,

u ; unknown nodal displacement vector of the structure,

p ; unknown nodal pressure vector of the fluid.

As described in the paper by MacNeal et al.,[5] the standard MSC/NASTRAN solid elements can be directly used to represent the pressure field of the fluid.

To avoid solving eigenvalues of the large unsymmetric matrix derived from Eqs. (3) and (4), it is assumed that the coupled eigen modes of the structure are approximately expanded in the finite series of the eigen modes obtained by the structure without the contained fluid.

The matrix A in Eqs. (3) and (4) represents the fluid-structure factor and input through NASTRAN DMIG bulkdata card. As NASTRAN's default option for mass matrix is of lumped mass, at first DMAP sequences were introduced to express the interaction terms in a lumped form, so that the nodal pressure of the fluid acts on only the corresponding nodal point of the structure In the case of some simple problems adaquate results are obtained as shown in later section, but as for more complicated structures the lumped mass method occasionally fails in obtaining accurate modes.

From this reason, another DMAP has been developed corresponding to the consistent mass formulation. In the latter method, the pressure at one nodal point of the fluid element on contact surface affects the adjacent nodal points as well as the corresponding nodal point of the structure, vice versa.

In the first phase, eigen modes of the structure without the fluid are obtained from the following equation,

$$(K_s - \omega^2 M_s) u = 0 \qquad (5)$$

where ω is the circular frequency. The interaction and the external load terms are omitted in Eq. (2).

From the assumption, the coupled eigen mode of the structure with the fluid is approximated by the linear superposition of the n modes from Eq. (5) as follows,

$$u = \sum_{i=1}^{n} \Phi_i C_i = \Phi C \qquad (6)$$

where

Φ_i ; i-th "empty mode" of the structure,

C_i ; i-th generalized coordinate.

Substituting Eq. (6) into the equation which is derived by eliminating pressure from Eqs. (3) and (4), the following equation is obtained.

$$\left[\Phi^t K_s \Phi - \bar{\omega}^2 (\Phi^t M_s \Phi + \Phi^t A^t K_f^{-1} A \Phi)\right] C = 0$$

where the external load vector is neglected.

In the second phase, this reduced eigenvalue equation is solved. In above equation, the third term is called virtual mass or added mass. $\Phi^t K_s \Phi$ and $\Phi^t M_s \Phi$ are diagonal matrices.

Once the structural components of the coupled fluid-structure modes are obtained, corresponding pressure components in the fluid can be derived from Eq. (4) and it is straightforward to incorporate these modes into seismic response and/or response spectrum analysis using the standard rigid formats available in MSC/NASTRAN. The simplified flow diagrams of these runs are shown in Fig. 1 for reference.

NUMERICAL EXAMPLES

Three illustrative numerical examples are shown in this section. Though practical problems are usually much more complex than those examples, same procedure can be applied.

Coupled Free Vibration of Infinite Beam Periodically Simply Supported

As shown in Fig. 2, the first numerical example is the coupled vibration of the infinite beam which has only one side wetted. Considering the boundary conditions shown in Figs. 2 and 3, the analytical solution is easily obtained. The ratio of "empty frequency" (ω) to "coupled frequency" ($\overline{\omega}$) is, for the i-th mode,

$$\frac{\omega}{\overline{\omega}} = \left(1 + \frac{\rho_f}{\rho_s A}\frac{l\,t}{n\pi}\coth\frac{n\pi}{l}b\right)^{\frac{1}{2}}$$

In the above ρ_f and ρ_s stand for the density of the fluid and the structure respectively and A for the cross sectional area of the beam. Though the lumped mass formulation is adopted,

Table 1 shows good agreements with exact solutions. Fig. 4 shows the pressure modes corresponding to the coupled modes of the beam.

<u>Coupled Vibration of Two Cantilevers Facing Each Other in Container Filled with Water</u>

The analytical model is shown in Fig. 5. The lumped mass method is used. Fig. 6 shows the mesh division. In the empty modes as shown in Fig. 7, two cantilvers move independently, but in the coupled modes those act on each other through the water as shown in Fig. 8. Symmetry and anti-symmetry modes are clearly distinguishable. Corresponding pressure modes are shown in Fig. 9.

<u>Coupled Vibration of Bottom Plate of Box Which Contains Water</u>

The fluid which has the free surface is enclosed with one elastic bottom plate and four rigid side panels, as shown in Fig. 10. The coupled vibrations of the elastic bottom plate are analysed by Rayleigh Ritz method using analytical functions and two DMAPs proposed in this paper.

Fig. 11 shows the mesh division of the fluid. For the empty modes, comparisons of calculated eigen frequencies and exact solutions are in Fig. 12. In Fig. 13, the eigen frequencies of the coupled motion are shown, compared with the solutions obtained by Rayleigh Ritz procedure. The generalized coordinates of Eq. (6) are shown in Table 2. Good agreements of these methods are observed.

CONCLUSIONS

Eulerian representation of fluid by conventional solid elements of MSC/NASTRAN can put a dynamic modal response analysi of a coupled containment fluid-structure system to practical use with a modest DMAP modification to the rigid formats. Some numerical examples show the validity of the approach.

REFERENCES

1) O.C.Zienkiewicz and P.Bettess,'Fluid-Structure Dynamic Interaction and Wave Forces. An Introduction to Numerical Treatment', Int. J. Meth. Engng., Vol. 13, No. 1, 1979, pp. 1.

2) MSC/NASTRAN Users' Manual, MSC, Los Angeles, CA, Febr., 1981.

3) The Nastran Theoreical Manual (Level 15.5), MSC, Los Angeles, CA, Dec., 1972.

4) O.C.Zienkiewicz and R.E.Newton,'Coupled Vibrations of a Structure Submerged in a Compressible Fluid', Proc. Int. Symp. on Finite Element Techniques', Stuttgart, 1969, pp. 361.

5) R.H.Macneal, R.Citerley and M.Chargin,'A New Method for Analyzing Fluid -Structure Interaction Using MSC/NASTRAN', SMIRT, 1979, B4/9.

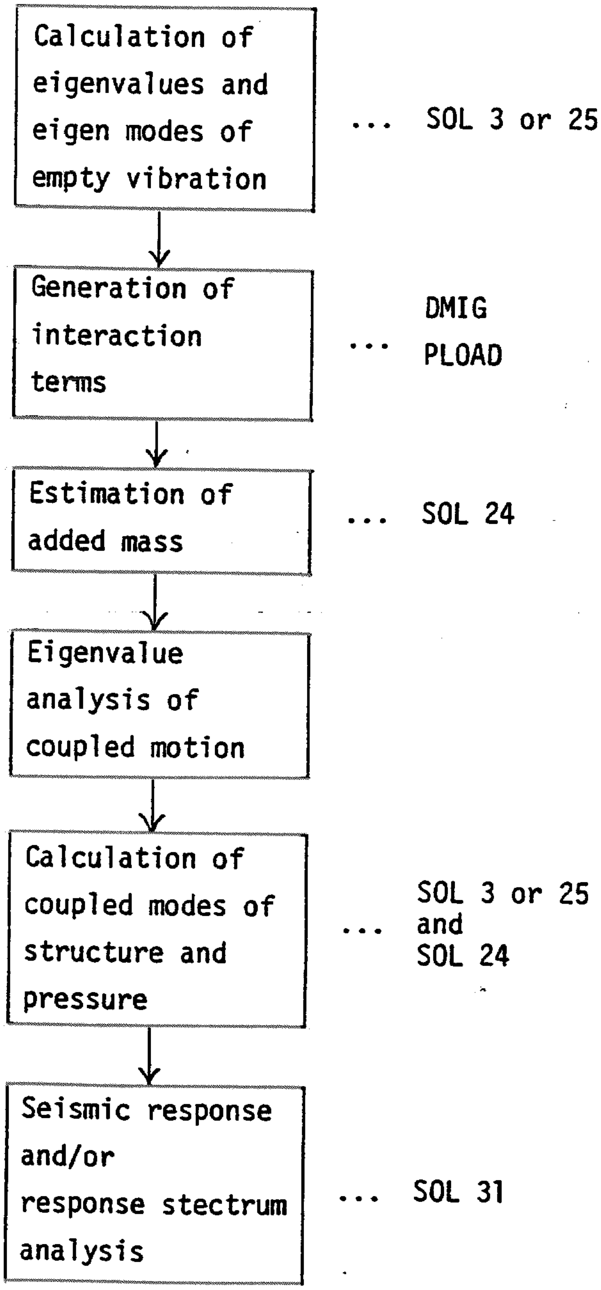


Fig. 1 Simplified flow diagrams for analysis of coupled fluid-structure response

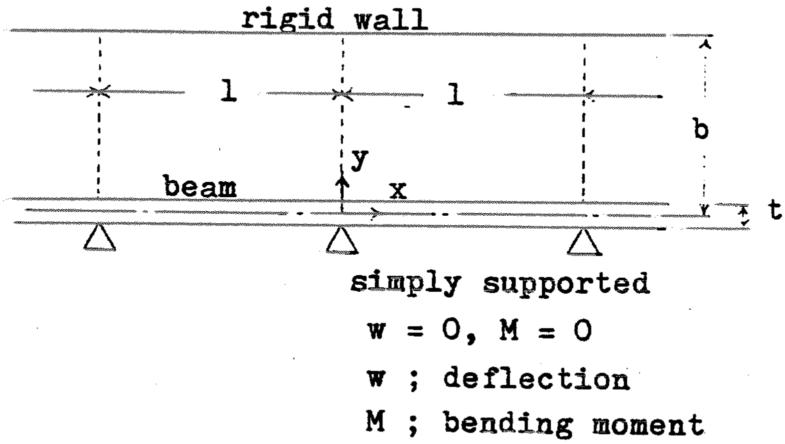


Fig. 2 Infinite beam having one wetted side.

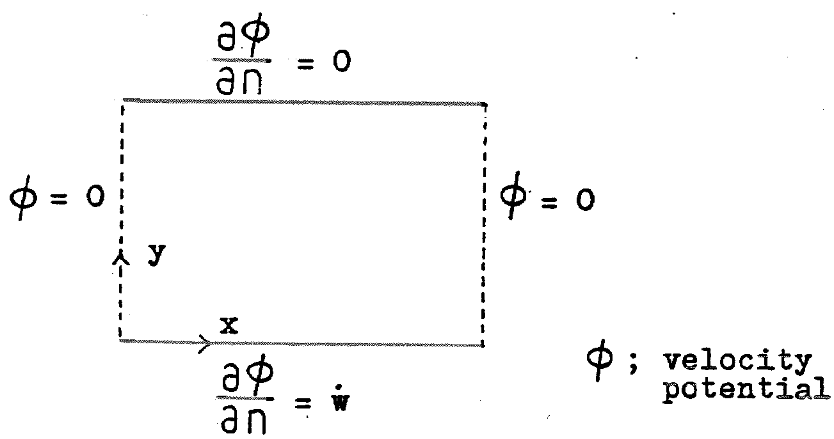


Fig. 3 Boundary conditions of potential.

Table 1 Analytical and numerical solutions of infinite beam periodically simply supported.

Analytical circular frequencies (rad./s)

mode	empty(ω)	coupled($\bar{\omega}$)	$\omega/\bar{\omega}$
1	1.3945×10^3	1.1641×10^3	1.1979
2	5.5779×10^3	5.0916×10^3	1.0955
3	1.2550×10^4	1.1791×10^4	1.0644

Numerical circular frequencies (rad./s)

mode	empty(ω)	coupled($\bar{\omega}$)	$\omega/\bar{\omega}$
1	1.3945×10^3	1.1625×10^3	1.1995
2	5.5773×10^3	5.0701×10^3	1.1000
3	1.2542×10^4	1.1703×10^4	1.0717

Specimen of Fig. 1 and material constants are as follows,

$E = 1.95\times10^6$ kg/cm^2 (Young modulus),
$\rho_s = 8.16\times10^{-6}$ kg/cm$^4\cdot$s^2,
$\rho_f = 1.\ \times10^{-6}$ kg/cm$^4\cdot$s^2,
t = 10cm,
A = 100cm,
l = 100cm,
b = 50cm.

