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1.0 INTRODUCTION

This paper describes a practical application of the use of multilevel/external superelements using MSC/NASTRAN through a typical example and Fairchild's related experience. The objective is to demonstrate the effectiveness of superelement analysis with regard to time and cost savings through an analysis by parts approach. This paper does not address the theoretical development and/or use of superelements, that information is plentiful and readily available. Several sources of superelement theory have been listed in section 9.0 at the end of this paper for those who are interested.

1.1 THE EXAMPLE PROBLEM

The finite element models shown in figures 1 through 4 in section 8.0 represent the assembly of structures to be analyzed. This assembly can be broken into five separate parts: the propulsion system structure, figure 2, the Space Transportation System (STS) interface structure, figure 3 and the payload which is further subdivided into three identical structures, figure 4. Table 1 assigns a superelement designation to each of these five structures and lists some parameters which describe the relative size of each.

The objective of the analysis is multifold. First a static analysis must be performed which assumes the assembly supported at the STS interface points. The resulting loads and stresses will be used to establish preliminary sizing of the structures. A dynamic analysis including sine and random vibration must then be performed to establish necessary stiffness and mass distribution requirements as well as to provide refinement to the preliminary member sizing.

At this point a component modes model must be formulated which represents the entire assembly. This model will then be used to perform a dynamic coupled loads analysis. The dynamic model responses will then be used for data recovery operations to establish final member sizing.

An additional analysis requirement is to provide for updating the analytical model with test data. This could be accomplished by two methods. One is to make adjustments to the structural properties of the analytical model until analysis and test data agree. This could be a time consuming and expensive process when one considers that stiffness, mass and damping properties must all be coordinated together. Another approach would be to take actual frequency, mode shape, and damping data from the test, perform a response analysis on it and evaluate the difference between analytical and test response data.

1.2 AVAILABLE RESOURSES

The aim in software selection was to make maximum use of available software. In finding a solution approach, Fairchild's choice for an analysis program was MSC/NASTRAN, version 61A. NASTRAN provides general purpose static and dynamic analysis capability and specifically can perform a random vibration response analysis. In addition, it has multilevel superelement capability for most solution methods, is well documented, MSC provides excellent support, and the program is readily available for a variety of hardware.

Pre- and post-processing was achieved through the use of PDA/PATRAN-G. This is an advanced high-speed color graphics pre- and post-processor for NASTRAN. In the pre-processor mode, PATRAN can quickly and accurately define simple or complex geometrical regions in space. These regions are then filled with appropriate finite elements to produce a discrete continuum representative of the physical structure. This mathematical model is then translated into the NASTRAN data format and the analysis is performed. Once the analysis has been completed the output can be transformed through a results translator for interactive post-processing. Deformed geometry, stress, strain, failure criteria and other parameters can then be graphically displayed. Of particular importance is PATRAN's color capability. Parameters such as element type, material type, geometric location, stress and displacement contours, strain energy, displacements, etc. can be color coded and displayed in various configurations. This aids greatly in model debugging and interpretation of results. PDA/PATRAN-G was used to produce all plots of structures in this paper.

The computer hardware used to run PATRAN and NASTRAN was a DEC VAX 11/780 engineering dedicated system. Fairchild's configuration had 4 megabytes of main memory, 2 RP06 disk drives and 2 RM05 disk drives for a total online storage capacity of 1.7 million blocks (1). Figure 5 shows the computer system block diagram in more detail. This data, as well as some run statistic data given in section 7.0, is included to demonstrate the large computing capacity available from a relatively inexpensive software/hardware system and how the system provides for maximum usage of available resources.

The analytical system approach is diagrammed in figure 6. This depicts the general analysis flow from data preparation to final reports.

(1) 1 block = 512 16 bit bytes

2.0 SUPERELEMENT APPLICATION

Multilevel superelements in MSC/NASTRAN Version 61A are mathematically equivalent to substructures which were available in previous NASTRAN releases. The primary differences involve the new cost saving technique of multilevel reduction, several user enhancements which make the procedure much more convenient to use, and the expansion of the technique to a much broader class of problems. The following is a brief discussion on some of the advantages and restrictions of the current superelement procedures and in addition, some suggestions for modified usage which alleviate most of the major restrictions.

2.1 ADVANTAGES

Multilevel reduction provides for the successive reduction and elimination of portions of the structure. In particular, a superelement does not have to represent any physical or geometric boundary. It can represent a portion of the structure to which minimal changes have to be made, or indeed a previous structure which has been tried and tested to which a new appendage is being attached. The possibilities and combinations of old and new structures are endless but it all leads to time and cost savings for the analyst.

The superelement approach itself provides natural boundaries between different structures and therefore, between different people or groups developing those structures. There is no longer a need for one person or group to develop large unwieldy models which are expensive to run. Superelements allow analysis by parts and in most cases lead to substantial cost savings.

From an operational standpoint the heart of the superelement analysis technique is the random access data base. All data blocks necessary to perform the requested analysis are stored on this data base as they are computed, and then purged from memory, only to be recalled at a later time when they are again necessary. This file can, and in fact should be used instead of the NPTP/OPTP for restarting, since the superelement sequences make no reference to NPTP/OPTP except for storage of the bulk data cards.

The superelement solution sequences are broken into three distinct phases: data generation and reduction, residual structure solution, and data recovery. While it is not necessary to perform three separate runs (all three phases can be completed in one run) it is often convenient to stop, examine intermediate results, and restart, rather than complete an erroneous analysis.

Lastly, superelement solution sequences are now available for a wide range of problems and solution techniques (see Table 2). These solution sequences are also available for non-superelement runs. This means you can run a standard NASTRAN deck through the appropriate superelement sequence to take advantage of the random access data base and data base manager techniques.

2.2 RESTRICTIONS

It would seem from the above discussion that superelements will solve a world of problems in a timely and cost effective manner. However, several restrictions exist with the current solution techniques. Several of the more important ones will now be discussed.

First and most important, duplicate grid or element numbers can not exist within or between superelements. This means that close coordination between separate groups doing different portions of the structure must take place. Each section of the structure must be given a block of grid and element numbers in order to insure that duplicate id's will not exist.

A further restriction is the requirement for the entire data deck to exist for the phase I definition. The current proceedure implies that the entire model must be present, broken into individual superelements, reduced, analyzed and data recovered all in one run. While it is true that different parts of the structure can be assembled and checked out at different times, the standard final phase I run requires all data to be present and all matrix operations to be repeated for final assembly.

The data base also presents some problems. Since it does not keep a copy of the bulk data deck the entire deck must be resubmitted when making a change to geometry, element, or property data. In addition, since the data base contains all data blocks required to perform an analysis in random access form, it tends to be quite large. Experience has shown for a 1000 degree of freedom problem approximately 10000 blocks are required for the data base. These files can become quite expensive to save for extended periods of time and in some cases, depending on the amount of available on-line storage capacity, can even cause computer system degradation or failure.

Secondary superelements can often be used to avoid some of the restrictions given above. However, even secondary superelements when used as documented can cause their own problems. For example, data recovery for secondary superelements is limited since all the data blocks necessary for data recovery are not created by the CSUPER card.

2.3 MODIFIED USAGE

Most of the aforementioned restrictions can be reduced and/or eliminated through proper planning and the use of some of the now standard superelement options. Specifically, the use of secondary/external superelements and the use of multiple data base files eliminates the need for the entire model to exist in a single phase I run, the need for unique grid point and element numbers, and the need for close coordination between individual superelements. Furthermore, the use of multiple data base files eliminates the need for a single large file and through proper use of the data base manager, case control and restart parameters, efficient restart and full data recovery operations can be accomplished.

A secondary superelement is one which is created from a primary and for which no specific bulk data exists. It can be created by use of a CSUPER bulk data card which can be used to apply appropriate translation and/or rotation, or it can be created from a copy of the primary data base. All portions of a structure that exhibit symmetry or similar construction are potentials for secondary superelements. It is recommended that mirror image and rotation of similar portions make use of the CSUPER card to create a secondary. Proper use of the CSUPER card is explained in the existing NASTRAN documentation. Portions of a structure which exhibit only translation of a similar portion are best created through data base equivalencing of the primary superelement.

Data base equivalencing is accomplished through the appropriate data base manager calls. Data base manager calls are placed in the Executive Control Deck as DMAP alters and have the following form:

```
     DBMGR   // n / pi / NAMEi  $

     where n - defines operation
          pi - parameters which control the operation
          NAMEi - file or data block names which are to be
                  operated on
```

The operations (n) available are listed in volume II of the User's Manual. The procedure would be to first make a duplicate copy of the primary data base using standard computer system software. Then an equivalence (n=7) of all existihg data blocks would be made equivalencing the primary SEid to the secondary SEid. This would be followed by a purge (n=5) of all primary data blocks. The resulting data base will then exist as if it were created by a duplicate primary bulk data deck with a different SEid. All this will be accomplished without a single matrix operation being performed.

An external superelement is one which exists in matrix form on a data base, ie., it has completed phase I processing. Fairchild has found that the neatest, cleanest, and most effective practical way of performing a superelement analysis is to maintain all large and/or identical superelements on individual data bases. This provides for efficient operation due to decreased data base size and facilitates proper data recovery for secondaries.

The reading of externals back into the analysis stream is always done through the CSUPER card where the SSID field contains the superelement identification number of the incoming superelement and the PSID field contains a zero. This causes a search of all files contained in the attached data base for a superelement number of SSID. Its external degrees of freedom are then attached to the grid points contained in the remaining fields of the CSUPER card, assuming six degrees of freedom at each grid point.

The creation and use of external superelements as outlined above implies the existence of multiple data bases. NASTRAN provides the capability to assemble a global data base as the union of individual data base files. In addition, individual data base files can be assigned to specific data base sets within the global data base. The data base sets can be used to define which files are read or write only, which files contain upstream data and which contain downstream data. The definition of data base files and sets is accomplished through keywords on the NASTRAN card. An example would be:

```
NASTRAN FILES = (DB02,DB03,DB04,DB05,DB07,DB08,DB09),
                DBSET 1 = (DB02, DB04), DBSET 2 = (DB03, DB05),
                DBSET 15 = (DB07, DB08, DB09)
```

The resulting data base is shown in block diagram form in figure 7.

3.0 STATIC ANALYSIS

Static analysis was performed using solution sequence 61. The planning, formulation of models, and analysis using superelements proceeds in exactly the same way as for conventional analysis. In fact, this analysis began with a conventional solution 24 model for the propulsion system structure. Later it was decided to take advantage of the superelement capability. By the inclusion of an SESET card the conventional model became a superelement model. Of particular importance to not only static, but all superelement analysis, is the Superelement Map (SEMAP). The SEMAP should reflect the analyst's superelement tree, listing all interior grid points and elements, as well as all boundary points. The SEMAP should be read and understood for each superelement run and its importance can not be overstressed.

3.1 PLANNING

In considering the assembly of structures to be analyzed it was obvious that the physical boundaries provided a natural application for superelement analysis. In addition, since the structural requirements for each assembly were in various stages of development, it made sense to plan the analyses based on the anticipated definition of requirements. The superelement tree shown in figure 8 was then established as the sequence. This allowed separate analysis of each major component, provided an independence between components, and allowed starting with the components which had the most definition. Since the payload models were known to be identical it was decided to use identical superelements, through data base duplication which would also facilitate data recovery for each. This lead directly to the use of multiple data bases.

