ROTOR-DISK SYSTEM GYROSCOPIC EFFECT IN MSC/NASTRAN DYNAMIC SOLUTIONS

By

Herand Bedrossian
Dynamics Engineer

Nicholas Veikos
Dynamics Engineer

Avco Lycoming Division, Stratford, Conn.

ABSTRACT

The analysis examines the nonlinear equations of motion governing a rotating structure and determines how they can be accurately represented using MSC/NASTRAN dynamic solutions. In particular, methods are developed and described which generate a skew-symmetric matrix and couple this to the MSC/NASTRAN analysis using direct matrix input in order to simulate the disk gyroscopic effect. These techniques will now allow for the analysis of nonsynchronous as well as synchronous whirl in a rotor-disk system using MSC/NASTRAN. The techniques are demonstrated using three different NASTRAN solutions - NORMAL MODES (SOL:3), DIRECT COMPLEX EIGENVALUE (SOL:28), AND DIRECT FORCED RESPONSE (SOL:26). The results show excellent correlation with closed form solutions for forward and reverse precession as well as nonsynchronous whirl frequencies.

INTRODUCTION

In the analysis of systems involving a rotor-disk assembly spinning with a constant velocity, it is essential to include the effect of disk gyroscopic moment for the rotor critical speed solution. The MSC/NASTRAN finite element program is designed to solve numerous structural dynamic problems. However, it does not provide a direct method of including the gyroscopic stiffening effect in its dynamic solutions.

The analysis presented here examines the equations of motion governing a rotating structure and presents a method for their solution using a linear finite element program such as MSC/NASTRAN. In particular, methods are developed and described which generate a skew-symmetric "gyroscopic cross-coupling" matrix and combine this to the NASTRAN generated stiffness and mass matrices by means of direct matrix input (DMIG) in order to simulate the gyroscopic effect.

These techniques were demonstrated utilizing MSC/NASTRAN for the critical speed solution of a rotor-disk system. The results showed excellent correlation with the closed form solution for both forward and reverse precession critical frequencies.

ANALYSIS

The equations of motion for a disk spinning with a constant angular velocity Ω, (see Figure 1), can be written as

$$m_d \ddot{x} = -Q_x$$

$$m_d \ddot{y} = -Q_y$$

$$I_p \ddot{\phi}_y - I_d \Omega \dot{\phi}_x = -M_y$$

$$I_p \ddot{\phi}_x + I_d \Omega \dot{\phi}_y = -M_x$$

Where the gyroscopic terms are $-I \Omega \dot{\phi}_x$ and $I \Omega \dot{\phi}_y$

m_d = mass of disk

I_p = polar moment of inertia of disk

I_d = moment of inertia of disk about a diameter

Q_x, Q_y = sum of external force acting on the disk in x, y directions, respectively

M_x, M_y = sum of external moments acting on the disk in ϕ_x, ϕ_y directions, respectively.

If we replace Q_x, Q_y and M_x, M_y with the elastic and friction forces, the set of equations of a spinning rotor system having n degrees of freedom can be written in matrix form as:

$$\underline{M}\,\ddot{\underline{u}} + \underline{B}\,\dot{\underline{u}} + \underline{G}\,\dot{\underline{u}} + \underline{K}\,\underline{u} = \underline{P}(t) \qquad (1)$$

where

- $\underline{M}$ is the system mass matrix
- $\underline{K}$ is the system stiffness matrix
- $\underline{P}(t)$ is the set of all applied external loads
- $\underline{B}$ is the matrix of damping coefficients
- $\underline{G}$ is the rotor gyroscopic matrix
- $\underline{u}(t)$ is the time dependent set of nodal displacements

Mathematically, the gyroscopic behavior of each stage is defined by equation (2).

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_p\Omega & 0 \\ 0 & 0 & 0 & -I_p\Omega & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_x \\ \dot{u}_y \\ \dot{u}_z \\ \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{bmatrix} \qquad (2)$$

where

I_p = Polar mass moment of inertia of stage

Ω = Rotor rotational velocity (rad/sec)

The procedure employed to obtain speed-dependent natural frequencies (λ_i), is the solution of set of equations in the form of;

$$\underline{M}\,\ddot{\underline{u}} + \underline{B}\,\dot{\underline{u}} + \underline{G}\,\dot{\underline{u}} + \underline{K}\,\underline{u} = 0 \qquad (3)$$

For an undamped system, equation (3) would yield,

$$\underline{M}\,\ddot{\underline{u}} + \underline{G}\,\dot{\underline{u}} + \underline{K}\,\underline{u} = 0 \qquad (4)$$

The frequencies of the natural vibrations are determined for different speeds of rotation in accordance with equation (4). The dependence of the natural frequencies λ on the speed Ω is shown for a typical case in figure (3).

Solution of equation (4) requires a nonsymmetric eigenvalue solver such as "Q-R" algorithm suitable for calculating general complex eigenvalues and eigenvectors, and yields rotor natural frequencies for a specified running speed Ω. This calculation must be repeated over a range of running speeds Ω_i to determine a critical speed; i.e., the coincidence of a running speed with a natural frequency. The direct calculation of the rotor critical speeds with a symmetric matrix solver is the subject of the following discussion.

The inertial symmetry of the rotor suggests possible transformation of equation (4) into a symmetric linear set of equations. For isotropic bearing stiffness there is complete circumferential symmetry, and the resultant steady state motion consists of purely circular orbits. Forward and backward critical speeds of the rotor system can be obtained by solution of equation (5) in the following matrix form.

$$\underline{M}_D \ddot{\underline{u}} + \underline{K}\,\underline{u} = 0 \qquad (5)$$

where

$$\underline{M}_D = \begin{bmatrix} \underline{M}_g & & & \\ & \underline{M}_g & & \\ & & \ddots & \\ & & & \underline{M}_g \end{bmatrix} \quad \text{and } \underline{M}_g = \begin{bmatrix} m & & & & & \\ & m & & & & \\ & & m & & & \\ & & & 3I_d = (I_p + I_d) & & \\ & & & & -I_d = (I_p - I_d) & \\ & & & & & I_p \end{bmatrix}$$

where

$\underline{M}_D$ = System mass matrix (nxn)

m = Mass at grid point

$\underline{M}_g$ = Nodal mass matrix with gyro-stiffening effect included.