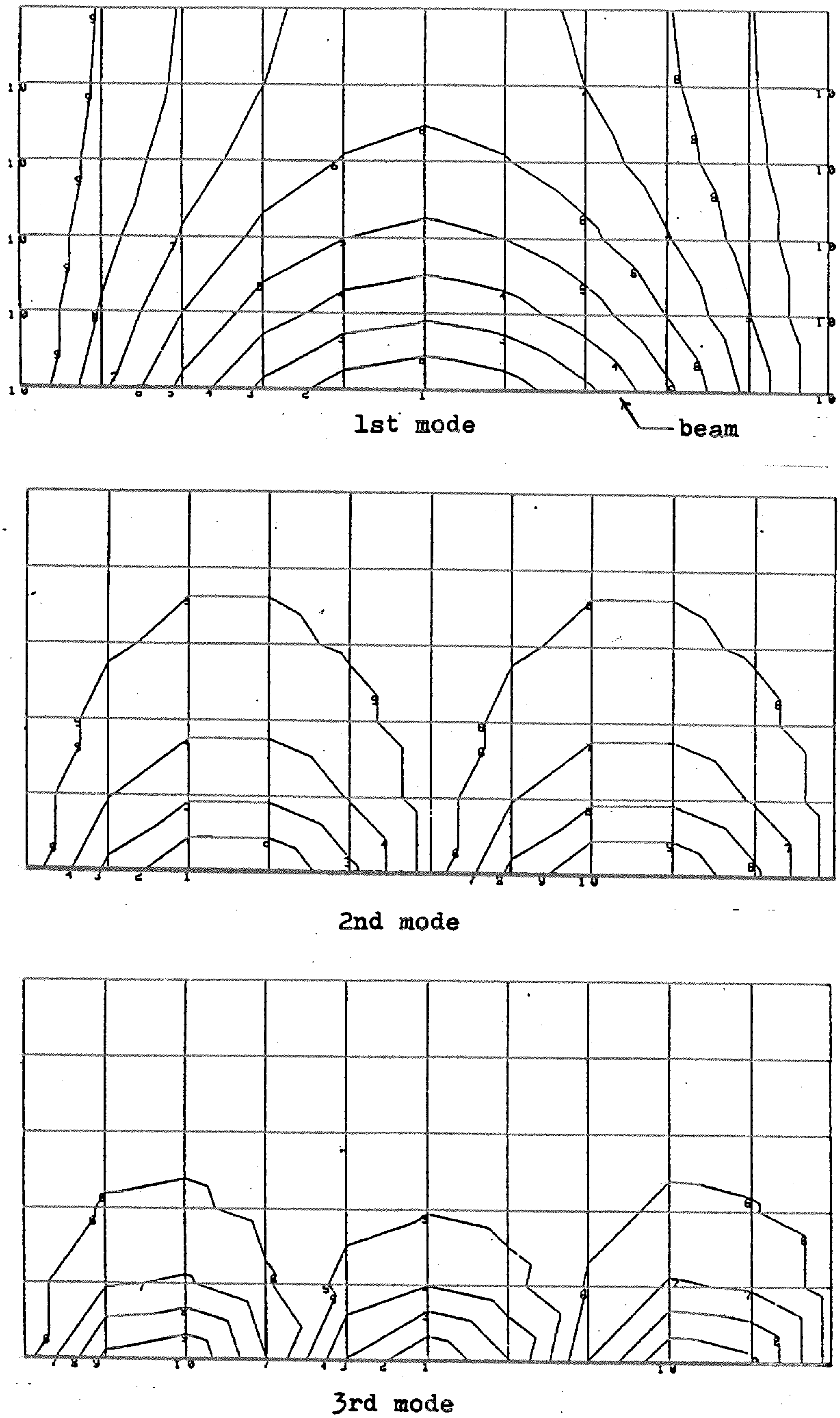


Fig. 4 Contours of pressure obtained by MSC/NASTRAN

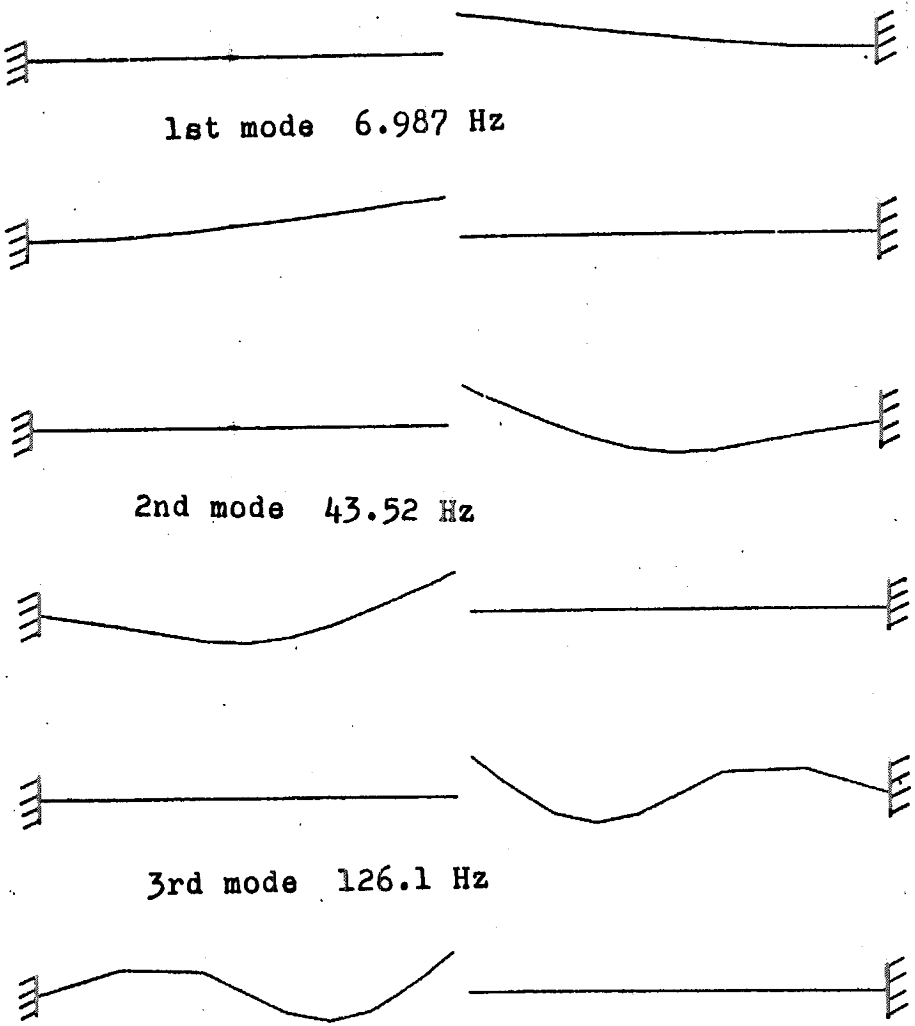


Fig. 7 Empty modes, moving independently.

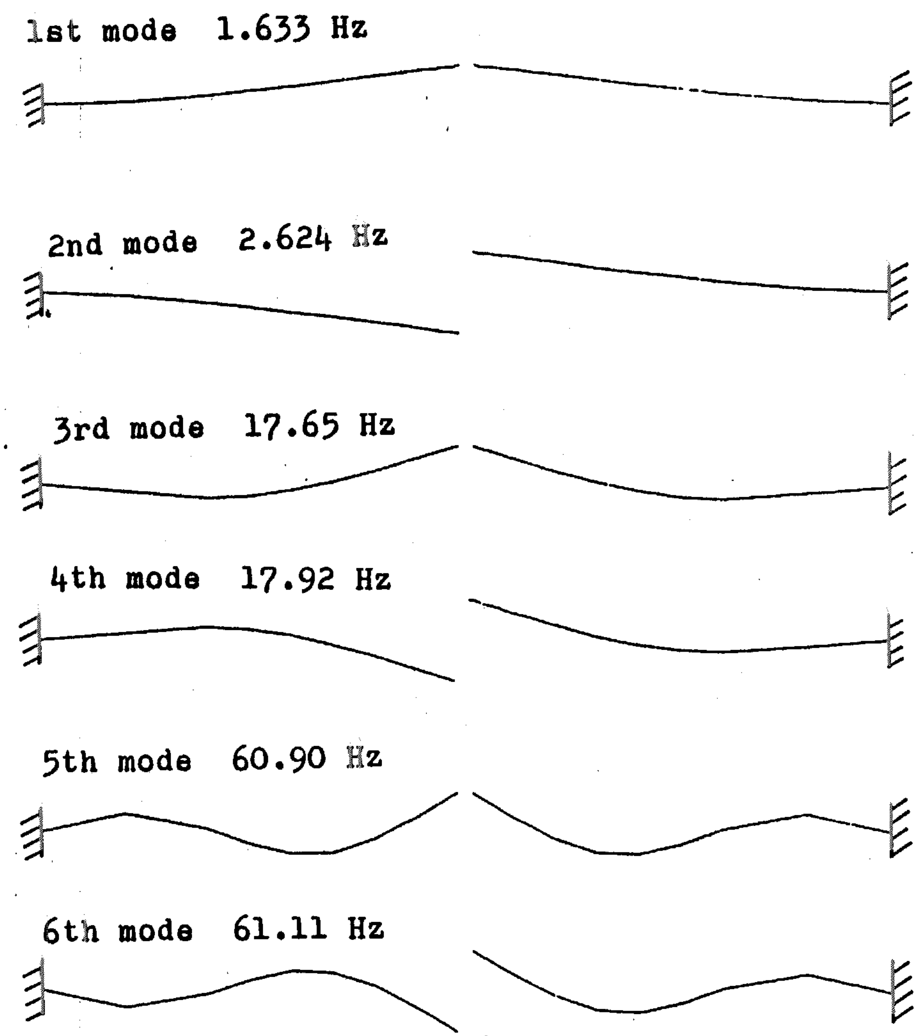


Fig. 8 Coupled modes, moving together.

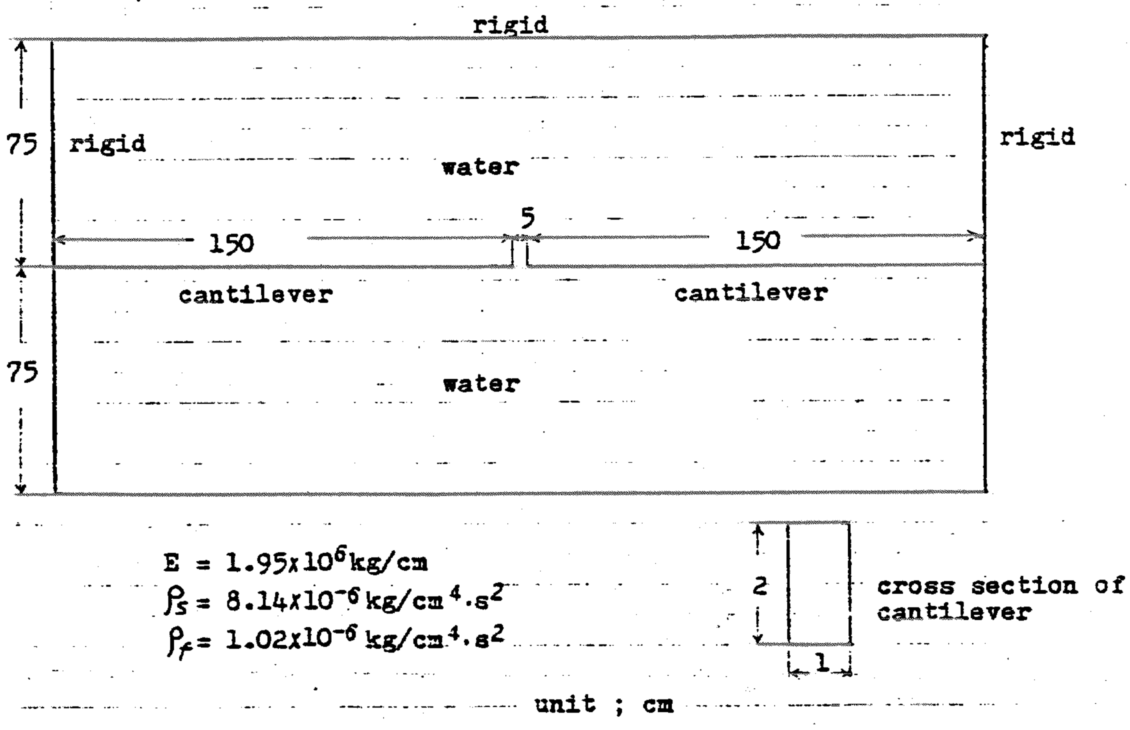


Fig. 5 Two cantilevers facing each other in the water

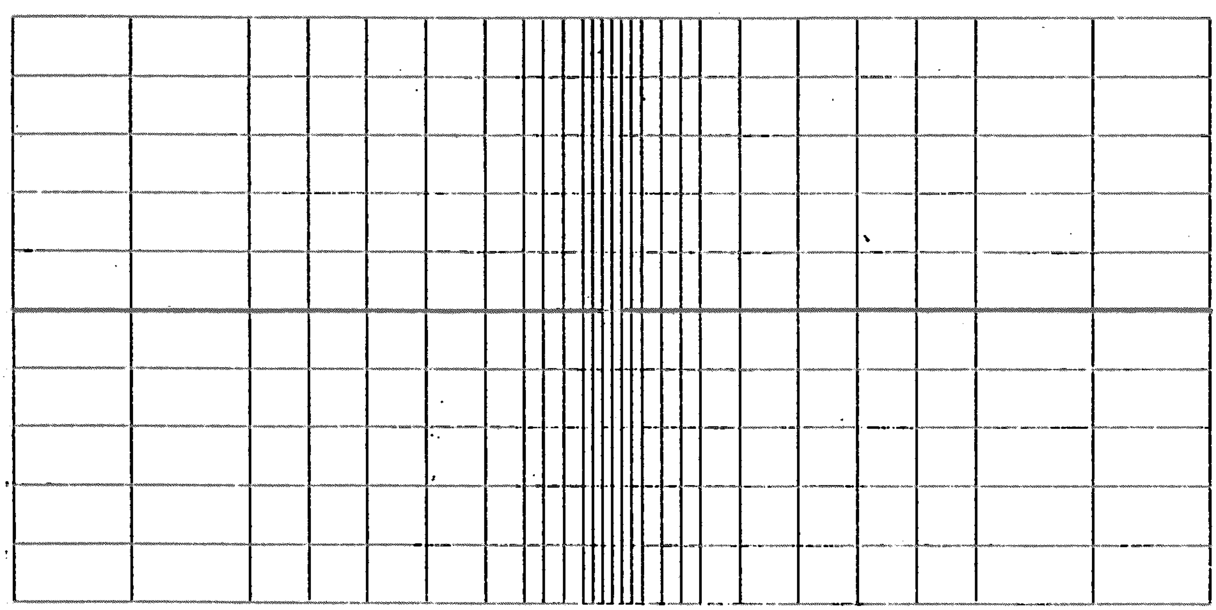
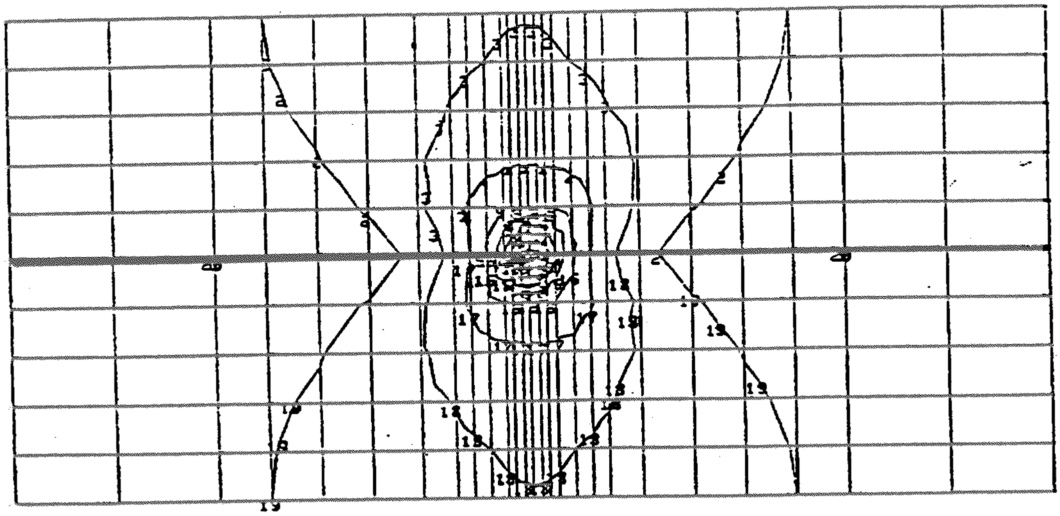
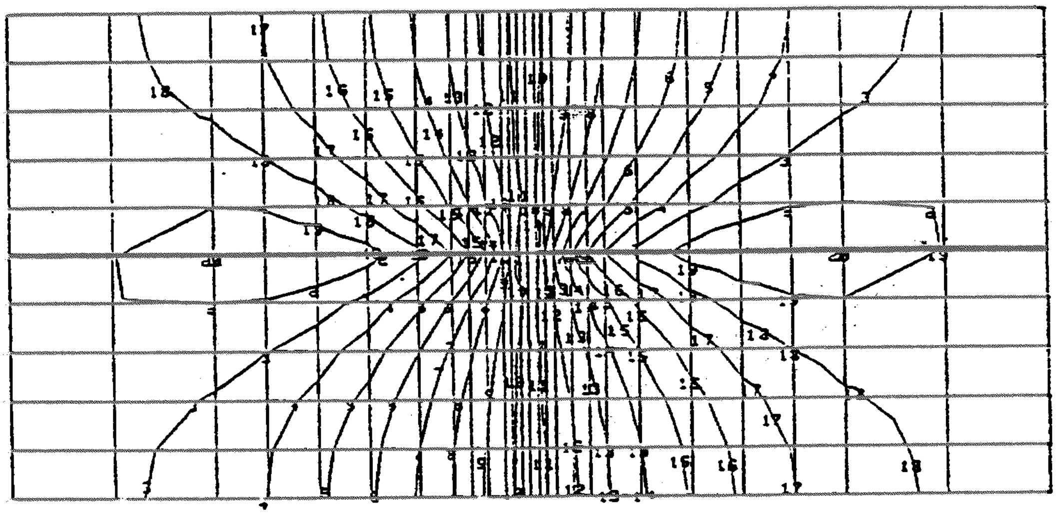