3.2 PROCESSING

The first task was to establish and assemble a static analysis model for the propulsion system, SE10. This structure was assembled as a superelement and a static analysis was performed assuming a fixed boundary and rigid mass loading for the remaining structures. This enabled initial member sizing and a rough approximation to the fundamental frequencies based on static deflections. This also provided a fully verified model and complete data base to be used for further downstream processing. The STS interface structure was handled in a similar manner.

The payload models were identical and merely translated in space. SE31 was developed as a primary and run through phase I processing for model verification and establishment of the data base. SE32 and SE33 were created as duplicates of SE31 through a file copy and appropriate data base manipulation. Each payload model was reduced to 8 boundary points assembled to form a stack. A second level reduction was performed, condensing out the middle payload to form a total stack with 8 boundary points.

At this point all phase I processing was complete. There were a total of six data bases created; one for each of the five superelements, plus one for the second level reduction and creation of SE30. However, only four data bases were ever required at one time, ie., SE30, SE31, SE32, and SE33.

Phase II, residual structure processing, was performed next. Again, four data bases were required: SE10, SE20, SE30 and the residual structure. The residual structure itself consisted of 34 grid points, boundary elements and CONM2's for the boundary grid points. SPC's were applied at the STS interface such as to provide seven degrees of constraint. Unit gravity loading was applied to each of the superelements during phase I processing. This estalblished equivalent boundary load vectors for each superelement subcase. Unit gravity loading was again applied to the residual structure so the effect of added residual structure elements and masses would be included in the final residual structure load vector.

Data recovery was performed systematically, one superelement at a time. This required attaching two data bases at one time, one containing the downstream solution data and one for the correct superelement being processed for data recovery. This operation proceeded up the superelement tree until all data recovery was complete.

3.3 SUPERELEMENT EXPERIENCE

a) CSUPER - (External Superelement Retrieval) - Reduced superelement data is stored on the data base in matrix form for the boundary grid points. Each boundary grid point is assumed to have six degrees of freedom. Deleted degrees of freedom contain null columns and rows. The order of the stored terms is according to sequential grid point number, lowest to highest. (Note that boundary points are not resequenced.) If there is any question about the stored order the SEMAP should be reviewed. For the retrieval of external superelement matrices, the order of grid points on the CSUPER card does not matter. Data is always retrieved from lowest to highest grid point number. Therefore, the CSUPER card provides no control over the attachment of external superelements. This means the relative geometric position and sequential order must be consistent between interfaces. (Use care as many internal checks are not performed for external superelements.)

b) External Superelement Connection - Normal attachment of superelements occurs between common grid points. Rigid elements and/or MPC equations can be used between (but not across) superelement boundaries. If attachment between common location grid points is desires, the RELEASE card can be used to provide pinned connections.

c) Mass Matrix for Statics - The mass matrix is not carried downstream, nor applied to the superelement boundaries for statics. For gravity loading an equivalent load vector is computed and translated to the boundary. This provides an exact load transformation and works well. However, results from the grid point weight generator are erroneous and load factors cannot be changed downstream.

d) Boundary Elements - Elements (including CONM's) appearing on a superelement boundary are not passed downstream. Boundary elements are those connected between downstream structure grid points. If at least one grid point attached to the element is an interior point, that element will be considered in the corresponding superelement. Therefore, do not place elements on the superelement boundary. If you must, 1. use the SEELT card to force processing with the current superelement, or 2. input these elements when processing the downstream structure.

e) Loadings - Loads which are anticipated to change should be placed on residual structure grid points. This allows the change to be assessed through relatively inexpensive processing of the residual structure. This policy should also be applied to grid point locations or elements which may change.

f) Executive Control - Cards of particular importance to superelements are as follows:

NASTRAN FILES = to define files to be used in the global data base

for this run.

NASTRAN DBSET i = to define subsets of the global data base

DBSET 1 = read/write upstream data

DBSET 2 = read/write downstream data

DBSET 14 = read protection

DBSET 15 = write protection

DIAGS 8 - Print matrix trailers
14 - Print DMAP Sequence
20 - Print DBMGR fetch/store messages
37 - Disable SE congruence test

ALTERS - Data base Create
Print Data base Dictionary (before & after)
Print boundary Load Vectors

g) Case Control - Table 3 has been reproduced from the NASTRAN documentation and presents a summary of case control cards for superelement analysis. Some additional explanation is given below.

SPC - if an SPC exists above the subcase level for one SE run, it must exist for all runs.

OUTPUT - requests serve no function during phase I processing

SUBCASE Definition

```
    Subcase i
    Load =
       .              r load cases for residual structure
       .
       .
    Subcase j
       .
    Subcase k
    Super p, 1
    Load =
       .              r subcases for SEp
       .
       .
    Subcase l                                  r subcases for
    Super p, r                                   each superele-
    Load =                                       ment in the
       .                                         current run
    Subcase m
    Super q, 1
    Load =
       .              r subcases for SEq
       .
       .
    Subcase n
    Super q, r
```

LOAD = , applies to internal SE

SUPER a, b, applies to SE boundary
a = SE number
b = load sequence number of residual structure

Total number of subcases =

(number of SE's + 1) * number of loadcases

h) Bulk Data - Only one bulk data card is necessary to create a superelement deck from a normal bulk data deck.

SESET - assigns grid points to a particular superelement

The remaining bulk data cards concerned with superelement statics, ie., RELEASE, CSUPER, and SEELT have already been discussed.

i) Update Runs - Update runs which modify the stiffness matrix require submission of the entire bulk data deck used to create the information contained on the data base. However, processing can be directed to specific superelements through the use of the SEMA and SEMG case control cards. Regeneration of load vectors or a change in load factors can be accomplished with minimal bulk data input and the proper use of the associated parameters. Updates to superelement runs are not like OPTP/NPTP restarts. The data base does not contain a copy of the bulk data and therefore updates must be controlled through proper use of case control cards (SEMG, SEMA, SELG, SELA) and parameters. For example, to change loading in the residual structure the following cards would be submitted:

```
NASTRAN FILES = (DBOi,DBOj,etc.),
        DBSETi = (DBOm,DBOn,etc.)
    .
    .
    .
CEND
TITLE =
    .
    .
    .
SELG =
SELA =
    .
    .
    .
SUBCASE
    .
LOAD =
    .
    .
BEGIN BULK
PARAM, NODATA, -1
PARAM, DLOAD, -1
PARAM, RESDUAL, 1
LOAD =
GRAV =
ENDDATA
```

DLOAD = -1; deletes old load cases

NODATA = -1; all data except loading data, on the data base is correct

RESDUAL = 1; jump directly to residual structure processing

j) Multiple Data Bases - Figure 9 shows a block diagram representation of the data base scheme used to perform the static analysis. It also gives the appropriate file assignments and use of the NASTRAN card. Figure 10 shows the database scheme and file assignments used for data recovery.

The most important discovery made concerning multiple data base use was the potential misuse of DBSET1 and DBSET2. DBSET1 contains upstream data, namely the blocks required to perform data recovery for the superelements it contains. DBSET2 contains the downstream data on the data blocks for the boundary grid points. In performing a preliminary analysis, data recovery was performed by assigning DBSET1 to the phase I data base for the particular superelement and DBSET2 was assigned to the residual structure data base. The data recovery was successful. The loading on the residual structure was then changed and a second data recovery was attempted, again using DBSET1 and DBSET2. In this case the run was sucessful but the results were wrong. It turned out that during the first data recovery the solution vector (ie., boundary displacements) for the particular superelement was written to DBSET1. During the second data recovery the program checked for the existance of a solution vector on DBSET1 and found the previous was stored there, writing the new solution vector to DBSET2. The results therefore used the boundary displacements of the old solution and the internal displacements of the new load case.

It is our recommendations therefore, to use DBSET15 (read only data base) and DBSET14 (write only data base) to control reading and writing of data bases.

4.0 SUPERELEMENTS IN DYNAMIC ANALYSIS

The analysis techniques for the use of superelements in dynamic analysis are very similar to those used for static analysis. These procedures use the same concept of breaking a large structure into sub-components to allow better use of resources. The main operations are the same for manipulating the database files as was previously described. One major difference between static analysis and dynamic analysis is the normal technique of reducing the analysis set (ASET) through Guyan reduction which eliminates analyst chosen degrees of freedom from the solution. This technique is an approximation, and accordingly can add various degrees of error to the analysis. An alternate technique called Generalized Dynamic Reduction (GDR) is now available which removes the burden of choosing important degrees of freedom from the user. An extension of Generalized Dynamic Reduction called Component Modal Synthesis (CMS) is also available to minimize the size of matrices required to pass information on the dynamic properties of a superelement downstream. An extension of CMS in MSC/NATRAN is now available that allows the transformation of test data into matrices to represent a structure by Component Modal Synthesis.

4.1 FORMS OF DYNAMIC REDUCTION

In order to represent a structure for a dynamic analysis it is advantageous to have a model smaller than that required for a static analysis, mainly to reduce processing time and cost. In many cases it is convenient to use the same model for static and dynamic analysis. The typical approach is to use the technique known as Guyan reduction. In an MSC/NASTRAN analysis this requires the analyst to manually choose points in the model, and their degrees of freedom, that are felt will significantly contribute to the dynamic response. Criteria for this choice of degrees of freedom is typically based on regions of large relative mass or expected high deflection. These points are listed along with their appropriate degrees of freedom on an ASET card. The resulting solution, after partitioning, is an ASET x ASET matrix manipulation. Typically the choice of these ASET points is a tedious task. The process of Guyan reduction transforms the stiffness terms with no loss in accuracy, however, due to the reduction of the mass matrix the solution is an approximation. Often, more points must be included due to uncertanty of effects on overall response of the model. If too few points are chosen, a loss in accuracy is observed.

In the case of superelements, it is also required that points be chosen in a similar manner, when using Guyan reduction. This means that the boundary points plus interior points of superelements must be passed down to the next level during processing. The resulting residual structure with Guyan reduction in superelements is the same size as that for a non-superelement solution of the same model. At this point it is seen that using Guyan reduction with superelements offers minimal benefit to conventional analysis techniques.

The technique of Generalized Dynamic Reduction offers a solution to some of the previously mentioned problems. This method generates an estimate of mode shapes and frequencies for a model and uses them to produce generalized coordinates to represent the dynamic characteristics of the structure. These generalized coordinates are then used to generate the solution to the eigenvalue problem. The user supplies a cutoff frequency allowing the GDR calculations to stop after supplying enough generalized coordinates for the frequency range of interest. This performs two tasks; it relieves the burden of choosing ASET points from the user, while minimizing the number of degrees of freedom to appropriately represent the structure. This technique requires minimal alteration to a typical NASTRAN input deck, and can be used in standard and superelement solutions sequences. The calculations of the approximate eigenvectors can require a sizable portion of the total cpu time for the soution. In the case of some medium sized models it may take more time to perform an analysis with GDR than one with typical ASET methods, however, the analyst need not spend preparation time determining aset points using GDR. In most cases the technique of generalized dynamic reduction offers better accuracy at a reasonable cost.

In the case of superelements, approximate generalized coordinates are used to solve the individual component eigenvector solution after which the actual generalized coordinates are coupled to the boundary points as a total representaion of the dynamic properties of the superelement. This total procedure of producing corrected generalized coordinates and coupling with the boundary for individual superelements to be passed to another structure is called Component Modal Synthesis. This procedure reduces the size of the matrices passed down in a superelement analysis when compared to Guyan reduction methods. This typically produces a final residual structure with many less degrees of freedom for the final solution than similar ASET techniques. Through the use of multilevel reduction, even more savings in size can be realized.

