USING NASTRAN TO SOLVE SYMMETRIC STRUCTURES WITH NONSYMMETRIC LOADS

by

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BUTLER ANALYSES

This paper deals with reflective dihedral symmetry; not rotational nor cyclic symmetry. There is nothing new about this until we talk about nonsymmetrical loads on symmetrical structures. In principal this too is an old topic, but I choose to talk about it, because I get a look of disbelief from clients when I tell them that I can legitimately confine such analyses to a half, quarter, or an octal segment. I tried to give these people a reference to consult and found none. Such reactions convinced me that it was worth while to air this topic in the open literature.

For completeness sake, I plan to start with a simple example, then explore those areas where the analyst must make his decisions, and conclude with a rather involved application.

In order to convince you that this approach to analysis is legitimate, I offer an argument from strength of materials based on a nonsymmetrically loaded simply supported prismatical beam.

LAW: For reflective dihedral symmetry in linear structures under non-symmetric loads, the response in any part of the whole structure can be determined by solving a unit segment of the symmetrical structure for all permutations of symmetric and antisymmetric boundary conditions with the nonsymmetric load in place for all of them by properly combining results of the permuted solutions.

The nub of this law rests on another law from tensor analysis that says that any nonsymmetric load can be decomposed into a symmetrical set plus an antisymmetrical set.

While in the process of defining, it would be well to distinguish anti-symmetrical from non-symmetrical and to be specific about the kind of symmetry being dealt with.

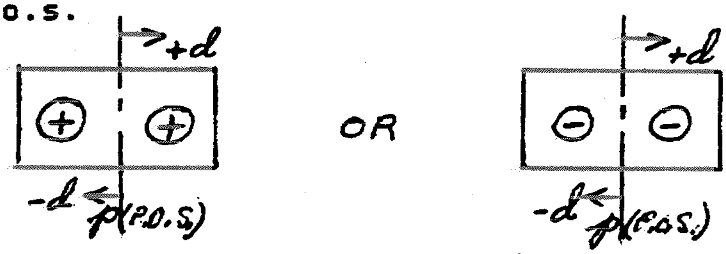
First of all our concern is for symmetry of elastic deformation. As a consequence, such actions as axial, bending, membrane, and torsional deformations need examining. As such, we will talk in terms of modes of deformation. Modes can be described in general by using "+" and "-" signs for opposite actions For example, in bending "+" can stand for tension and "-" can stand for compression, or in axial deformation "+" can stand for condensation and "-" can stand for rarefaction, etc.

Symmetry maps the elastic deformations on one side of the plane of symmetry (p.o.s.) into like values on the opposite side of the p.o.s. at corresponding distances perpendicular to the p.o.s.

D[x=(p-d),y,z] = D[x=(p+d),y,z] will be written in abbreviated notation as

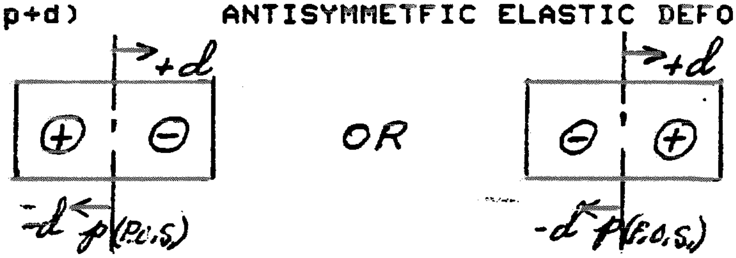
D(p-d) = D(p+d)   SYMMETRIC ELASTIC DEFORMATION

where D can represent any elastic deformation; p is the location of the p.o.s.; and d is the distance away from the p.o.s., all measured perpendicular to the p.o.s.



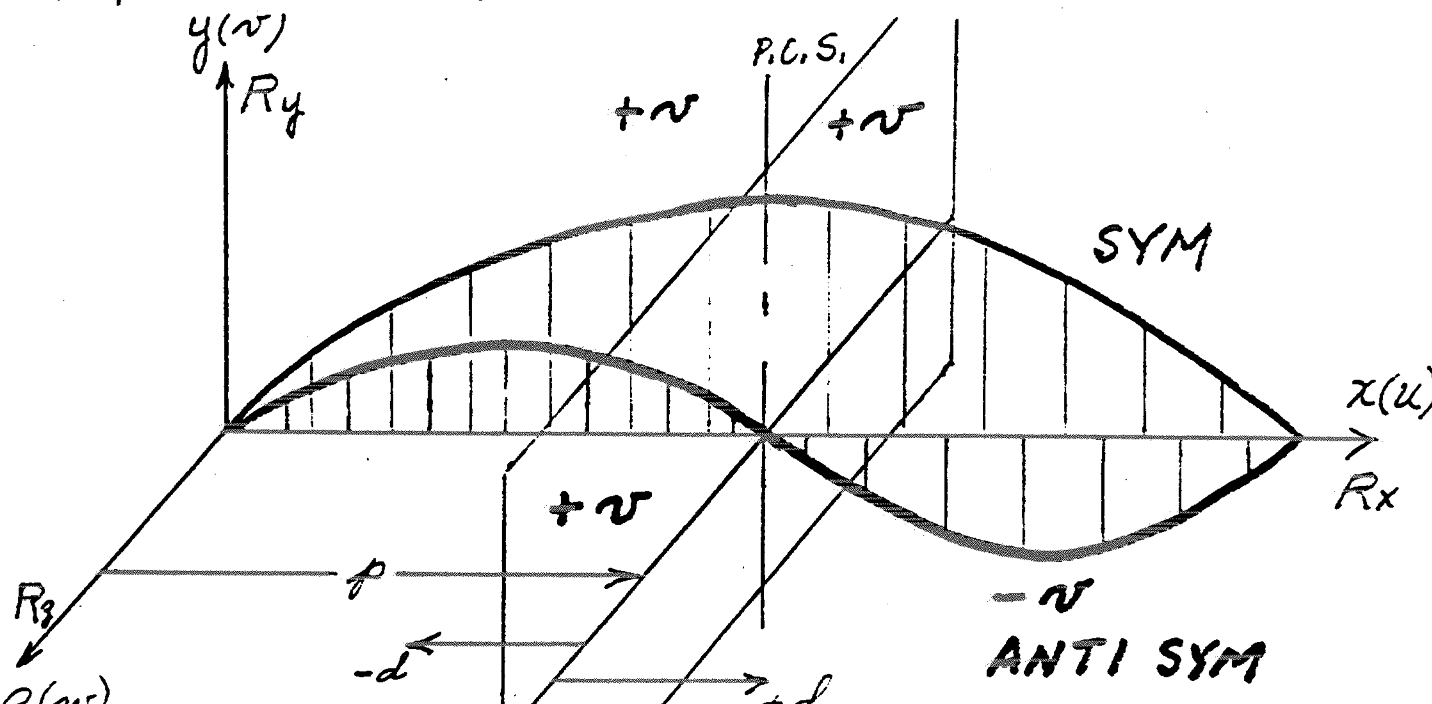
Antisymmetry maps the elastic deformations on one side of the p.o.s. into like values -- but of opposite sign -- on the opposite side of the p.o.s. at corresponding distances perpendicular to the p.o.s.