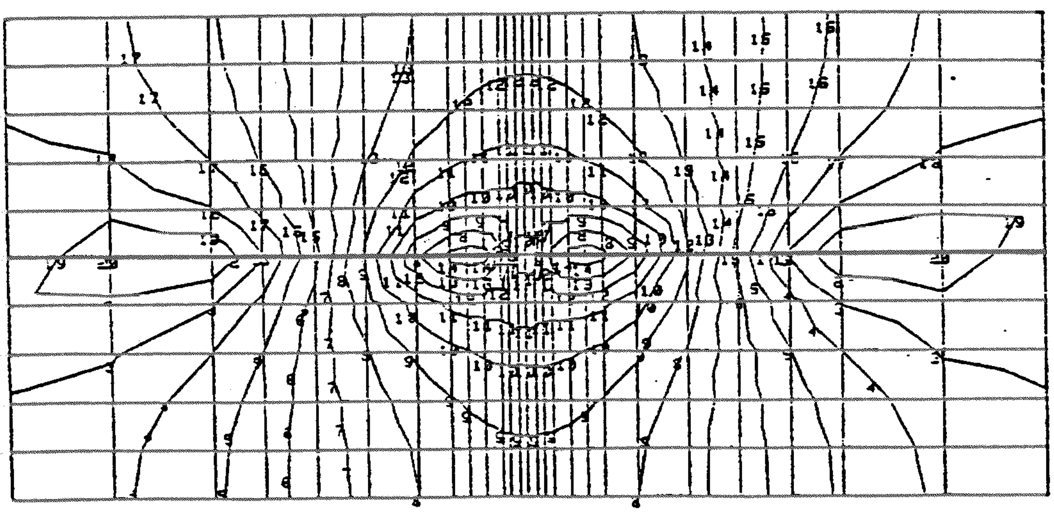
Fig. 6 Mesh division of fluid



1st mode

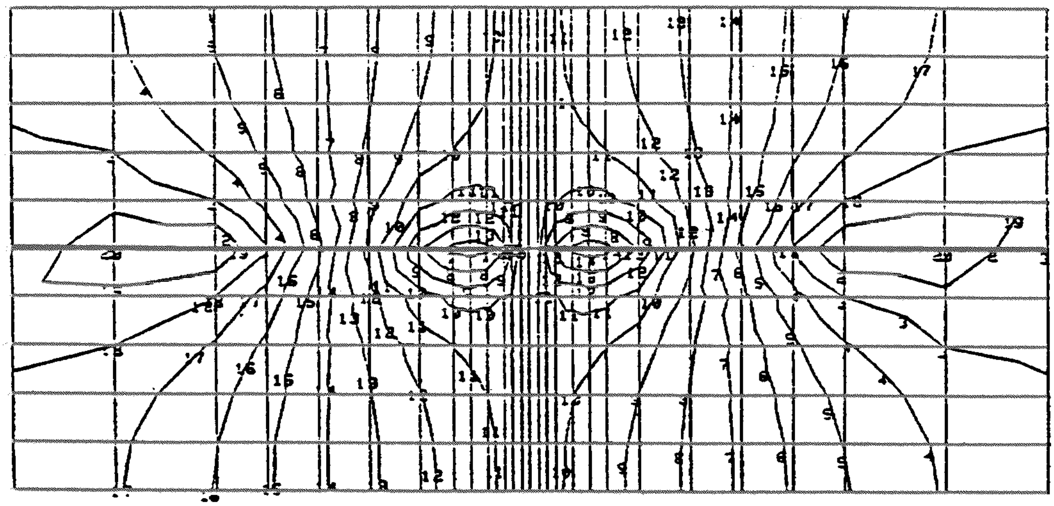


2nd mode

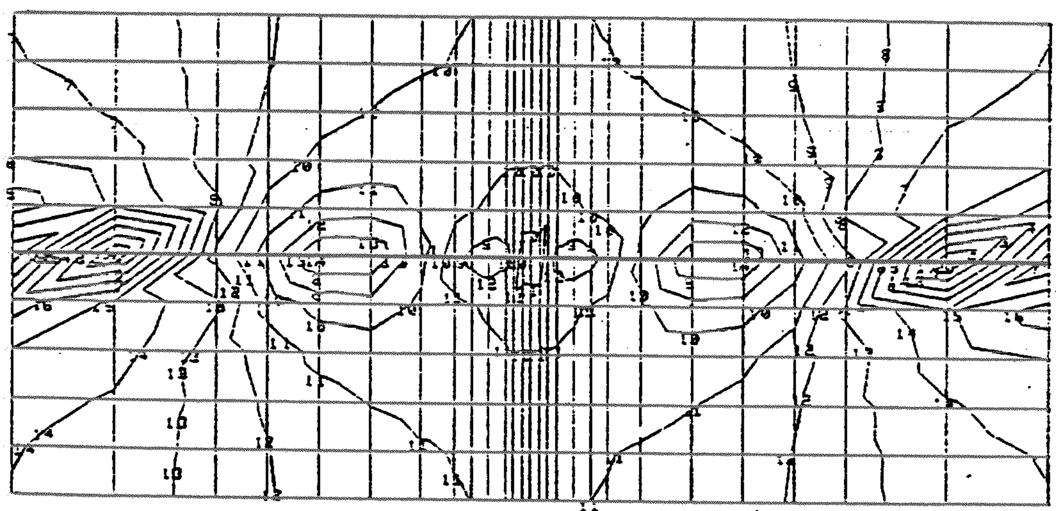


3rd mode

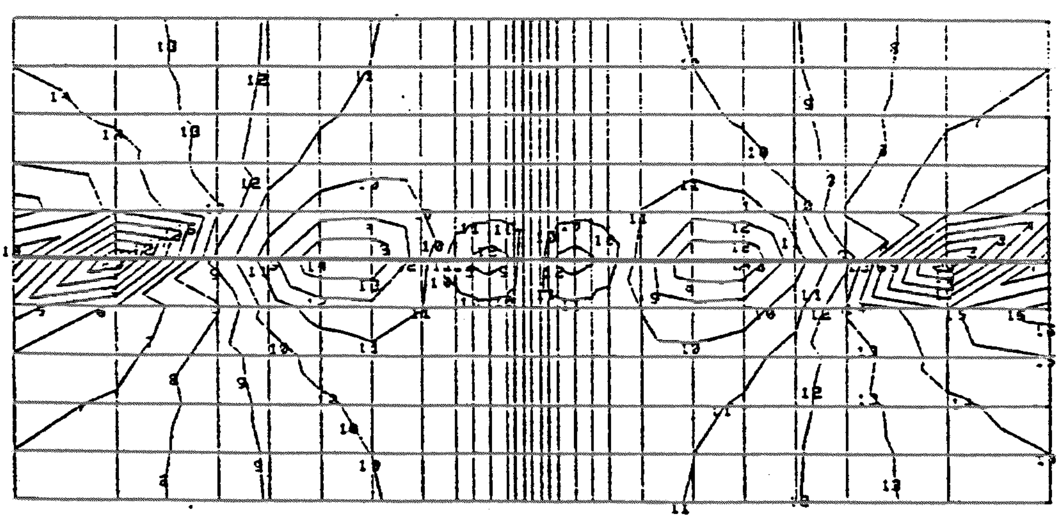
Fig. 9 Contours of pressure corresponding to the coupled modes shown in Fig. 8 (cont.).



4th mode

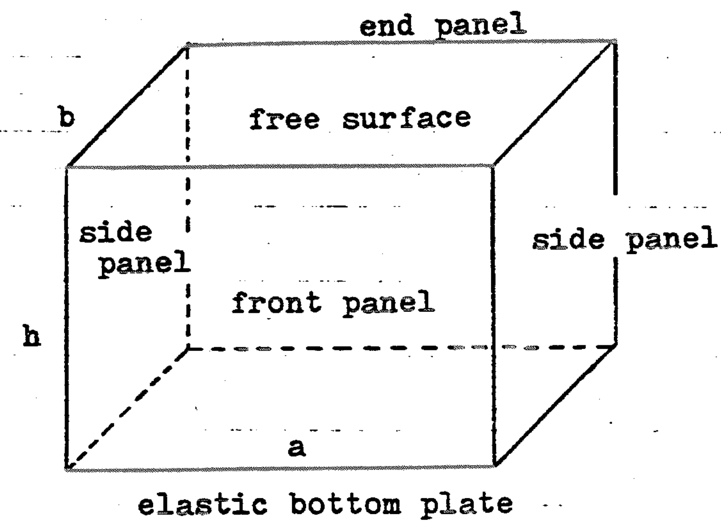


5th mode



6th mode

Fig. 9 Contours of pressure corresponding to the coupled modes shown in Fig. 8.



a = 200cm

b = 100cm

h = 80cm

Fig. 10 Fluid enclosed with a elastic bottom plate and 4 rigid panels.

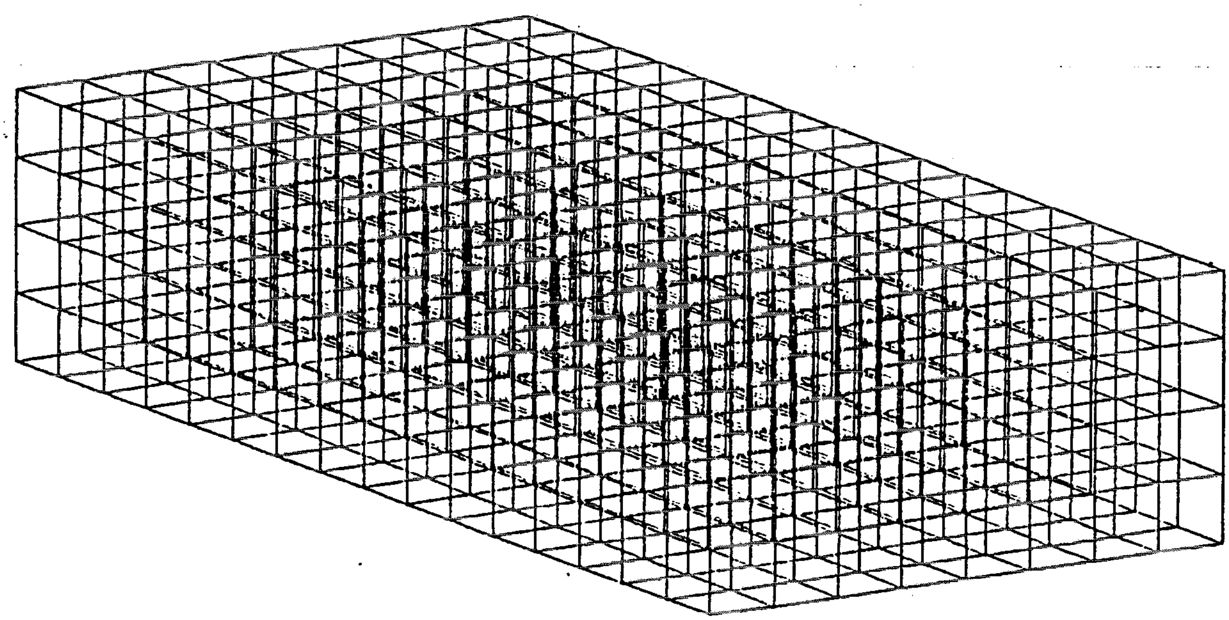


Fig. 11 Mesh division of fluid using 512 solid elements.