4.2 INPUT DECK CHANGES FOR GENERALIZED DYNAMIC REDUCTION

The additions to the NASTRAN input deck include a DYNRED case control card which chooses a DYNRED bulk data card to control the reduction. The case control deck must have the DYNRED card placed in the subcase for the superelement to be reduced. In the case of the use of GDR in non-superelement analysis, the DYNRED case control card must appear above the subcase level.

The bulk data deck requires a DYNRED card as well as cards to allocate and partition the generalized coordinates produced by the process. The bulk data DYNRED card states the cutoff frequency for the modes of interest in the analysis. Additional parameters effect the accuracy of the approximate eigenvectors produced. Default values are recommended, reducing the parameters required to the highest frequency of interest. SPOINT cards must be added to the deck to generate scalar degrees of freedom to be used as generalized coordinates. SEQSET (QSET for the residual structure) cards must specify the degrees of freedom to be used as generalized coordinates. ASET cards must also be used in the residual structure to insure a non-null O-set. A method for this is to place the QSET degrees of freedom on the ASET card. The EIGR card used to specify the method of eigenvalue extraction should use the same frequency range as that specified on the DYNRED card. If the mass method of normalization is used the generalized mass produced and passed down in the superelement process will be unity for all modes. Accordingly, the generalized stiffness will be the eigenvalue or the radian frequency squared. The required number of SPOINT's is typically one and one half times the number of modes that will occur below the cutoff frequency. It is up to the user to provide sufficient SPOINT coordinates. If too few are provided a fatal message will result and the number of required generalized coordinate locations will be output. If too many SPOINT's are provided, unused points are constrained by the AUTOSPC feature when used.

The additional required input for Generalized Dynamic Reduction can be simplified to the following list:

a) Case Control

o DYNRED card with set id referencing each DYNRED bulk data card for each superelement to be reduced. Must be in the individual subcase for superelements, but above the subcase for non-superelement analysis.

b) Bulk Data

o DYNRED card with highest frequency (Hz) of interest. Default values can be used for other parameters with good results.

o SPOINT cards to generate enough degrees of freedom to fulfill the requirements for the frequency range. This value is typically 1.5 times the number of modes below the cutoff frequency. It is sufficient to provide a large number of spoints as unused degrees of freedom are automatically removed by AUTOSPC.

o SEQSET (QSET for residual) cards with listing of SPOINT id numbers to be used as generalized coordinates.

o ASET (residual structure only) cards with listing of SPOINT id numbers to be used as generalized coordinates. Same numbers as QSET card.

o EIGR card should have frequency range that agrees with the range on the DYNRED card. Choosing a number of requested modes will sometimes cause an error. By using mass normalization the generalized mass and generalized stiffness passed down to the residual structure will be unity and the eigenvalue for each mode respectively.

o PARAM AUTOSPC YES should be used to remove extra generalized degrees of freedom.

4.3 MATRICES PRODUCED BY COMPONENT MODAL SYNTHESIS

There are four matrices that are generated to represent the dynamic characteristics of the individual superelement. These are MAA, MLAA, KAA, and KLAA. The MAA matrix contains the boundary mass characteristics and the coupling terms between the boundary and the generalized coordinates. The MLAA matrix contains the generalized mass terms. If the mass method of normalization is used the terms in the matrix are unity along the diagonal for the columns of generalized coordinates. Both MAA and MLAA are the same size, boundary degrees of freedom plus generalized coordinates (used and unused) in sequential order. The KAA matrix contains the boundary stiffness. The KLAA matrix contains the generalized stiffness for the model, which are also the eigenvalues for each mode if mass normalization is used. Both KAA and KLAA are the same size as MAA and MLAA described earlier. The topology of these matrices is distinct in that the regions of the matrices for the generalized degrees of freedom are null in the MAA and KAA matrices, while the boundary degree of freedom locations are null in the MLAA and KLAA matrices. The total representation of the model is internally obtained by adding MAA and MLAA together to form the total mass matrix while adding KAA and KLAA together to form the total stiffness matrix. A diagram of the topology of these matrices is shown on figure 11.

When using the release feature for superelements an extra matrix, MAPS, is also required. This matrix removes selected degrees of freedom from the boundary at the time of assembly with a downstream superelement.

These matrices are normally produced by a superelement assembly and reduction phase but can be generated by an outside code or DMAP sequence. Examples of this are the procedures used in NASTRAN solutions 41, 42 and 43 to take test data and produce the required matrices and data blocks on a database to be assembled as external superelements.

4.4 MULTILEVEL DYNAMIC REDUCTION

In the technique of Component Modal Synthesis each superelement is processed through the Generalized Dynamic Reduction phase to produce approximate eigenvectors for particular boundary conditions. Procesing continues for the individual superelement and the actual eigenvalue solution is performed using the approximate eigenvectors. Produced from this solution are a set of corrected eigenvectors that are to be passed on to the next level of superelement processing. These eigenvectors are the component modes for the superelement. These corrected eigenvectors are used to generate coupling terms between the boundary degrees of freedom and the generalized coordinates. This means that a separate eigenvalue analysis is performed for each component based upon the conditions of the boundary. These boundary conditions are altered from the default clamped condition by using the SEBSET and SECSET cards during the reduction of the individual components. The next level of external superelement processing that combines with the previously CMS reduced superelement must use the CSUPER card to assemble the boundary properties as well as the upstream generalized coordinates. Under normal circumstances, this would mean a CSUPER card in the downstream superelement with GRID point id's for the placing of boundary mass and stiffness, as well as SPOINT id's for the generalized coordinates to be passed down. The total number of degrees of fredom on the CSUPER card must agree with the size of the matrices of the upstream superelement. These matrix sizes can be observed by looking at the matrix trailers printed by DIAG 8.

When using SPOINT's for generalized coordinates on the CSUPER card all degrees of freedom representing the dynamics of the upstream superelements are passed on to the residual structure. This is due to the fact that SPOINT's cannot be specified as being internal to an individual superelement, and are therefore part of the residual structure. This may not be advantageous in a multilevel strategy, where it may be possible to reduce the number of generalized coordinates passed to the residual structure. In superelement reduction, only those points interior to the superelement are reduced. Boundary points and exterior points are not considered in the reduction.

In a multilevel strategy, it may be beneficial to use regular GRID points to supply degrees of freedom for the generalized coordinates passed down from a previously reduced external superelement, by the use of a CSUPER card. These GRID points can be declared interior to the present level superelement and considered in the current reduction. By placing these GRID id's on the CSUPER card for an external superelement there is effectively a double reduction performed, where the current level superelement is considered along with upper level dynamic representations.

The final matrices used in the residual structure will consist of physical points manually placed in the residual structure as well as generalized coordinates passed down from upstream superelements.

4.5 COMPONENT MODAL SYNTHESIS

Generalized Dynamic Reduction can be used in both Direct and Modal formulations of NASTRAN dynamic analysis. Component Modal Synthesis is an extension of GDR when using modal solutions. CMS in MSC/NASTRAN is merely an extension of Generalized Dynamic Reduction performed by correcting the generalized coordinates to be passed downstream in the next level of processing. With additional inputs, CMS allows the boundary points of a superelement to be considered to be free or fixed during generalized coordinate generation. By default GDR/CMS considers all boundary points to be fixed. These boundary points are implicitly placed in the B-set (fixed during GDR/CMS). CMS input allows selected degrees of freedom to be explicitly placed in the B-set (fixed) or the C-set (free). Typically, points on the boundary of the assembled model that are considered to act freely with respect to the nearby structure should be placed in the CSET. Points that merely follow surrounding motions most probably should be placed in the BSET. By using the default BSET assuption considerable time is saved in the solution as the analysis procedure is simplified. The choice of these conditions is up to the analyst. For the majority of the time, the default values will generated good results.

4.6 FREQUENCY AND RANDOM RESPONSE ANALYSIS

All typical forms of dynamic response analysis can use the results from Generalized Dynamic Reduction and Component Modal Synthesis. In the case of Frequency and Random Response analysis only those cards required to generate loads, damping, and output frequencies need be in the case control and bulk data deck. By using parameters, the solution sequence skips directly to the point of generating and applying the frequency dependent loads and generating the response output. Data recovery similar to that described for static superelement analysis is performed. Again, when using external superelements, this procedure would commonly benefit from multiple database usage. In general the following steps would be taken in producing a dynamic modal response analysis using superelements:

1. Generate a component modal solution for each individual superelement, using a separate database for each.

2. Process combinations of superelements in a multilevel strategy to combine superelements and possibly perform multiple reductions where possible, again placing new information on new databases.

3. Process the residual structure generating the modal solution for the entire assembly producing another database.

4. Run the response problem for the residual structure placing the response information on another residual structure database.

5. Perform a data recovery on individual superelements using only the databases with downstream solution data or and the database for the individual.

One item that is sometimes overlooked in non-superelement random analysis is the requirement for proper subcase numbers to be referenced by the RANDPS card. In the case of superelements the subcase callout on the RANDPS card must reference the proper subcase for the appropriate superelement.

The savings in the use of component modal synthesis for frequency response analysis is that a greatly reduced model is solved as the residual structure without the penalty of error or uncertanty as that common when using Guyan Reduction. Data recovery for the upstream superelements is performed by following the superelement tree back up to the desired component. This recovery, as in static analysis, carries the interface reactions upstream to allow extraction of all common output quantities for each superelement.

5.0 SHUTTLE PAYLOAD EXAMPLE

In the case of the model that was used for this analysis, the structure was broken up in to the same sections for dynamic analysis as that for static analysis. Indeed, the same models were used. Two reasons were behind the developement and analysis of the dynamic models. First the dynamic characteristics of the assembly were required for design purposes in order to check the sizes of previously static defined structural members. Secondly, it was necessary to generate a model representation, in matrix form, to be passed on to perform a coupled loads analysis.

Due to the desire to use the same model for dynamics as was used in the static analysis, the method of Generalized Dynamic Reduction was chosen. As stated earlier, this method frees the analyst from the task of choosing an ASET for the individual models while minimizing the number of degrees of freedom required to adequately represent the model. Since modal methods of analysis were chosen the actual technique is called Component Modal Synthesis. Because of the ease in analysis the majority of the default values were used for the dynamic reduction.

Test cases were used to check the question of the use of explicit SEBSET and SECSET selection for the individual superelements. This investigation revealed little sensitivity to the difference in boundary conditions during component mode generation and assembly for the configuration in question. For this reason, and the simplification in the analysis, the default characteristic of implicit BSET for all boundary degrees of freedom was chosen.

5.1 PRODUCTION OF COMPONENT MODAL MODEL

Due to the necessity of producing a modal model for coupled loads analysis it was also required that consideration be made on how to generate the required matrices to be transfered, as it was desireable not to transfer the entire model. In order to generate the required information, the basic structure of the superelement tree was modified. The tree structure of the dynamic analysis is shown in figure 12. The tree shows the three payload models generated from one database, as all superelements for the payloads were identical images. These used the method of database copying, however, instead of the typical method of identical superelement generation by use of the CSUPER card. This was done to generate full and separate databases for each payload model to allow easier bookkeeping. The propulsion system model was also run as a separate model. These four models were all tip superelements in the dynamic tree. These all used Generalized Dynamic Redution and Component Modal Synthesis to reduce the matrices to the size of the individual boundaries with generalized coordinates to represent the individual superelements to a maximum cutoff frewquency of interest of 2000 Hz. These models all used default characteristics for the reduction, including the implicit inclusion of boundary points in the BSET. Spoint cards were used to define the degrees of freedom to be used as generalized coordinates. These four models were assembled into the STS interface structure. This interface structure was considered as an individual superelement with only five GRID points in the residual structure.