D(p-d) = (-1)D(p+d)   ANTISYMMETFIC ELASTIC DEFORMATION



BOUNDARY CONDITIONS AT THE PLANE OF SYMMETRY

The first task is to translate these elastic actions into boundary conditions on displacements at the p.o.s. for the unit segment of the structure. Start with bending in the XY plane for a beam colinear with the x-axis.



The elastic actions in XY bending require for symmetry that $v(p-d) = v(p+d)$. In the limit as corresponding points approach the p.o.s.

$$\lim_{d \to 0} [v(p-d) = v(p+d)] \implies v(p) \neq 0$$

Since elastic continuity is maintained at the p.o.s., there must be continuity in the slope of v. Corresponding slopes in symmetry are of opposite sign:

$$\frac{\partial v}{\partial x}(p-d) = (-1)\frac{\partial v}{\partial x}(p+d)$$

In the limit as corresponding slopes approach the p.o.s.

$$\lim_{d \to 0} \left[\frac{\partial v}{\partial x}(p-d) = (-1)\frac{\partial v}{\partial x}(p+d)\right] \implies \frac{\partial v}{\partial x}(p-0) = (-1)\frac{\partial v}{\partial x}(p+0)$$

This can be satisfied only if $\frac{\partial v}{\partial x}(p) = 0$. This is equivalent to prescribing that the rotational d.o.f. about the Z axis at the p.o.s. is zero; i.e. $Rz(p) = 0$.

The elastic actions in XY bending require for antisymmetry that

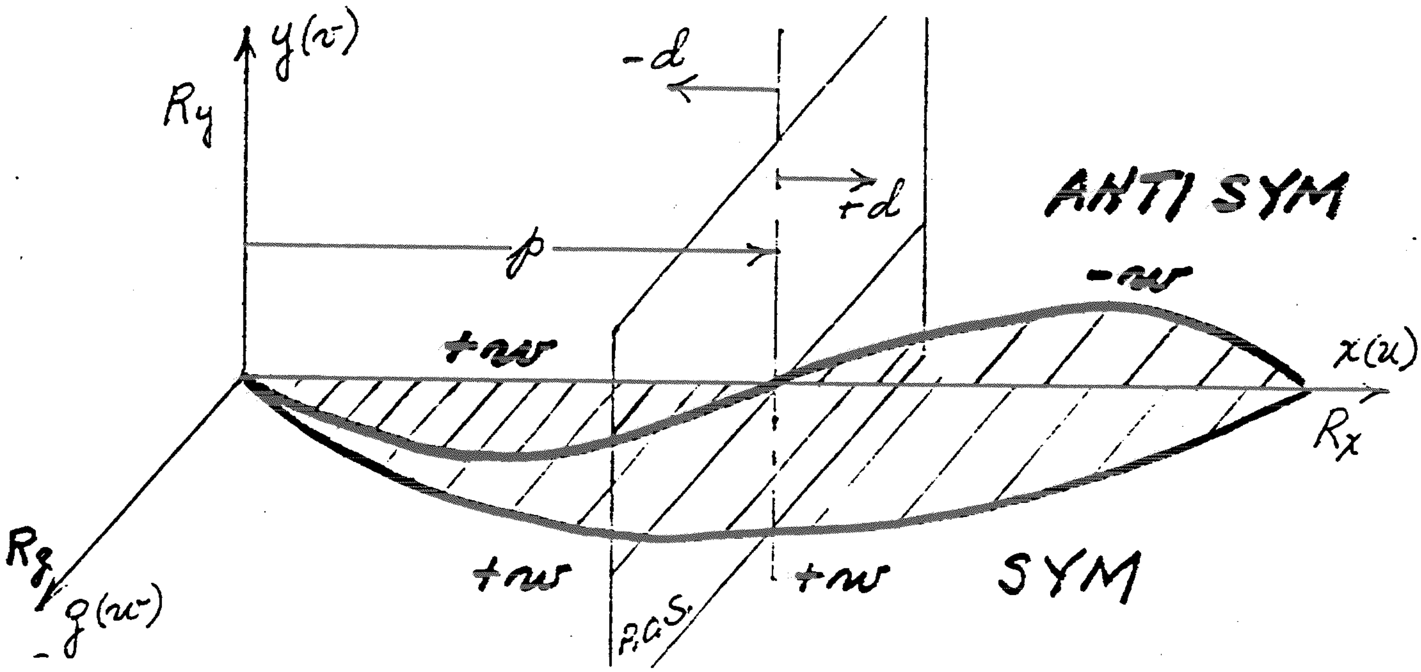
$$v(p-d) = (-1)v(p+d).$$

In the limit at p as corresponding points approach the p.o.s.

$$\lim_{d \to 0} [v(p-d) = (-1)v(p+d)] \implies v(p-0) = (-1)v(p+0).$$

This can be satisfied only if $v(p) = 0$. Intuitively one can affirm that the slope $\frac{\partial v}{\partial x}$ must be allowed to be non-zero at p; therefore the rotational d.o.f. about Z at the p.o.s. must be non-zero; i.e. $Rz(p) \neq 0$.

Turn next to bending in the XZ plane.



The elastic actions in XZ bending require for symmetry that $w(p-d) = w(p+d)$. In the limit as corresponding points approach the p.o.s.

$$\lim_{d \to 0} [w(p-d) = w(p+d)] \implies w(p) \neq 0.$$

Since elastic continuity is maintained at the p.o.s., there must be continuity in the slope of w. Corresponding slopes in symmetry are of opposite sign:

$$\frac{\partial w(p-d)}{\partial x} = (-1)\frac{\partial w(p+d)}{\partial x}$$

In the limit as corresponding slopes approach the p.o.s.

$$\lim_{d \to 0} \left[\frac{\partial w(p-d)}{\partial x} = (-1)\frac{\partial w(p+d)}{\partial x}\right] \implies \frac{\partial w(p-0)}{\partial x} = (-1)\frac{\partial w(p+0)}{\partial x}$$

This can be satisfied only if $\frac{\partial w(p)}{\partial x} = 0$; therefore $Ry(p) = 0$.

The elastic actions in XZ bending require for anti-symmetry that $w(p-d) = (-1)w(p+d)$.