□ ; consistent mass

■ ; lumped mass

m ; no. of waves in the longitudinal direction

n ; no. of waves in the transverse direction

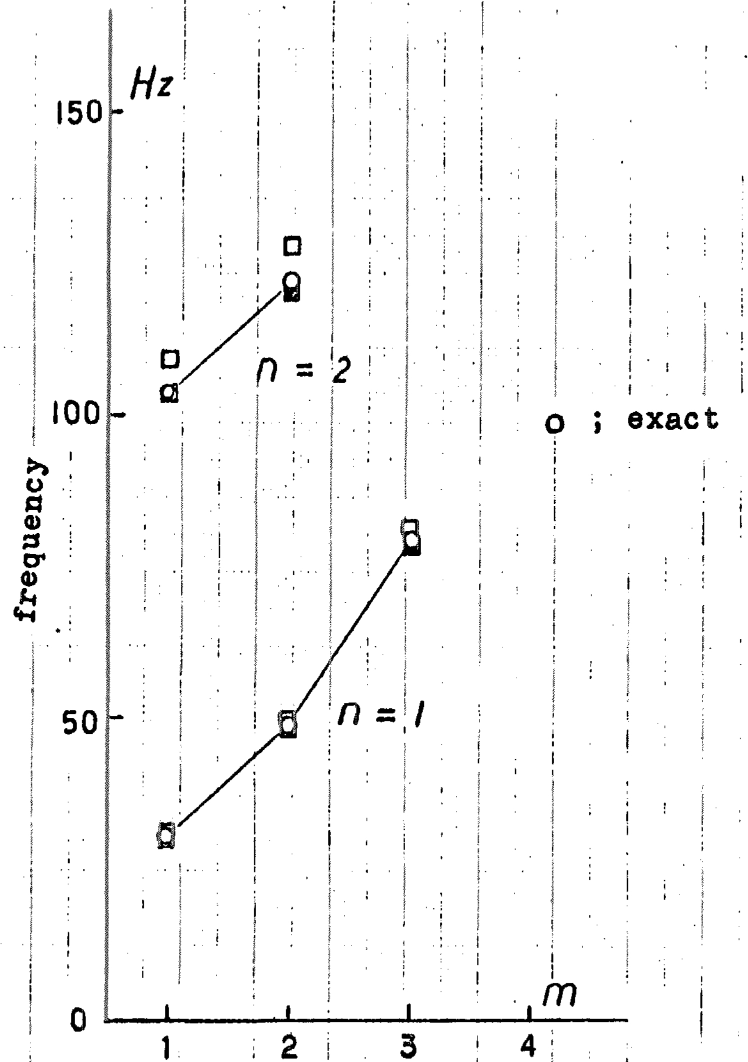


Fig. 12 Eigen frequencies of empty modes of bottom plate.

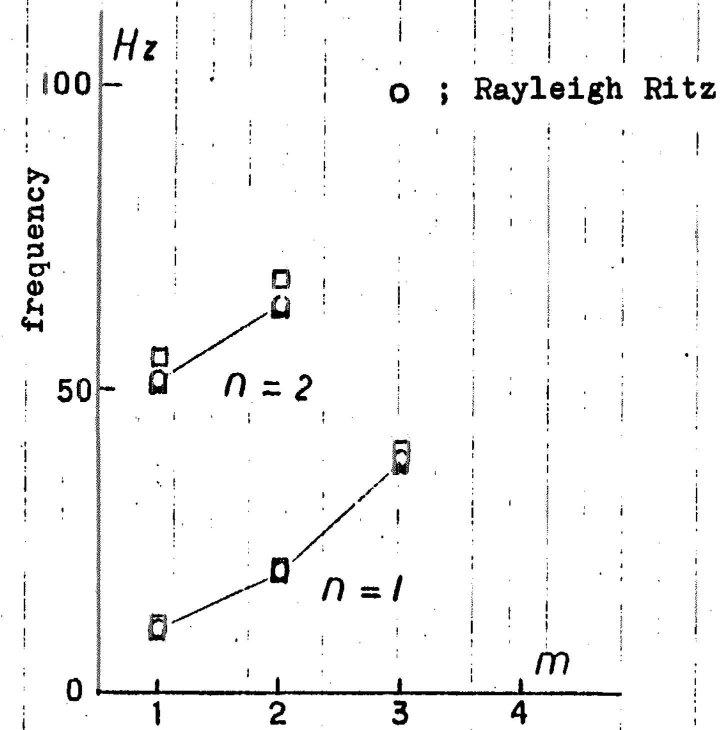


Fig. 13 Eigen frequencies of coupled modes of bottom plate.

Table 2 Generalized coordinates

(m, n)
m ; number of waves in the longitudinal direction
n ; number of waves in the transverse direction

lumped
consistent
Rayleigh Ritz

empty mode / coupled mode	(1, 2)	(3, 1)	(5, 1)	(1, 3)
(1, 1)	1. 1. 1.	.0227 .0236 .0234	.00274 .00287 .00298	.00336 .00356 .00391
(3, 1)	-.1525 -.1588 -.1617	1. 1. 1.	.0253 .0286 .0280	.00339 .00331 .00394
(5, 1)	-.0763 -.0781 -.0878	-.1287 -.1454 -.1427	1. 1. 1.	.0291 .0312 .0470
(1, 3)	-.1763 -.1867 -.2021	-.0261 -.0245 -.0260	-.0496 -.0533 -.0777	1. 1. 1.

empty mode / coupled mode	(2, 2)	(4, 2)	(1, 2)	(3, 2)	(2, 1)	(4, 1)
(2, 2)	1. 1. 1.	.0313 .0367 .0341	—	—	—	.0324 -.0159 0.0
(4, 2)	-.0763 -.0895 -.0872	1. 1. 1.	—	—	—	-.00247 .00142 0.0
(1, 2)	—	—	1. 1. 1.	.0294 .0330 .0304	—	—
(3, 2)	—	—	-.0601 -.0675 -.0657	1. 1. 1.	—	—
(2, 1)	-.0008 .0004 0.0	—	—	—	1. 1. 1.	.0246 .0265 .0268
(4, 1)	-.0324 .0159 0.0	—	—	—	-.1531 -.1644 -.1674	1. 1. 1.

Modal Analysis of Coupled Fluid - Structure Response

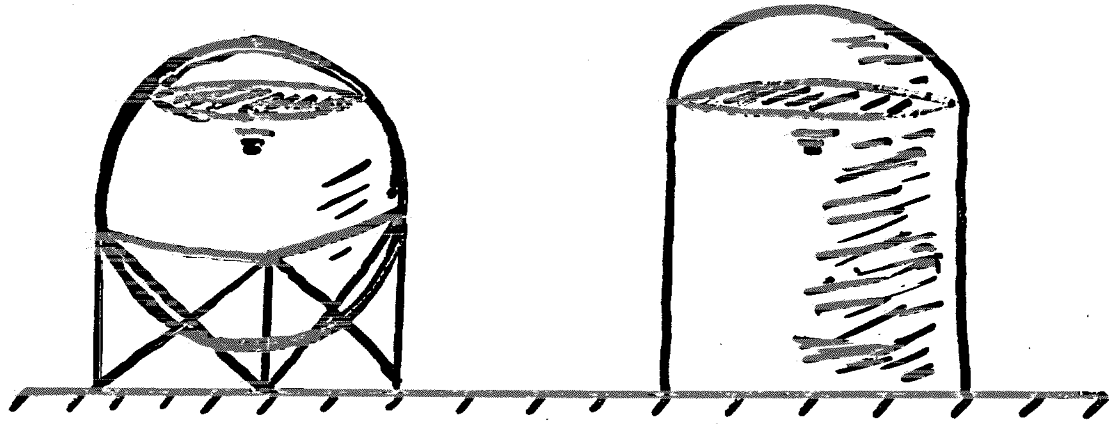
Masaaki Watanabe

Hiroshi Saito

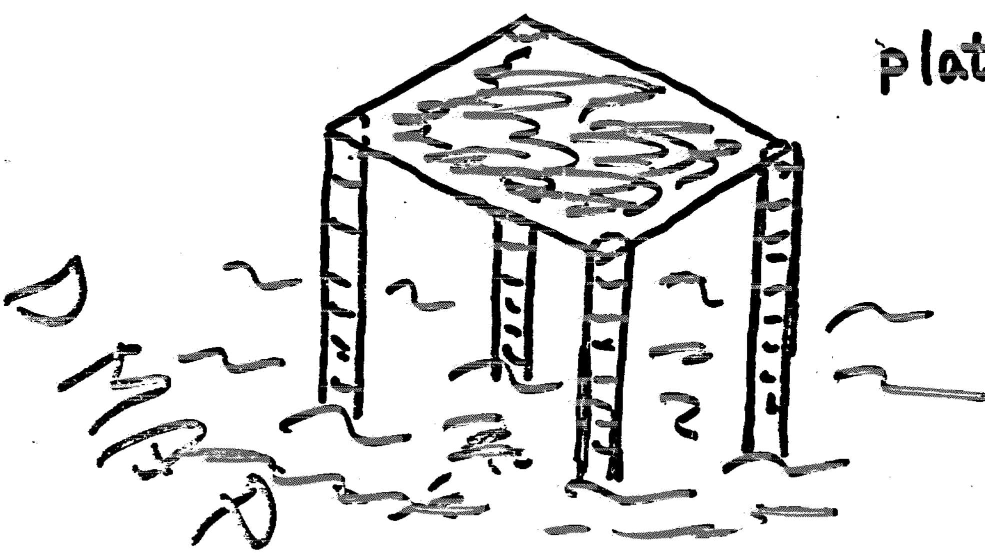
Mitsubishi Research Institute, Inc.