In order to produce the minimum size matrices to be transfered for the coupled loads analysis, a method of double reduction was employed. This method used GRID points as locations for placing the upstream generalized coordinates from the four models. These grid points were defined to be in the interior of the interface model. Because of this inclusion the dynamic characteristics of the upstream models were considered in the reduction. The resulting reduced model consisted of the five interface points to be tied to the STS, along with generalized coordinates coupled to the five points to represent the dynamic characteristics of the entire assembly of superelements. The parameter FIXEDP was used to output mode shapes for the natural frequencies of the reduced model. Figure 13 shows the assembly of these superelements and the use of multiple databases.

In order to prepare the information to represent the structure for coupled loads analysis four matrices MAA, MLAA, KAA, and KLAA were fetched from the database. Using an external DMAP run these matrices were combined to form a system stiffness matrix from the sum of KAA and KLAA and a system mass matrix from MAA and MLAA. Further manipulation was performed to strip the matrices of null rows and columns. These resulting matrices were printed out along with their degree of freedom association by the use of DMAP module MATGPR. The final matrices were output to file through the OUTPUT2 and OUTPUT4 modules allowing the group performing the coupled loads analysis to use either format for reading the data. The final package to be sent to represent the structure consisted of a tape with the system mass and stiffness matrices in component modal form in two formats. Along with this tape a description of the file format and a listing of the matrices with their degree of freedom association was provided. This data was presented in the most simplified form due to the desire to allow any code to use these matrices, without limitation to the use of NASTRAN only. This assumption of a non-NASTRAN solution also limited the possibilities for the methods for resulting data to be returned for data recovery. Figure 14 details the combination of the superelement matrices for transfer between groups.

As stated earlier the residual structure consisted of the five STS attatchment points. This residual structure can be used for the application of dynamic loads on the reduced model in a non-shuttle coupled model for the investigation of dynamic loads. This residual structure is used to perform data recovery on the STS interface model, and then the tip superelements of the propulsion system and payloads.

5.2 RECOVERY OF DATA

The data returned from the contractor can be handled in two ways depending on the analysis technique. If the models were all analyzed solely through the use of MSC/NASTRAN superelement analysis then all work would be simplified to appropriate communication and transfer of reduced databases between parties. In our case a more general approach was used. Due to the assumption that any code could be used to solve the system dynamics for the coupled loads analysis, it was not possible to rely on properly formatted matrices for the solution vector of the residual structure. It was decided to request time history data of acceleration and displacement for the points connected to the boundary. In the case of the transient loads analysis this data would be applied to the residual structure in a duplicated transient solution. By using the STS coupled analysis interface output as input, the analysis conditions are reproduced without the need for the STS matrices. This technique need only be performed for the boundary points and need not concern itself with the generalized coordinates for the reduced model, as the only possible load path is that through the boundary.

6.0 CONCLUSIONS

The following is a list of general conclusions drawn from the topics presented in this paper:

1. Multiple database usage allows greater user control over the locations used for storage of data to be passed between superelements. Qualifiers on the NASTRAN card allow write protection (DBSET 15) or read protection (DBSET 14) of any of the asigned databases to insure that old databases are not overwritten.

2. The use of random access database files is more efficient than the old OPTP/NPTP restart files but more user interaction is required. By the use of case control cards and parameters in the superelement run, the user has increased control over operations that are performed.

3. By using external superelements it is possible for separate analysts to work on adjoining components without worry of redundant GRID or element id numbers. The only information that must be given to each analyst before the model generation is the description of the boundary between structures.

4. By running the analysis in parts, the matrices that are used are reduced in size, requiring less time for the sum of the superelement runs than for one large model solution.

5. By performing phase-I analysis on the external superelements one at a time, the analyst can check results before proceeding, to keep from running an erroneous analysis. It is also possible by this method for models to be generated under a different schedule, checked out, and saved for later combination with downstream models.

6. Using Guyan Reduction in superelements produces the same sized system solution as that for a conventional analysis since degrees of freedom to represent the dynamic properties of the superelements must be passed to the residual structure.

7. The technique of Generelaized Dynamic Reduction frees the analyst from the task of choosing an ASET for dynamic analysis. The only required parameter for the GDR is the maximum frequency of interest. Other parameters can be specified for more user control.

8. Generalized Dynamic Reduction produces a more accurate dynamic representation of a model than the same sized ASET representation. Generalized Dynamic Reduction does take more time for solution than ASET methods for small and medium sized models, but less analyst time is required in preparation.

9. By using Generalized Dynamic Reduction in superelement modal solutions a technique called Component Modal Synthesis is accomplished. This technique generates a set of generalized coordinates to represent the superelement using less degrees of freedom than conventional methods.

10. Through the use of multi-level reduction the generalized coordinates of upstream superelements can be absorbed into downstream structures minimizing the size of the residual structure.

11. By stopping the superelement process before reaching the residual structure, four matrices can be fetched from the database to represent the dynamic characteristics of the model. These four matrices can be combined using simple DMAP statements into a mass matrix and stiffness matrix that can be passed on to another analysis code for coupled analysis.

7.0 RUN TIME STATISTICS

The following table decribes several runs of varying size and solution sequence. The database size is added as information for the amount of space that had to be saved after each run in order to continue the superelement assembly or data reduction.

NO.	MODEL	SOLUTION TYPE	DEGREES OF FREEDOM	NUMBER OF ELEMENTS	CPU TIME (S)	DATABASE SIZE (BLOCKS)
1.	PAYLOAD	61 PH-I	1248	384	508	6391
2.	STS INT STR	61 PH-I	2232	1016	1554	16390
3.	PROP SYSTEM	61 PH-I	2514	762	2706	18403
4.	RESID STR	61 PH-I PH-III	204	28	120	1353
5.	STS INT STR	61 PH-III	2232	1016	806	19371
6.	PROP SYSTEM	61 PH-III	2514	762	769	20372
7.	PAYLOAD	63 PH-I GDR	1398	384	1887	7634
8.	STS INT STR	63 PH-I GDR	3153	1016	16721	30756
9.	PROP SYSTEM	63 PH-I GDR	2589	762	8150	20306
10.	DOUBLE REDUCTION	63 PH-II GDR	983	0	6513	12309
11.	RESID STR	63 PH-II	137	0	402	1188
12.	PAYLOAD	63 PH-III	1398	384	1168	10527
13.	STS INT STR	63 PH-III	3153	1016	1559	35013
14.	STS INT STR	63 PH-I GUYAN	2232	1016	5492	36685
15.	TOTAL MODEL	3	8490	2930	36000	-
16.	MODEL 1	3 GDR	3578	482	5089	-
17.	MODEL 1	24	3078	482	2728	-
18.	MODEL 2	3 GUYAN	6528	1207	16072	-
19.	MODEL 2	24	6528	1207	4575	-

8.0 FIGURES

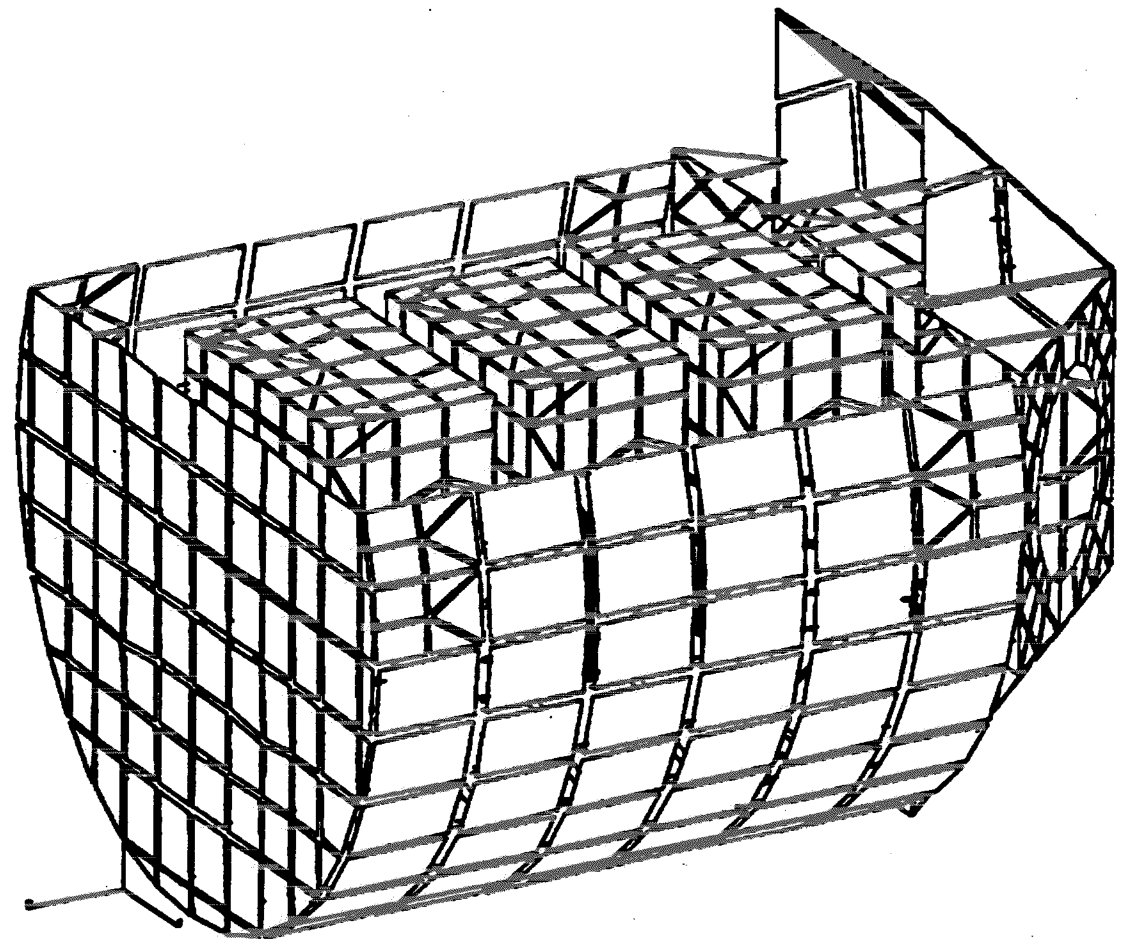


FIGURE 1

FULL STRUCTURE

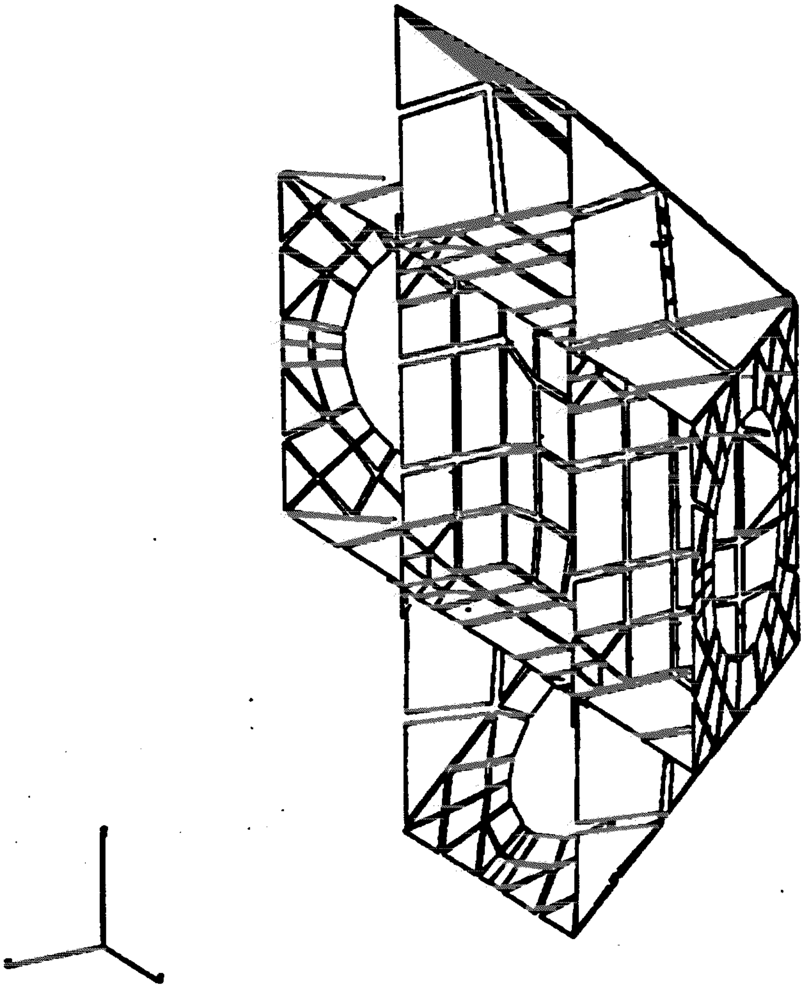


FIGURE 2
SUPERELEMENT 10 (SE 10)