The real eigenvalue solution of equation (5) would yield both the forward and reverse precession critical speeds. However, the mass matrix $\underline{M}_D$ would be non-positive definite.

The foregoing outlined procedure may readily be employed to calculate the forced-response solution of a rotor-disk system incorporating the gyroscopic stiffening effect. This can be simply achieved by rearranging equation (1) and rewriting it in the form of equation (6), as following:

$$\underline{M}^*_D \, \ddot{\underline{u}} + \underline{B} \, \dot{\underline{u}} + \underline{K} \, \underline{u} = \underline{P}(t) \qquad (6)$$

where

$$\underline{M}^*_D = \begin{bmatrix} \underline{M}^*_g & & & \\ & \underline{M}^*_g & & \\ & & \ddots & \\ & & & M^*_g \end{bmatrix} \text{ and } \underline{M}^*_g = \begin{bmatrix} m & & & & & \\ & m & & & & \\ & & m & & & \\ & & & I_d & jI_p & \\ & & & -jI_p & I_d & \\ & & & & & I_p \end{bmatrix}$$

where

M^*_D = System mass matrix (nxn)

M^*_g = Nodal mass matrix with gyro-stiffening effect included

$j = \sqrt{-1}$

DISCUSSION

The rotor-disk system in figure (2), which is identical to that in reference (1) for which an analytical solution exists, was used to determine the accuracy of the methods developed here.

A simple lumped-mass model of the rotor system was generated using bar elements to represent the shaft. The mass and inertia of the disk were lumped accordingly at the disk attachment grid location by means of CONM2 bulk data cards. A plot of natural frequency (λ) versus speed (Ω) for the rotor system of figure (2) is shown in figure (3).

Using the Direct Complex Eigenvalue Solution (RF: 28), the speed dependent gyroscopic damping terms ($I_p\Omega$) were included in the system equations by means of Direct Matrix Input (DMIG) bulk data cards. Figure 4 shows the NASTRAN Case Control and Bulk Data deck for the rotor-disk system model at a rotational speed of $\Omega = 100$ rad/sec. The calculation was repeated over a range of running speeds ($\pm\Omega_i$) to determine the critical speed of the rotor-disk system.

The HESS method of eigenvalue solution was used and the frequencies obtained were in excellent agreement with the exact analytical solution presented in reference 1. A combination of generalized dynamic reduction and HESS method of complex eigenvalue solution was found to be more economical.

The critical speeds of the rotor system (points at which $\lambda = \pm\Omega$) were also computed using Rigid Format 3 (Normal Modes Solution). To include the gyroscopic effect, the inertia matrix was modified to the form of equation (5) and the corresponding disk inertia terms were added to the mass matrix by means of CONM2 bulk data cards. The NASTRAN bulk data deck for the test case is shown in Figure (5). The INV method of eigenvalue solution was used to calculate the forward and reverse precession critical frequencies. A combination of generalized dynamic reduction and MGIV method of eigenvalue solution could not be used due to the non-positive definite form of the mass matrix.

The gyroscopic effect was also included in the frequency response solution in the form of equation (6) which has complex off-diagonal terms in the mass matrix.

The NASTRAN input deck is shown in figure (6). The imaginary off-diagonal inertia terms were introduced into the system inertia matrix at the disk attachment grid location, through the use of DMIG bulk data cards. Using the Direct Frequency Response solution (RF:26), the rotor response due to a unit unbalance load acting at the disk grid location was computed and the frequencies identified. The response curves for grid point 10 are shown in figure (7).

The critical speeds obtained by the Direct Complex Eigenvalue and Normal Modes solutions, as well as the frequencies identified from the response curves of the Direct Frequency Response solution were compared with the exact solution presented in reference (1) in addition to an in-house lumped parameter rotor dynamics program. These results are tabulated in table (1), which proved to be in excellent agreement.

In order to further examine the above procedures, a finite element model of the test case was constructed. Plate elements were used to model the shaft and the disk (figure 8). The first two critical speeds of the shaft were determined using the Normal Modes Solution at zero speed. The inertia matrix was then altered through the use of a CONM2 bulk data card located at the disk center of mass to include the gyroscopic effect. The NASTRAN deck for this example is shown in figure (9) and the natural frequencies obtained are tabulated in table (2). Figure (10) shows the mode shapes.

CONCLUSION

A method has been developed which allows the MSC/NASTRAN user to more accurately predict the behavior of a rotor-disk system by including the effect of the gyroscopic moments. This is achieved by altering the damping or inertia matrices at grid points which have proportionally large mass moments of inertia. The procedure is relatively easy to implement and has produced excellent results for actual rotor-disk systems as well as the test case presented here.