Tokyo Japan

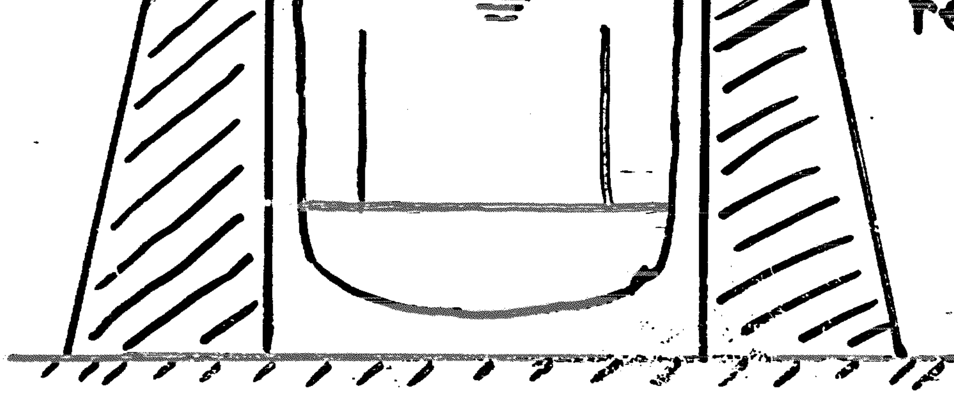
Oil tanks



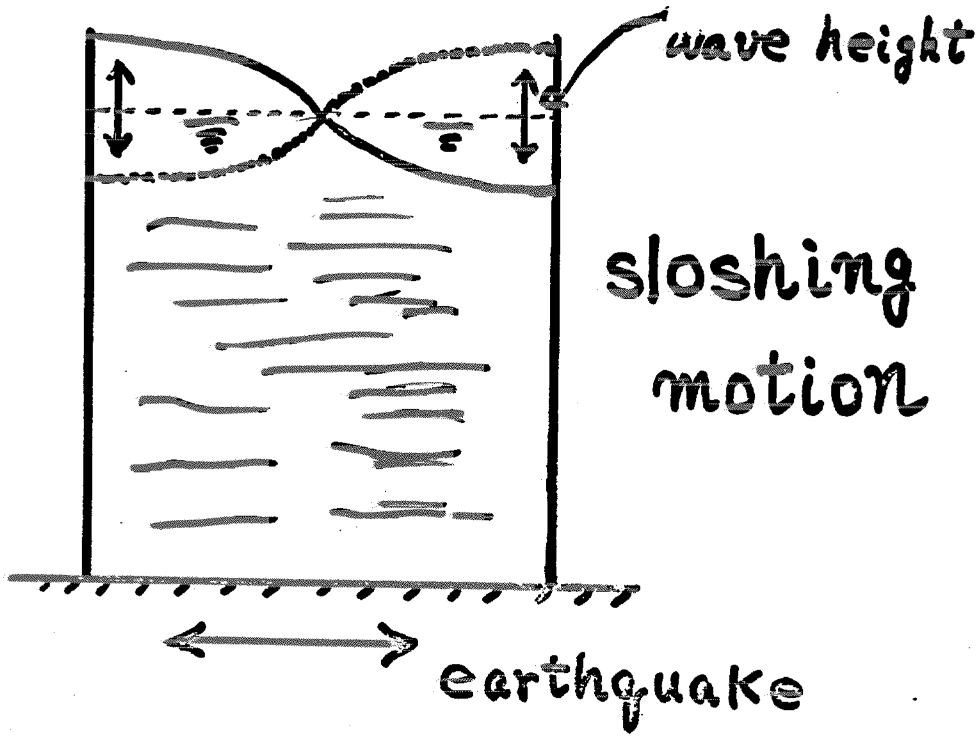
platform

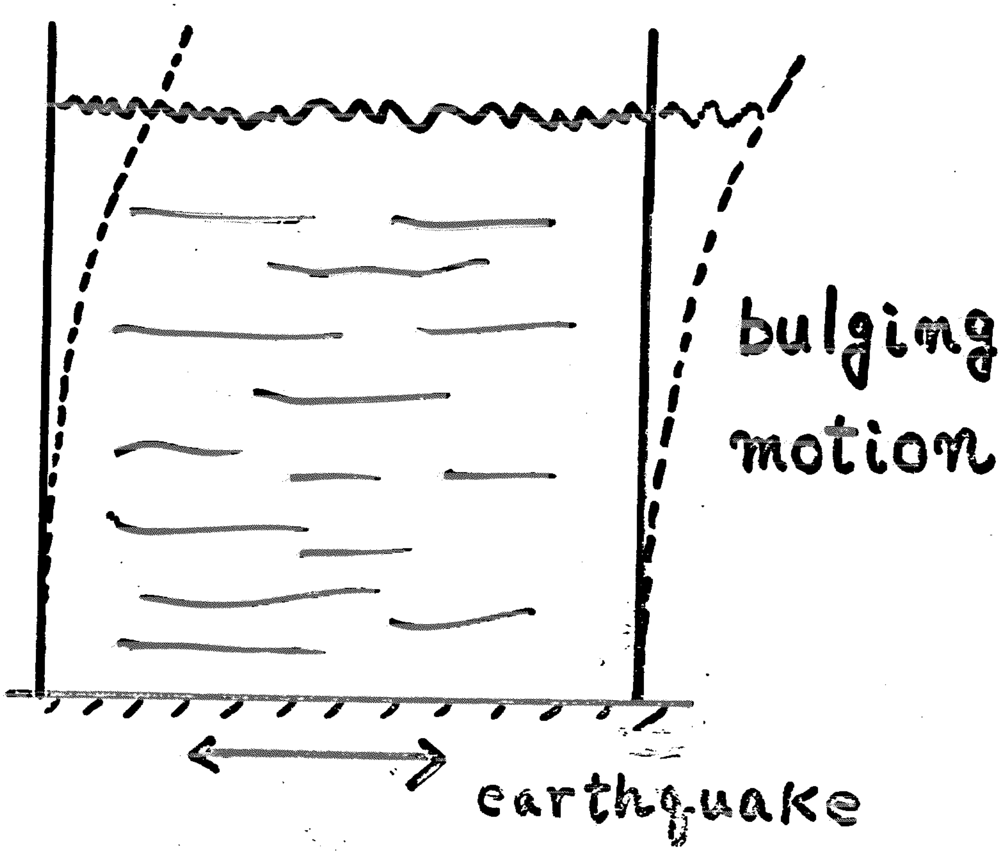


nuclear reactor



Two types of fluid motion





Numerical Methods of the coupled fluid-structure dynamic behaviours

① Lagrangian approach

movable configuration
elastic solid

② Eulerian approach

unmovable configuration
velocity potential

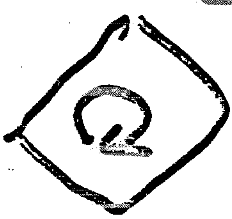
- Linear theory
- non viscous flow

Lagrangian apploach

fluid ...... elastic solid

finite bulk modulus
very small shear modulus

rotatry modes generated

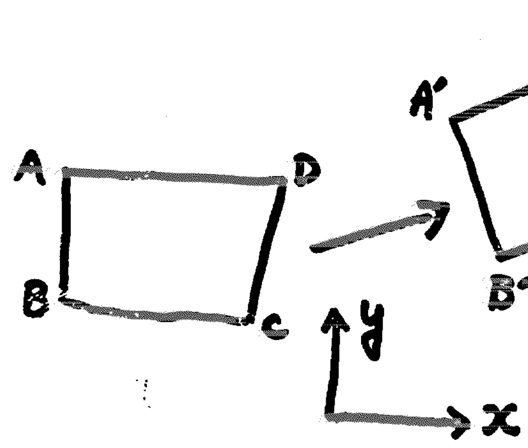


↑

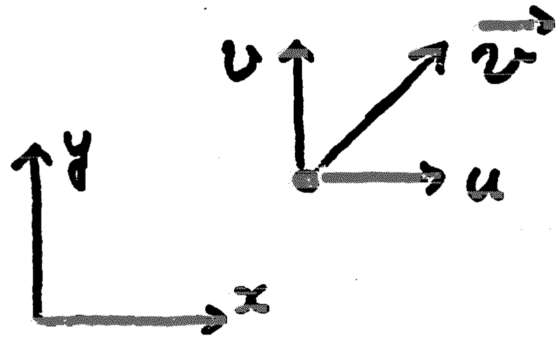
hybrid variational principle

and/or

penalty method



Eulerian approach



introducing velocity potential ϕ

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y} \quad \cdots\cdots \quad (A)$$

or pressure field P

$$u = -\frac{1}{\rho}\frac{\partial P}{\partial x}, \quad v = -\frac{1}{\rho}\frac{\partial P}{\partial y} \quad (B)$$

irrotational flow

continuity equation

(incompressible flow)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \cdots\cdots \quad (C)$$

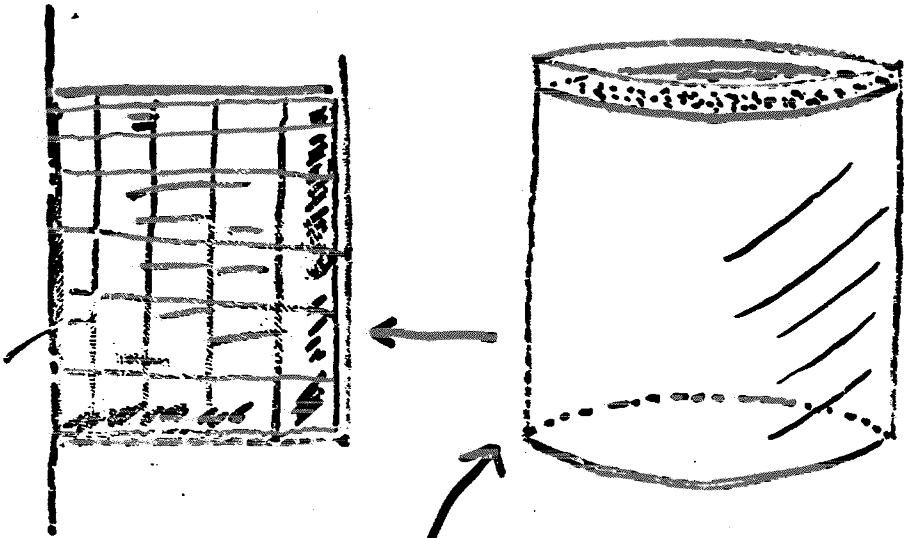
substituting (A), (B) into (C)

Laplace's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad \frac{1}{\rho}\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) = 0$$

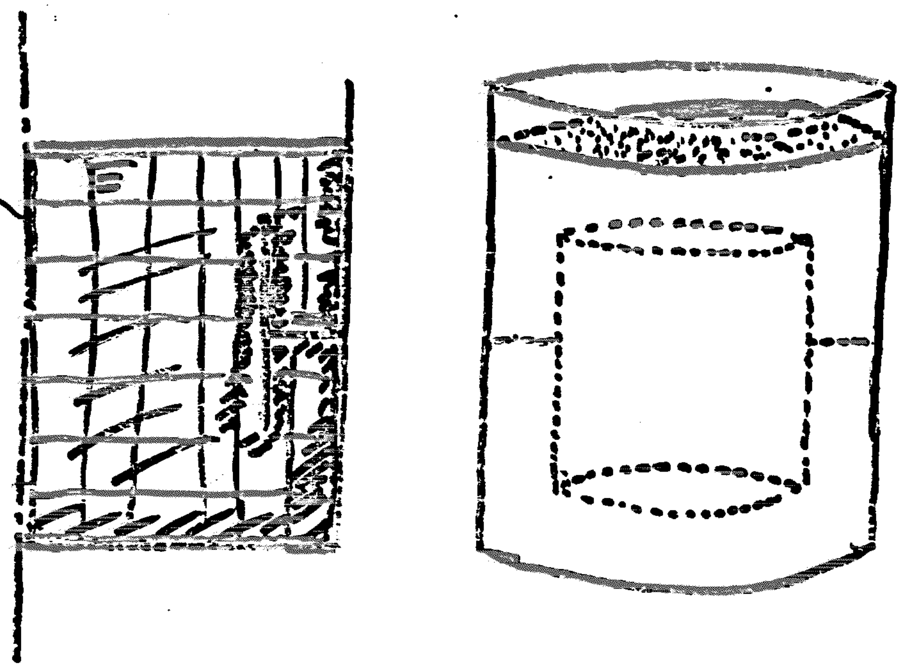
MSC/NASTRAN's virtual mass method

finite element



available for simple shaped structure

finite element



new DMAP modification

available for complex shaped structure

Variational Functional

$$\iiint_{V_e} (\sigma_{ij}\,\delta\varepsilon_{ij} + \rho_s \ddot{u}_i\,\delta u_i)\,dV - \iint_{S_s} p\,n_i\,\delta u_i\,dS - \iint_{S_F} \bar{T}_i\,\delta u_i\,dS = 0$$

for the structure

$$\iiint_{V_f} \frac{1}{\rho_f} p_{,i}\,\delta p_{,i}\,dV + \iint_{S_s} \ddot{u}_i\,n_i\,\delta p\,dS = 0$$

for the fluid

where u_i ; displacement

p ; pressure

.....

etc.

FEM

structure

$$K_s u + M_s \ddot{u} - \underline{A}^t P = F$$

pressure (fluid)

$$K_f P + \underline{A} \ddot{u} = 0$$

free vibration of coupled mode

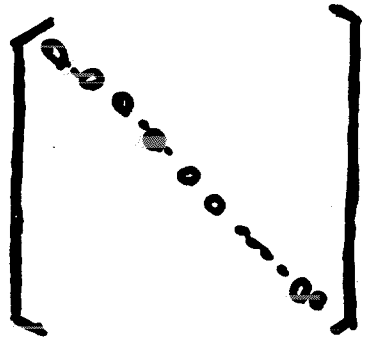
$$\left[\begin{array}{c|c} M_s & 0 \\ \hline A & 0 \end{array}\right] \left\{\begin{array}{c} \ddot{u} \\ \hline \ddot{P} \end{array}\right\} + \left[\begin{array}{c|c} K_s & -A^t \\ \hline 0 & K_f \end{array}\right] \left\{\begin{array}{c} u \\ \hline P \end{array}\right\} = 0$$

↑

large, unsymmetric eigen value problem

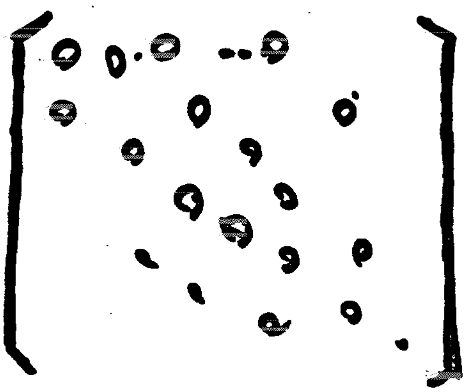
lumped mass method

DMAP



NASTRAN default

Consitent mass method



PARAM COUPMASS

Interaction matrix A should be estimated corresponding to the mass matrix.

lumped mass A ←---- DMIG

consistent mass A ←--- DMIG PLOAD

the first phase

free vibration of empty mode

$$(K_s - \omega^2 M_s) U_s = 0 \qquad \text{(D)}$$

Coupled eigen mode

by the linear superposition

of empty eigen modes

$$U = \sum_{i=1}^{n} \Phi_i C_i$$

$$= \Phi C \qquad \text{(E)}$$

Φ_i ; i-th empty mode of the structure

C_i ; i-th generalized coordinate

the second phase

Basic equation of coupled fluid-structure problem

$$\left[\Phi^t K_s \Phi - \omega^2 (\Phi^t M_s \Phi + \Phi^t A^t K_f^{-1} A \Phi)\right] C = 0$$

added mass term

(F)

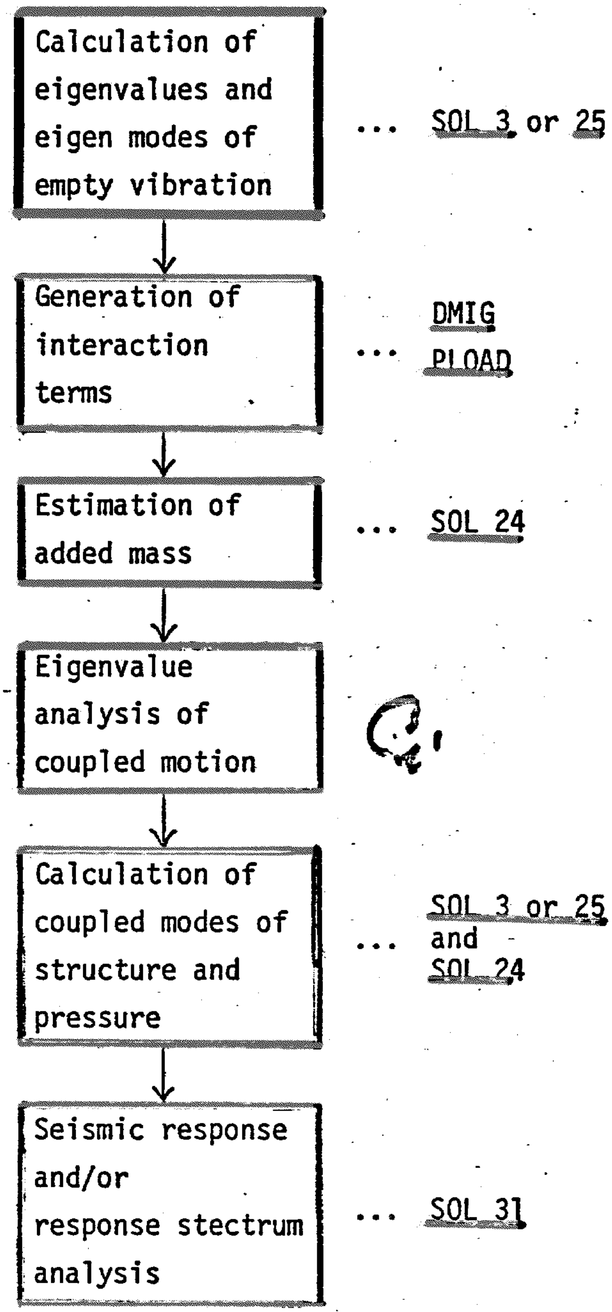


Fig. 1 Simplified flow diagrams for analysis of coupled fluid-structure response

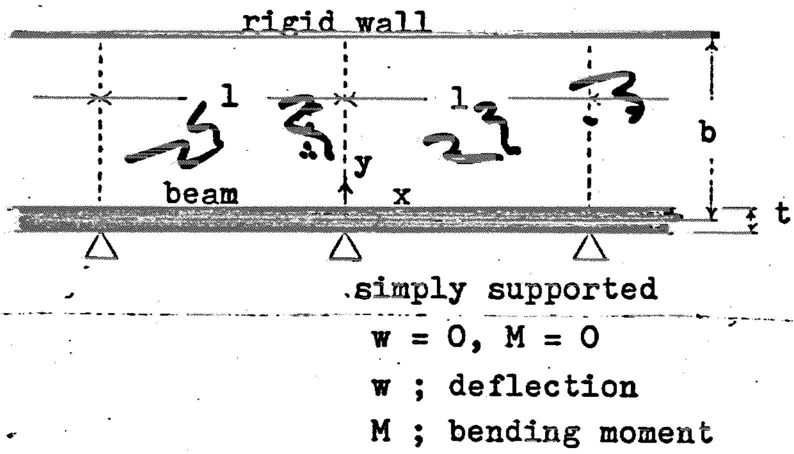


Fig. 2 Infinite beam having one wetted side.