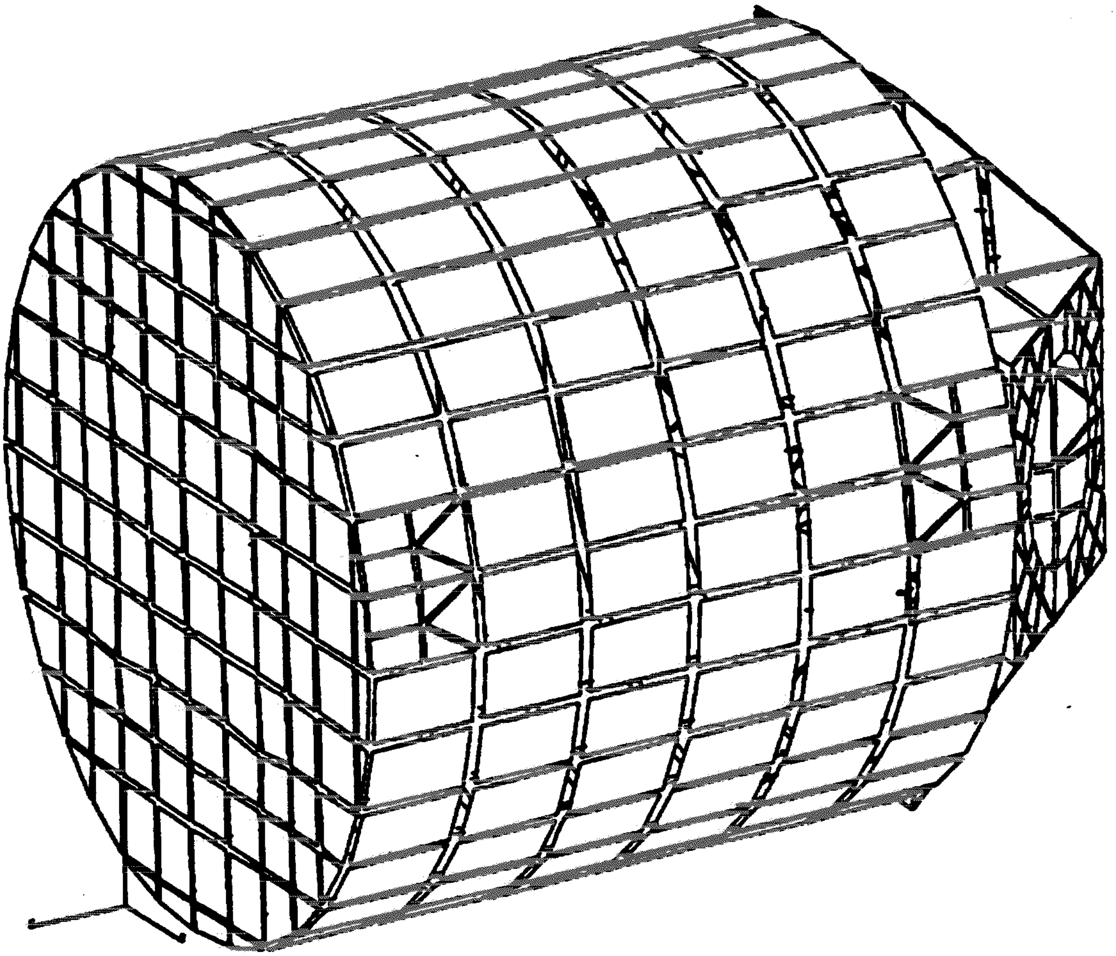


FIGURE 3
SUPERELEMENT 20 (SE 20)

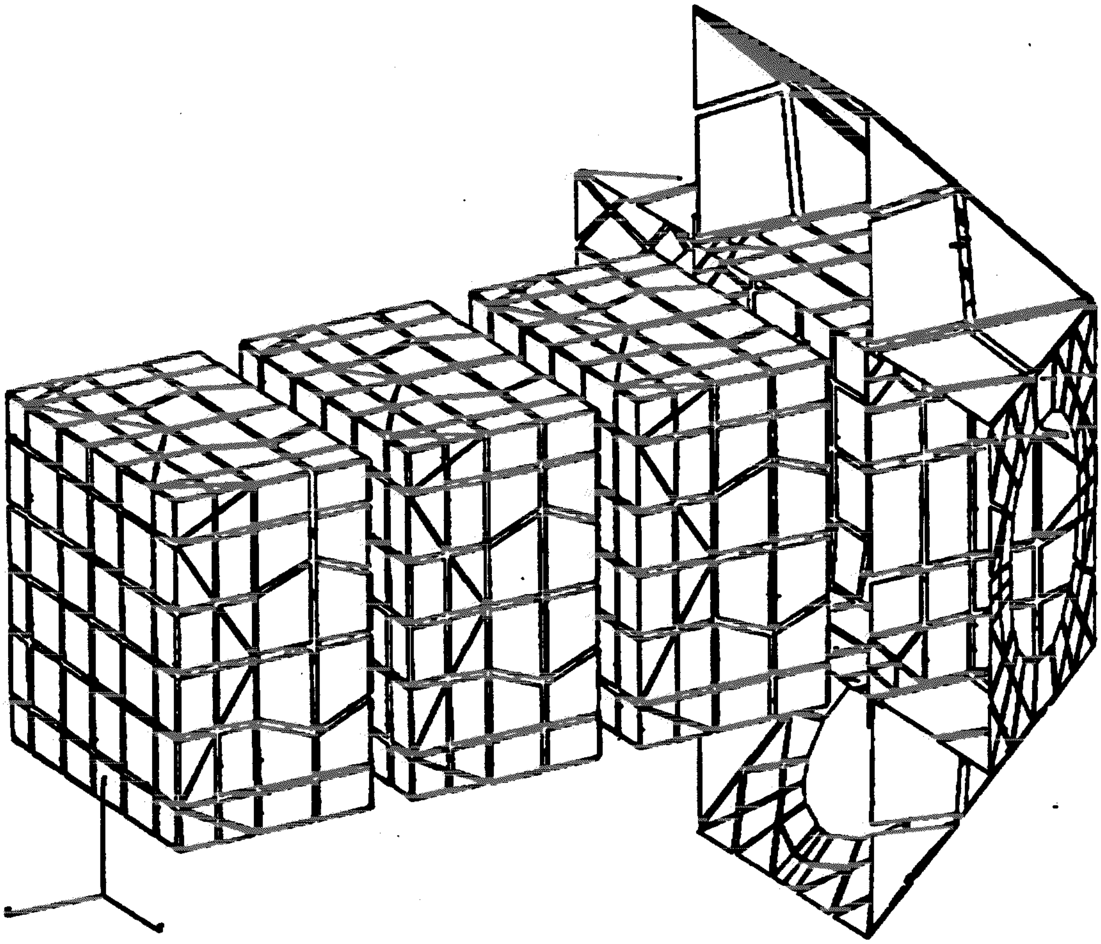


FIGURE 4

SUPERELEMENTS 31, 33 and 33 (SE 31, 32 and 33)