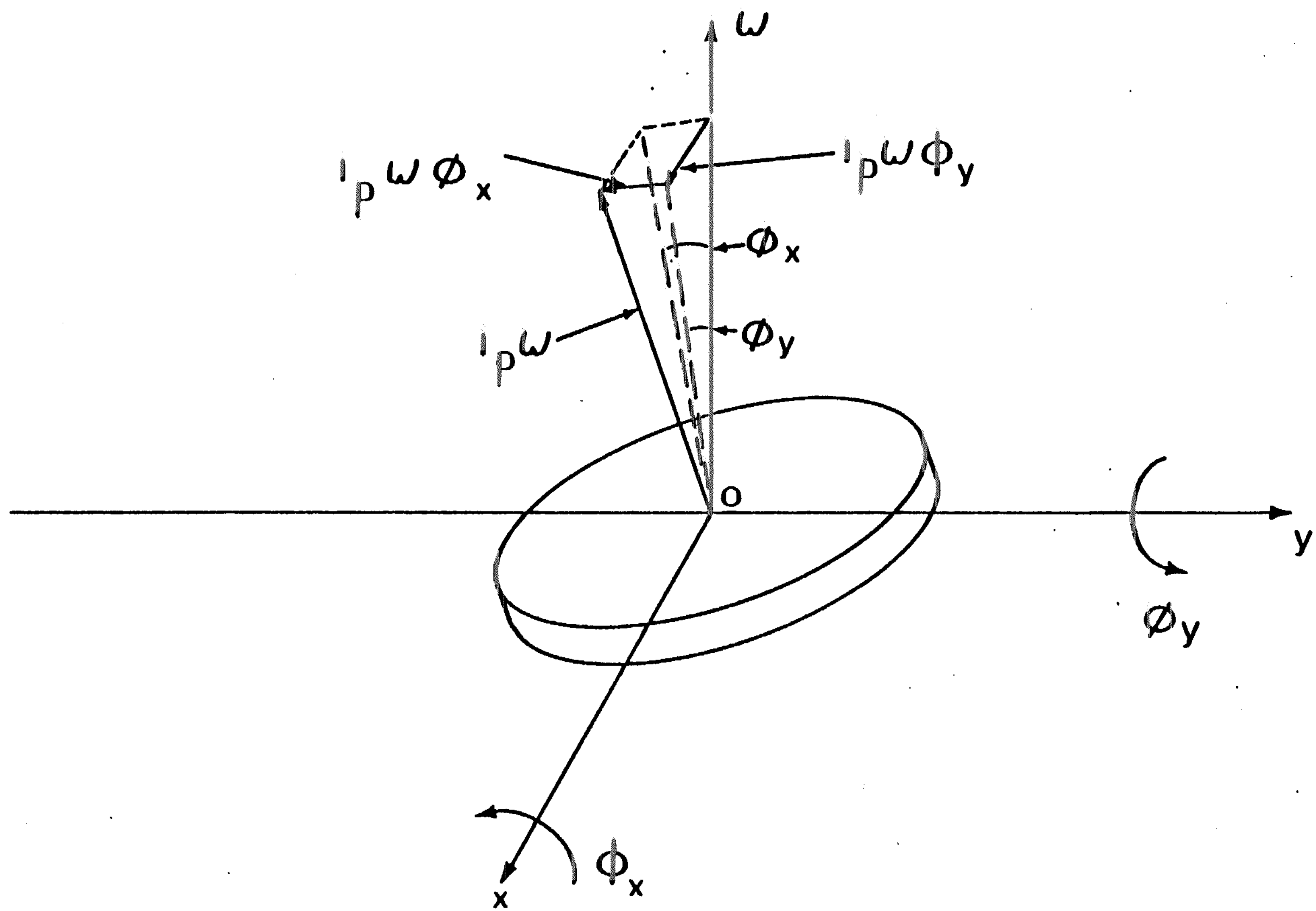
This paper has examined only a few of the possible methods of representing the gyroscopic effect in MSC/NASTRAN dynamic solutions. Additional investigation is necessary to further develop the techniques presented here. For example, automated procedures can be developed in MSC/NASTRAN that allow for the specification of the gyroscopic effect in a straightforward, efficient manner at each grid location.

REFERENCES

1. Dimentberg, F. M., "Flexural Vibrations of Rotating Shafts", London: Butterworths, 1961, pp. 61-65.

2. MSC/NASTRAN User's and Application Manuals.

DISK GYROSCOPIC EFFECT



BB1002
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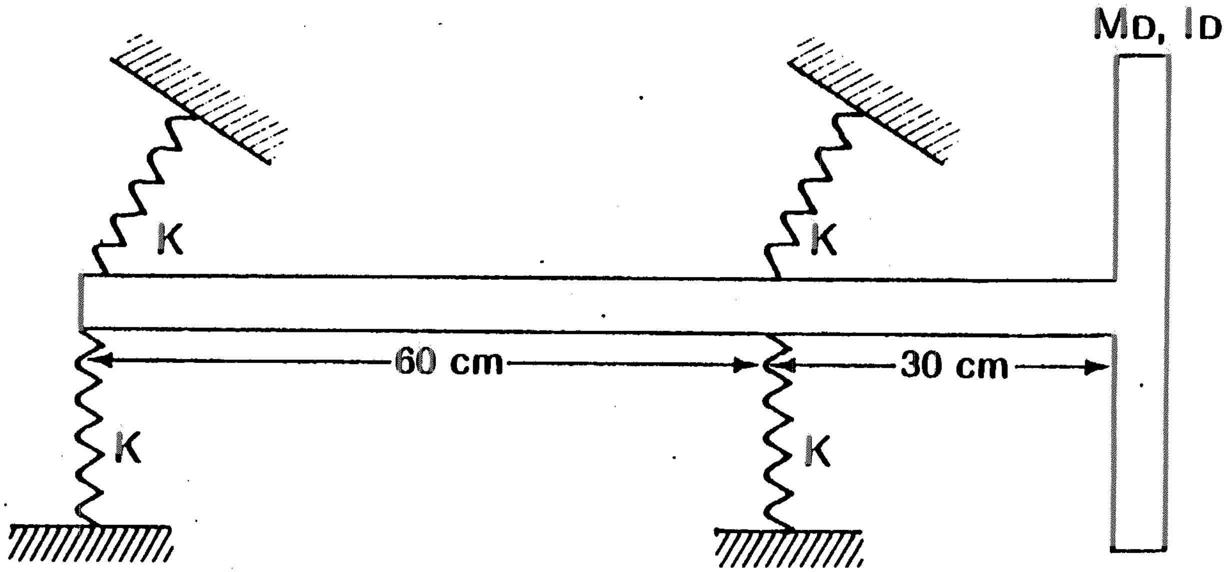
FIGURE 1.