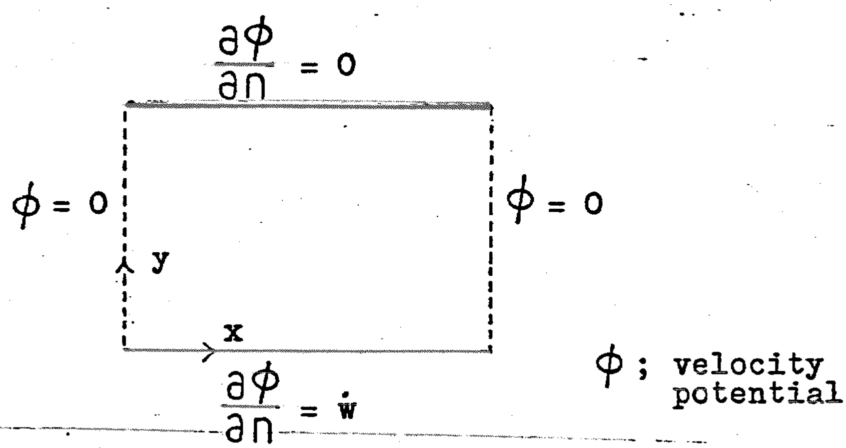


Fig. 3 Boundary conditions of potential.

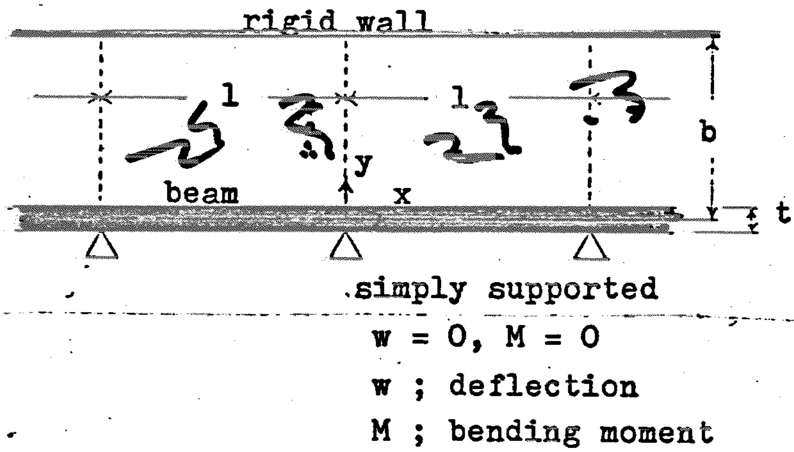


Fig. 2 Infinite beam having one wetted side.

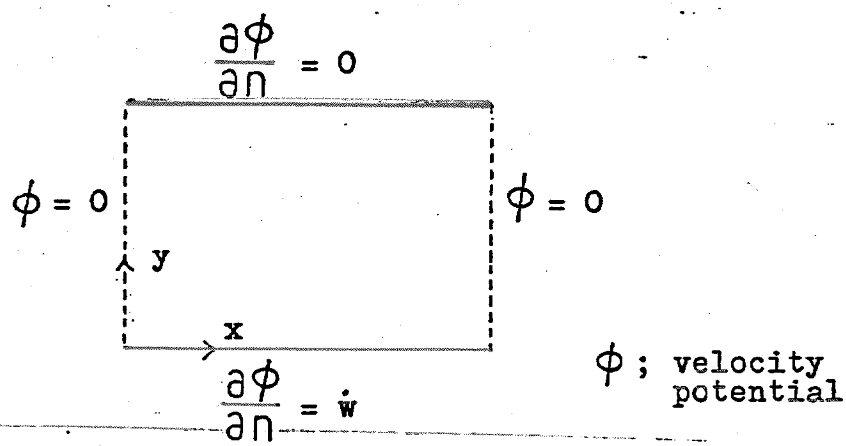


Fig. 3 Boundary conditions of potential.