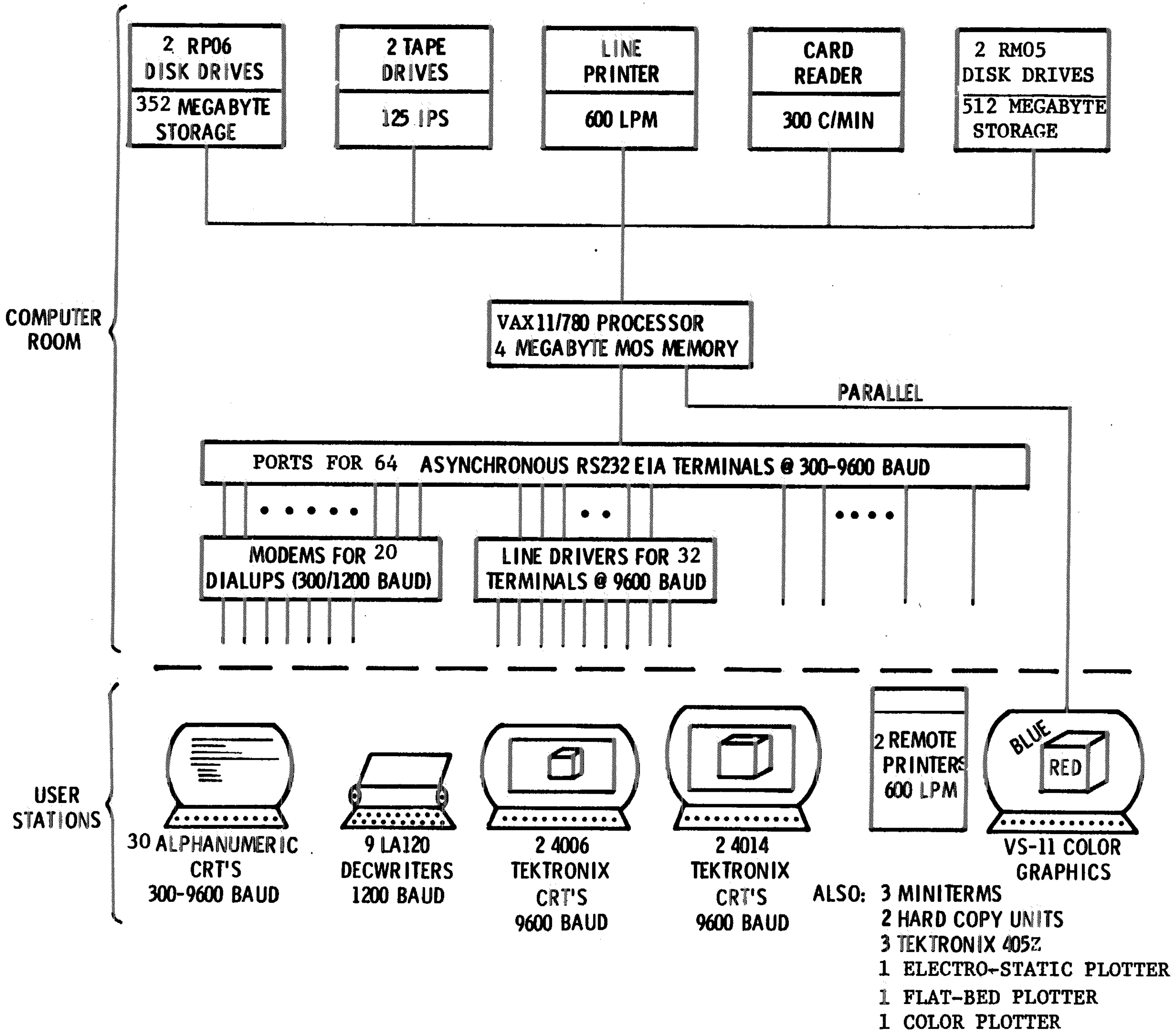


FIGURE 5

COMPUTER SYSTEM HARDWARE

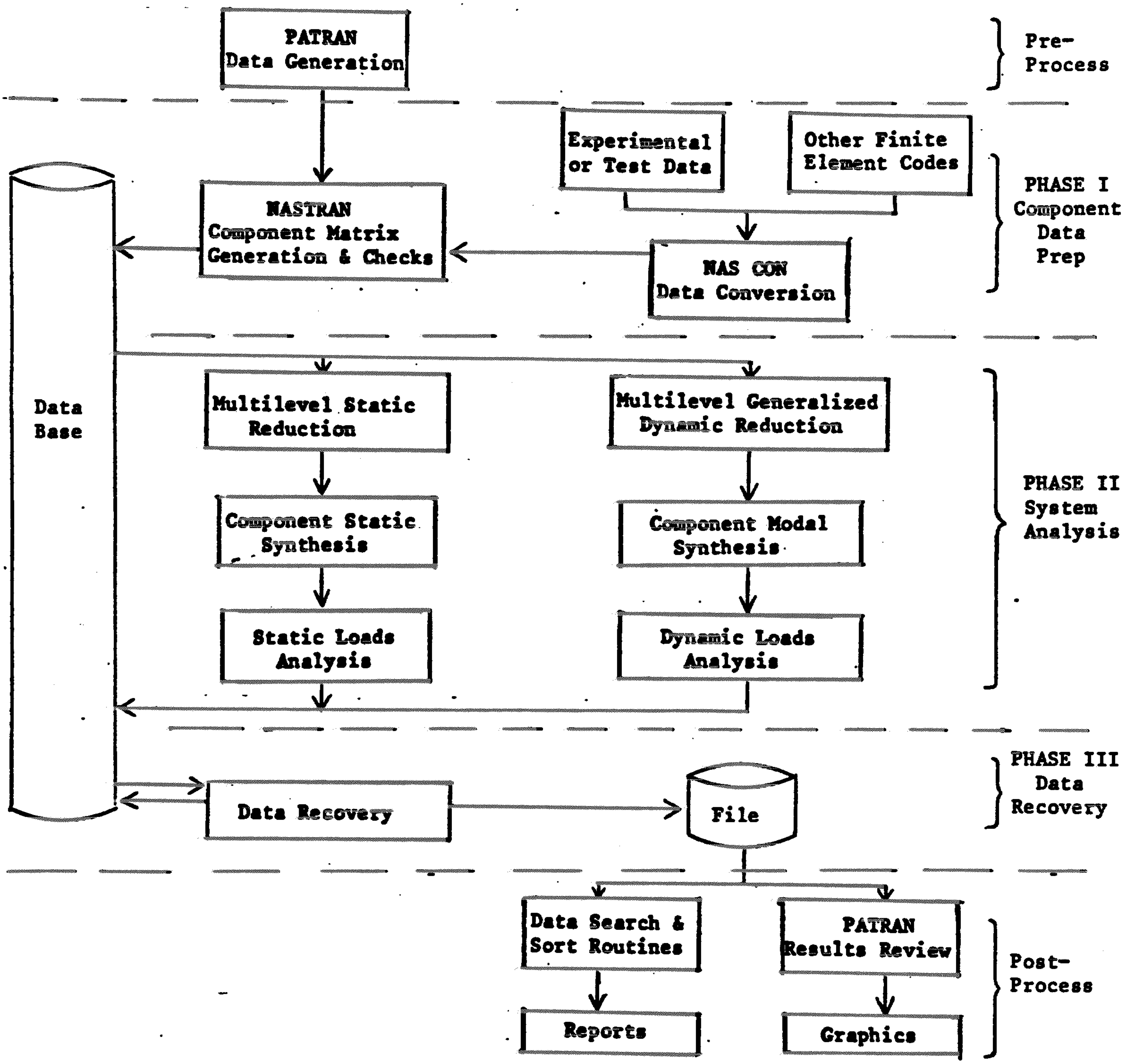


FIGURE 6
ANALYSIS APPROACH

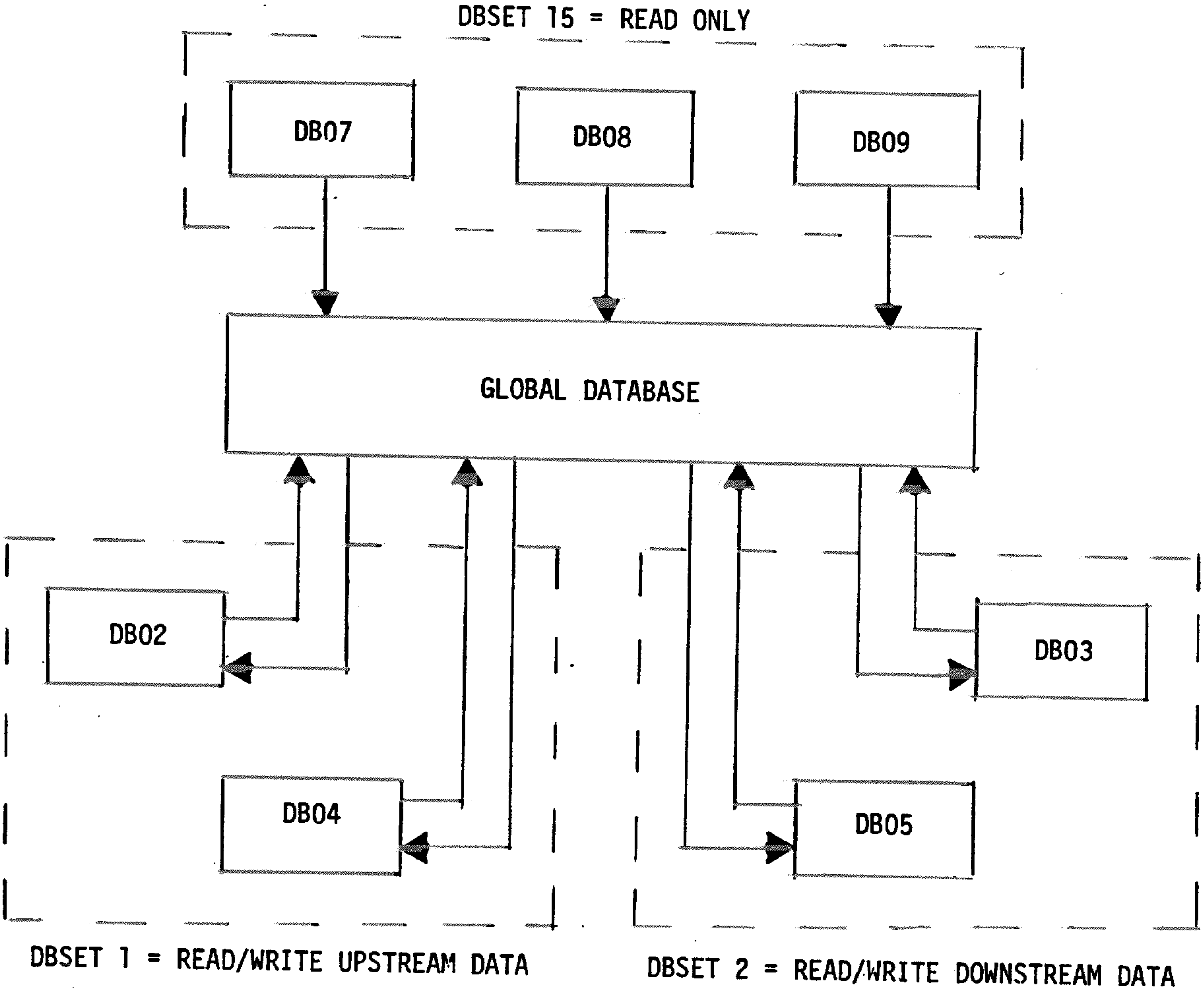


FIGURE 7
MULTIPLE DATABASE EXAMPLE

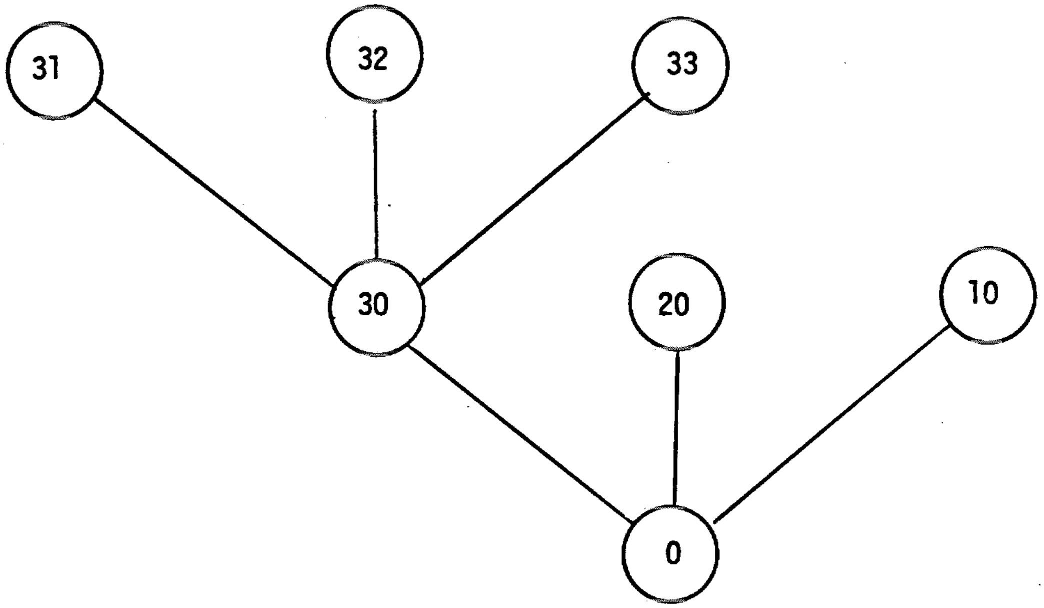
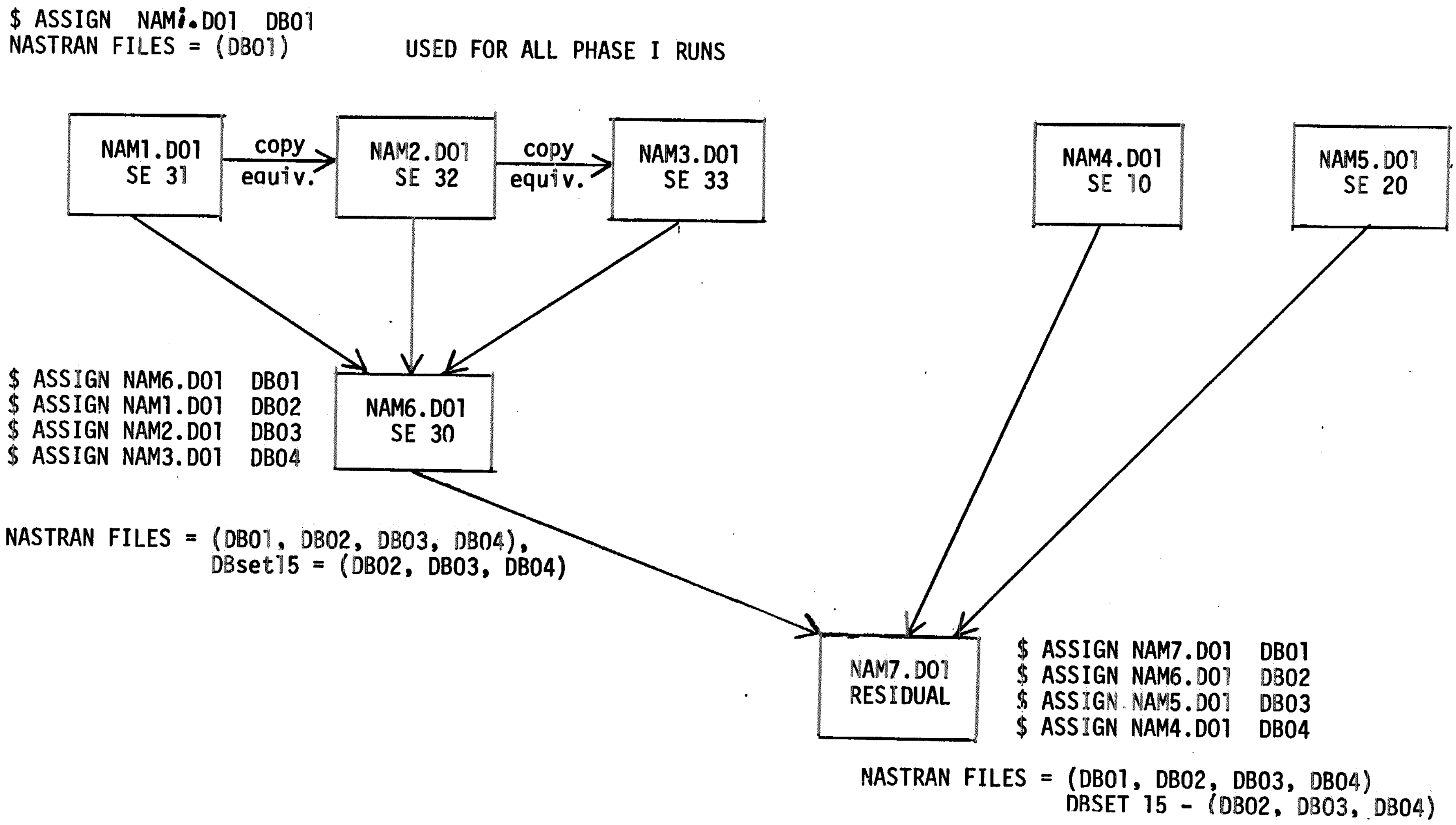


FIGURE 8
SUPERELEMENT TREE



```
$ ASSIGN  NAMi.D01  DB01
NASTRAN FILES = (DB01)        USED FOR ALL PHASE I RUNS
```

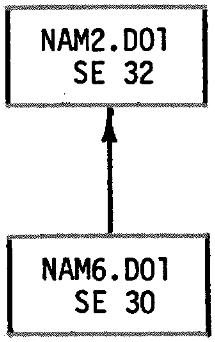
```
$ ASSIGN NAM6.D01   DB01
$ ASSIGN NAM1.D01   DB02
$ ASSIGN NAM2.D01   DB03
$ ASSIGN NAM3.D01   DB04

NASTRAN FILES = (DB01, DB02, DB03, DB04),
                DBset15 = (DB02, DB03, DB04)
```

```
$ ASSIGN NAM7.D01   DB01
$ ASSIGN NAM6.D01   DB02
$ ASSIGN NAM5.D01   DB03
$ ASSIGN NAM4.D01   DB04

NASTRAN FILES = (DB01, DB02, DB03, DB04)
                DBSET 15 - (DB02, DB03, DB04)
```

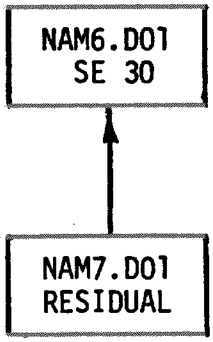
FIGURE 9

MULTIPLE DATABASES (PHASE I AND II)

DATA RECOVERY FOR SE 32

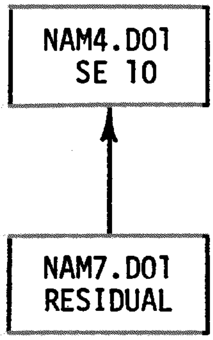


```
$ ASSIGN NAM2.D01  DB01
$ ASSIGN NAM6.D01  DB02

NASTRAN FILES = (DB01, DB02),
                DBSET 15 =(DB02)
```



```
$ ASSIGN NAM6.D01  DB01
$ ASSIGN NAM7.001 (DB02)

NASTRAN FILES = (DB01, DB02),
                DBSET 15 =(DB02)
```

DATA RECOVERY FOR SE 10



```
$ ASSIGN NAM4.D01  DB01
$ ASSIGN NAM7.D01  DB02

NASTRAN FILES = (DB01, DB02),