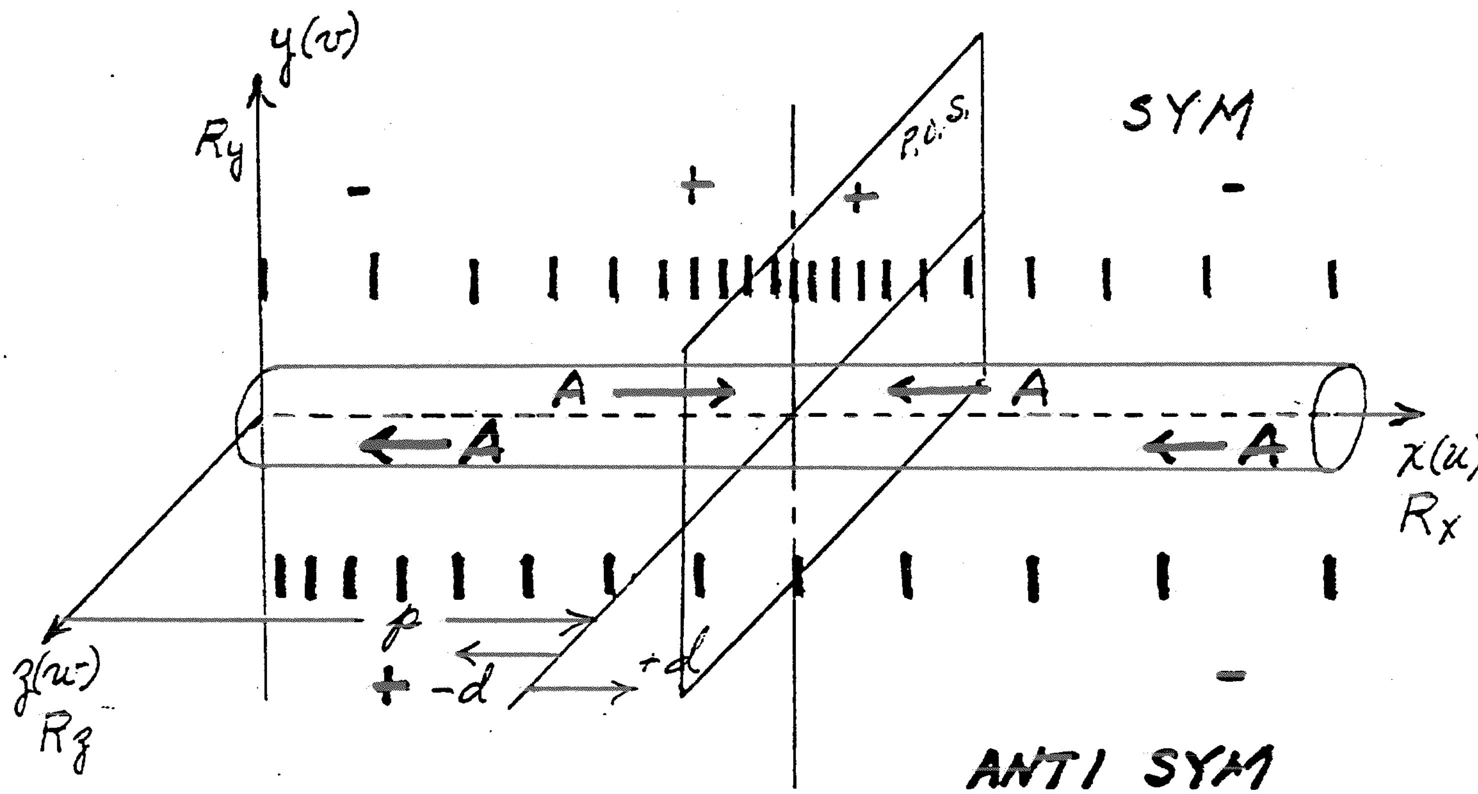
In the limit as corresponding points approach the p.o.s.

$$\lim [w(p-d) = (-1)w(p+d)] \implies w(p-0) = (-1)w(p+0).$$

This can be satisfied only if $w(p) = 0$.

The slope $\frac{\partial w(p)}{\partial x}$ must be allowed to be non-zero at the p.o.s.; which is equivalent to the condition $Ry(p) \neq 0$.

Turn next to longitudinal deformation in X



The longitudinal actions in X require for symmetry that both sides be either in condensation or in rarefaction; therefore axial forces, A, are equal and opposite

$$A(p-d) = (-1)A(p+d),$$

but this implies that axial deformations are likewise equal and opposite

$$u(p-d) = (-1)u(p+d).$$

In the limit as corresponding points approach the p.o.s

$$\lim_{d \to 0} [u(p-d) = (-1)u(p+d)] \implies u(p-0) = (-1)u(p+0).$$

this can be satisfied only if $u(p) = 0$.

The longitudinal actions in X require for anti-symmetry that one side be in condensation while the other side is in rarefaction; therefore axial forces, A, are equal

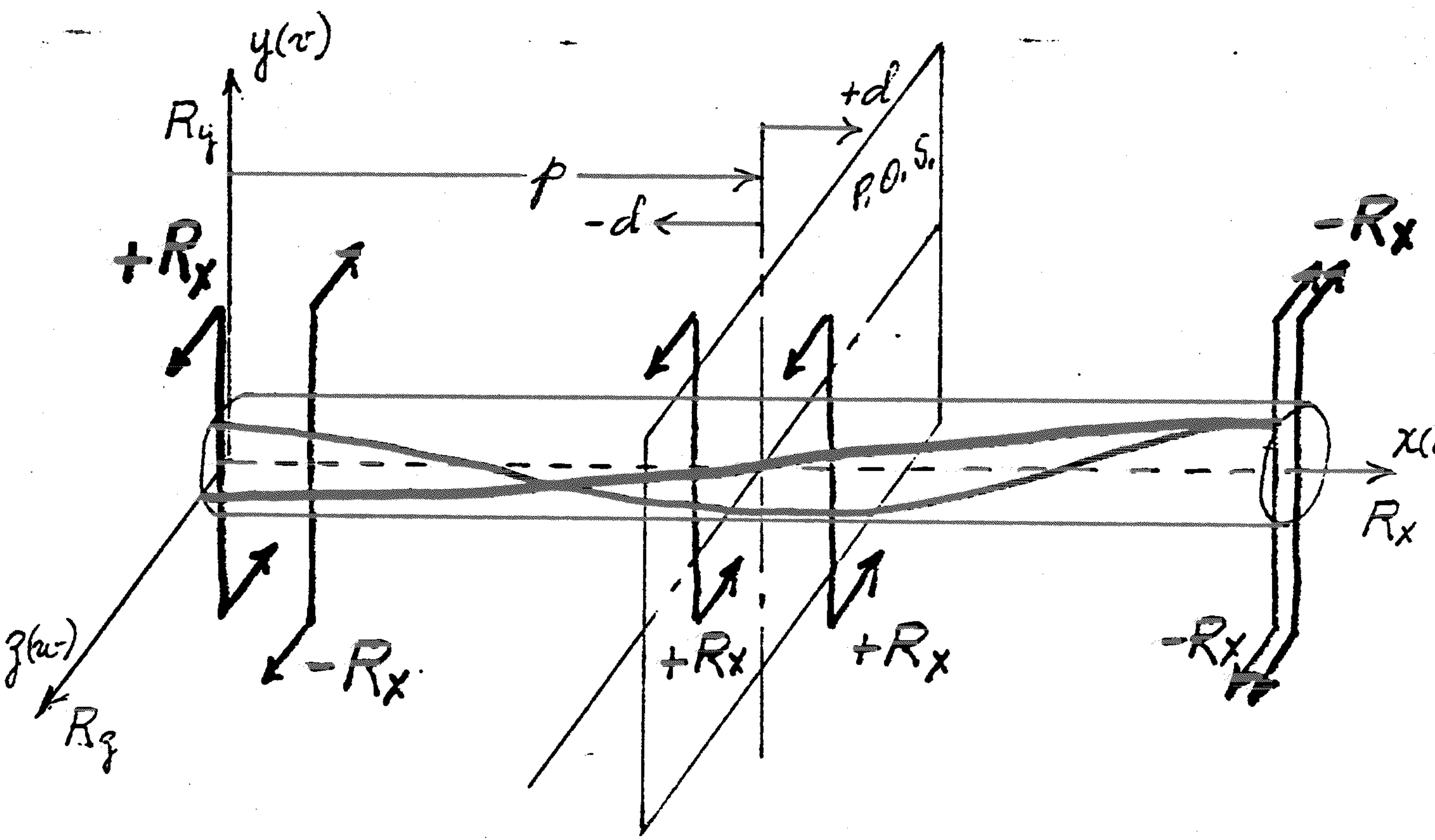
$A(p-d) = A(p+d)$; but this implies that axial deformations are likewise equal