SIMPLY SUPPORTED ROTOR-DISK SYSTEM



$K = \infty$

$M_D = 0.0157$ kg sec²/cm

$I_D = 2.45$ kg sec²cm

$I_p = 2\ I_D$

$EI = 1.6477 \times 10^6$ kg cm²

SHAFT ASSUMED MASSLESS

BB1001
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FIGURE 2.

CHARACTERISTIC SOLUTION OF THE ROTOR SYSTEM OF FIGURE 2

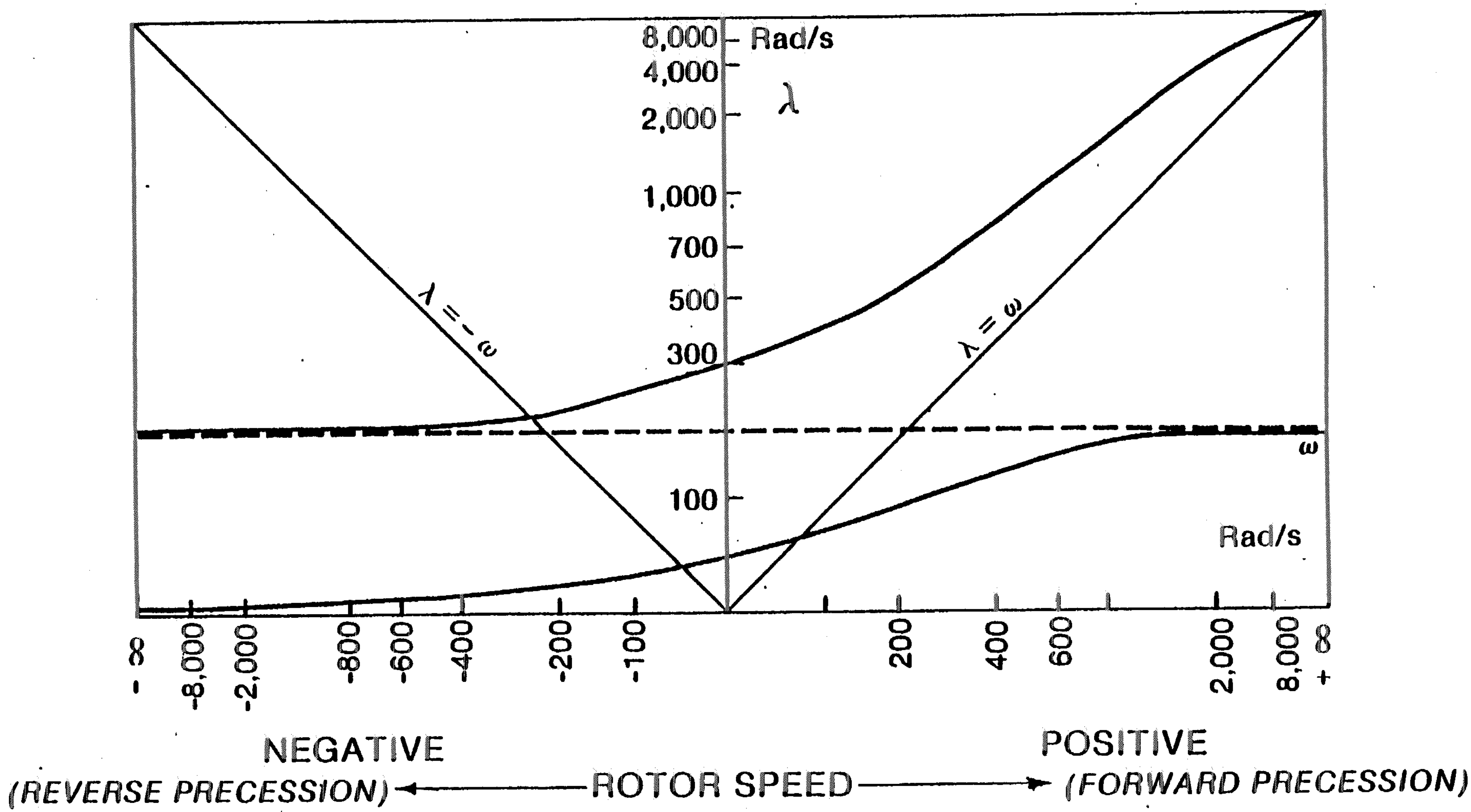

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FIGURE 3.

N A S T R A N   E X E C U T I V E   C O N T R O L   D E C K   E C H O

```
ID [illegible]NAST
APP DISP
SOL 28
DIAG 8,13,14
TIME  3
```

DIRECT COMPLEX EIGENVALUE

C A S E   C O N T R O L   D E C K   E C H O

```
CARD
COUNT
  1      TITLE=  COMPLEX EIGENVALUE
  2      SUBTITLE=  ROTOR SYSTEM GYROSCOPIC EFFECT(W= 100 RAD/SEC)
  3      SPC=2222
  4      B2PP=DAMP
  5      CMETHOD=1
  6      DISP(PHASE)=ALL
  7      BEGIN COMPLEX EIGEN VALUE
```