                DBSET 15 =(DB02)
```

FIGURE 10

MULTIPLE DATA BASES (PHASE III)

$$MAA = \begin{bmatrix} \begin{bmatrix} \text{BOUNDARY} \\ \text{MASS} \\ \\ \end{bmatrix} & \begin{bmatrix} \text{BOUNDARY TO} \\ \text{GENERALIZED} \\ \text{COORDINATED} \\ \text{COUPLING} \end{bmatrix} \\ \begin{bmatrix} \text{BOUNDARY TO} \\ \text{GENERALIZED} \\ \text{COORDINATE} \\ \text{COUPLING} \end{bmatrix} & \begin{bmatrix} \\ \text{NULL} \\ \\ \end{bmatrix} \end{bmatrix}$$

$$MLAA = \begin{bmatrix} \begin{bmatrix} \text{NULL} \end{bmatrix} & \begin{bmatrix} \text{NULL} \end{bmatrix} \\ \begin{bmatrix} \text{NULL} \end{bmatrix} & \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 & \searrow \end{bmatrix} \end{bmatrix}$$

$$KAA = \begin{bmatrix} \begin{bmatrix} \text{BOUNDARY} \\ \text{STIFFNESS} \end{bmatrix} & \begin{bmatrix} \text{NULL} \end{bmatrix} \\ \begin{bmatrix} \text{NULL} \end{bmatrix} & \begin{bmatrix} \text{NULL} \end{bmatrix} \end{bmatrix}$$

$$KLAA = \begin{bmatrix} \begin{bmatrix} \text{NULL} \end{bmatrix} & \begin{bmatrix} \text{NULL} \end{bmatrix} \\ \begin{bmatrix} \text{NULL} \end{bmatrix} & \begin{bmatrix} \lambda 1 & & & \\ & \lambda 2 & & \\ & & \lambda 3 & \\ & & & \ddots \end{bmatrix} \end{bmatrix}$$

FIGURE II
MATRIX TOPOLOGY

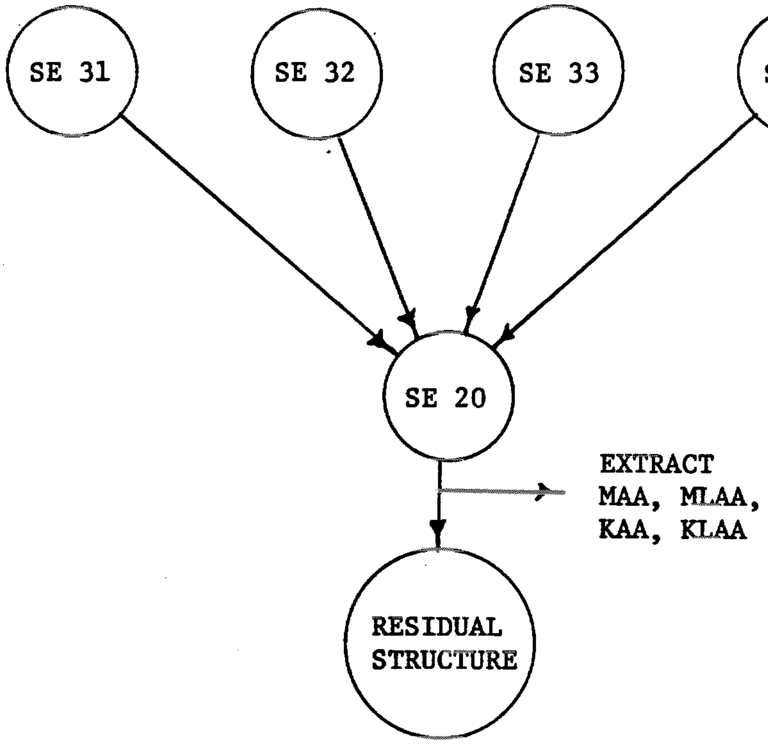
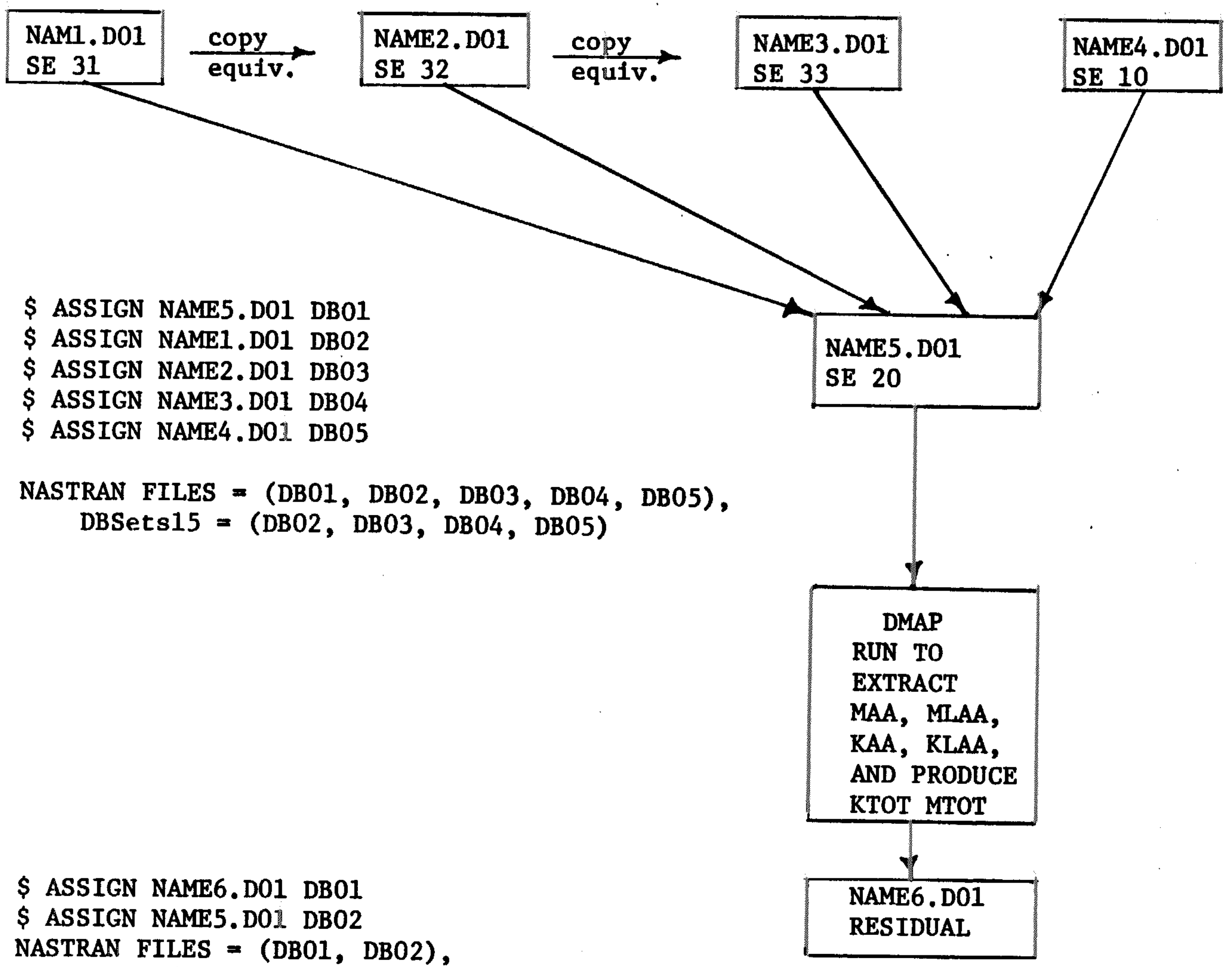