$$u(p-d) = u(p+d).$$

In the limit as corresponding points approach the p.o.s.

$$u(p-0) = u(p+0) \implies u(p) \neq 0.$$

Finally turn to torsion about X.



The torsional actions about X for symmetry require that $Rx(p-d) = Rx(p+d)$. In the limit as corresponding points approach the p.o.s.

$$\lim_{d \to 0} [Rx(p-d) = Rx(p+d)] \implies Rx(p) \neq 0.$$

For anti-symmetry

$$\lim_{d \to 0} [Rx(p-d) = (-1)Rx(p+d)] \implies Rx(p-0) = (-1)Rx(p+0).$$

This can be satisfied only if $Rx(p) = 0$.

All six degrees of freedom have been examined for symmetric and antisymmetric boundary condition requirements at the p.o.s. The results are summarized in a table below.

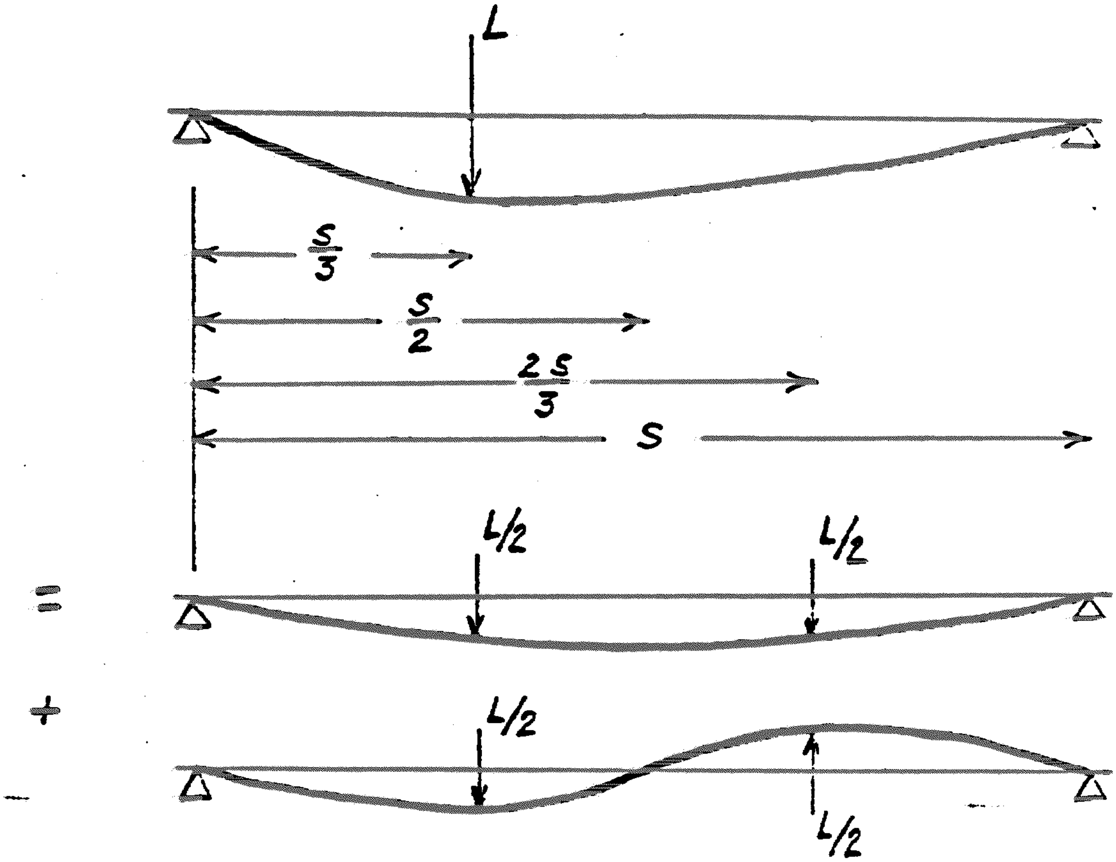
BOUNDARY CONDITION @ YZ P.O.S.				
D.O.F. No.	SYM	ANTISYM	P.O.S. ORIENTATION	BOUNDARY CONSTRAINT FORCE
1	$u(p)=0$	$u(p)\neq 0$	Perp to p.o.s.	Axial in X
2	$v(p)\neq 0$	$v(p)=0$	In p.o.s.	Trnsv in Y
3	$w(p)\neq 0$	$w(p)=0$	In p.o.s.	Trnsv in Z
4	$Rx(p)\neq 0$	$Rx(p)=0$	About normal to	Tors about X
5	$Ry(p)=0$	$Ry(p)\neq 0$	About axis in	Moment about Y
6	$Rz(p)=0$	$Rz(p)\neq 0$	About axis in	Moment about Z
Total	1,5,6=0 2,3,4≠0	1,5,6≠0 2,3,4=0		

Several generalizations can be noted:

1. When a constraint requirement in a d.o.f. is zero for one condition, it is non-zero for the other condition.
2. For either condition there are always 3 zero and 3 non-zero constraint requirements.
3. Constraint forces at the p.o.s. are those forces that are transmitted to the adjacent segment in the whole structure.
4. Rather than deal in rote quantities for each change of coordinate system, the boundary condition requirements can be expressed in terms of axes imbedded in the p.o.s.: two axes lying in the p.o.s. and an axis normal to the p.o.s.