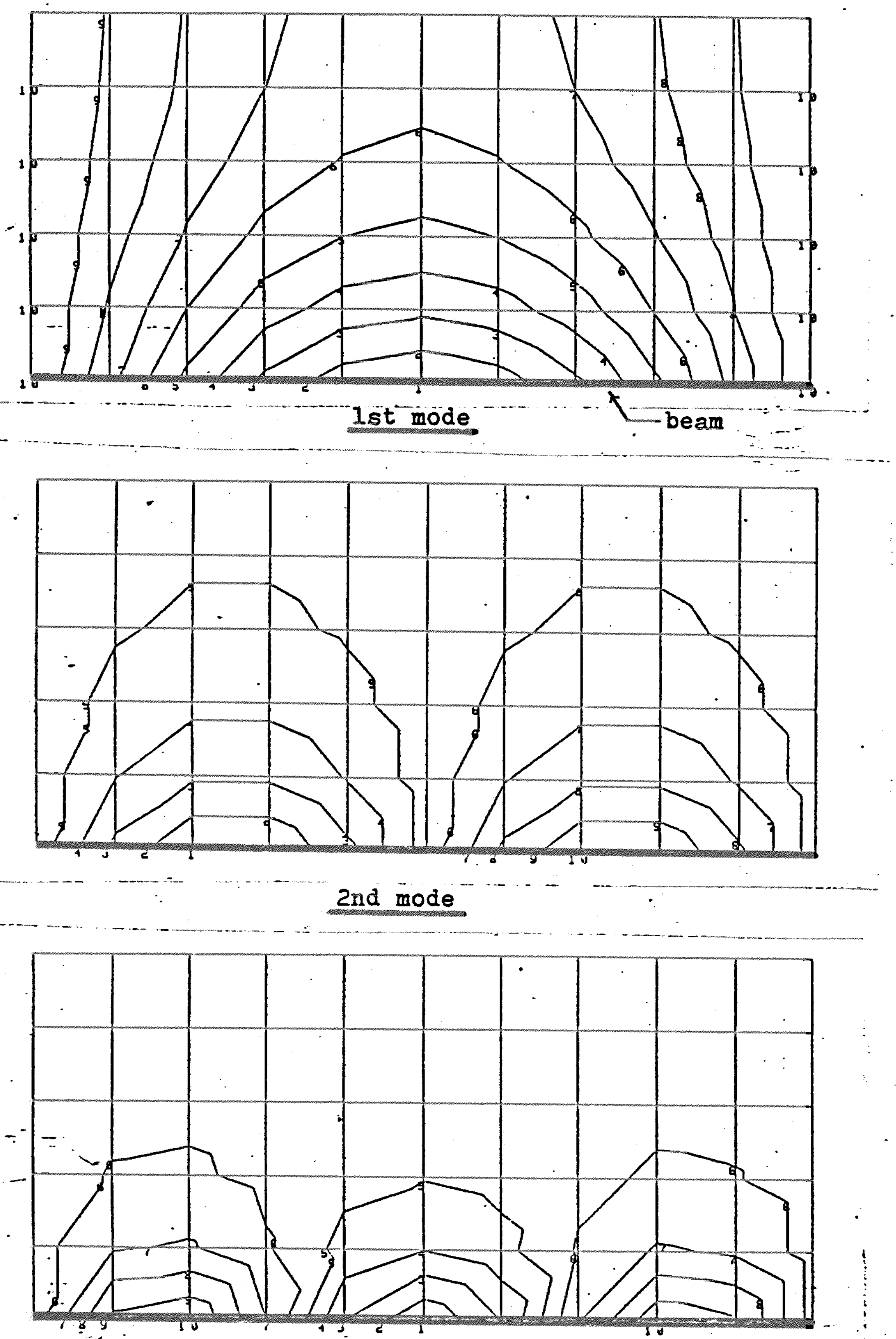


Fig. 4 Contours of pressure obtained by MSC/NASTRAN

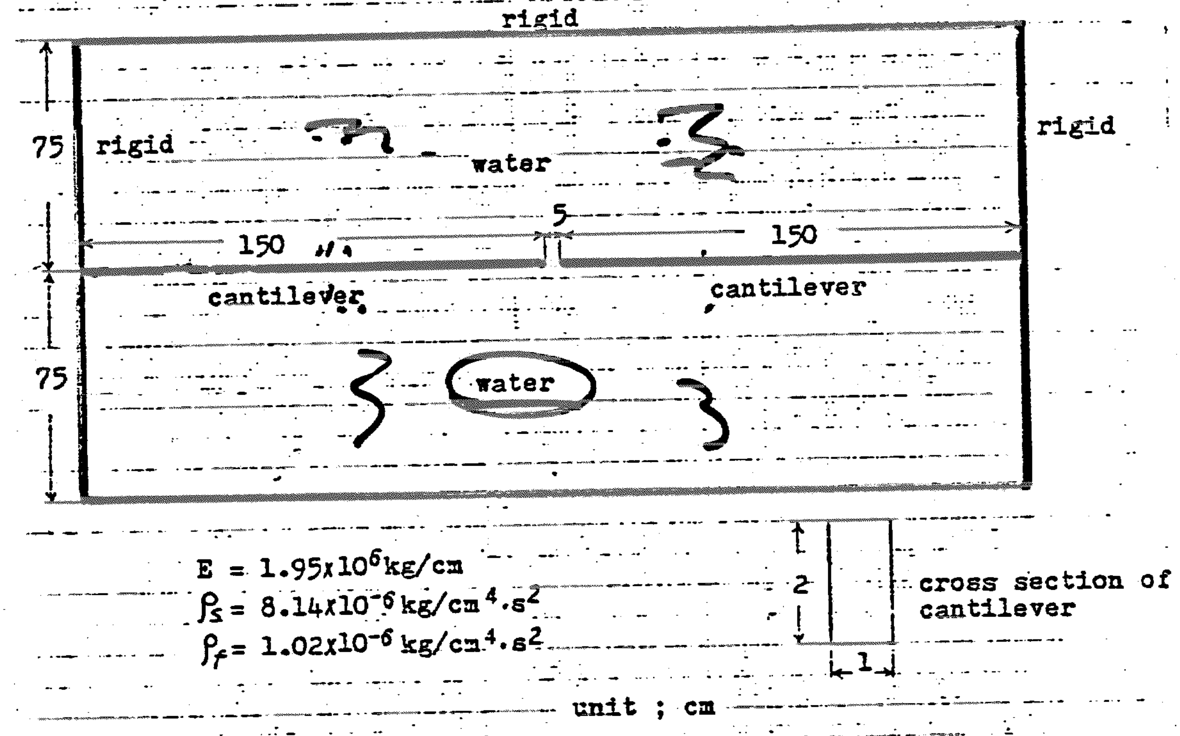


Fig. 5. Two cantilevers facing each other in the water

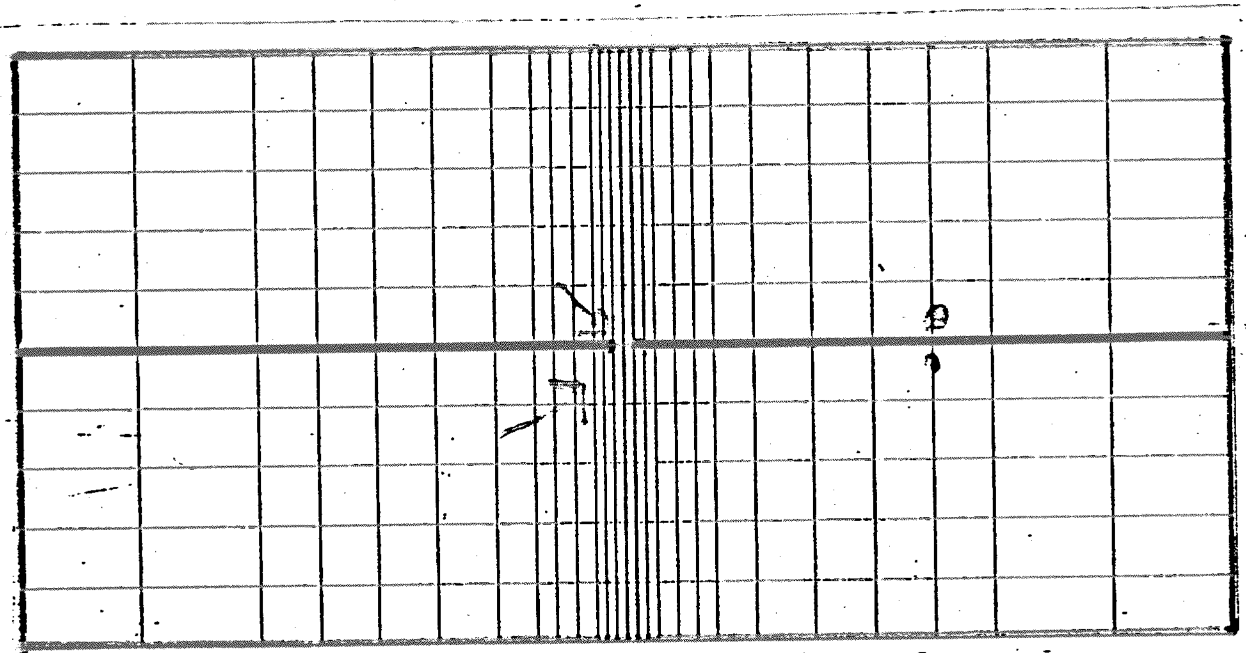


Fig. 6 Mesh division of fluid

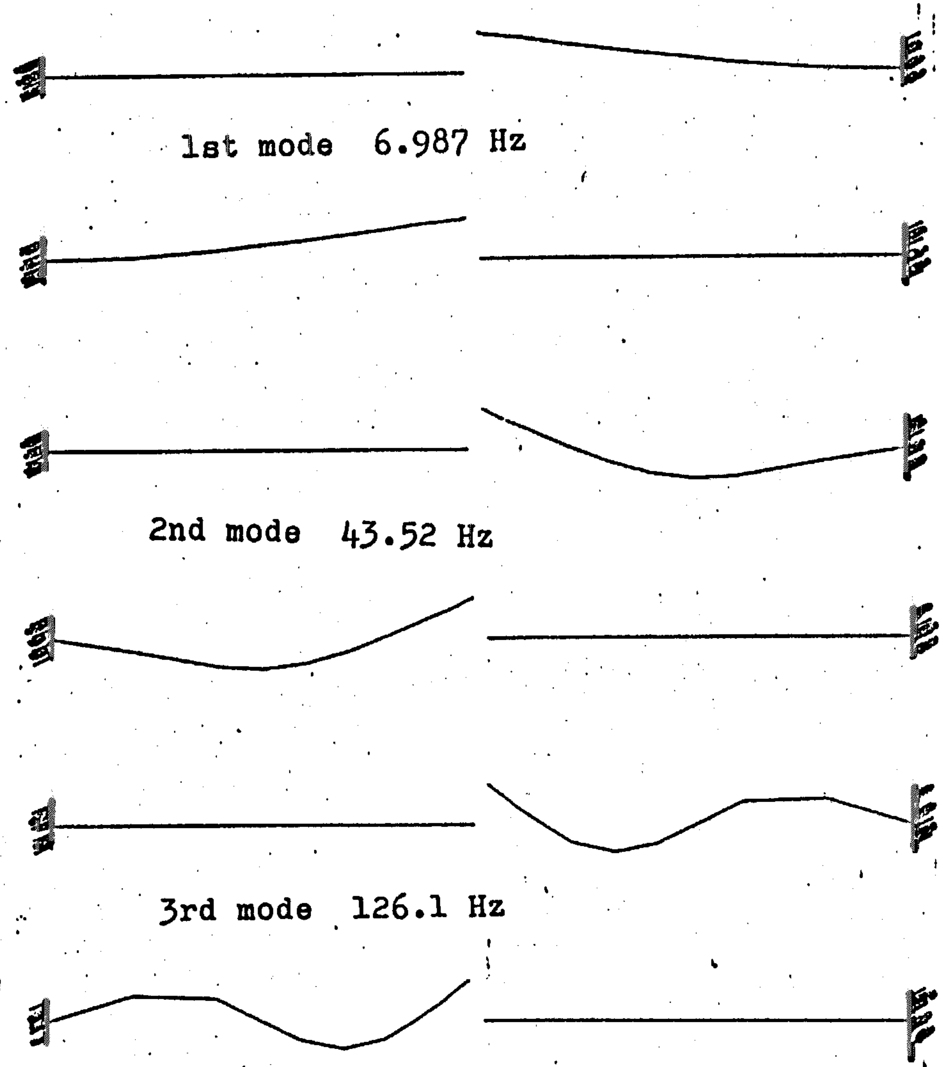


Fig. 7 Empty modes, moving independently.

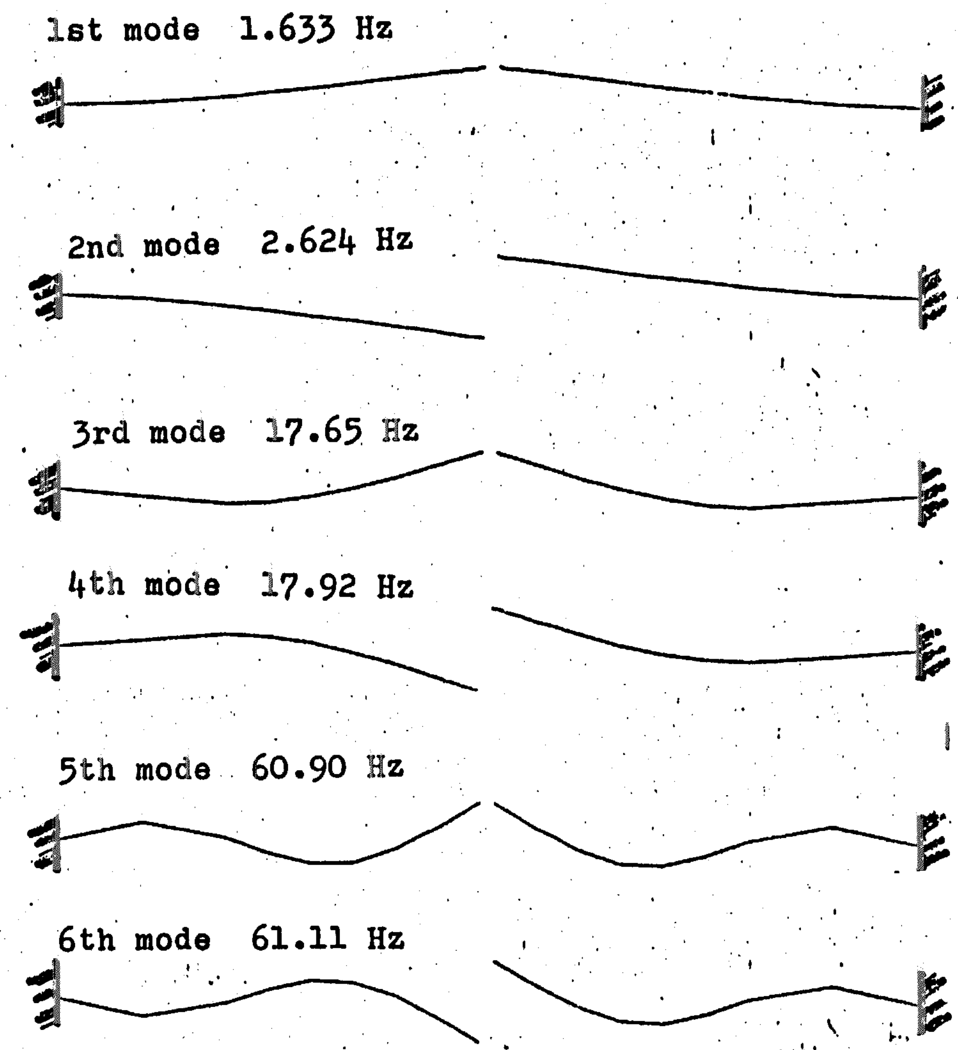
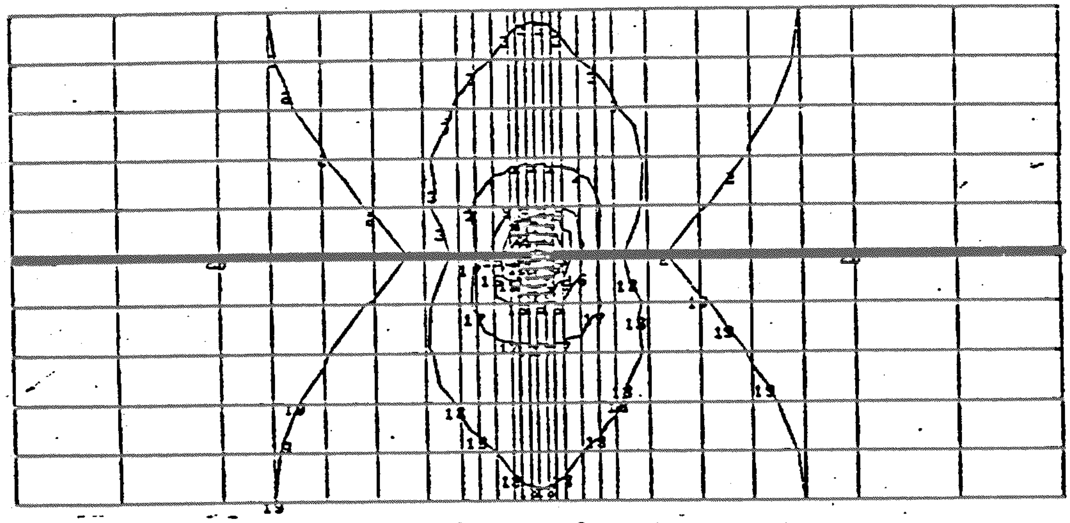
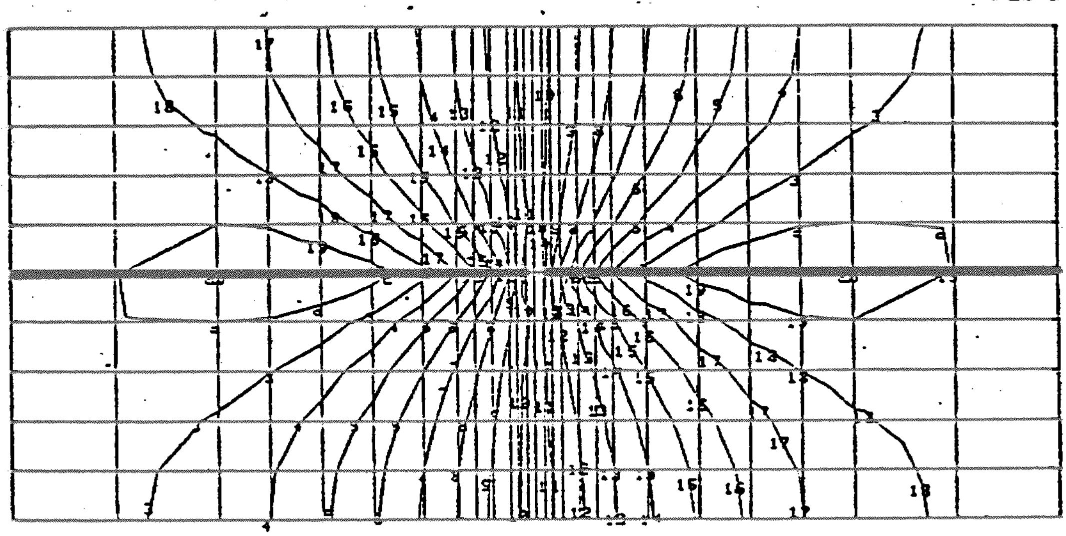


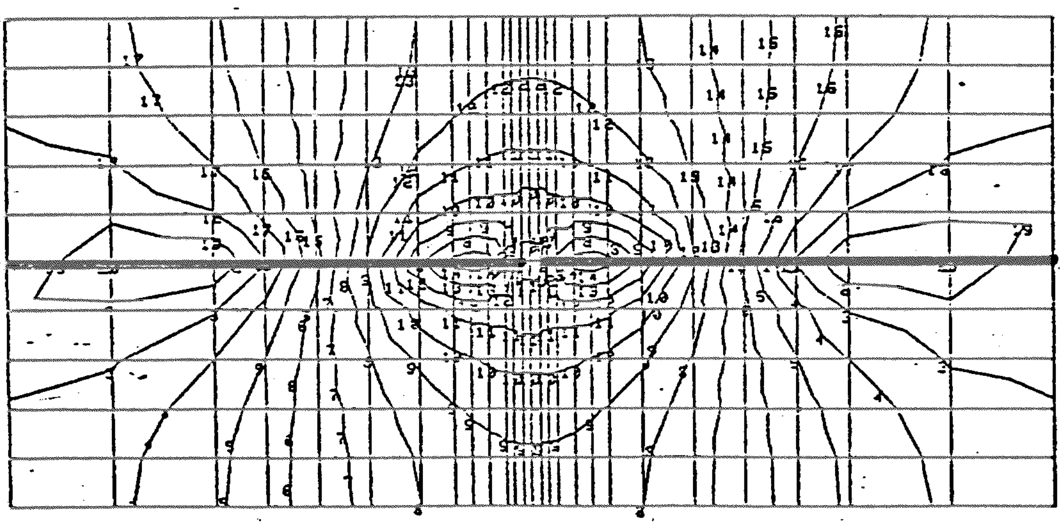
Fig. 8 Coupled modes, moving together.