DMIG AT GRID POINT

S O R T E D   B U L K   D A T A   E C H O

```
CARD
COUNT    .   1  ..   2  ..   3  ..   4  ..   5  ..   6  ..   7  ..   8  ..   9  ..  10  .
   1-      CBAR    1       1       1       2       7001
   2-      CBAR    2       1       2       3       7001
   3-      CBAR    3       1       3       4       7001
   4-      CBAR    4       1       4       5       7001
   5-      CBAR    5       1       5       6       7001
   6-      CBAR    6       1       6       7       7001
   7-      CBAR    7       1       7       8       7001
   8-      CBAR    8       1       8       9       7001
   9-      CBAR    9       1       9       10      7001
  10-      CONM2   77771   10              157.E-4
  11-      +CM33   2.45            2.45                                                    +CM33
  12-      DMIG    DAMP    0       1       1       0
  13-      DMIG    DAMP    10      4               10      5       -490.
  14-      DMIG    DAMP    10      5               10      4       490.
  15-      EIGC    1       HESS    MAX                                                     +123
  16-      +123    0.      500.    0.      0.      30.     1       3                       +223
  17-      GRID    1               0.      .0      0.              36
  18-      GRID    2               0.      .0      10.             6
  19-      GRID    3               0.      .0      20.             6
  20-      GRID    4               0.      .0      30.             6
  21-      GRID    5               0.      .0      40.             6
  22-      GRID    6               0.      .0      50.             6
  23-      GRID    7               0.      .0      60.             6
  24-      GRID    8               0.      .0      70.             6
  25-      GRID    9               0.      .0      80.             6
  26-      GRID    10              0.      .0      90.             6
  27-      GRID    7001            10.     .0      100.            123456
  28-      MAT1    100     1.0E06          .30     1.000E-9
  29-      PARAM   ASING   1
  30-      PARAM   COUPMASS1
  31-      PARAM   GRDPNT  0
  32-      PBAR    1       100     10.0    1.6477061.647706
  33-      SPC1    2222    12      1       7
           ENDDATA
```

GYROSCOPIC MATRIX

$I_p \Omega$

C O M P L E X   E I G E N V A L U E   S U M M A R Y

EIGENVALUE (REAL)	EIGENVALUE (IMAG)	FREQUENCY (CYCLES)	DAMPING COEFFICIENT
1.292249E-07	3.835486E+01	6.104365E+00	-6.738306E-09
1.292249E-07	7.045030E+01	1.121251E+01	-3.668541E-09
3.385185E-02	2.307337E+02	3.672240E+01	-2.934276E-04
2.600582E-01	3.996965E+02	6.361360E+01	-1.301270E-03

FIGURE 4. DIRECT COMPLEX EIGENVALUE SOLUTION FOR TEST CASE INCLUDING GYROSCOPIC EFFECT. Ω = 100 RAD/SEC

```
ID NENAST
APP DISP
SOL 3                                   NORMAL MODES
DIAG 8,13,14
TIME  2
CEND
```

CASE CONTROL DECK ECHO

```
CARD
COUNT
  1      TITLE= NORMAL MODES
  2      SUBTITLE=  ROTOR SYSTEM GYROSCOPIC EFFECT INCLUDED
  3      SPC=2222
  4      METHOD=1
  5      DISP=ALL
  6      BEGIN NORMAL MODES
```

SORTED BULK DATA ECHO

```
CARD
COUNT    .  1  ..  2  ..  3  ..  4  ..  5  ..  6  ..  7  ..  8  ..  9  ..  10  .
  1-     CBAR    1       1       1       2       7001
  2-     CBAR    2       1       2       3       7001
  3-     CBAR    3       1       3       4       7001
  4-     CBAR    4       1       4       5       7001
  5-     CBAR    5       1       5       6       7001
  6-     CBAR    6       1       6       7       7001
  7-     CBAR    7       1       7       8       7001
  8-     CBAR    8       1       8       9       7001
  9-     CBAR    9       1       9       10      7001
 10-     CONM2   77771   10              157.E-4                                 +CM33
 11-     +CM33   -2.45           +7.35
 12-     EIGR    1       INV     0.      100.    3                               +123
 13-     +123    MAX
 14-     GRID    1               0.      .0      0.              36
 15-     GRID    2               0.      .0      10.             6
 16-     GRID    3               0.      .0      20.             6
 17-     GRID    4               0.      .0      30.             6
 18-     GRID    5               0.      .0      40.             6
 19-     GRID    6               0.      .0      50.             6
 20-     GRID    7               0.      .0      60.             6
 21-     GRID    8               0.      .0      70.             6
 22-     GRID    9               0.      .0      80.             6
 23-     GRID    10              0.      .0      90.             6
 24-     GRID    7001            10.     .0      100.            123456
 25-     MAT1    100     1.0E06          .30     1.000E-9
 26-     PARAM   ASING   1
 27-     PARAM   GRDPNT  0
 28-     PBAR    1       100     10.0    1.6477061.647706
 29-     SPC1    2222    12      1       7
         ENDDATA
```

$-I_D$ $+3I_D$

REAL EIGENVALUES

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	3	2.186738E+03	4.676257E+01	7.442494E+00	2.920580E-02	6.364674E+01
2	2	4.989539E+03	7.063667E+01	1.124218E+01	1.245173E-02	6.212837E+01
3	1	4.347046E+04	2.084957E+02	3.318312E+01	2.899526E-02	1.260437E+03
4	4	-5.715493E+04	2.390711E+02	3.804933E+01	-6.018072E-02	3.439628E+03
5	5	7.011850E+04	2.647991E+02	4.214407E+01	-6.018072E-02	-4.219781E+03
6	6	1.125430E+05	3.354744E+02	5.339240E+01	-6.018072E-02	-6.772918E+03
7	7	1.337552E+05	3.657256E+02	5.820703E+01	-6.018072E-02	-8.049584E+03
8	8	1.464826E+05	3.827305E+02	6.091344E+01	-6.018072E-02	-8.815430E+03
9	9	1.549675E+05	3.936592E+02	6.265280E+01	-6.018072E-02	-9.326055E+03

NOTE: NEGLECT MODES WITH A NEGATIVE GENERALIZED STIFFNESS

FIGURE 5. NORMAL MODES SOLUTION FOR TEST CASE INCLUDING GYROSCOPIC EFFECT

```
            N A S T R A N    E X E C U T I V E    C O N T R O L    D E C K    E C H O

                 ID MSNAST
                 APP DISP
                 SOL 26
                 DIAG 6,13,14
                 TIME  5
                 CEND
```