- Symmetry
  - Zero
    - Translation normal to p.o.s.
    - Rotation about 2 axes lying in p.o.s.
  - Non-zero
    - Translation along 2 axes lying in p.o.s.
    - Rotation about normal to p.o.s.
- Anti-Sym
  - Zero
    - Translation along 2 axes lying in p.o.s.
    - Rotation about normal to p.o.s.
  - Non-zero
    - Translation normal to p.o.s.
    - Rotation about 2 axes lying in p.o.s.

The law that assymetric loads on symmetric structures can be represented by the superposition of partial load conditions can be illustrated by a simply supported prismatic beam with an off-center load.



Plan of Analysis Using One Symmetric Half Only

Ingredient case 1. L/2 @ s/3:sym bc @ s/2. Can represent either side.

2. L/2 @ s/3:asym bc @ s/2. Can represent either side.

LH behavior Sym + Asym. Sum F=L/2 + L/2=L. Sum M=2xL/2xs/3=sL/3

RH bahavior Sym - Asym. Sum F=L/2 - L/2=0. Sum M=L/2xs/3-L/2xs/3=0

These statements will now be verified by (1) solving for the slope and deflection of the whole beam, (2) solving for the slope and deflection of the LH half using half load, under symmetric boundary conditions, (3) solving for slope and deflection of the LH half using half load under antisymmetric boundary conditions, (4) evaluating slope and deflection formulas @ S/4, s/2, and 3s/4, (5) adding the symmetric and antisymmetric response and comparing with the response of the LH half of the whole beam, and (6) subtracting the antisymmetric response from the symmetric response and comparing with the response of the RH half of the whole beam.

The slope and deflection of the S-S beam with load L @ s/3 and EI = 1 are:

	x=s/4	x=s/2	x=3s/4
y'(x)	$\frac{-53Ls^2}{48x27}$	$\frac{5Ls^2}{24x27}$	$\frac{101Ls^2}{48x54}$
y(x)	$\frac{-71Ls^3}{72x72}$	$\frac{-23Ls^3}{36x36}$	$\frac{-119Ls^3}{2x72x72}$

The slope and deflection of the left hand half with half load @ s/3 under symmetric boundary conditions at the plane of symmetry are and evaluated at s/4 and s/2 are:

Sym y'(x)	$\frac{-23Ls^2}{36x16}$	0
Sym y(x)	$\frac{-29Ls^3}{72x32}$	$\frac{-23Ls^3}{36x36}$

The slope and deflection of the left hand half with half load under anti-symmetric boundary conditions at the plane of symmetry are and evaluated at s/4 and s/2 are:

Anti y'(x)	$\frac{-5Ls^2}{64X81}$	$\frac{5Ls^2}{24x27}$
Anti y(x)	$\frac{-23Ls^3}{96x216}$	0

To depict the left half full scale behavior @ x=s/4

sum Sym y'(s/4) & Anti y'(s/4) $= \frac{-23Ls^2}{36x16} - \frac{5Ls^2}{64X81} = \frac{-53Ls^2}{48x27}$

and

sum Sym y(s/4) & Anti y(s/4) $= \frac{-29Ls^3}{72x32} - \frac{23Ls^3}{96x216} = \frac{-71Ls^3}{72x72}$

At x = s/2 the sum also checks; e.g.

sum Sym y'(s/2) & Anti y'(s/2) $= 0 + \frac{5Ls^2}{24x27} = \frac{5Ls^2}{24x27}$

and

sum Sym y(s/2) & Anti y(s/2) $= \frac{-23Ls^3}{36x36} + 0 = \frac{-23Ls^3}{36x36}$

Differences are taken in order to use results of the analysis from the left hand half to predict the behavior of the right hand half. Compare differences of the LH half @ x = s/4 with the full scale @ x = 3s/4; e.g.

subtract Sym y'(s/4) & Anti y'(s/4) = $\frac{-23Ls^2}{36x16} + \frac{5Ls^2}{64X81} = \frac{-101Ls^2}{36x72}$

Note: The sign is opposite to that of y'(3s/4), because the calculated slope is in a reflected position wrt plane of sym.

subtract Sym y(s/4) & Anti y(s/4) = $\frac{-29Ls^3}{72x32} + \frac{23Ls^3}{96x216} = \frac{-119Ls^3}{2x72x72}$

In summary for the beam analysis:

Half symmetry was used.
Non-symmetric load was split into 1/2 Sym & 1/2 Anti-Sym.
L.H. side was the solution segment.
Ingredient S/C 1; Load=L/2 with symmetric bound. cond.
Ingredient S/C 2; Load=L/2 with antisymmetric bound. cond.
Sol'n whole L.H. = 1.0 (S/C 1) + 1.0 (S/C 2)
Sol'n whole R.H. = 1.0 (S/C 1) - 1.0 (S/C 2)
In solving just one segment with 2 different boundary conditions, the same half load is used in both cases.

The representation of the whole structure comes about during the combining of the ingredients. How they should be combined depends upon which segment was chosen as the solution segment, which segment contained the nonsymmetric load, and what part of the whole structure is being synthesized. This will be illustrated for two variations:

SOL'N SEGMENT = LH, LOAD SEGMENT = LH
- LH Whole for defl. & slope = Sym + Antisym
- RH Whole for defl. = Sym + Antisym
- RH Whole for slope = (-1){Sym - Antisym}

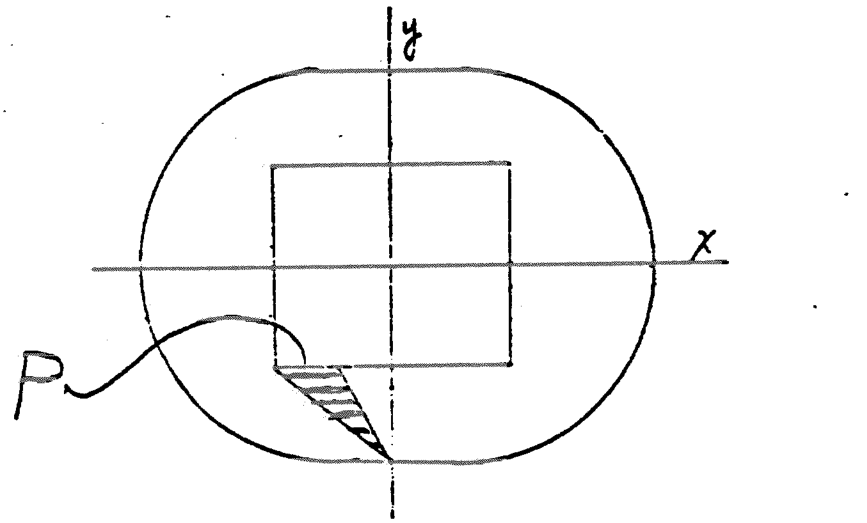
SOL'N SEGMENT = RH, LOAD SEGMENT = LH
- LH Whole for defl. = Sym + Antisym
- LH Whole for slope = (-1){Sym + Antisym}
- RH Whole for defl. & slope = Sym - Antisym

New wrinkles get introduced when there is more than one p.o.s. When there was one p.o.s. there were 2 ingredient subcases. For two p.o.s.'s the solution segment is a quadrant, and there will be 4 ingredient subcases to solve. For three p.o.s.'s the solution segment is an octant, and there will be 8 ingredient subcases to solve.