1st mode

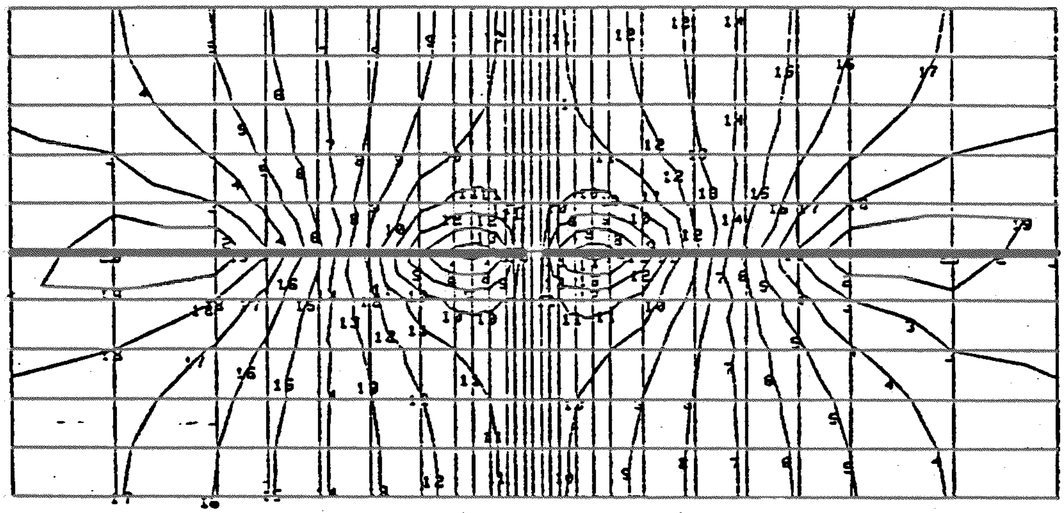


2nd mode

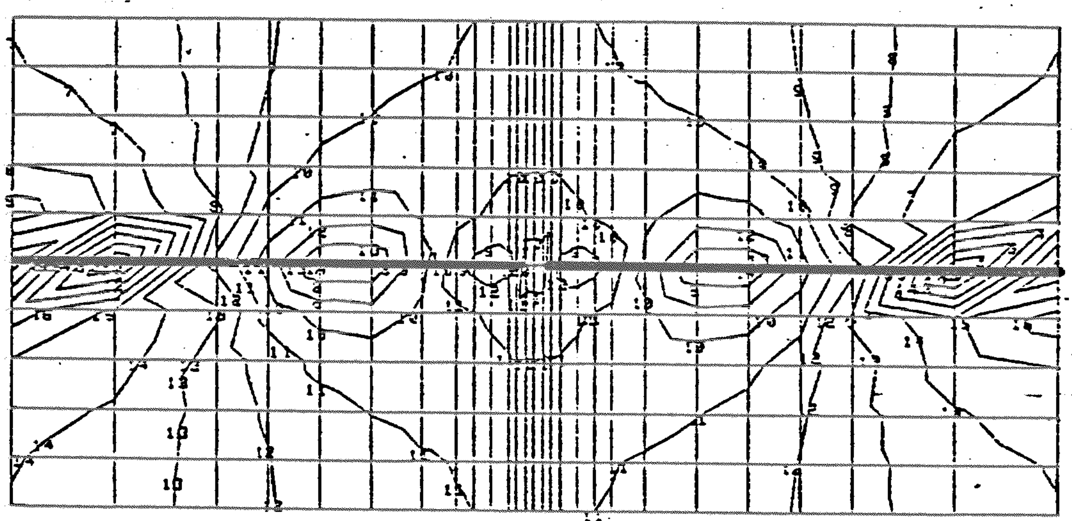


3rd mode

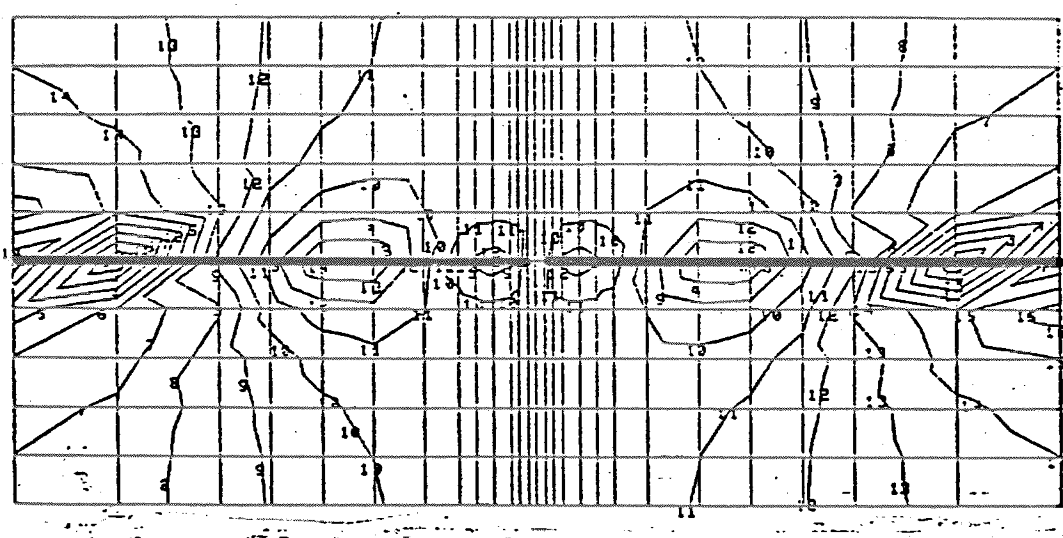
Fig. 9 Contours of pressure corresponding to the coupled modes shown in Fig. 8 (cont.).



4th mode

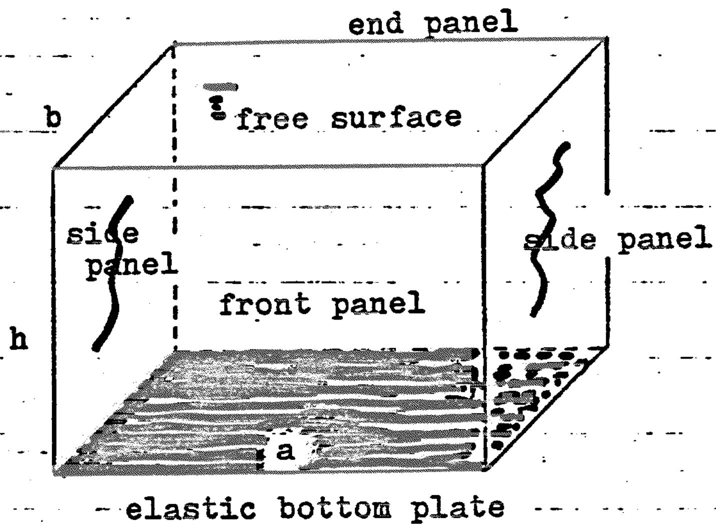


5th mode



6th mode

Fig. 9 Contours of pressure corresponding to the coupled modes shown in Fig. 8.



a = 200cm

b = 100cm

h = 80cm

Fig. 10 Fluid enclosed with a elastic bottom plate and 4 rigid panels.

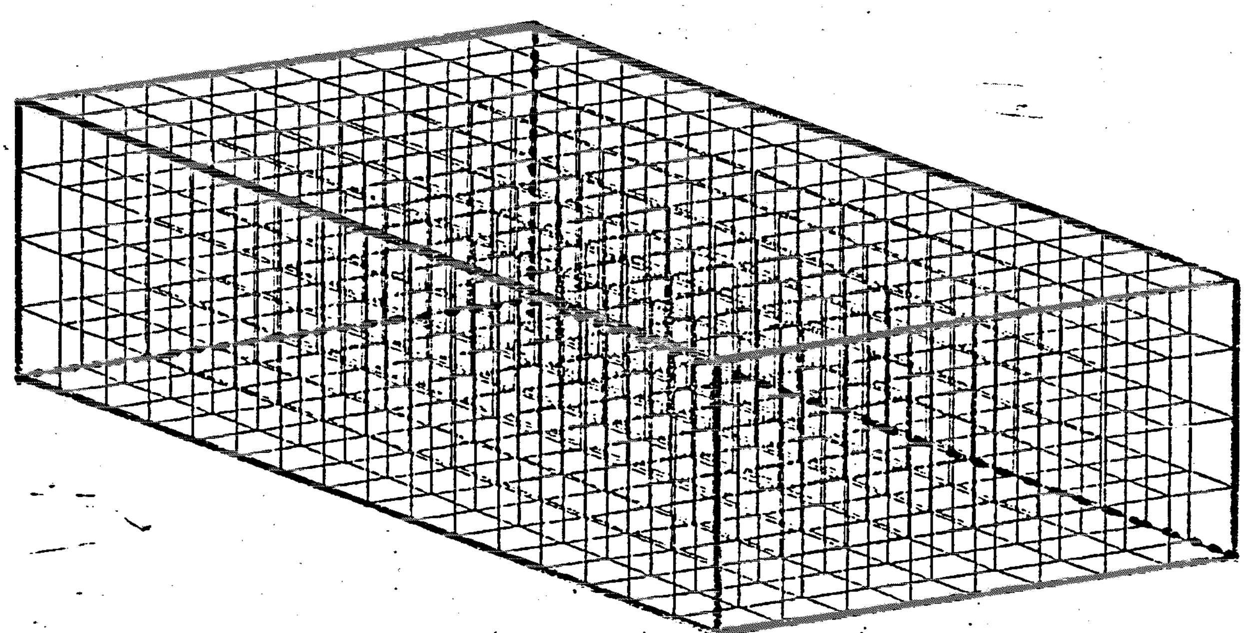


Fig. 11 Mesh division of fluid using 512 solid elements.

HEXA

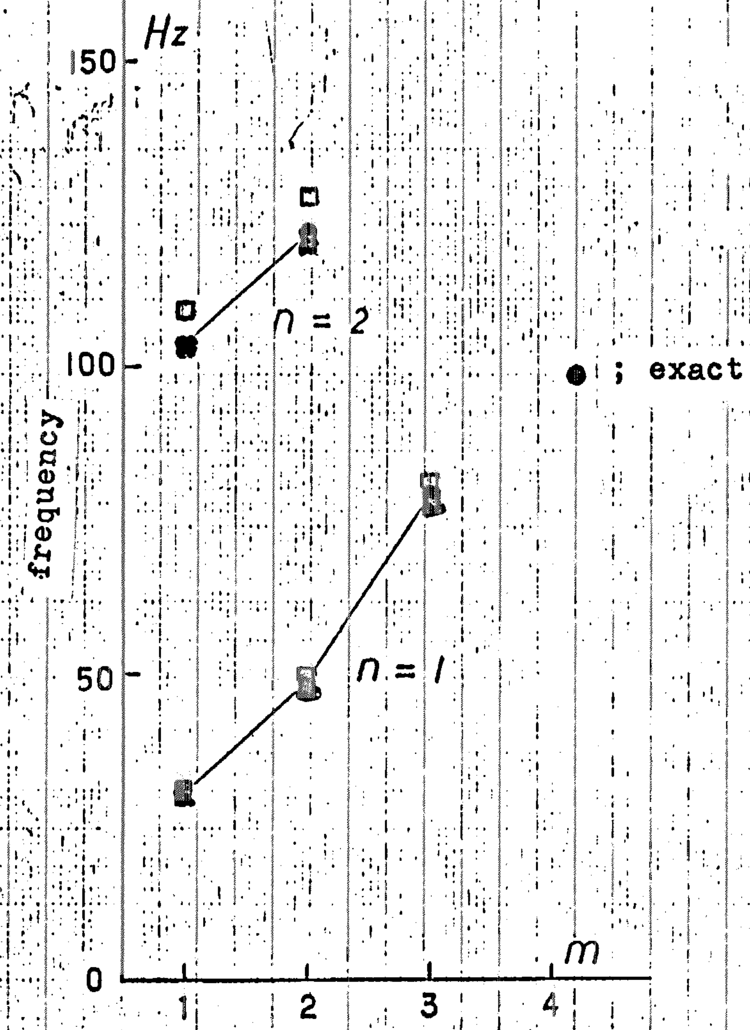


Fig. 12 Eigen frequencies of empty modes of bottom plate.

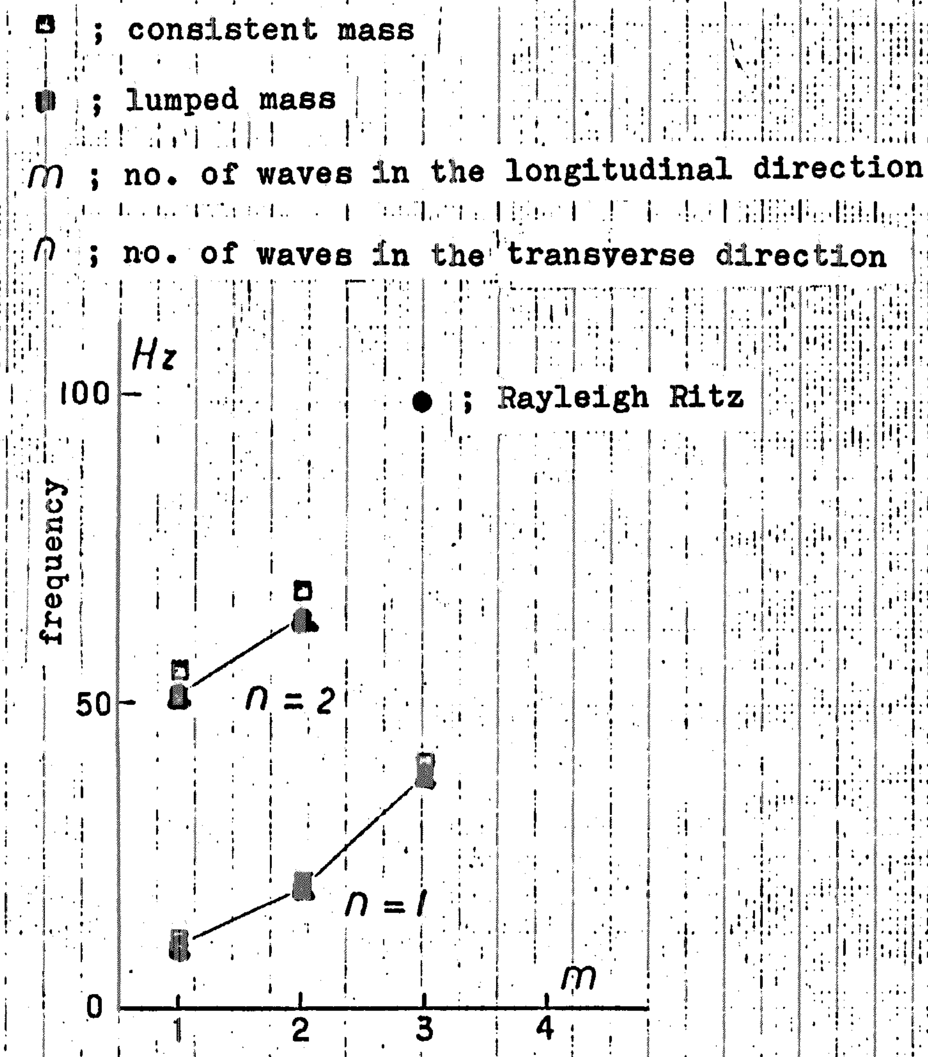


Fig. 13 Eigen frequencies of coupled modes of bottom plate.