DIRECT FREQUENCY RESPONSE

```
                          C A S E    C O N T R O L    D E C K    E C H O
        CARD
        COUNT
          1            TITLE=  FORCED RESPONSE SOLUTION
          2            SUBTITLE=  ROTOR SYSTEM GYROSCOPIC EFFECT INCLUDED
          3            SPC=2222
          4            M2PP=MASS
          5            FREQ=20
          6            DLOAD=15
          7            ECHO=BOTH
          8            SET 1=10,4
          9            DISP(SORT2,PLOT,PHASE)=1
         10            OUTPUT(XYPLOT)
         11            PLOTTER NASTPLT MODEL D,0
         12            XPAPER=28.3
         13            YPAPER=25.
         14            XAXIS=YES
         15            YAXIS=YES
         16            XLOG=YES
         17            YLOG=YES
         18            XTITLE=                                                       FREQUENCY(CPS)
         19            TCURVE=  POINT 10 VERTICAL DISP
         20            XYPLOT,XYPRINT DISP /10(T1)
         21            TCURVE=  POINT 10 LATTERAL DISP
         22            XYPLOT,XYPRINT DISP /10(T2)
         23            TCURVE=  POINT 4 VERTICAL DISP
         24            XYPLOT,XYPRINT DISP /4(T1)
         25            TCURVE=  POINT 4 LATTERAL DISP
         26            XYPLOT,XYPRINT DISP /4(T2)
         27            BEGIN FORCED RESPONSE SOLUTION
```

DMIG AT GRID POINT

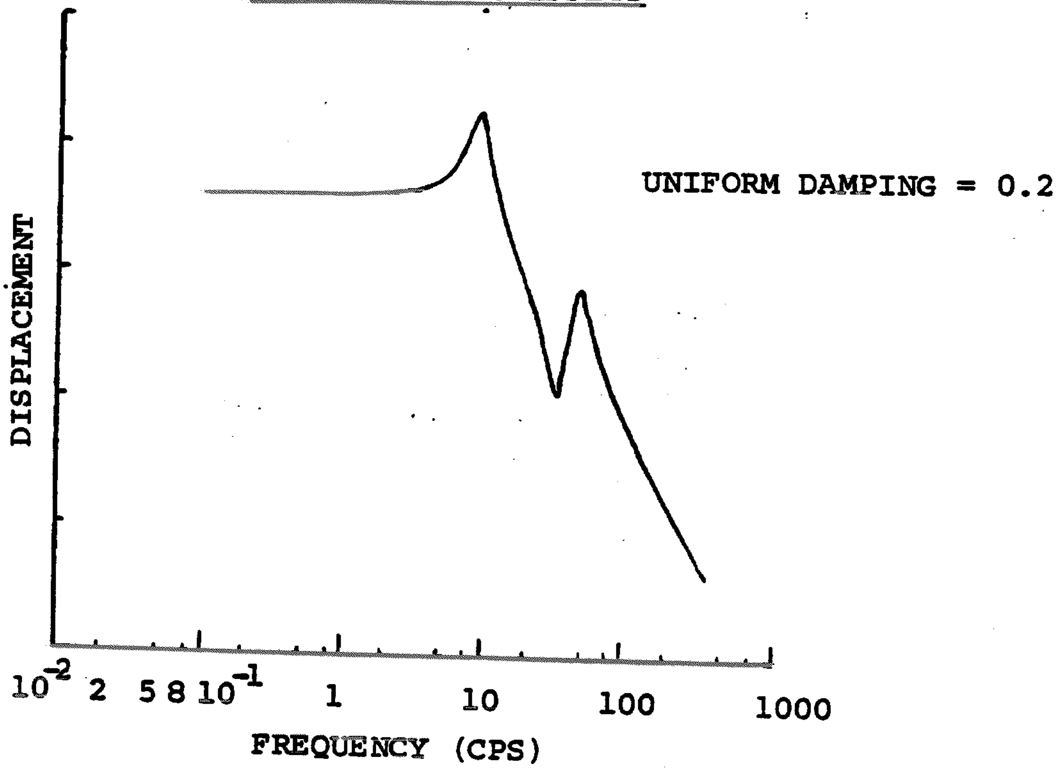
```
                                S O R T E D    B U L K    D A T A    E C H O
      CARD
      COUNT        .   1  ..   2  ..   3  ..   4  ..   5  ..   6  ..   7  ..   8  ..   9  ..  10  .
        1-        CBAR     1       1       1       2       7001
        2-        CBAR     2       1       2       3       7001
        3-        CBAR     3       1       3       4       7001
        4-        CBAR     4       1       4       5       7001
        5-        CBAR     5       1       5       6       7001
        6-        CBAR     6       1       6       7       7001
        7-        CBAR     7       1       7       8       7001
        8-        CBAR     8       1       8       9       7001
        9-        CBAR     9       1       9       10      7001
       10-        CONM2    77771   10              157.E-4
       11-        +CM33    2.45            2.45                                                   +CM33
       12-        DAREA    16      10      1       1.00E+0
       13-        DMIG     MASS    0       1       3       0
       14-        DMIG     MASS    10      4               10      5               4.9
       15-        DMIG     MASS    10      5               10      4               -4.9
       16-        FREQ1    20      0.1     1.      400
       17-        GRID     1               0.      .0      0.              36
       18-        GRID     2               0.      .0      10.             6
       19-        GRID     3               0.      .0      20.             6
       20-        GRID     4               0.      .0      30.             6
       21-        GRID     5               0.      .0      40.             6
       22-        GRID     6               0.      .0      50.             6
       23-        GRID     7               0.      .0      60.             6
       24-        GRID     8               0.      .0      70.             6
       25-        GRID     9               0.      .0      80.             6
       26-        GRID     10              0.      .0      90.             6
       27-        GRID     7001            10.     .0      100.            123456
       28-        MAT1     100     1.0E06          .30     1.000E-9
       29-        PARAM    GRDPNT  0
       30-        PBAR     1       100     10.0    1.6477061.647706
       31-        RLOAD1   15      16                      18
       32-        SPC1     2222    12      1       7
       33-        TABLED1  18
       34-        +9000    0.      1.      5000.   1.      ENDT                                   +9000
                  ENDDATA
```