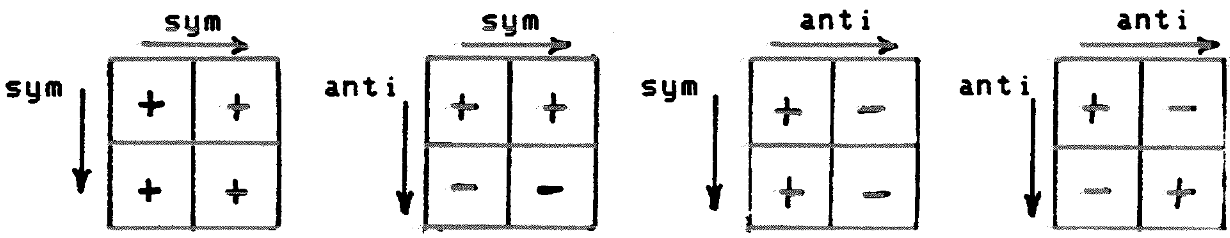
Expanding on quarter symmetry there are two possible boundary conditions over the section of the structure at each of two planes of symmetry. Four permutations of combined boundary conditions are possible:

	XZ PLANE OF SYMMETRY	YZ PLANE OF SYMMETRY	ABBREV
1.	Symmetric	Symmetric	SX, SY
2.	Anti-symmetric	Symmetric	AX, SY
3.	Symmetric	Anti-symmetric	SX, AY
4.	Anti-symmetric	Anti-Symmetric	AX, AY

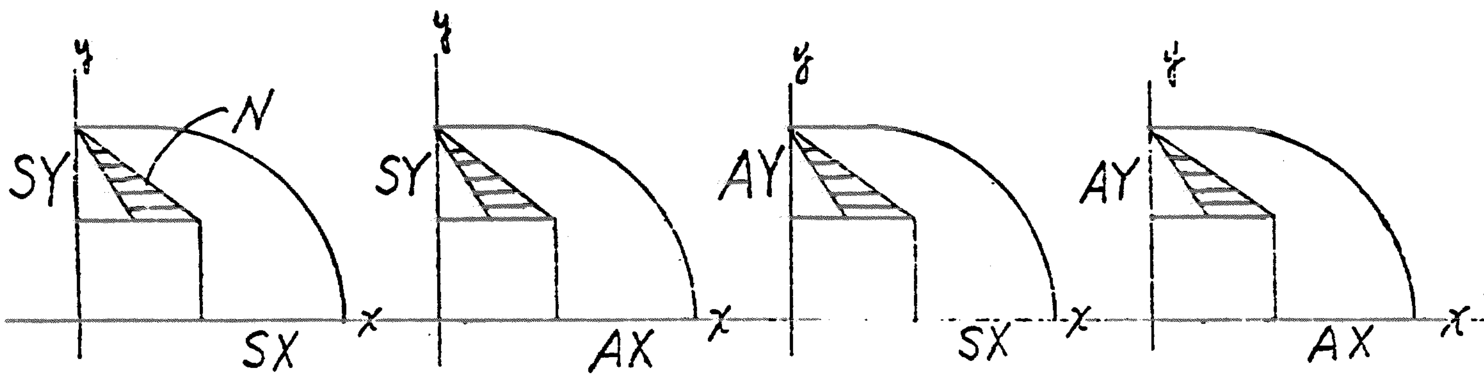
An example is taken with a non-symmetric load in the 3rd quadrant. The load, P, on the whole structure is shown with a shading



The modes associated with the establishment of boundary conditions for 2 planes of symmetry can be illustrated simply as



Let quadrant one be the model-quadrant. Four ingredient cases are constructed from the model quadrant with the 4 combinations of boundary conditions and each is given the same nominal load N.



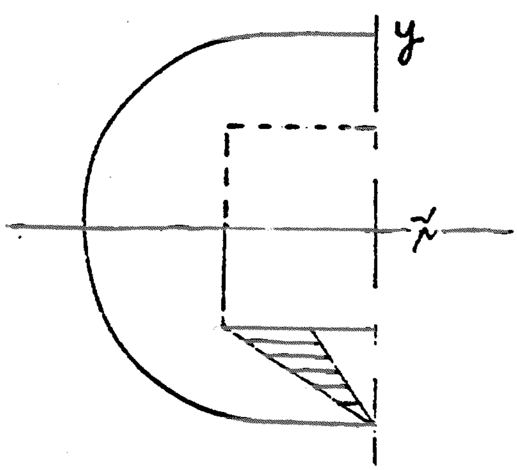
Since I found no guidelines in the literature I had to arrive at my own rules for combining ingredient subcases to represent any quadrant of the structure. There will always be one loaded quadrant and 3 non loaded quadrants. If a given non-symmetric load spans more than one quadrant, take the load as it appears in one quadrant at a time and do a 4 segment decomposition on each and solve the eight subcases then combine the 8 parts together in SUBCOM. The logic used in arriving at the signs to be applied to the coefficients of ingredient subcases for representing the responses in the 4 different quadrants will be taken up as one topic. The logic used in arriving at the magnitudes of of these coefficients will be taken up as another topic. The logic will be developed with respect to a particular example of a skewed load acting in the third quadrant of the structure. The representative structure will be modeled in the first quadrant. I asked myself "What are the signs to use for the ingredients to represent the nonloaded quadrants?" I don't remember the evolution of my thinking, but after I hit on the correct rule, I remember feeling ashamed of myself for being temporarily content with an inexact method earlier on. The nice thing is that the correct rule is a simple rule. The bothersome part in digging out this rule was for the quadrant which was located diagonally across from the loaded quadrant. This nagging problem disciplined my thinking until I finally clicked.

The RULE OF SIGNS is:
There is no change of sign when the response from a given quadrant is reflected about a symmetric boundary to yield the response in an adjacent quadrant. The sign is changed when the response from a given quadrant is reflected about an antisymmetric boundary to yield the response in an adjacent quadrant.

Loads from the 4 component cases must add up to 4N when representing the loaded quadrant which in this case is quadrant 3. Loads from these same 4 component cases must add up to zero in the other 3 quadrants (1, 2, and 4). Therefore the sign of the scale factors must be positive when combining Subcases to represent quadrant 3;

QUAD 3
+SX, SY
+AX, SY
+SX, AY
+AX, AY

The reflections for quadrant 2 with respect to the signed array from quadrant 3 are about the X axis.