FIGURE 12
DYNAMIC SUPERELEMENT TREE FOR ASSEMBLY



```
$  ASSIGN NAME i.D01 DB01
NASTRAN FILES = (DB01)
```

```
$ ASSIGN NAME5.D01 DB01
$ ASSIGN NAME1.D01 DB02
$ ASSIGN NAME2.D01 DB03
$ ASSIGN NAME3.D01 DB04
$ ASSIGN NAME4.D01 DB05

NASTRAN FILES = (DB01, DB02, DB03, DB04, DB05),
    DBSets15 = (DB02, DB03, DB04, DB05)
```

```
$ ASSIGN NAMES, D01 DB01
NASTRAN FILES = (DB01),
DBSet15 = (DB01)
```

```
$ ASSIGN NAME6.D01 DB01
$ ASSIGN NAME5.D01 DB02
NASTRAN FILES = (DB01, DB02),
DBSet15 = (DB02)
```

FIGURE 13
DYNAMIC MODEL REDUCTION AND SOLUTION

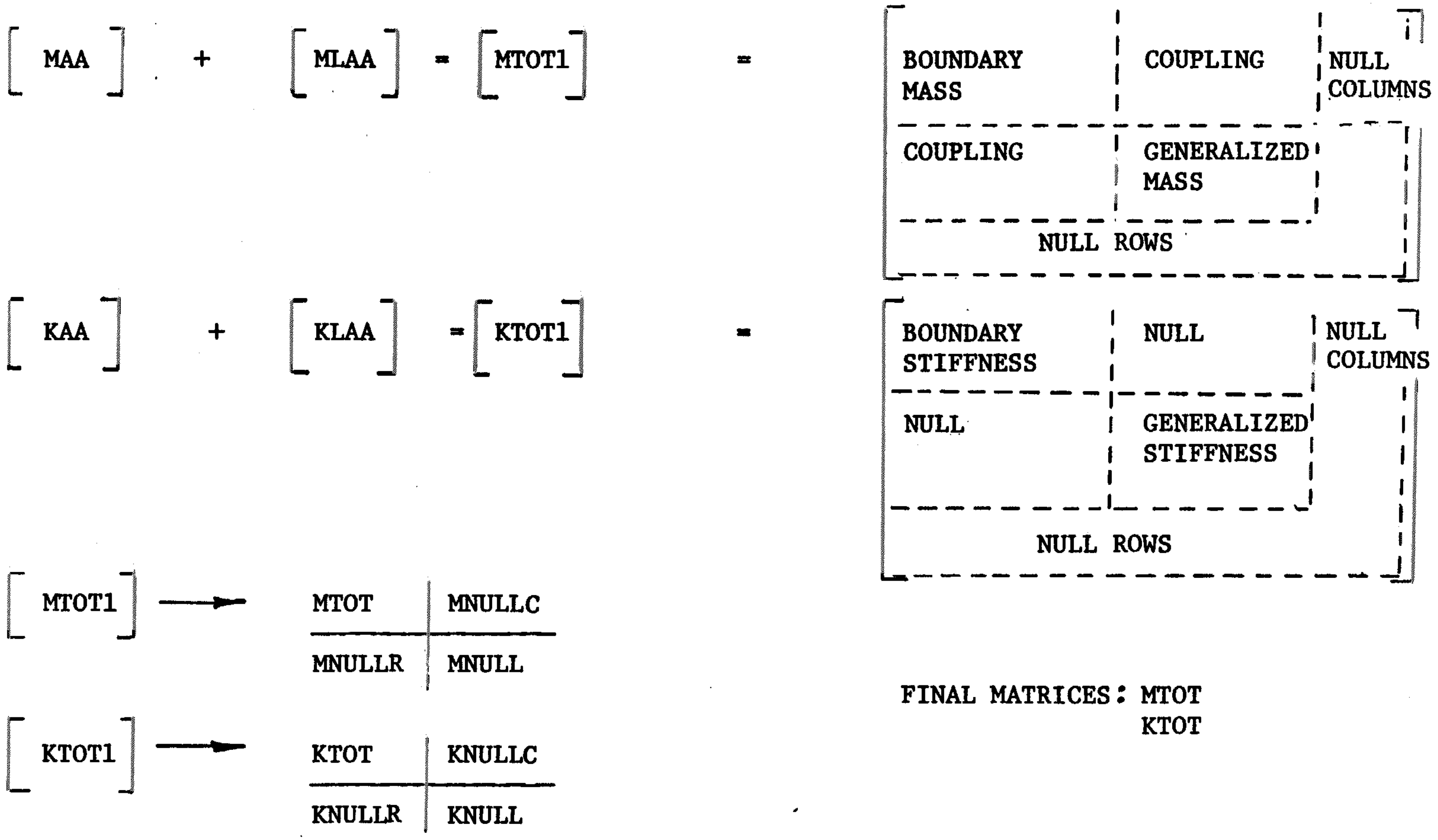


FIGURE 14
TOTAL SYSTEM MATRIX PRODUCTION

8.1 TABLES

TABLE 1
SUPERELEMENT IDENTIFICATION

Name	Superelement	Number of Grid Points	Number of Elements	Number of Boundry Grid Points
Propulsion System Structure	SE 10	419	762	12
STS Interface Structure	SE 20	372	1016	17
Payload #1	SE 31	208	384	8
Payload #2	SE 32	208	384	8
Payload #3	SE 33	208	384	8
Residual Structure	SE 0	34	28	34

TABLE 2
SUPERELEMENT SOLUTION SEQUENCES

Flexibility input format	41
Hybrid input format	42
Stiffness input format	43
Model checkout	60
Statics or linear heat transfer analysis	61
Statics with cyclic symmetry	81
Statics - alternate solution	62
Normal modes	63
Normal modes with cyclic symmetry	83
Direct complex eigenvalue	67
Direct frequency response	68
Direct frequency with cyclic	88
Direct transient response	69
Modal complex eigenvalue	70
Modal frequency response	71
Modal transient response	72
Transient heat transfer	89

TABLE 3

SUMMARY OF CASE CONTROL CARDS FOR SUPERELEMENT ANALYSIS

Mnemonic	Purpose	Location
SEMG	Specifies superelements for which stiffness, mass and damping matrices are to be generated.	Above subcase level
SEMA	Specifies superelements for which stiffness matrices are to be assembled and reduced.	Above subcase level
SELG	Specifies superelements for which static loads are to be generated.	Above subcase level
SELA	Specifies superelements for which loads, mass and damping matrices are to be assembled and reduced.	Above subcase level
SEFINAL	Specifies which superelements are to be processed last.	Above subcase level
SEEXCLUDE	Specifies superelements whose structural matrices and load vectors are not to be assembled into the matrices of downstream superelements.	Above subcase level
SUPER	Defines subcases that reference individual superelement.	Within subcase that reference individual superelements
SEPLOT	Defines individual superelements for which undeformed or deformed plots are requested.	Within plot portion of Case Control Deck

TABLE 3 (Continued)

Mnemonic	Purpose	Location
SEALL	Specifies the superelement identification numbers for which all matrices and loads will be generated and assembled.	
SEDR	Specifies the superelement identification numbers for which data recovery will be performed.	
SEKR	Superelement stiffness matrix assembly and reduction.	
SELR	Superelement load assembly and reduction.	
SEMR	Superelement mass oval clamping assembly and reduction.	

9.0 REFERENCES FOR SUPERELEMENT ANALYSIS

1. MSC/NASTRAN Applcations Manual,
Jerrard A. Joseph, Editor, the MacNeal-Schwendler Corp.
    - a) Section 1.3
    - b) Section 2.20
    - c) Section 2.4
    - d) Application Note June/July 1978
    - e) Application Note October/November 1979

2. MSC/NASTRAN User's Manual, Version 61,
Caleb W. McCormick, Editor, the MacNeal-Schwendler Corp.
    - a) Section 1.12
    - b) Section 2.2
    - c) Section 2.3
    - d) Section 2.4
    - e) Section 3.1
    - f) Section 3.2.3
    - g) Section 3.3.21
    - h) Section 3.3.22
    - i) Section 5.4

3. MSC/NASTRAN Theoretical Manual, Level 15.5,
Richard H. MacNeal, Editor, the MacNeal-Schwendler Corp.,
December 1972
    - a) Section 1.2.4
    - b) Section 1.3.1
    - c) Section 3.5
    - d) Section 4.4
    - e) Section 10.5
    - f) Section 14.1

4. MSC/NASTRAN Handbook for Superelement Analysis,
the MacNeal-Schwendler Corp., M. A. Gockel, Editor, November 1981

5. Component Mode Synthesis for External Superelements,
the MacNeal-Schwendler Corp., F. D. Bellinger, May 1981

6. "Coupling of Substructures for Dynamic Analysis",
R. R. Craig, M. C. Bampton, AIAA Journal, Vol. 6, No. 7,
July 1968

7. "Matrix Structural Analysis of Substructures", J. S. Przemieniecki,
AIAA Journal, Vol. 1, No. 1, January 1963, pp. 138-147.

8. "Automated Stress Analysis Using Substructures", I. C. Taig,
October 1965, paper presented at Conference on Matrix Methods in
Structural Mechanics, Wright-Patterson Air Force Base, Ohio.

9. "Dynamic Analysis of Structural Systems Using Component Modes",
W. C. Hurty, AIAA Journal, Vol. 3, No. 4, April 1965, pp. 678-685

10. "Dynamic Analysis of Structural Systems Using Component Mode
Synthesis", W. C. Hurty, Rept. 32-530, 1964, Jet Propulsion Lab,
Pasedena, California.

11. "Branch Mode Analysis of Vibrating Systems", G. M. L. Gladwell, Journal of Sound Vibration, Vol 1, 1964, pp. 41-59