GYROSCOPIC MASS MATRIX

jI_p

FIGURE 6. DIRECT FREQUENCY RESPONSE SOLUTION FOR TEST CASE INDLUCING GYROSCOPIC EFFECT

FREQUENCY RESPONSE CURVE FOR TEST CASE
NO GYROSCOPIC EFFECT



FREQUENCY RESPONSE CURVE FOR TEST CASE
GYROSCOPIC EFFECT INCLUDED

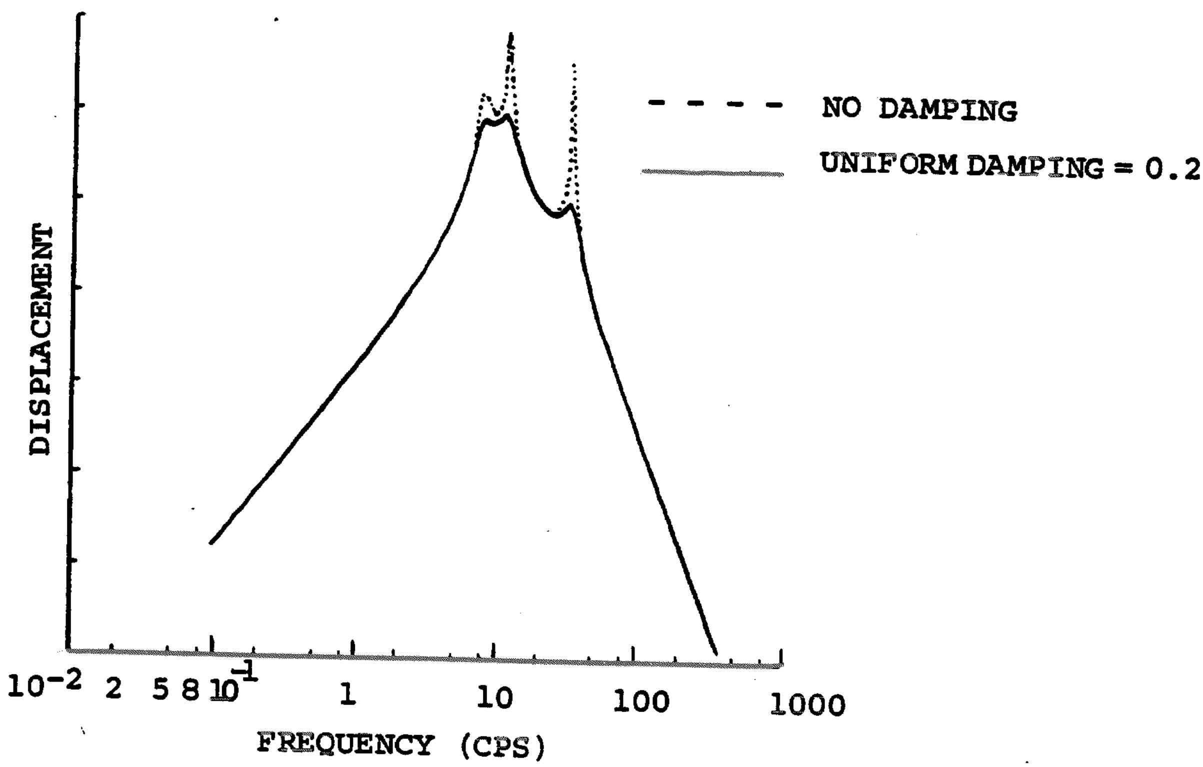


FIGURE 7.

WHIRL FREQUENCIES FOR TEST CASE

SOLUTION	WHIRL FREQUENCY (RADIANS/SECOND)			
	FORWARD PRECESSION		REVERSE PRECESSION	
CLOSED FORM (REF. 1)	70	238j	46.4	208.5
NORMAL MODES (SOL: 3)	70.6	-239	46.8	208.5
DIRECT COMPLEX EIGENVALUE (SOL: 28)	70.6	-239	46.8	208.5
DIRECT FORCED RESPONSE (SOL:26)	70	-	45	207
IN-HOUSE ROTOR DYNAMICS PROGRAM	70.6	-239	-	-

TABLE: 1

NASTRAN PLATE MODEL OF TEST CASE

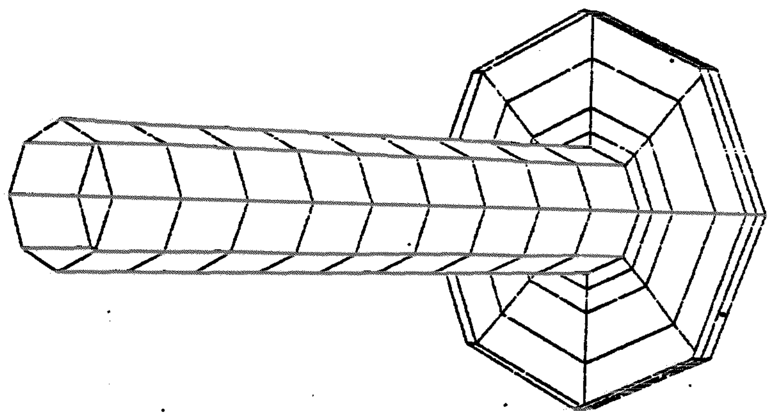


FIGURE 8.