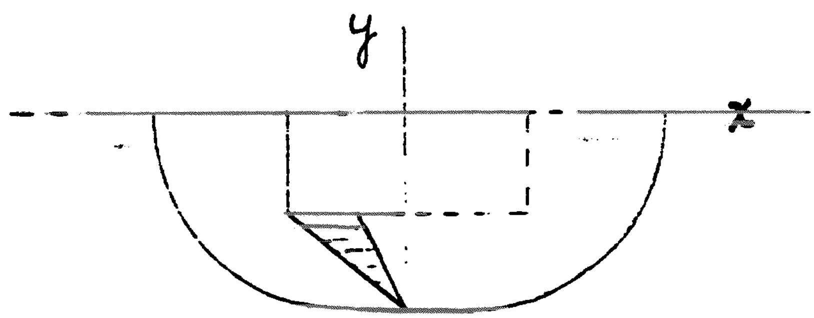


The first case SX,SY keeps a positive sign, because it is a symmetric reflection. The second case AX,SY changes sign and becomes negative, because the reflection with respect to the X axis is antisymetric. Similarly, the third case SX,AY has a symmetric boundary with respect to X and an antisymmetric boundary with respect to Y. But since the reflection is with respect to X in going from quad 3 to quad 2, there is no change of sign. This implies that the antisymetric conditions along Y remain the same in quadrant 2 as they did in quadrant 3 when it is associated with a symmetric reflection in X. Applying this logic to case four gives rise to a change in sign for the AX,AY response in quadrant 2. The coefficient signs for qaudrant 2 are tabulated as

QUAD 2
+SX,SY
-AX,SY
+SX,AY
-AX,AY

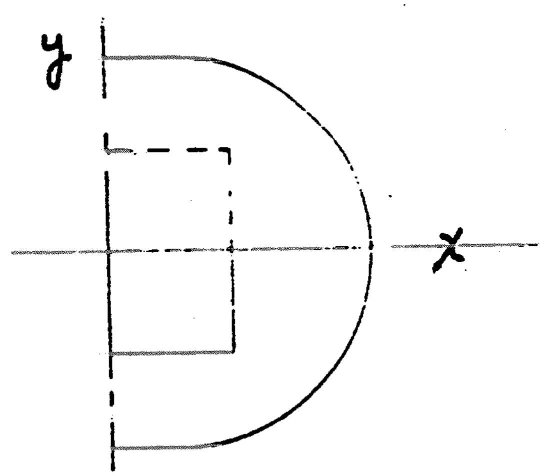
The reflections from quadrant 3 to quadrant 4 are about the Y axis so that anti-symmetric reflections with respect to Y change sign accordingly. The tabulation for quadrant 4 is

QUAD 4
+SX,SY
+AX,SY
-SX,AY
-AX,AY



The case for the response in quadrant 1 can be stated in terms of reflections from quadrant 4 or from quadrant 2. The same results are obtained in either case. Referring to the signed tabulation for quadrant 4, the results for quadrant 1 become, as a result of reflecting the signed quantities of quadrant 4 about the X axis

QUAD 1
+SX,SY
-AX,SY
-SX,AY
+AX,AY



The magnitude of the coefficient, C, depends on the type of symmetry (e.g. half or quarter or octant); the magnitude of the nominal load, N, used in the ingredient subcases; and the magnitude of the ultimate loading, U, as it appears in the whole structure. The magnitude of the coefficient is the same for all components in all four quadrants. The equation to use to have all four components sum to the ultimate in the loaded quadrant for quarter symmetry is:

$$CN(SX,SY) + CN(AX,SY) + CN(SX,AY) + CN(AX,AY) = U$$

$$4CN = U$$

$$C = U/(4N)$$

For the half symmetry case with respect to X, the formula to have both components sum to the ultimate for the loaded half is:

$$SN(SX) + CN(AX) = U$$

$$2CN = U$$

$$C = U/(2N)$$

The extension to octal symmetry becomes $C = U/8N$.

If r stands for the number of reflections then the general formula for RULE OF MAGNITUDE of the coefficient is:

$$C = U/[2^{r}N],$$

where r is 1 for the one plane of half symmetry, r is 2 for the two planes of quarter symmetry, and r is 3 for the three planes of octal symmetry.

The observations made so far with one and two dimensional elements can easily be extended to three dimensional elements. When modeling with polyhedra there are only 3 translational d.o.f. per grid point. The question arises as to whether the general rules which were developed for grid points having rotational as well as translational d.o.f.'s will apply for polyhedral models. Since pairs of contrained grid points on a solid boundary will produce couples, the zero slope effect will be achieved, so it is possible to satisfy the boundary requirements with only sets of translational cponstraints at the p.o.s. Look at the table of Boundary Conditions on page 6. For symmetry the only translational zero constraint is on u. But the condition that Ry and Rz be zero must be satisfied in a macro sense if not at the grid point level. Polyhedral elements will have at least a pair of points in each of 3 coordinates, so that a pair of constraints on u when separated by a span in z will produce a restraining couple about Y which will have the effect of constraining Ry to zero. Similarly, a pair of constraints on u when separated by a span in y will produce a restraining couple about Z which will have the effect of constraining Rz to zero. Apply this same reasoning to anti-symmetry and you will be reassured that the displacement constraints on v & w will contain Rx at zero.

The extension of the rule of signs on page 11 to octal symmetry is accomplished by changing the word "quadrant" to "segment". There are two possible boundary conditions over the section of the structure at each of the three planes of symmetry. Eight permutatins of these boundary conditions is possible. Consequently, for the loaded octal segment the signs to combine the eight ingredient subcases are all positive.

Sx, Sy, Sz
Sx, Sy, Az
Sx, Ay, Sz
Sx, Ay, Az
Ax, Sy, Sz
Ax, Sy, Az
Ax, Ay, Sz
Ax, Ay, Az

Changing signs when reflecting across an antisymmetric boundary successively until the octal segment diagonally opposite to the loaded segment is reached, the array of signs become

+Sx, Sy, Sz
-Sx, Sy, Az
-Sx, Ay, Sz
+Sx, Ay, Az
-Ax, Sy, Sz
+Ax, Sy, Az
+Ax, Ay, Sz
-Ax, Ay, Az.

NASTRAN PROCEDURE

Before proceeding to an application we should determine what the implication for all these observations are for setting up a problem within NASTRAN.

The analyst must model a segment with a fraction of the nonsymmetric load in place. If the loaded segment is chosen as the solution segment, the load is oriented as it is specified. If the solution segment is other than the loaded segment the load must be in a properly reflected orientation.

If n is allowed to represent the number of p.o.s.'s, set up 2^n sets of Single Point Constraints (SPC's) for all permutations of symmetric and anti-symmetric boundary conditions at the planes of symmetry.

Set up one load set.

Set up 2^n subcases each with the same load but each with a different one of the SPC sets.

Request OLOAD for checking on the correct assembly of load.

Request SPCFORCES for checking on equilibrium for checking on equilibrium and for the magnitudes of loading across the planes of symmetry.

Arrange the subcases in a logical sequence that will be manageable for the assigning of coefficients when the results are to be combined.

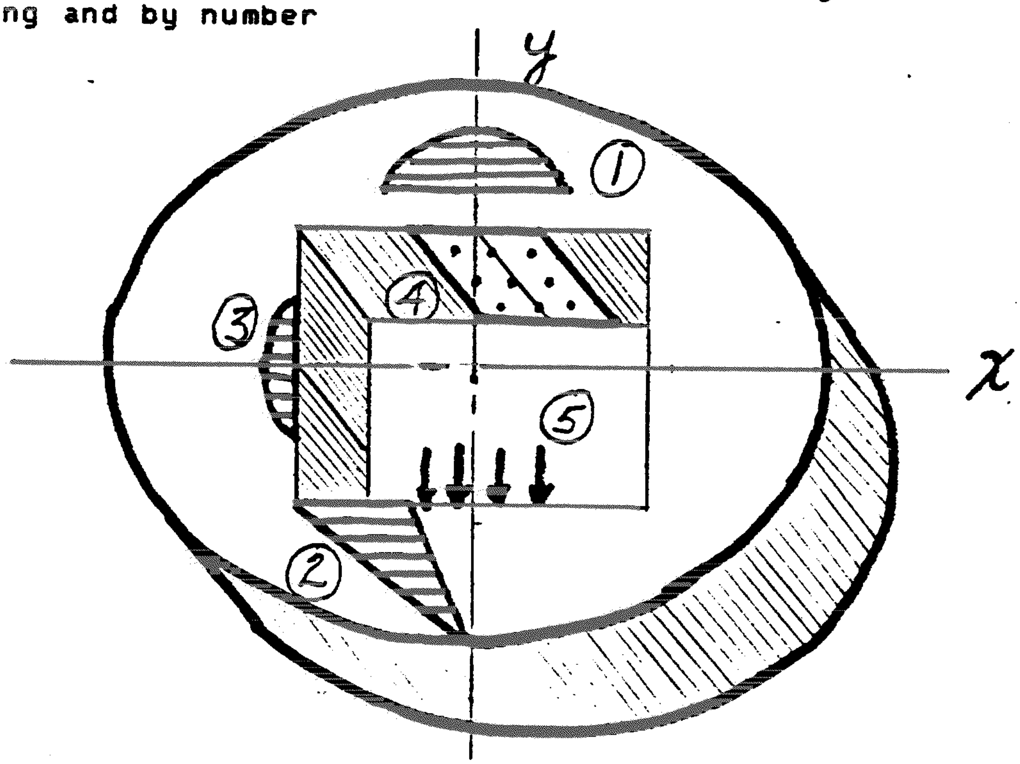
Refer to the RULE OF MAGNITUDE on page 14 for computing the combining coefficients. Refer to the RULE OF SIGNS on page 11 for setting the signs of combining coefficients.

Set up the output sets for the results. If output from individual as well as combined subcases is desired, the Case Control option of SUBCOM should be used. If output from only combined cases is desired, the Case Control option of SYMCOM can be used. A separate combining case is set up for each segment from which results are desired.

The values of scaling coefficients are transcribed to NASTRAN through the Case Control entry of SUBSEQ for SUBCOM cases and SYMSEQ for SYMCOM cases. NASTRAN reads the string of coefficients from left to right and assigns them as amplifiers to the results of preceeding subcases in the order from top to bottom as they appear in the Case Control Packet. This can take on such an abstract appearance that care is needed to ensure that they are in correct sequence. The burden is entirely on the analyst to exercise tidy bookkeeping to ensure proper correspondence.

APPLICATION

These techniques were applied to a thick structure having quarter symmetry which was modeled with polyhedra. Five distinct loads were applied to the structure with varying degrees of symmetry, such that 12 subcases were needed to represent all of the ingredient conditions. A sketch of the loadings is shown by shading and by number



Their symmetry characteristics were as tabulated below:

LOAD NO	RELATION TO P.O.S. XZ	YZ	NUMBER OF INGREDIENT SUBCASES
1	Unsym	Sym	2
2	Unsym	Unsym	4
3	Sym	Unsym	2
4	Unsym	Sym	2
5	Unsym	Sym	2

A quadrant was modeled with a fine mesh of 5 courses of solids through the thickness to obtain detailed stress distributions for studying stress concentrations and fatigue failure. Loads 1 and 4 were combined for one result and loads 2, 3, and 5 were combined for another result. The analyses were successful in that the patterns and levels of stresses were quite reasonable.

CONCLUSION

What does all this special business buy you? A way to quantify the merits is to set up measures such as results per unit cost or results per manhour or results per computer hour. Such management schemes were not used, but some areas are explored that can be useful to evaluate relative merits. Decomposition time varies as the square or the cube of the matrix order N, depending on the density and/or band and/or wave front. A comparison follows.

ESTIMATES OF DECOMPOSITION TIME							
TYPE SYM	LEAST PER INGREDIENT	# ING	NET	MOST PER INGREDIENT	# ING	NET	SAVING
FULL	NXN	1	NXN	NXNXN	1	NXNXN	0
HALF	NXN/4	2	NXN/2	NXNXN/8	2	NXNXN/4	2:1<S<4:1
QTR	NXN/16	4	NXN/4	NXNXN/64	4	NXNXN/16	4:1<S<16:1
OCT	NXN/64	8	NXN/8	NXNXN/512	8	NXNXN/64	8:1<S<64:1

If on the other hand one wanted as detailed a model that could be relied upon to be free of ill-conditioning, then the order of the segment model would be approximately 8,000 d.o.f. Using symmetry would give results comparable to a full scale model of order 16,000 for half symmetry; of order 32,000 for quarter symmetry; and of order 64,000 for octal symmetry. This indicates that as one goes to higher order symmetry in an analysis, one gets more results and uses less time to do the job by taking advantage of symmetry.

Another factor to look at is the analyst's preparation time. It takes less time to set up a smaller solution segment. It takes more time to set up more boundary conditions. It takes more time to set up more subcases and combining cases with coefficients. Once the model is complete, however, the entire job can be run as a single submittal so the handling is no more demanding. The evaluation of the results of the job becomes easier because the parceling of results from the combining cases makes the job more tractable.

A disadvantage to call to your attention is in getting a report on the load as it is associated in a combining subcase. OLOAD does not report on the equivalent combined load from a combining subcase. The user must engage in one of two routes which are not completely free of human error in order to get a report on the equivalent combined load. He can set up a dummy subcase and do the combining of the load with the bulk data LOAD card and by assigning amplifying coefficients according to those used in the combining sequence statement and call for OLOAD in this dummy subcase.
Alternatively, he can take the output of the PG vector from successive loops through module SSG1, and write a DMAP packet to scale the components and sum them to produce a combined load to check.

It appears that the advantages outweigh the disadvantages so that I would recommend that if you find that this paper has reduced the running of a symmetry problem to a useful routine, you might employ it more and enjoy some of its benefits.
One can be assured that his solution is not compromised for having used symmetry. All parts of the solution are contained in the result after the combining has been accomplished.

T. G. Butler March 5, 1982.