N A S T R A N   E X E C U T I V E   C O N T R O L   D E C K   E C H O

```
ID MSNAST,ROTORDY
APP DISP
SOL 3  ------------------  NORMAL MODES
DIAG 8,13,14
TIME 5
CEND
```

C A S E   C O N T R O L   D E C K   E C H O

```
CARD
COUNT
  1      TITLE=  SAMPLE CASE FOR ROTOR SYSTEM GYRO EFFECT
  2      SUBTITLE=  REAL EIGENVALUE SOLUTION
  3      METHOD=3333
  4      SPC=2222
  5      DISP=ALL
  6      ECHO=BOTH
  7      BEGIN NORMAL MODE ANALYSIS
```

I N P U T   B U L K   D A T A   D E C K   E C H O

```
.  1  ..  2  ..  3  ..  4  ..  5  ..  6  ..  7  ..  8  ..  9  ..  10  .
$$$$$$$$$$$$$$$$
EIGR    3333    INV         0.      100.        5                      +EIGR
+EIGR   MAX
$$$$$$$$$$$$$$$$
PARAM   TINY    0.5
PARAM   ASING   1
PARAM   GRDPNT  0
PARAM   AUTOSPC YES
MAT1    100     3.00E07         .30     1.000E-9
MAT1    200     9.0E09          .30     1.6000-5      -2I_D
MAT1    300     9.0E09          .30     1.000E-9               +2I_D
PQUAD4       1      100   .08880      100
PQUAD4       2      200   .50000      200
CONM2   77771    10                 1.0E-9
+CM33   -4.70             +4.70                                     +CM33
SPC1     2222     12      101     201     301     401     501     601
SPC1     2222     12      701       1       7     107     207     307
SPC1     2222     12      407     507     607     707
$$$$$$$$$$$$$$$$
GRID       1      22  0.5806    0.0    10.0000    22              0
GRID       2      22  0.5806    0.0    20.0000    22              0
GRID       3      22  0.5806    0.0    30.0000    22              0
GRID       4      22  0.5806    0.0    40.0000    22              0
GRID       5      22  0.5806    0.0    50.0000    22              0
GRID       6      22  0.5806    0.0    60.0000    22              0
GRID       7      22  0.5806    0.0    70.0000    22              0
GRID       8      22  0.5806    0.0    80.0000    22              0
GRID       9      22  0.5806    0.0    90.0000    22              0
GRID      10      22  0.5806    0.0   100.0000    22              0
GRID      11      22  6.8306    0.0   100.0000    22              0
GRID      12      22 13.0306    0.0   100.0000    22              0
GRID      13      22 19.3306    0.0   100.0000    22              0
GRID      14      22 25.2000    0.0   100.0000    22              0
```

R E A L   E I G E N V A L U E S

EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
3.682635E+00	1.919020E+00	3.054215E-01	1.435187E-02	5.285272E-02
2.112021E+03	4.595673E+01	7.314240E+00	2.322870E-02	4.905951E+01
5.072242E+03	7.121968E+01	1.133496E+01	1.116662E-02	5.663982E+01
4.399121E+04	2.097408E+02	3.338129E+01	2.110285E-02	9.283396E+02
-5.418514E+04	2.327770E+02	3.704761E+01	-2.911943E-03	1.577840E+02

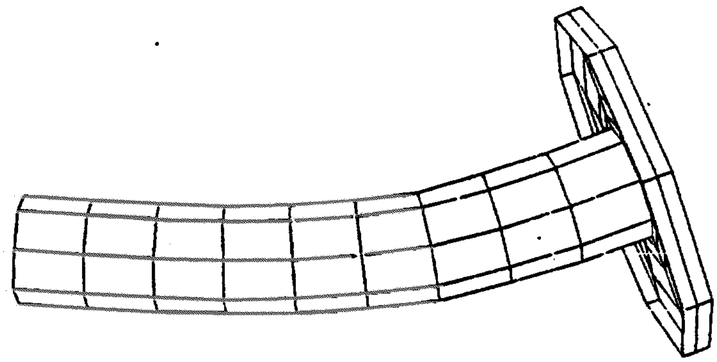
FIGURE 9. NORMAL MODES SOLUTION FOR PLATE MODEL INCLUDING GYROSCOPIC EFFECT

COMPARISON OF NASTRAN PLATE MODEL SOLUTION WITH CLOSED FORM SOLUTION FOR TEST CASE

SOLUTION	WHIRL FREQUENCY (RADIANS/SEC)					
	NO GYROSCOPIC EFFECT		FORWARD PRECESSION		REVERSE PRECESSION	
CLOSED FORM (REF. 1)	55.4	302.4	70.0	238j	46.4	208.5
NORMAL MODES (RF:3) - PLATE MODEL	55.3	302.7	71.2	232.8j	45.9	203.7

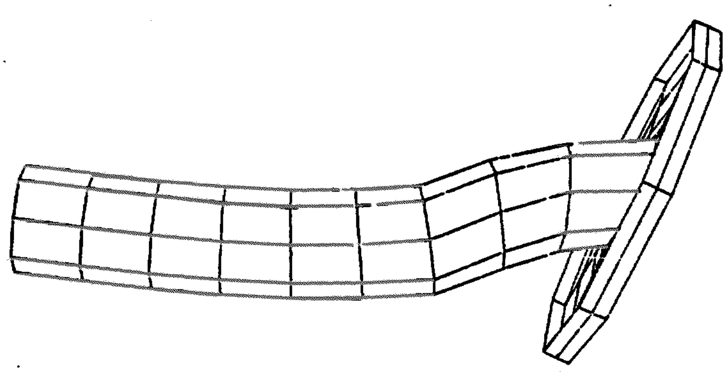
TABLE 2.

NASTRAN PLATE MODEL - MODE SHAPES



MODE 1

FORWARD PRECESSION - f = 11.3 Hz   REVERSE PRECESSION - f = 7.3 Hz



MODE 2

REVERSE PRECESSION - f = 33.4 Hz

FIGURE 10.